
Supplementary information

Table S1: Comparison between numerical (Num.) and analytical results for dimensionless concentration of substrate $u(\zeta)$ for various values of parameter φ_1 when $\gamma_1 = \gamma_2 = 0.1$ and $\alpha_1 = 5$.

ζ	$\varphi_1 = 10$ and $m = 0.7285$					$\varphi_1 = 15$ and $m = 0.6101$				
	Num.	AGM Eq.(20) or HPM [15] or ADM [30]	TSM Eq. (35)	Error % of AGM Eq. (20)	Error % of TSM Eq. (35)	Num.	AGM Eq.(20) or HPM [15] or ADM [30]	TSM Eq. (35)	Error % of AGM Eq. (20)	Error % of TSM Eq. (35)
0	0.73	0.72	0.73	1.37	0.00	0.60	0.58	0.61	3.33	1.67
0.2	0.74	0.73	0.74	1.35	0.00	0.62	0.59	0.63	4.84	1.61
0.4	0.77	0.76	0.77	1.30	0.00	0.67	0.65	0.67	2.98	0.00
0.6	0.83	0.82	0.83	1.20	0.00	0.75	0.73	0.75	2.67	0.00
0.8	0.90	0.90	0.90	0.00	0.00	0.86	0.85	0.86	1.16	0.00
1	1.00	1.00	1.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00
	Average error %			0.87	0.00	Average error %			2.50	0.55

Table S2: Comparison between numerical and analytical results for dimensionless concentration of substrate $u(\zeta)$ for various values of parameter γ_1 when $\varphi_1 = \gamma_2 = 0.1$ and $\alpha_1 = 5$.

ζ	$\gamma_1 = 7$ and $m = 0.8052$					$\gamma_1 = 15$ and $m = 0.6101$				
	Num.	AGM Eq. (20) or HPM [15]] or ADM [30]	TSM Eq. (35)	Error % of AGM Eq. (20)	Error % of TSM Eq. (35)	Num.	AGM Eq. (20) or HPM [15]] or ADM [30]	TSM Eq. (35)	Error % of AGM Eq. (20)	Error % of TSM Eq. (35)
0	0.80	0.80	0.80	0.00	0.00	0.60	0.58	0.61	3.33	1.67
0.2	0.81	0.81	0.81	0.00	0.00	0.62	0.59	0.63	4.84	1.61
0.4	0.84	0.83	0.84	1.20	0.00	0.67	0.65	0.67	2.98	0.00
0.6	0.88	0.87	0.88	1.14	0.00	0.75	0.73	0.75	2.67	0.00
0.8	0.93	0.93	0.93	0.00	0.00	0.86	0.85	0.86	1.16	0.00
1	1.00	1.00	1.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00
	Average error %			0.39	0.00	Average error %			2.50	0.55

Table S3: Comparison between numerical and analytical results for dimensionless concentration of substrate $u(\zeta)$ for various values of parameter γ_2 when $\gamma_1 = \varphi_1 = 0.1$ and $\alpha_1 = 5$.

ζ	$\gamma_2 = 5$ and $m = 0.8399$					$\gamma_2 = 7$ and $m = 0.7623$				
	Num.	AGM Eq. (20) or HPM [15]	TSM Eq. (35)	Error % of AGM Eq. (20)	Error % of TSM Eq. (35)	Num.	AGM Eq. (20) or HPM [15]	TSM Eq. (35)	Error % of AGM Eq. (20)	Error % of TSM Eq. (35)
0	0.84	0.86	0.84	2.38	0.00	0.77	0.80	0.76	3.90	1.30
0.2	0.85	0.86	0.85	1.18	0.00	0.78	0.81	0.77	3.85	1.28
0.4	0.87	0.88	0.87	1.15	0.00	0.81	0.83	0.80	2.47	1.23
0.6	0.90	0.91	0.90	1.11	0.00	0.86	0.87	0.85	1.16	1.16
0.8	0.95	0.95	0.94	0.00	1.05	0.92	0.93	0.92	1.09	0.00
1	1.00	1.00	1.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00
	Average error %			0.97	0.17	Average error %			2.09	0.83

Table S4: Comparison between numerical and analytical results for dimensionless concentration of product $v(\zeta)$ for various values of parameter φ_2 when $\gamma_1 = 1, \gamma_2 = 1, \gamma_3 = 0.1, \gamma_4 = 1, \alpha_1 = 5$ and $m = 0.9158$.

ζ	$\varphi_2 = 20$ and $l = 0.4445$					$\varphi_2 = 35$ and $l = 0.0321$				
	Num.	AGM Eq. (26) or HPM [15]] or ADM [30]	TSM Eq. (41)	Error % of AGM Eq. (26)	Error % of TSM Eq. (41)	Num.	AGM Eq. (26) or HPM [15]] or ADM [30]	TSM Eq. (41)	Error % of AGM Eq. (26)	Error % of TSM Eq. (41)
0	0.44	0.44	0.44	0.00	0.00	0.03	0.02	0.03	33.33	0.00
0.2	0.47	0.46	0.47	2.13	0.00	0.07	0.06	0.07	14.29	0.00
0.4	0.53	0.53	0.53	0.00	0.00	0.19	0.18	0.19	5.26	0.00
0.6	0.65	0.64	0.65	1.54	0.00	0.39	0.38	0.39	2.56	0.00
0.8	0.81	0.80	0.81	1.23	0.00	0.66	0.66	0.66	0.00	0.00
1	1.00	1.00	1.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00
	Average error %			0.82	0.00	Average error %			9.24	0.00

Table S5: Comparison between numerical and analytical results for dimensionless concentration of product $v(\zeta)$ for various values of parameter γ_3 when $\gamma_1 = 1, \gamma_2 = 1, \varphi_1 = 0.1, \gamma_4 = 1, \alpha_1 = 10$ and $m = 0.9542$.

ζ	$\gamma_3 = 40$ and $l = 0.39268$					$\gamma_3 = 65$ and $l = 0.01502$				
	Num. Eq. (10)	AGM Eq. (26) or HPM [15]] or ADM [30]	TSM Eq. (41)	Error % of AGM Eq. (26)	Error % of TSM Eq. (41)	Num Eq. (10)	AGM Eq. (26) or HPM [15]] or ADM [30]	TSM Eq. (41)	Error % of AGM Eq. (26)	Error % of TSM Eq. (41)
0	0.39	0.39	0.39	0.00	0.00	0.01	0.01	0.01	0.00	0.00
0.2	0.42	0.42	0.42	0.00	0.00	0.05	0.05	0.05	0.00	0.00
0.4	0.49	0.49	0.49	0.00	0.00	0.18	0.17	0.18	5.55	0.00
0.6	0.62	0.61	0.62	1.61	0.00	0.38	0.37	0.38	2.63	0.00
0.8	0.79	0.79	0.79	0.00	0.00	0.66	0.66	0.66	0.00	0.00
1	1.00	1.00	1.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00
	Average error %			0.27	0.00	Average error %			1.36	0.00

Table S6: Comparison between numerical and analytical results for dimensionless concentration of product $v(\zeta)$ for various values of parameter γ_4 when $\gamma_1 = 1, \gamma_2 = 1, \gamma_3 = 0.1, \varphi_1 = 1, \alpha_1 = 5$ and $m = 0.915769$.

ζ	$\gamma_4 = 10$ and $l = 0.7029$					$\gamma_4 = 20$ and $l = 0.4104$				
	Num. Eq. (10)	AGM Eq. (26) or HPM [15]] or ADM [30]	TSM Eq. (41)	Error % of AGM Eq. (26)	Error % of TSM Eq. (41)	Num Eq. (10)	AGM Eq. (26) or HPM [15] or ADM [30]	TS M Eq. (41)	Error % of AGM Eq. (26)	Error % of TSM Eq. (41)
0	0.70	0.72	0.70	2.86	0.00	0.41	0.44	0.41	7.32	0.00
0.2	0.71	0.73	0.71	2.82	0.00	0.43	0.46	0.43	6.98	0.00
0.4	0.75	0.76	0.75	1.33	0.00	0.51	0.53	0.51	3.92	0.00
0.6	0.81	0.82	0.81	1.23	0.00	0.63	0.64	0.63	1.59	0.00
0.8	0.90	0.90	0.90	0.00	0.00	0.80	0.80	0.80	0.00	0.00
1	1.00	1.00	1.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00
	Average error %			1.37	0.00	Average error %			3.30	0.00

Table S7: Comparison between numerical and analytical normalized steady-state source terms of liquid phase ψ_l for various values of parameter φ_1 and α_1 when $\gamma_1 = \gamma_2 = 0.1$.

φ_1	$\alpha_1 = 0.1$					$\alpha_1 = 0.5$				
	Num. Eq. (13)	AGM Eq. (27) or HPM [15]] or ADM [30]	TSM Eq. (43)	Error % of AGM Eq. (27)	Error % of TSM Eq. (43)	Num. Eq. (13)	AGM Eq. (27) or HPM [15]] or ADM [30]	TSM Eq. (43)	Error % of AGM Eq. (27)	Error % of TSM Eq. (43)
0	0.06	0.06	0.06	0.00	0.00	0.04	0.04	0.04	0.00	0.00
0.1	0.09	0.09	0.09	0.00	0.00	0.07	0.07	0.07	0.00	0.00
0.5	0.20	0.21	0.21	5.00	5.00	0.15	0.15	0.15	0.00	0.00
1	0.34	0.37	0.34	8.82	0.00	0.26	0.27	0.26	3.85	0.00
5	1.25	1.56	1.33	24.8	6.40	1.00	1.15	1.01	15.00	1.00
10	2.08	3.09	2.42	48.56	16.35	1.74	2.27	1.90	30.46	9.19
50	5.64	15.2	5.23	169.7	7.27	4.81	11.15	5.22	131.8	8.52
100	7.75	30.4	5.66	291.7	26.97	7.31	22.27	5.66	204.6	22.57
	Average error %			68.68	7.75	Average error %			48.22	5.16

Table S8: Comparison between numerical and analytical normalized steady-state source terms of gas phase ψ_g for various values of parameter φ_2 and α_1 when $\gamma_1 = \gamma_2 = 1, \gamma_3 = \gamma_4 = 0.1, \varphi_1 = 1$ and $\omega = 1$.

φ_2	$\alpha_1 = 0.1$					$\alpha_1 = 0.5$				
	Num. Eq. (14)	AGM Eq. (28) HPM [15], or ADM [30]	TSM Eq. (44)	% Error of AGM Eq. (28)	% Error of TSM Eq. (44)	Num. Eq. (14)	AGM Eq. (28) or HPM [15]] or ADM [30]	TSM Eq. (44)	% Error of AGM Eq.(28)	% Error % of TSM Eq. (44)
0	0.08	0.09	0.09	12.50	12.50	0.06	0.07	0.06	16.67	0.00
0.1	0.12	0.14	0.12	16.67	0.00	0.10	0.10	0.10	0.00	0.00
0.5	0.27	0.32	0.28	18.52	3.70	0.22	0.23	0.22	4.54	0.00
1	0.47	0.54	0.47	14.87	0.00	0.37	0.40	0.37	8.11	0.00
5	2.02	2.36	2.03	16.83	0.49	1.58	1.73	1.60	9.49	1.27
10	3.96	4.64	3.98	17.17	0.50	3.10	3.40	3.14	9.68	1.29
50	19.5	22.8	19.55	17.21	0.41	15.26	16.73	15.42	9.63	1.05
100	38.9	45.5	39.02	17.19	0.41	30.46	33.40	30.78	9.65	1.05
	Average error %			16.37	2.25	Average error %			8.47	0.58