



Article

Theoretical and Experimental Decomposition of Mechanical Error Propagation in Optical Rotary Encoders

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Abstract

This paper presents a combined theoretical and experimental methodology for analysing and validating mechanical error propagation in optical rotary encoders. A vector-based mathematical model is experimentally verified using a three-stage methodology comprising measurement of shaft centre motion, grating disc motion and encoder angular error. Independent measurements of the individual mechanical error sources are combined to predict the encoder angular error and compared with traceable angular comparator measurements. The proposed methodology achieved correlation coefficients up to 0.984 for two different grating disc mounting conditions, demonstrating good agreement between the predicted and measured angular error profiles. The results confirm that encoder angular error is governed by the combined contribution of multiple mechanical error sources and demonstrate that, once the dominant eccentricity component is minimised, optimisation of the circumferential optical readhead position provides an effective means of reducing the residual angular error. The proposed methodology provides a practical framework for analysing, predicting and optimising mechanical error sources during the design, assembly and calibration of high-accuracy optical rotary encoders.

Keywords: optical rotary encoder; angular error; grating disc eccentricity; shaft centre motion; readhead position

1. Introduction

The error sources in optical rotary encoders can be broadly classified into three categories: (I) signal level errors caused by imperfect signal interpolation, harmonic distortion, and noise in the electronic readout chain [1–3]; (II) graduation errors introduced during manufacture of the grating disc, including irregular line spacing and pitch variation; and (III) mechanical installation errors arising from imperfect alignment of the encoder disc relative to the rotation axis [4]. Understanding and reducing mechanical error propagation is therefore the central challenge for achieving high angular accuracy in practical encoder installations.

A comprehensive review by Zhao et al. [5] identifies mechanical installation eccentricity of the grating disc as consistently the dominant error source for optical encoders, significantly exceeding graduation and interpolation contributions. The eccentricity of the grating pattern centre relative to the nominal rotation axis has a well-understood geometric effect: a fixed offset of magnitude between the grating centre and the axis produces a

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first harmonic (once-per-revolution) sinusoidal angular error. Behrani et al. [6] developed a refined three-centre variable eccentricity model that better captures the actual phase relationships observed in high-precision angular sensor fine angle signals, attributing the eccentricity to a combination of manufacturing disc runout, clearance fit mounting, and bearing wobble contributions. Online eccentricity compensation strategies have been explored by several groups, including the cobweb-structured grating scale approach of Zhao et al. [7], which encodes radial displacement information directly onto the grating to enable real-time eccentricity parameter estimation; and the single readhead method proposed by Fang et al. [8] for online identification of coupled eccentricity and tilt in polar coordinate encoders. Hardware embedded self-calibration solutions for angle encoders have also been reported [9].

While eccentricity-based first harmonic errors are most readily detected and compensated, high-accuracy applications require attention to higher-order mechanical error components that become significant once the dominant first harmonic is suppressed. These higher-order contributions arise primarily from non-circular bearing motion, that is, the two-dimensional trajectory traced by the shaft centre in the plane of the grating as the shaft rotates [10,11]. Bearing-induced radial runout, axial wobble, and cage frequency disturbances all generate Fourier harmonics at integer and non-integer multiples of the rotation frequency. The relationship between shaft centre trajectory harmonics and the resulting angular error harmonics depends on the circumferential position of the optical readhead, creating a position-dependent transfer function that has not been systematically characterised in the literature. Jia et al. [12] analysed the angle measurement error mechanism caused by rotor runout and demonstrated the importance of its directional projection. However, the dependence of the resulting angular error on the circumferential readhead position has received considerably less attention.

The characterisation of rotary axis motion is well established in spindle metrology. Error separation methods based on reversal, multi-probe and multi-step measurements enable the spindle motion to be distinguished from reference artefact form errors [13]. Subsequent developments have achieved sub-nanometre error separation [14], simultaneous radial and tilt error measurement [15], and multi-degree-of-freedom spindle characterisation [16]. Despite these advances, shaft motion metrology and encoder error analysis have generally been treated separately, and the measured shaft centre trajectory has not been systematically incorporated into experimental prediction of encoder angular error.

The effect of grating disc installation on encoder accuracy extends beyond simple eccentricity to include disc tilt, mounting-induced deformation, and uneven fixing forces. Yu et al. [17] demonstrated that grating disc inclination generates a characteristic double-cycle angular error harmonic and requires at least three readheads for complete suppression. Li et al. [18] investigated the angular error introduced by taper-mount installation, showing through finite element analysis that screw-induced disc deformation generates m cycle harmonics whose order depends on the number of fastener screws, and proposed a comprehensive model that addresses deformation, eccentricity, and inclination simultaneously. Another comprehensive analytical model incorporating rotary axis errors, eccentricity, inclination, graduation errors and readhead installation errors has been developed to describe their combined influence on angular measurement [4,10].

Current encoder error compensation strategies can broadly be classified into offline calibration, in which systematic errors are identified using a high-accuracy reference instrument and stored as correction tables; and online self-correction, where multiple readheads or auxiliary sensing techniques continuously estimate and compensate systematic errors during operation. Hsieh et al. [19,20] proposed high-accuracy calibration procedures based on shift-angle and dual-autocollimator methods, enabling precise generation of encoder correction tables using traceable reference measurements. Ishii et al. [21]

developed multi readhead encoder systems capable of real-time suppression of systematic angular errors by exploiting redundancy between multiple detecting readheads, while Watanabe et al. [22] introduced the SelfA concept for achieving ultra-high resolution and accuracy through self-calibration. In parallel, Jia et al. [23] proposed a Fourier expansion-back-propagation neural network optimised using a genetic algorithm, reducing encoder errors in articulated coordinate measuring machine applications. Although these approaches significantly improve encoder accuracy, they compensate for the resulting angular error rather than addressing the underlying mechanical error propagation mechanisms responsible for its generation.

More broadly, recent developments outside conventional grating encoders further demonstrate the increasing role of model-assisted measurement in optical metrology. Structured and vortex light has been investigated for rotation and angular displacement sensing, while reconstructive optical instruments combine compact optical encoding hardware with computational reconstruction [24,25]. Although these approaches employ different physical principles from conventional rotary encoders, they reflect a broader movement towards combining optical measurements with physical or computational models to extract metrological information.

Despite extensive research on encoder eccentricity modelling, spindle error separation and multi readhead self-calibration, a unified experimental framework that independently characterises the shaft-bearing system, grating disc mounting, and readhead position, and quantitatively links these factors to the final angular positioning accuracy through a validated mechanical model, has not yet been reported. Existing studies typically treat eccentricity as a single, lumped error source or investigate shaft motion and encoder accuracy independently, without experimentally verifying their individual and combined contributions to the resulting angular error. Furthermore, the effect of circumferential readhead position on the projection of rotor system motion into encoder angular error has not been systematically investigated.

These limitations are particularly important for open-type optical encoders, in which the grating disc is mounted directly on the application shaft and the stationary readhead is attached to the supporting structure. In such configurations, the final angular accuracy is determined not only by the intrinsic performance of the encoder but also by the complete mechanical assembly, including shaft-bearing motion, grating disc mounting errors, and readhead placement. Consequently, encoder performance specified or characterised independently of the application mechanics may not fully represent the angular accuracy achieved after integration.

Direct calibration of the complete rotary axis after encoder integration can address this uncertainty, but it generally requires specialised metrology equipment, such as a precision polygon with an autocollimator, an angular interferometer, or a high-accuracy rotary comparator. Such equipment may not be readily available during the design, assembly, and optimisation stages. This creates a practical need for an experimentally validated mechanical framework capable of relating independently measurable assembly parameters to the resulting angular error, thereby enabling the final angular accuracy to be assessed and potentially improved without relying exclusively on full system angular calibration.

This study addresses these limitations by developing and experimentally validating a three-stage mechanical error propagation framework for open-type optical rotary encoders. The proposed framework represents shaft centre motion and grating disc eccentricity as separate two-dimensional displacement vectors. These vectors are combined to obtain the resultant grating centre motion, which is then projected onto the local tangential direction at the readhead position and converted into angular error. The shaft centre motion, grating disc motion and encoder angular error are measured independently using

orthogonal inductive probes, digital microscopy and a traceable angular comparator, respectively. These independently obtained measurements are quantitatively linked through the proposed model to predict the encoder angular error profile. Experimental validation is performed under two grating disc mounting conditions with different eccentricity levels, enabling the individual and combined contributions of the measured mechanical error sources to be evaluated.

The work has two related but distinct objectives. The first is to experimentally validate the proposed mechanical error propagation framework by comparing the predicted angular error profiles with independent angular comparator measurements and by examining how the measured parameters combine to produce the resulting encoder error. The second is to investigate, through numerical analysis using the experimentally measured rotor trajectories, the influence of circumferential readhead position on the residual angular error. This analysis identifies readhead placement as an additional design parameter that can potentially be used to minimise residual error in open-type encoder installations.

2. Theoretical Model of Error Propagation

The operating principle of an optical rotary encoder is based on the relative motion between a rotating optical grating and a stationary optical readhead. Ideally, the centre of the grating pattern coincides with the rotation axis, resulting in a purely rotational motion of the grating track. In practical encoders, however, the grating centre follows a non-ideal trajectory due to rotor system kinematic motion and imperfect grating disc centring. This trajectory gives rise to a tangential displacement of the grating pattern at the readhead position, which is interpreted as an angular measurement error [10,12].

As illustrated in Figure 1, the grating centre trajectory is decomposed into two displacement vectors. The vector $\vec{e}_1(\varphi)$ describes the instantaneous displacement of the shaft centre relative to the nominal rotation axis fixed to the encoder housing. The vector $\vec{e}_2(\varphi)$ describes the eccentricity of the grating pattern centre relative to the shaft centre. The resulting displacement of the grating pattern centre is therefore:

$$\vec{e}(\varphi) = \vec{e}_1(\varphi) + \vec{e}_2(\varphi), \quad (1)$$

where φ is the shaft angular position.

In Figure 1, the point O denotes the origin of the stationary coordinate system OXY , fixed to the encoder housing. The point O_1 represents the instantaneous shaft centre position, while O_2 denotes the centre of the grating pattern. The vector $\vec{e}_1(\varphi)$ is associated with bearing imperfections, rotor geometry, assembly deviations, loading, thermal effects and other kinematic influences. Its magnitude and direction may vary during one revolution. By contrast, the magnitude of the grating eccentricity vector $\vec{e}_2$ is assumed constant because the grating disc is rigidly connected to the shaft, while its direction rotates together with the rotor.

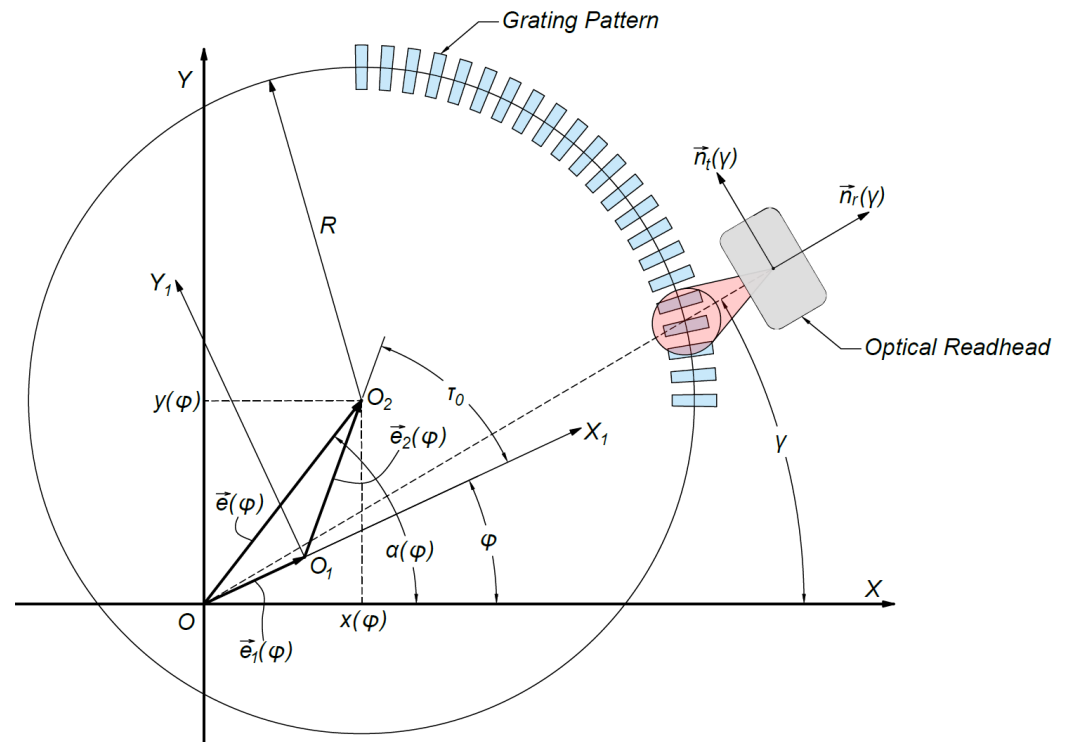


Figure 1. Vector decomposition of the grating pattern centre motion and its projection at the optical readhead.

2.1. Grating Centre Motion Decomposition

A rotating coordinate system $O_1X_1Y_1$ is attached to the shaft. The shaft centre displacement in the stationary coordinate system is written as:

$$\vec{e}_1(\varphi) = \begin{bmatrix} x_{o_1}(\varphi) \\ y_{o_1}(\varphi) \end{bmatrix}, \quad (2)$$

The initial grating eccentricity in the shaft-fixed coordinate system is:

$$\vec{e}_{2,0} = e_2 \begin{bmatrix} \cos \tau_0 \\ \sin \tau_0 \end{bmatrix}, \quad (3)$$

where e_2 is the eccentricity magnitude and τ_0 is its initial orientation relative to the X_1 -axis. During shaft rotation, this vector is transformed into the stationary coordinate system by the rotation matrix:

$$M(\varphi) = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix}, \quad (4)$$

The resultant grating centre displacement is therefore:

$$\vec{e}(\varphi) = \vec{e}_1(\varphi) + M(\varphi)\vec{e}_{2,0}, \quad (5)$$

Equation (5) represents the combined planar trajectory of the grating centre. It combines the arbitrary translational motion of the shaft centre with the rotating eccentricity of the grating disc and therefore forms the basis for predicting the mechanical contribution to the encoder angular error.

In Cartesian form, the grating centre coordinates are:

$$x(\varphi) = x_{o_1}(\varphi) + e_2 \cos(\tau_0 + \varphi), \quad (6)$$

$$y(\varphi) = y_{o_1}(\varphi) + e_2 \sin(\tau_0 + \varphi), \quad (7)$$

The first terms in Equations (6) and (7) describe the shaft centre trajectory, whereas the second term describes the once-per-revolution motion caused by grating eccentricity. The resultant displacement magnitude and direction are

$$e(\varphi) = \sqrt{x^2(\varphi) + y^2(\varphi)}, \quad (8)$$

$$\alpha(\varphi) = \text{atan2}[y(\varphi), x(\varphi)] \quad (9)$$

Since the shaft rotates periodically, its trajectory is also periodic and can be represented by a truncated Fourier series [23]:

$$x_{O_1}(\varphi) = a_{x,0} + \sum_{n=1}^N [a_{x,n} \cos(n\varphi) + b_{x,n} \sin(n\varphi)], \quad (10)$$

$$y_{O_1}(\varphi) = a_{y,0} + \sum_{n=1}^N [a_{y,n} \cos(n\varphi) + b_{y,n} \sin(n\varphi)], \quad (11)$$

where N is the highest considered harmonic order; $a_{x,0}$ and $a_{y,0}$ are the mean offsets; $a_{x,n}$, $b_{x,n}$, $a_{y,n}$ and $b_{y,n}$ are the Fourier coefficients of the n -th harmonic.

2.2. Projection at the Optical Readhead

The optical readhead measures the local displacement of the grating track rather than the overall displacement magnitude of the grating centre $e(\varphi)$. Therefore, the resultant displacement vector must be projected onto the local radial and tangential directions of the readhead position. Let the readhead be located at the circumferential angle γ , measured from the positive X-axis. The local radial and tangential unit vectors are:

$$\vec{n}_r(\gamma) = \begin{bmatrix} \cos \gamma \\ \sin \gamma \end{bmatrix}, \quad (12)$$

$$\vec{n}_t(\gamma) = \begin{bmatrix} -\sin \gamma \\ \cos \gamma \end{bmatrix}, \quad (13)$$

The radial displacement at the readhead is obtained by projecting the grating centre displacement onto $\vec{n}_r$

$$u_r(\varphi, \gamma) = \vec{e}(\varphi) \cdot \vec{n}_r(\gamma), \quad (14)$$

or

$$u_r(\varphi, \gamma) = x(\varphi) \cos \gamma + y(\varphi) \sin \gamma, \quad (15)$$

Similarly, the tangential displacement could be expressed as:

$$u_t(\varphi, \gamma) = \vec{e}(\varphi) \cdot \vec{n}_t(\gamma), \quad (16)$$

which gives:

$$u_t(\varphi, \gamma) = -x(\varphi) \sin \gamma + y(\varphi) \cos \gamma, \quad (17)$$

The two components have different physical effects in an optical encoder. The radial component u_r changes the radial distance between the readhead and the grating track. Depending on the optical principle and readhead design, this may affect signal amplitude, contrast, modulation depth, focus, and interpolation quality. Although the radial component may influence the optical signal quality, its effect depends on the specific readhead design and optical transfer characteristics and is therefore outside the scope of the present model.

The tangential component u_t displaces the grating pattern along the sensing direction. This produces an apparent phase shift in the periodic optical signals and therefore a

corresponding angular measurement error. The present model considers this tangential effect as the primary direct mechanical contribution to the encoder error.

Substitution of Equations (6) and (7) into Equation (17) gives:

$$u_t(\varphi, \gamma) = -x_{o_1}(\varphi) \sin \gamma + y_{o_1}(\varphi) \cos \gamma + e_2 \sin(\tau_0 + \varphi - \gamma), \quad (18)$$

Thus, the tangential displacement consists of a shaft motion contribution and a grating eccentricity contribution. For a readhead positioned at $\gamma = 0$, the local tangential direction coincides with the positive Y-axis direction, and Equation (18) reduces to:

$$u_t(\varphi, 0) = y(\varphi), \quad (19)$$

2.3. Conversion of Tangential Displacement into Angular Error

Let R denote the effective radius of the optical grating track. For the micrometre-scale displacements considered in the optical encoders $|u_t| \ll R$. The angular displacement associated with the local tangential motion can therefore be evaluated using the small angle approximation [4]:

$$\Delta\theta(\varphi, \gamma) \approx \frac{u_t(\varphi, \gamma)}{R}, \quad (20)$$

where $\Delta\theta(\varphi, \gamma)$ denotes the predicted angular error at shaft angular position φ for a readhead located at circumferential position γ . By substituting Equation (18), the complete angular error relation is:

$$\Delta\theta(\varphi, \gamma) = \frac{-x_{o_1}(\varphi) \sin \gamma + y_{o_1}(\varphi) \cos \gamma + e_2 \sin(\tau_0 + \varphi - \gamma)}{R}, \quad (21)$$

Equation (21) is the principal angular error equation of the model, which explicitly relates the shaft centre motion, grating disc eccentricity, optical readhead position, and grating track radius to the resulting encoder error.

2.4. Influence of Optical Readhead Position

The general model Equation (21) shows that the same two-dimensional grating centre trajectory produces different angular error waveforms at different optical readhead positions. For the four cardinal positions:

$$\begin{aligned} \gamma = 0^\circ : u_t &= y, \\ \gamma = 90^\circ : u_t &= -x, \\ \gamma = 180^\circ : u_t &= -y, \\ \gamma = 270^\circ : u_t &= x, \end{aligned} \quad (22)$$

Consequently, the predicted angular error amplitude, RMS value, and harmonic content depend on the circumferential location of the readhead. This dependence is particularly relevant when the shaft trajectory is not circular or is dominated by higher-order directional components. A readhead located near a direction of minimum tangential projection may exhibit lower mechanically induced error than a readhead installed at another angular position.

3. Materials and Methods

3.1. Experimental Workflow

The objective of this study is to experimentally decompose the propagation of mechanical errors into the final angular measurement error of an optical rotary encoder. Since the measured encoder error is produced by the combined influence of rotor motion and grating geometry, these contributions cannot be separated using a single experimental method. Therefore, three complementary experimental stages were designed, each

emphasising a different part of the overall error propagation process. The complete methodology is summarised in Figure 2.

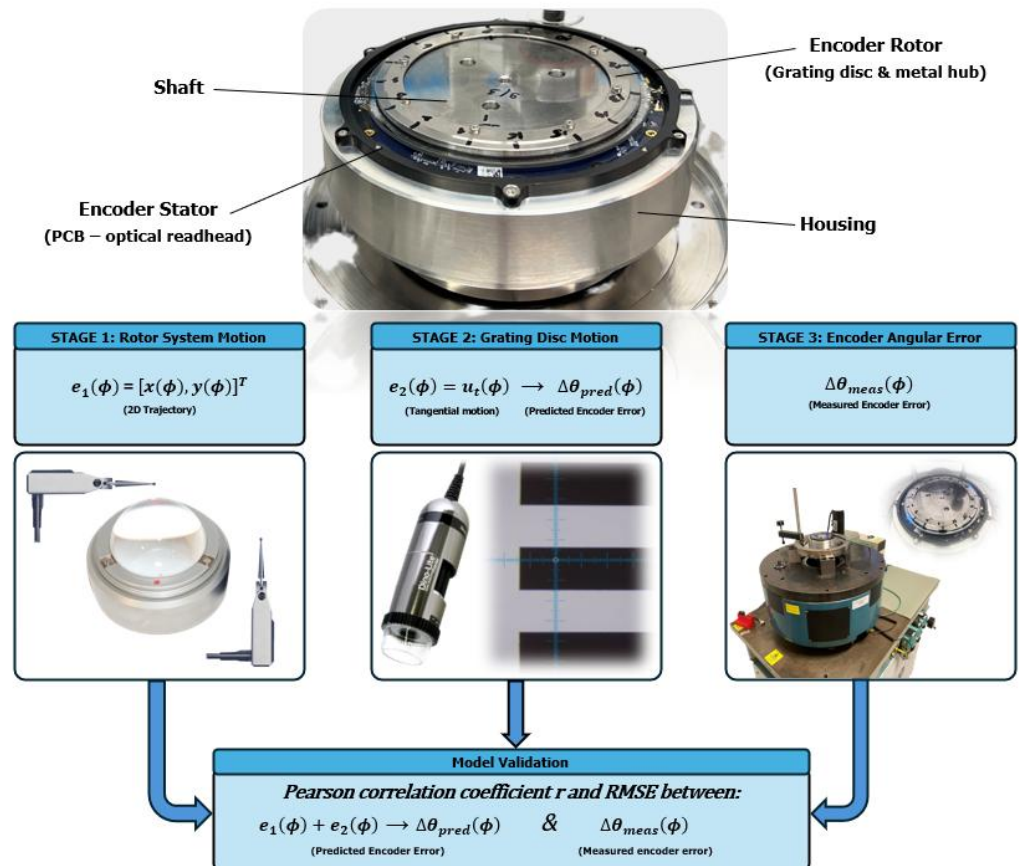


Figure 2. Experimental investigation and validation workflow.

All experiments were performed using the same experimental rotary encoder assembly. The assembly consisted of an aluminium housing supporting a stainless-steel shaft through two preloaded deep-groove ball bearings, forming the mechanical rotary system. An open-type 22-bit absolute optical encoder (JSC Precizika Metrology, Vilnius, Lithuania) was integrated into the assembly. The encoder rotor (metal hub and glass grating disc) was rigidly mounted on the shaft, whereas the encoder stator (printed circuit board with the optical readhead and signal-processing electronics) was fixed to the housing. The mounting arrangement allowed the grating disc to be re-centred with respect to the shaft axis while maintaining the same rotor system and stator throughout all experiments. Consequently, only the relative position of the grating disc changed between the investigated mounting conditions, whereas the optical readhead position remained unchanged. The nominal radius of the optical grating track is $R = 53.25$ mm. All measurements were performed in a temperature-controlled laboratory maintained at 20.0 ± 0.2 °C.

The experimental methodology consists of three consecutive stages. In Stage 1, the two-dimensional shaft centre trajectory is measured using a precision reference hemisphere and two orthogonally arranged inductive probes. The measured shaft centre motion is subsequently used to simulate its contribution to the encoder angular error at different circumferential readhead positions. In Stage 2, the motion of the actual optical grating disc relative to the stationary microscope is measured under two different disc mounting conditions with different eccentricity levels. In Stage 3, the angular error profiles predicted from Stage 1 and Stage 2 measurements are compared with direct calibration measurements obtained using a high-accuracy angular comparator. Together, these three

stages connect the measured mechanical motion of the encoder rotor system with the resulting angular measurement error.

3.2. Stage 1: Characterisation of the Rotor System Motion

The objective of the first experimental stage was to characterise the two-dimensional motion of an idealised grating centre induced by the encoder rotor system. Instead of an optical grating disc, a precision reference hemisphere was mounted on the encoder shaft to represent an ideal grating, in which grating pitch errors, local pattern distortions, edge form deviations, and axial disc runout were considered negligible. Under this assumption, the measured trajectory represents the combined motion of the idealised grating centre resulting from the rotor system and its initial centring relative to the shaft. Consequently, the experiment provides an experimental analogue of the displacement model introduced in Section 2 while eliminating the influence of actual grating manufacturing errors.

The experimental setup is shown in Figure 3. A precision 50 mm diameter polished glass reference hemisphere (Taylor Hobson Ltd., Leicester, United Kingdom) was rigidly mounted on the encoder shaft. The hemisphere had a form deviation of approximately $0.03\ \mu\text{m}$ and was calibrated with a certified uncertainty of less than 5 nm, making its geometric form error negligible compared with the micrometre-scale displacements investigated in this study. Consequently, the measured trajectory is dominated by the mechanical behaviour of the rotor system and the imposed centring condition rather than by imperfections of the reference artefact.

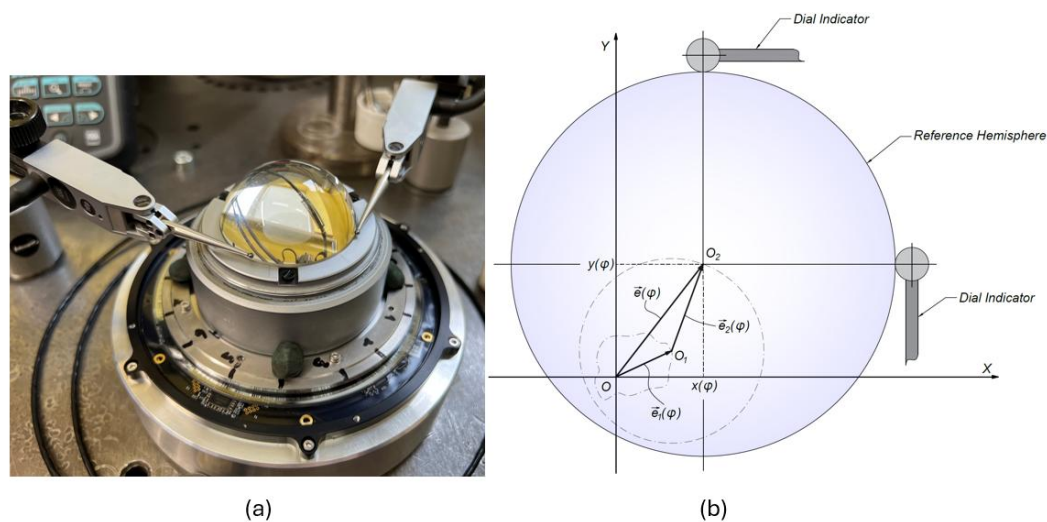


Figure 3. Stage 1 experimental setup: (a) reference hemisphere mounted on the encoder shaft; (b) schematic of the Cartesian displacement measurement.

Two TESA 32.10802 inductive displacement probes (TESA SA, Renens, Switzerland), connected to TESATRONIC® TWIN-T10 digital display units (with a digital resolution of $0.1\ \mu\text{m}$), were mounted orthogonally around the hemisphere and aligned with the stationary X and Y axes of the measurement coordinate system. The specified indication error of the measurement system is within 1% of the selected measuring range. The overall measurement uncertainty is additionally affected by probe alignment, mounting stability, and environmental conditions and is reflected by the repeatability obtained from the six repeated measurements.

The measuring axes intersected at the nominal centre of the hemisphere, enabling simultaneous measurement of the Cartesian displacement components $x(\varphi)$ and $y(\varphi)$ during shaft rotation. Unlike conventional radial runout measurements, the two-probe arrangement enabled reconstruction of the complete two-dimensional trajectory of the

reference hemisphere as a coupled planar displacement. The measured Cartesian trajectory represents an experimental analogue of the resultant displacement vector $e(\varphi)$, where the imposed hemisphere centring replaces the grating eccentricity term of the theoretical model.

$$e(\varphi) = \begin{bmatrix} x(\varphi) \\ y(\varphi) \end{bmatrix} \quad (23)$$

To investigate the influence of grating centring, two mounting conditions were intentionally prepared. In Condition A, the hemisphere was carefully centred with respect to the shaft axis, representing a well centred grating. In Condition B, a larger initial eccentricity was introduced before tightening, representing a poorly centred grating. Since both measurements were performed using the same rotor system, the principal difference between the two trajectories is the magnitude of the introduced centring component. This approach enables the influence of initial grating centring to be investigated independently of the intrinsic mechanical behaviour of the rotor system. Increasing the number of angular positions could improve trajectory reconstruction and higher-order harmonic estimation by reducing interpolation and aliasing effects. However, it would not change the displacement resolution of the inductive probe system at each individual position.

For each mounting condition, six complete shaft revolutions were measured at sixteen equally spaced angular positions over one revolution. The repeated measurements were used to evaluate repeatability, determine the mean two-dimensional trajectory and estimate the associated confidence intervals. This sampling scheme provided a practical compromise between angular resolution and the duration of the manually positioned measurements and repeated experimental series. It was sufficient to evaluate the harmonic content up to eight harmonics considered in this study.

Based on the theoretical model represented by Equations (10) and (11), the first harmonic contains the combined contribution of the rotor system first harmonic motion and the initial hemisphere centring eccentricity. Because Conditions A and B differ primarily in their initial centring, this dominant component was removed to enable comparison of the remaining higher-order motion associated with the rotor system. For each condition, the mean coupled two-dimensional trajectory was fitted by linear least squares using the sine and cosine terms corresponding to $n = 1$. The fitted first harmonic components were then subtracted component-wise from x and y displacement data of every measured repetition within the corresponding condition. Thus, the same first harmonic component, estimated from the mean trajectory of each condition, was removed from all its repetitions while preserving repeat-to-repeat differences.

Discrete Fourier decomposition was subsequently applied separately to quantify and compare the harmonic content before and after first harmonic removal. Comparison of the resulting trajectories from Conditions A and B provides an experimental assessment of the similarity of the remaining higher-order rotor motion under different initial centring conditions.

Numerical Investigation of Optical Readhead Position

The experimental measurements described in the previous subsection provide the complete two-dimensional trajectory of the idealised grating centre in the stationary Cartesian coordinate system. Since the trajectory is represented by both displacement components ($x(\varphi)$ and $y(\varphi)$), it can be numerically projected onto the tangential direction of an optical readhead located at an arbitrary angular position without requiring additional experiments.

According to the mathematical model presented in Section 2, the tangential displacement observed by an optical readhead positioned at an angular location γ is obtained by

projecting the measured displacement vector onto the local tangential direction using Equation (17). The corresponding angular measurement error is then determined from Equation (20). This approach enables the experimentally measured rotor system motion to be evaluated for any virtual readhead position around the encoder circumference while preserving exactly the same measured mechanical trajectory. Consequently, the influence of optical readhead location can be investigated independently of the rotor system itself, since only the projection direction changes whereas the measured displacement vector remains unchanged.

To evaluate this influence, the measured trajectories obtained under mounting Conditions A and B were projected onto optical readhead positions distributed around the full circumference. The projection angle γ was varied from 0° to 360° with a constant angular increment of 1° , and for each position the corresponding tangential displacement, predicted angular error, its peak-to-peak value and root-mean-square (RMS) value were calculated.

Because the initial centring error primarily contributes to the first harmonic of the displacement trajectory, the numerical analysis was performed for both the complete measured trajectories and the trajectories after removal of the first harmonic. This enabled the influence of optical readhead position to be investigated under two representative conditions: the total idealised grating centre motion, including the initial hemisphere centring error, and the residual rotor system motion after compensation of the dominant once-per-revolution component. Comparison of these two cases allows the contribution of initial centring to be distinguished from the influence of the intrinsic higher-order rotor system motion.

3.3. Stage 2: Measurement of Grating Motion Using a Digital Microscope

Following the characterisation of the shaft centre motion in Stage 1, the second experimental stage focused on the displacement of the optical grating itself. Unlike the reference hemisphere experiment, which isolated the mechanical motion of the rotating shaft, this stage investigated the combined effect of shaft motion and grating mounting eccentricity observed directly at the grating pattern. The objective was to experimentally determine the tangential displacement of the grating at the optical readhead position and to predict the corresponding angular error.

Figure 4 illustrates the experimental arrangement and the adopted measurement principle. The measurements were performed using a Dino-Lite AM7515MT8A digital microscope (AnMo Electronics Corporation, Taiwan, China) mounted above the encoder disc while the encoder shaft was rotated manually through one complete revolution. The microscope was rigidly fixed to the experimental frame to eliminate any relative movement between the optical system and the encoder housing during the measurements. The microscope was focused on the outer edge of the grating pattern, and the displacement of the same grating line was tracked throughout the rotation.

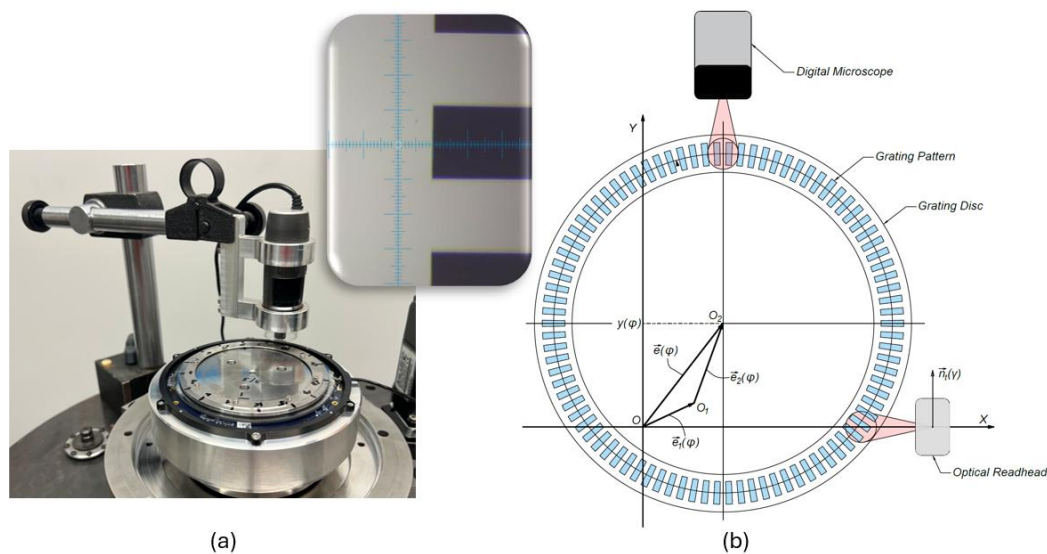


Figure 4. Stage 2 experimental setup for measuring grating pattern motion: (a) digital microscope used to measure the tangential displacement of the grating pattern; (b) schematic of the measurement principle and its relation to the optical readhead sensing direction.

The microscope observes only a small region of the optical grating; therefore, it measures the displacement of an individual grating line rather than the complete two-dimensional trajectory of the grating centre. To directly measure the tangential displacement relevant to encoder operation, the microscope was positioned at a circumferential location approximately 90° from the optical readhead. At this position, the local tangential direction of the circular grating coincides with the global Y -axis, allowing the tangential displacement to be measured directly as the vertical image displacement:

$$u_t(\varphi) = u_Y(\varphi) \quad (24)$$

This measurement configuration eliminates the need for any coordinate transformation of the recorded displacement while remaining fully consistent with the theoretical model. The measured tangential displacement was subsequently converted into the corresponding angular error using the relationship given by Equation (20). Thus, the microscope experiment provides a direct experimental estimate of the mechanical contribution of grating centre motion to the encoder angular error.

The microscope images were acquired using the same angular sampling scheme of sixteen equally spaced angular positions over one shaft revolution, including the repeated closing position at 360° . Five independent measurement series were performed for each investigated mounting condition. Two different grating disc mounting conditions were investigated. Condition D1 corresponded to the more centred mounting of the grating disc. To establish Condition D2, the disc was rotated to a different angular orientation relative to the shaft and deliberately remounted with a larger centring offset. Therefore, D1 and D2 differed in both the magnitude and initial orientation of the grating disc eccentricity vector relative to the rotor system. The shaft-bearing system, housing, and optical readhead position remained unchanged. These two conditions were subsequently used throughout numerical analysis and compared with the encoder calibration results obtained in Stage 3.

The displacement of the selected grating line edge was determined directly from the microscope images using the calibrated measurement scale displayed in the DinoCapture 2.0 software (AnMo Electronics Corporation, Taiwan, China). The displacement was evaluated relative to the initial angular position using the visible $1 \mu\text{m}$ measurement grid, which represented the principal limitation of the measurement resolution. Additional

uncertainty sources include local variations in the grating-line geometry (edge roughness, line-width variations and manufacturing non-uniformity), image noise, illumination and focus variations, edge identification repeatability, and residual mechanical instability of the experimental setup. Therefore, the repeatability obtained from the five repeated measurements provides a practical estimate of the combined uncertainty of the complete measurement procedure.

3.4. Stage 3: Angular Error Measurements

The final stage of the study was performed to experimentally validate the proposed mechanical error propagation model by comparing the predicted angular error with the actual error measured using an industrial rotary encoder calibration system (angle comparator developed by JSC Precizika Metrology, Vilnius, Lithuania). The comparator used, shown in Figure 5, consists of a precision air-bearing rotary table equipped with a high-accuracy reference encoder and is intended for traceable calibration of rotary encoders. A detailed description of the comparator design, calibration procedure, and metrological characteristics is available in our previous work [26].

The encoder under investigation was mounted on the comparator using the same mechanical assembly employed throughout Stages 1 and 2, ensuring that the shaft position, grating disc orientation and optical readhead configuration were maintained unchanged during the corresponding measurements. For each grating disc centring condition (D1 and D2), five complete revolutions were measured. The mean angular error profile and Fourier harmonic decomposition were then calculated.

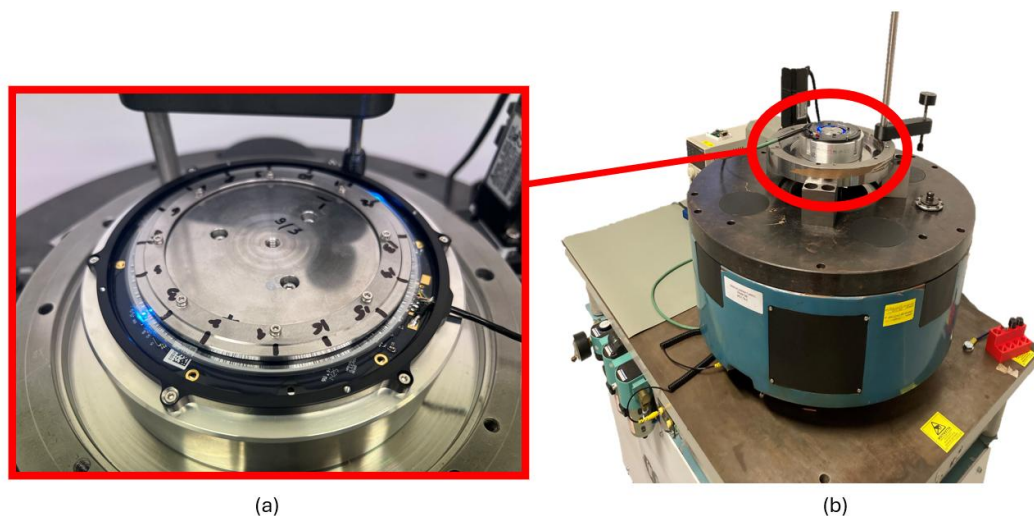


Figure 5. Stage 3 experimental setup for encoder angular error measurement: (a) magnified view of encoder under test; (b) angular comparator used for encoder calibration.

The comparator measurements were subsequently used as the experimental reference for validation of the proposed mechanical error propagation model. The measured encoder angular error profiles were compared with the angular error predicted from the grating disc motion (Stage 2) and with the combined mechanical model incorporating the higher-order rotor system motion identified in Stage 1. The agreement between the measured and predicted angular error profiles was quantified using Pearson's correlation coefficient:

$$r = \frac{\sum_{i=1}^N (\Delta\theta_{pred,i} - \bar{\Delta\theta}_{pred})(\Delta\theta_{meas,i} - \bar{\Delta\theta}_{meas})}{\sqrt{\sum_{i=1}^N (\Delta\theta_{pred,i} - \bar{\Delta\theta}_{pred})^2 \sum_{i=1}^N (\Delta\theta_{meas,i} - \bar{\Delta\theta}_{meas})^2}} \quad (25)$$

and the root-mean-square error (RMSE):

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (\Delta\theta_{\text{pred},i} - \Delta\theta_{\text{meas},i})^2} \quad (26)$$

where $\Delta\theta_{\text{pred}}$ and $\Delta\theta_{\text{meas}}$ denote the predicted and comparator measured angular errors respectively; $\overline{\Delta\theta_{\text{pred}}}$ and $\overline{\Delta\theta_{\text{meas}}}$ are their respective mean values; and N is the number of evaluated angular positions. Pearson's correlation coefficient characterises agreement in waveform shape, whereas RMSE quantifies the pointwise difference between the predicted and measured angular error profiles.

Furthermore, the higher-order residual waveforms and their harmonic content were compared to evaluate the individual contributions of the rotor system and grating disc geometry to the total encoder angular error. The repeatability of the comparator measurements was also evaluated to confirm the reliability of the experimental validation.

3.5. Statistical Analysis of Model Validation

Confidence intervals for the correlation coefficients were estimated using a non-parametric repeat level bootstrap procedure. The resampling unit was a complete measured revolution rather than an individual angular point, because the angular samples within each periodic waveform are not statistically independent. Treating individual angular positions as independent observations would therefore underestimate the uncertainty of the correlation coefficients.

The bootstrap analysis was performed separately for Conditions D1 and D2. In each bootstrap iteration, five complete comparator repetitions, five microscope repetitions, six hemisphere A repetitions, and six hemisphere B repetitions were independently sampled with replacement from their respective datasets. The number of sampled repetitions was thus equal to the number available in the original experiment. The selected repetitions were averaged pointwise, after which the combined model prediction and correlation calculation sequence was repeated. A total of 5000 bootstrap iterations were performed. The 95% confidence intervals were obtained from the 2.5th and 97.5th percentiles of the resulting bootstrap distribution [27,28].

The statistical significance of the RMSE reduction obtained by incorporating rotor system contributions was evaluated using a paired comparison between the microscope-only and combined model predictions. These predictions were evaluated against the same five comparator repetitions at identical angular positions and differed only by the addition of the rotor system contribution. For each comparator repetition, RMSE was calculated for both predictions, and the paired reduction was defined as:

$$\Delta\text{RMSE}_j = \text{RMSE}_{\text{microscope},j} - \text{RMSE}_{\text{complete},j} \quad (27)$$

where j denotes the comparator repetition and a positive value ΔRMSE_j indicates an improvement produced by incorporating the rotor system contribution. The null hypothesis that the mean paired RMSE reduction was zero was evaluated separately for Conditions D1 and D2 using a two-sided paired t -test. The 95% confidence interval of the mean RMSE reduction was additionally estimated from 5000 bootstrap resamples of the five paired differences. Statistical significance was evaluated at $\alpha = 0.05$.

All numerical processing, least-square fitting, Fourier decomposition, readhead position projection, statistical analysis and figure generation were performed using MATLAB R2022b (The MathWorks, Inc. Natick, MA, USA).

4. Results and Discussion

The results are presented according to the three-stage experimental procedure described in Section 3. Section 4.1 characterises the shaft centre motion using the reference hemisphere measurements described in Section 3.2. Section 4.2 evaluates the influence of circumferential readhead position using the numerical projection procedure introduced in Section Numerical Investigation of Optical Readhead Position. Section 4.3 examines the grating disc motion measured by microscope according to Section 3.3. Section 4.4 validates the mechanically predicted angular error profiles against the angular comparator measurements described in Section 3.4, with the associated statistical analysis performed according to Section 3.5. Finally, the practical implications, limitations and directions for future work are discussed in Section 4.5.

4.1. Separation of Rotor System and Eccentricity-Induced Motion (Stage 1)

Figure 6 presents the measured Cartesian displacement components $x(\varphi)$ and $y(\varphi)$ obtained under the two investigated centring conditions. For both conditions, the six repeated measurements show excellent agreement, resulting in narrow 95% confidence intervals over the entire revolution. The mean two-dimensional repeatability was $0.137\ \mu\text{m}$ for Condition A and $0.187\ \mu\text{m}$ for Condition B, confirming that the observed trajectories were highly repeatable and that the measured differences between the two conditions originate primarily from the imposed centring rather than from measurement uncertainty.

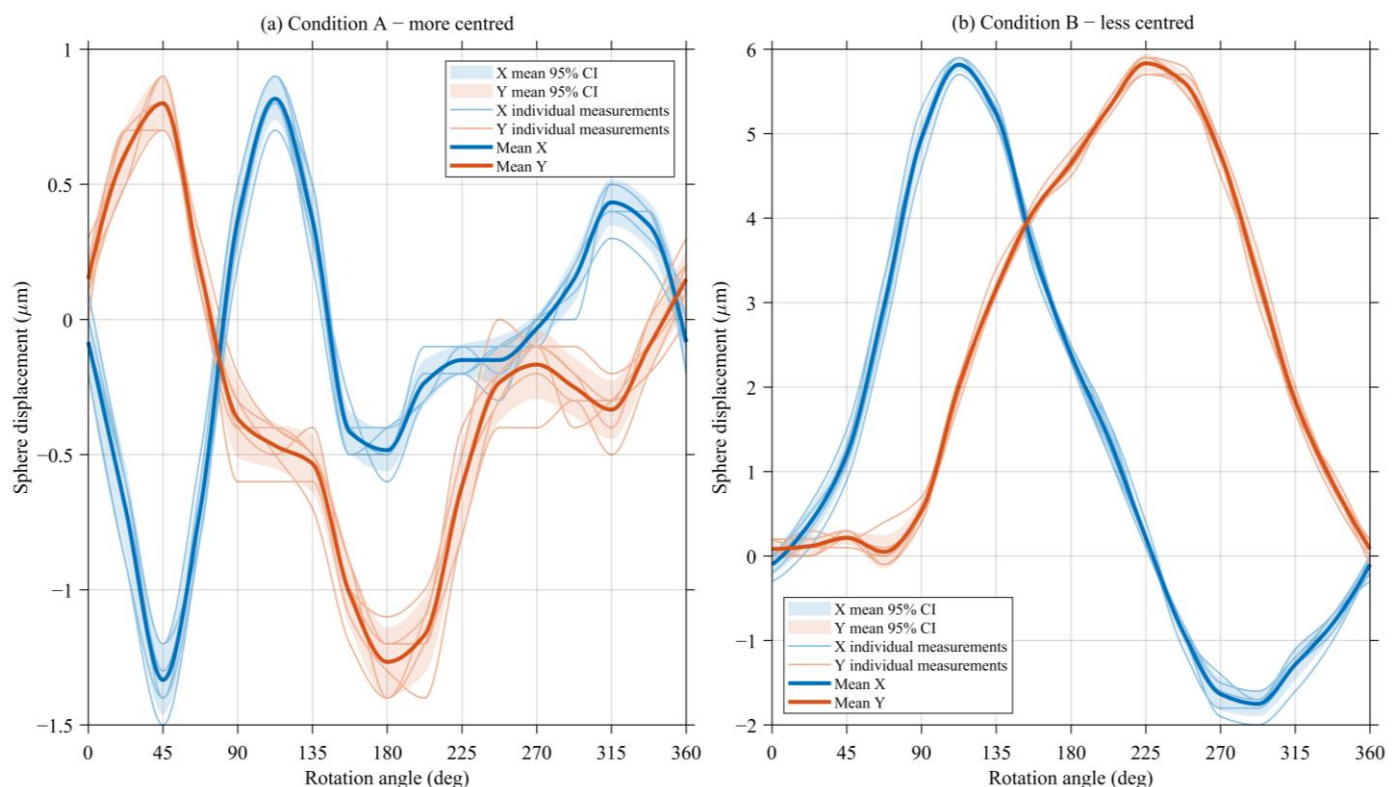


Figure 6. Measured Cartesian displacement trajectories for Conditions A (a) and B (b).

The principal characteristics of both investigated conditions are summarised in Table 1. The influence of the intentionally introduced centring error is clearly visible in the measured trajectories. The well-centred assembly Condition A produced a trajectory diameter of $2.50\ \mu\text{m}$, whereas the intentionally decentred assembly Condition B increased the trajectory diameter to $7.94\ \mu\text{m}$. Similarly, the combined first harmonic amplitude increased

from $0.69\ \mu\text{m}$ to $4.49\ \mu\text{m}$, indicating that the deterioration of centring is predominantly expressed through the first harmonic of the measured displacement.

Table 1. Summary of the measured motion characteristics for Conditions A and B.

Parameter	Condition A	Condition B
2D trajectory repeatability, (μm)	0.137	0.187
Mean trajectory diameter, (μm)	2.50	7.94
1st harmonic amplitude, (μm)	0.69	4.49
Residual trajectory diameter, (μm)	2.17	2.26

The centred trajectories together with their fitted first harmonic approximations are shown in Figure 7a, while the corresponding residual trajectories after first harmonic elimination are presented in Figure 7b. As anticipated from the theoretical model, removing the first harmonic suppresses the dominant contribution of the initial centring while preserving the higher-order components associated with the rotor system.

A comparison of the residual trajectories demonstrates that the remaining shaft centre motion is nearly identical for both centring conditions. The residual X and Y displacement components shown in Figure 7c,d exhibit very similar waveform shapes throughout the complete revolution. Quantitatively, the Pearson correlation coefficients between the residual displacement components reached 0.995 for the X direction and 0.965 for the Y direction. The overall two-dimensional trajectory cosine similarity was 0.988, while the residual two-dimensional RMSE between both trajectories was only $0.107\ \mu\text{m}$, with a maximum point-wise difference below $0.20\ \mu\text{m}$.

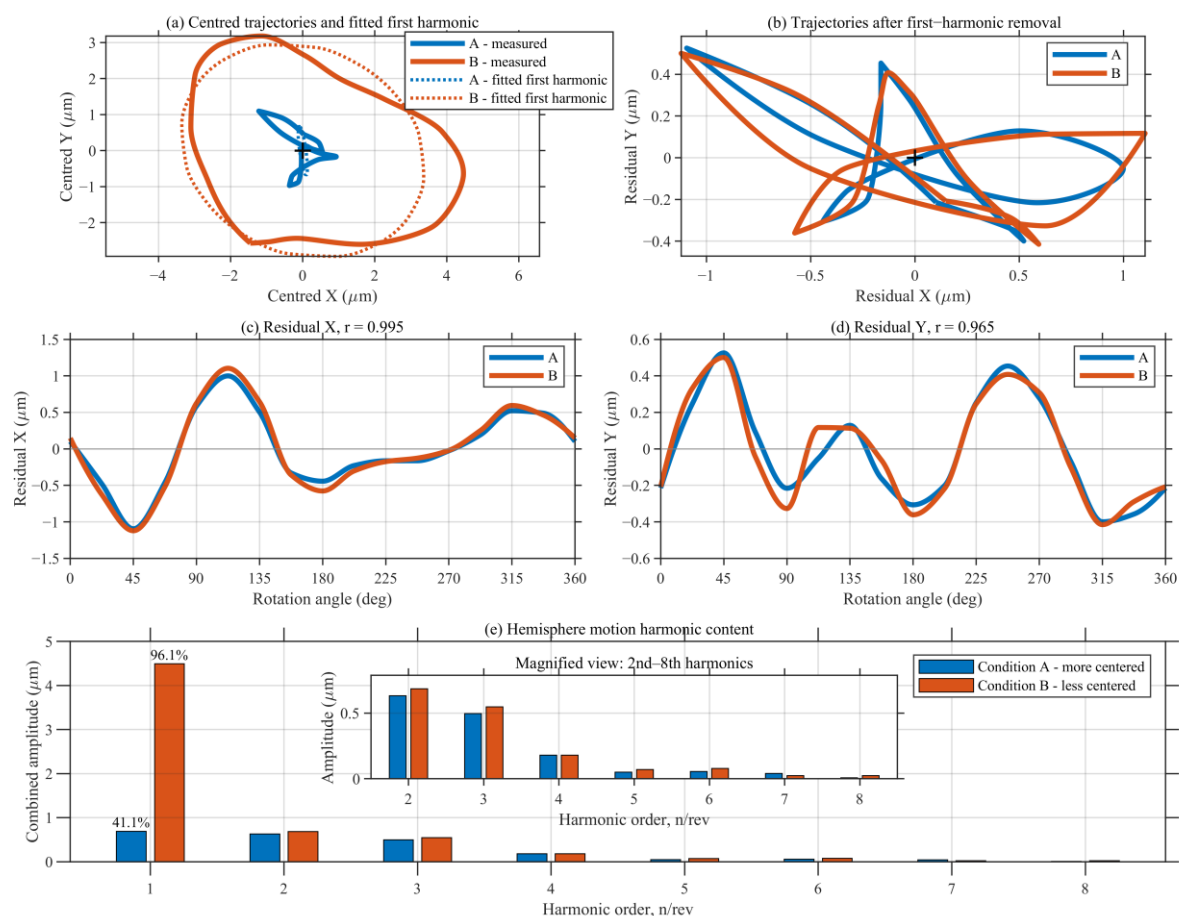


Figure 7. Experimental characterisation of the rotor system motion using the reference hemisphere: (a) measured trajectories and fitted first harmonic components; (b) residual trajectories after first

harmonic removal; (c) residual X direction displacement; (d) residual Y direction displacement; (e) harmonic content of the measured motion.

The harmonic spectra of the measured trajectories are compared in Figure 7e. The first harmonic represents the dominant component in both assembly conditions, accounting for approximately 41.1% of the total harmonic amplitude in Condition A and 96.1% in Condition B. In contrast, the amplitudes of the second and higher harmonics remain nearly identical despite the substantially different initial centring conditions. Consequently, the correlation between the residual harmonic spectra (second to eighth harmonic) reaches 0.998, with a cosine similarity of 0.999, confirming that the higher-order harmonic content is practically independent of the imposed centring.

Although the imposed centring condition strongly affects the magnitude of the first harmonic, the remaining higher-order shaft centre motion is an intrinsic characteristic of the encoder rotor system. Consequently, after suppressing the first harmonic, both assembly conditions converge to nearly identical residual trajectories. This observation provides the experimental basis for the subsequent investigation of optical readhead position, in which both the complete measured trajectories and their first harmonic free residuals are used to predict the resulting encoder angular error.

4.2. Influence of Readhead Position on the Predicted Angular Error

The predicted angular error for representative readhead positions is presented in Figure 8a,d,g,j. For each investigated case, three characteristic positions are shown: the reference readhead position ($\gamma = 0^\circ$), the numerically determined optimum position corresponding to the minimum peak-to-peak error, and the worst position producing the maximum error. Although the measured mechanical trajectory remained unchanged, the predicted angular error varied considerably with readhead position because at each different position, the optical readhead observes a different tangential projection of the same two-dimensional displacement vector. Consequently, identical mechanical motion can produce substantially different angular errors depending solely on sensor location.

The influence of readhead position is quantified by the peak-to-peak angular error distributions shown in Figure 8b,e,h,k, whereas the complete angular error fields over one shaft revolution are presented in Figure 8c,f,i,l. The peak-to-peak error exhibits a smooth periodic variation with readhead position, indicating that the sensitivity is governed by the spatial characteristics of the measured rotor motion rather than by random measurement variations. The heat maps further illustrate that changing the readhead position modifies the projection of the measured displacement trajectory without altering its fundamental mechanical behaviour.

The numerical optimisation results are summarised in Table 2. Before first harmonic removal, the influence of readhead position was relatively limited. For Condition A, the predicted peak-to-peak error varied from 5.61 arcsec to 9.63 arcsec, corresponding to an improvement ratio of 1.72 between the optimum and worst readhead positions. For Condition B, where a larger initial centring error was intentionally introduced, the predicted peak-to-peak error ranged from 20.63 arcsec to 31.12 arcsec, resulting in an improvement ratio of 1.51. These results indicate that large eccentricity-induced first harmonic motion masks the influence of readhead position. Under such conditions, relocation of the optical readhead provides only limited improvement because the overall angular error is dominated by the eccentricity component rather than by the higher-order rotor system motion.

A markedly different behaviour is observed after removing the first harmonic from the measured trajectories. The minimum predicted peak-to-peak error decreases to 3.12 arcsec and 3.23 arcsec for Conditions A and B respectively, whereas the corresponding maximum errors remain approximately 8.40 arcsec and 8.86 arcsec. Consequently, the

sensitivity to readhead position increases substantially, with improvement ratios of 2.69 and 2.75, respectively. These results demonstrate that, once the dominant centring-related component has been compensated, the remaining higher-order rotor system motion becomes strongly dependent on the projection direction, making the selection of the optical readhead position considerably more important.

An important observation is that, despite the substantially different initial centring conditions, the predicted optimal readhead positions become nearly identical after first-harmonic removal. The optimum positions differ by only 2° (353° for Condition A and 351° for Condition B), while the corresponding peak-to-peak sensitivity curves exhibit a Pearson correlation coefficient of 0.977. Similarly, the RMS distributions show a correlation coefficient of 0.980, indicating that the remaining position-dependent behaviour is almost entirely governed by the intrinsic higher-order motion of the rotor system rather than by the initial centring condition.

These results confirm the hypothesis established in Section 4.1 that the dominant low-order error is primarily associated with grating eccentricity, whereas the remaining higher-order motion represents an intrinsic characteristic of the rotor system. Consequently, once the eccentricity component has been minimised through precise mechanical centring or compensation, the residual encoder accuracy is governed predominantly by the rotor system motion. Under these conditions, the circumferential position of the optical readhead becomes an important design parameter because it determines the projection of the residual motion into the measured angular error. The proposed methodology therefore provides a practical tool for evaluating and optimising readhead placement during the design and assembly of high-accuracy optical rotary encoders.

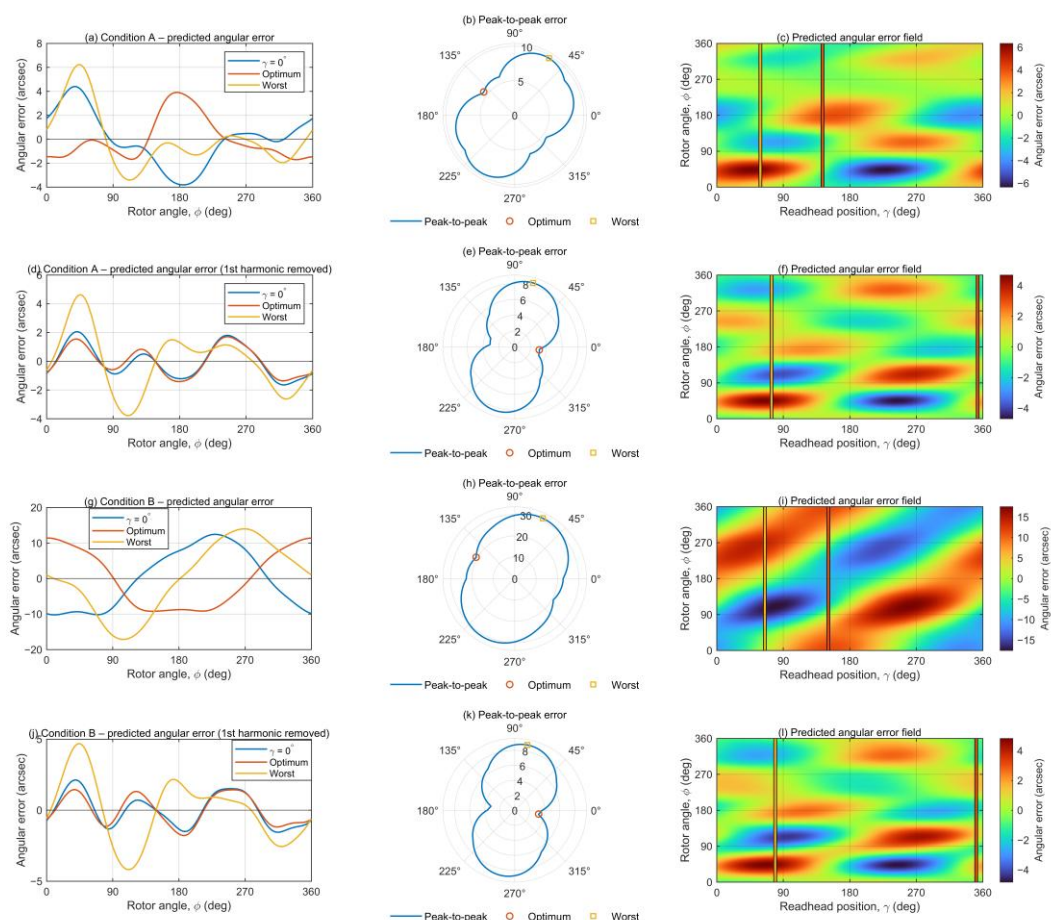


Figure 8. Numerical investigation of the influence of the optical readhead circumferential position on the predicted angular error: (a–c) Condition A with the complete rotor system motion; (d–f)

Condition A after first harmonic removal; (g–i) Condition B with the complete rotor system motion; and (j–l) Condition B after first harmonic removal. The left column shows the predicted angular error profiles for the reference ($\gamma = 0^\circ$), optimum, and worst readhead positions; the centre column shows the corresponding peak-to-peak error as a function of readhead position; and the right column presents the predicted angular error field over the complete range of rotor angles and readhead positions.

Table 2. Influence of optical read head position on the predicted angular error.

Parameter	Condition A	Condition A (1st Harmonic Re- moved)	Condition B	Condition B (1st Harmonic Re- moved)
Best readhead position γ , ($^\circ$)	143	353	151	351
Min. peak-to-peak error, (")	5.61	3.12	20.63	3.23
Worst readhead position γ , ($^\circ$)	59	74	65	79
Max. peak-to-peak error, (")	9.63	8.40	31.12	8.86
Improvement ratio	1.72	2.69	1.51	2.75

4.3. Grating Eccentricity and Predicted Angular Error (Stage 2)

The measured tangential displacement of the grating for the two investigated disc mounting conditions is presented in Figure 9a,b. Five repeated measurements were performed for each condition, and the mean displacement and corresponding 95% confidence interval were calculated at each angular position. The repeated curves show good agreement over the complete revolution. The maximum pointwise standard deviation was 0.71 μm for condition D1 and 0.84 μm for condition D2, while the mean standard deviations were 0.24 and 0.44 μm , respectively. These values are small relative to the measured peak-to-peak displacements and indicate that the differences between the two conditions are predominantly systematic rather than being caused by measurement scatter.

A pronounced difference was observed between the two different disc installations. The more centred condition D1 exhibited a tangential displacement of 6.4 μm peak-to-peak, whereas the less centred condition D2 reached 22.4 μm peak-to-peak. The displacement range therefore increased by a factor of approximately 3.5. The waveform measured for D2 was strongly dominated by a once-per-revolution component, indicating that the changed mounting condition introduced a substantially larger effective eccentricity contribution. Because the disc was also rotated relative to the shaft during remounting, the phase relationship between the disc eccentricity vector and the rotor system motion was different from that under D1.

The resulting error waveforms are shown in Figure 9c,d, together with the fitted first harmonic and the residual obtained after first harmonic removal. The complete predicted angular error was 24.79 arcsec peak-to-peak for D1 and 86.75 arcsec peak-to-peak for D2. The corresponding RMS values were 7.48 and 28.88 arcsec. Thus, the less centred installation increased the complete predicted peak-to-peak error by approximately 3.5 times and the RMS error by approximately 3.9 times.

The main difference between the two conditions was associated with the first harmonic. Its amplitude increased from 9.32 arcsec for D1 to 40.47 arcsec for D2, corresponding to an increase by a factor of approximately 4.3. The first harmonic accounted for 77.8% of the total fitted harmonic energy in D1 and 98.3% in D2. Consequently, the complete error of the less centred condition was almost entirely governed by the once-per-revolution component associated with the grating disc installation. After first harmonic removal, the two predicted error waveforms became considerably more similar. The residual peak-to-peak error was 13.65 arcsec for D1 and 15.54 arcsec for D2, while the corresponding RMS values were 3.53 and 3.92 arcsec. The residual peak-to-peak and RMS values

therefore differed by only approximately 14% and 11%, respectively, despite the much larger difference in the complete errors.

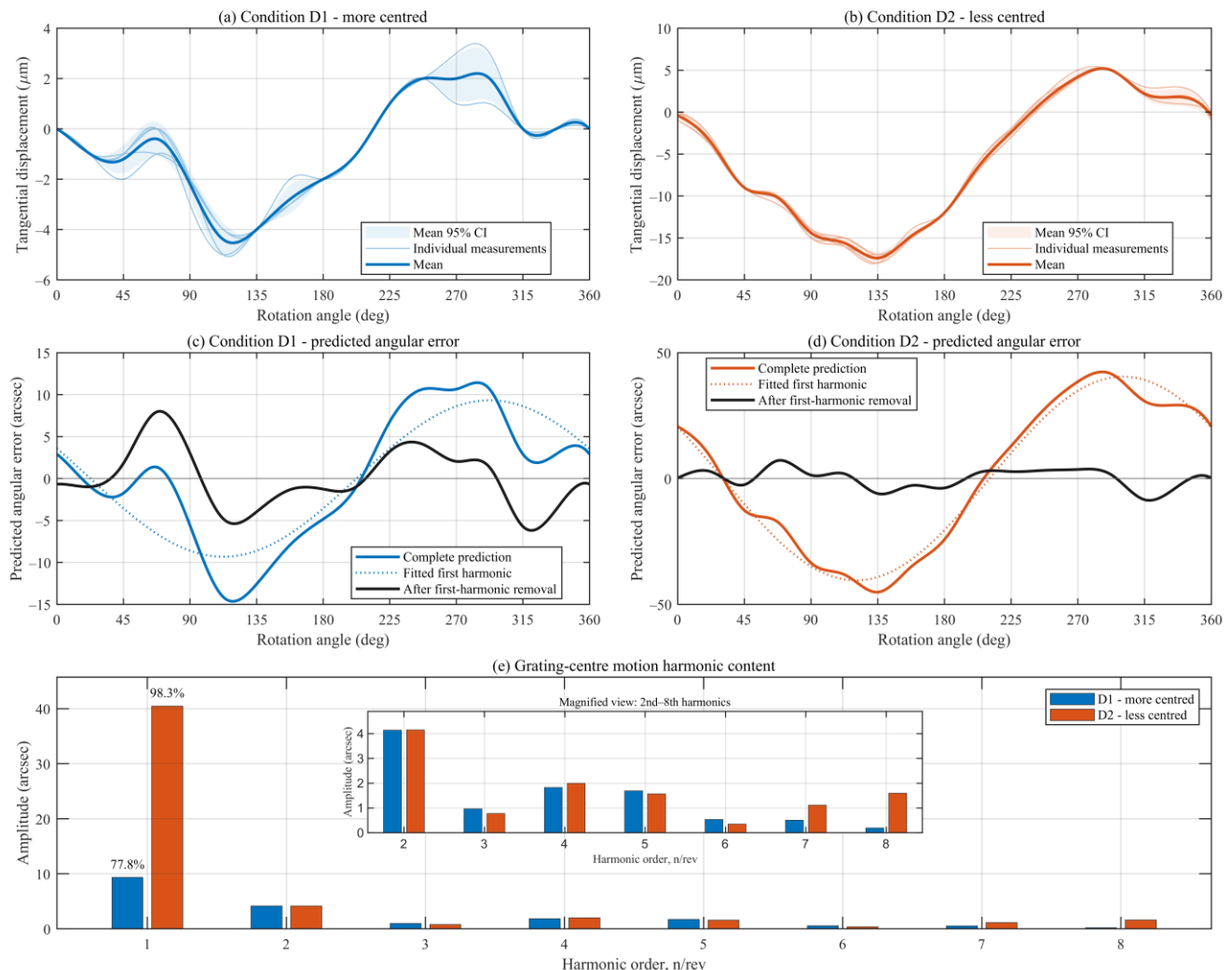


Figure 9. Experimental characterisation of grating disc motion for the two centring conditions: (a,b) measured tangential displacement, including individual measurements, mean profiles, and 95% confidence intervals for conditions D1 and D2, respectively; (c,d) corresponding predicted angular error profiles, fitted first harmonic components, and residual errors after first harmonic removal; and (e) harmonic content of the predicted grating centre motion.

It should be noted that D2 was produced not only by increasing the grating disc centring offset, but also by rotating the disc relative to the shaft. The two conditions therefore represent different vector superpositions of the disc fixed contributions and the common rotor system motion. First harmonic removal suppresses the dominant effect of the changed eccentricity magnitude and orientation, explaining why the residual profiles become substantially more similar. However, disc fixed higher-order components were also rotated relative to the shaft reference, while remounting may have introduced small changes in disc seating, inclination or deformation. These effects, together with microscope measurement uncertainty, explain why the residual amplitudes do not coincide exactly.

The magnified harmonic view in Figure 9e shows that the dominant higher-order terms were also of comparable magnitude for the two installations. The second harmonic amplitudes were nearly identical, reaching 4.14 arcsec for D1 and 4.15 arcsec for D2. The

fourth and fifth harmonic amplitudes were likewise similar, with values of 1.83 and 1.69 arcsec for D1 and 2.00 and 1.57 arcsec for D2, respectively. Larger relative differences occurred in some of the smaller components, particularly the seventh and eighth harmonics; however, their absolute amplitudes remained substantially lower than the first harmonic.

These results show that changing the disc mounting condition primarily affected the first harmonic component, whereas the residual higher-order structure remained comparatively stable. The higher-order components may contain contributions from repeatable shaft motion, local grating pattern geometry, and the microscope measurement process and therefore should not be attributed exclusively to a single physical source.

4.4. Experimental Validation Using an Angular Comparator (Stage 3)

The proposed mechanical error propagation model was experimentally validated by comparing the predicted encoder angular error with direct measurements obtained using a high-precision angular comparator. Figure 10 presents five repeated measurements for the two grating disc centring conditions. The measured peak-to-peak angular error was 27.91 arcsec for D1 and 86.57 arcsec for D2. The corresponding repeatability RMS values were 1.33 and 0.50 arcsec, respectively, indicating sufficient measurement stability for comparison with the mechanically predicted error profiles.

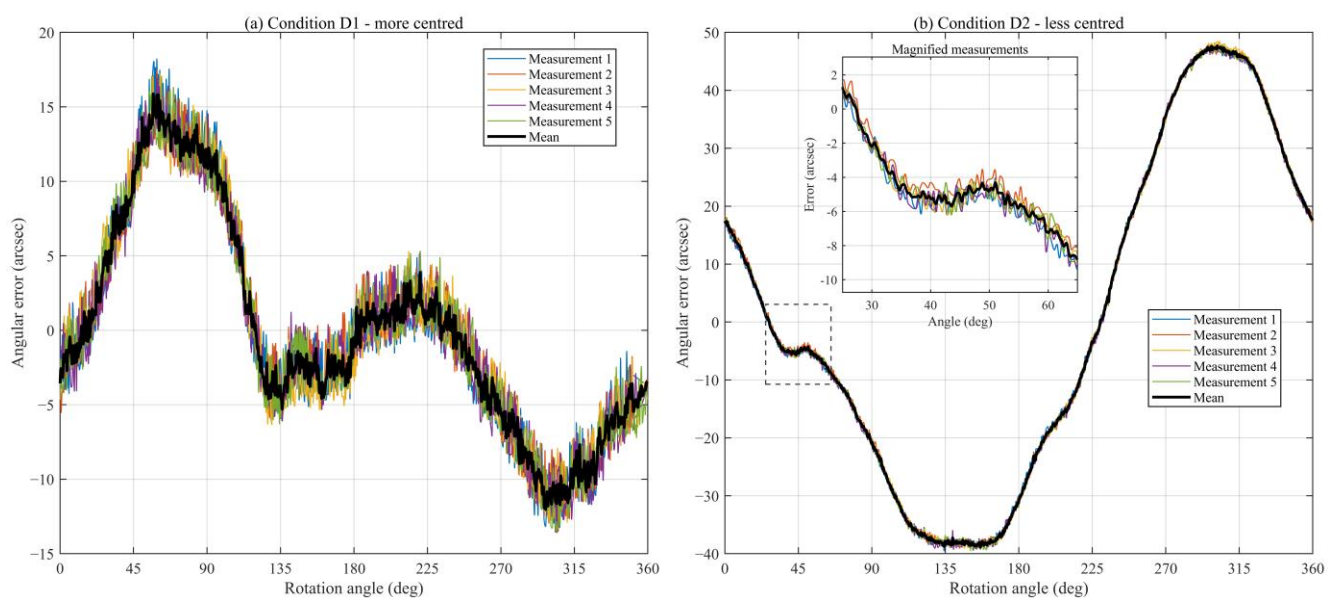


Figure 10. Repeated angular comparator measurements for the two investigated grating disc mounting orientations: (a) D1 and (b) D2.

Two levels of mechanical prediction were considered in the validation presented in Figure 11. The microscope prediction was obtained directly from the grating disc motion measured in Stage 2 and therefore represents the angular error predicted from the measured tangential motion of the actual grating pattern. The combined mechanical prediction retained the first and second harmonic components obtained from Stage 2 and incorporated the third and higher-order rotor system motion identified independently in Stage 1. Conditions A and B were not paired with Conditions D1 and D2. Instead, they represented two reference hemisphere centring conditions measured on the same shaft-bearing system. After removing the first and second harmonic components, the residual A and B trajectories, referenced to the same shaft angle position, were averaged pointwise to estimate the common higher-order motion of the rotor system. The same averaged rotor system contribution was used for both D1 and D2, whereas their condition-specific low-order components were obtained from the corresponding Stage 2 measurements. The tangential

component of the averaged Stage 1 residual at the optical readhead was converted into angular error and combined with the low-order Stage 2 prediction. The first two Stage 1 harmonics were excluded to prevent double counting because the Stage 2 grating motion measurements already contained the first and second harmonic contributions associated with the actual grating disc mounting condition.

Figure 11a,b compare the comparator measurements with the microscope and combined mechanical predictions. The repeat level bootstrap analysis showed that the microscope prediction reproduced the measured angular error profiles with correlation coefficients of 0.871 for D1, with a 95% confidence interval of (0.832–0.893), and 0.977 for D2, with a 95% confidence interval of (0.975–0.978). Incorporation of the independently measured higher-order rotor system contribution increased the correlations to 0.897 (95% CI: 0.870–0.910) and 0.984 (95% CI: 0.982–0.986), respectively. These intervals were obtained by resampling complete experimental repetitions rather than individual angular points and therefore account for the observed repeat-to-repeat variability while preserving the dependence of the samples within each periodic waveform.

The corresponding RMSE values, calculated from the mean comparator profiles, decreased from 3.68 to 3.22 arcsec for D1 and from 6.21 to 5.12 arcsec for D2 after the higher-order rotor system contribution was incorporated. The simultaneous increase in correlation and decrease in RMSE under both conditions indicate that the independently measured rotor system motion contains a measurable contribution to the final encoder error and improves the predictive capability of the proposed model. The consistency and statistical significance of this improvement across the individual comparator repetitions are assessed separately below.

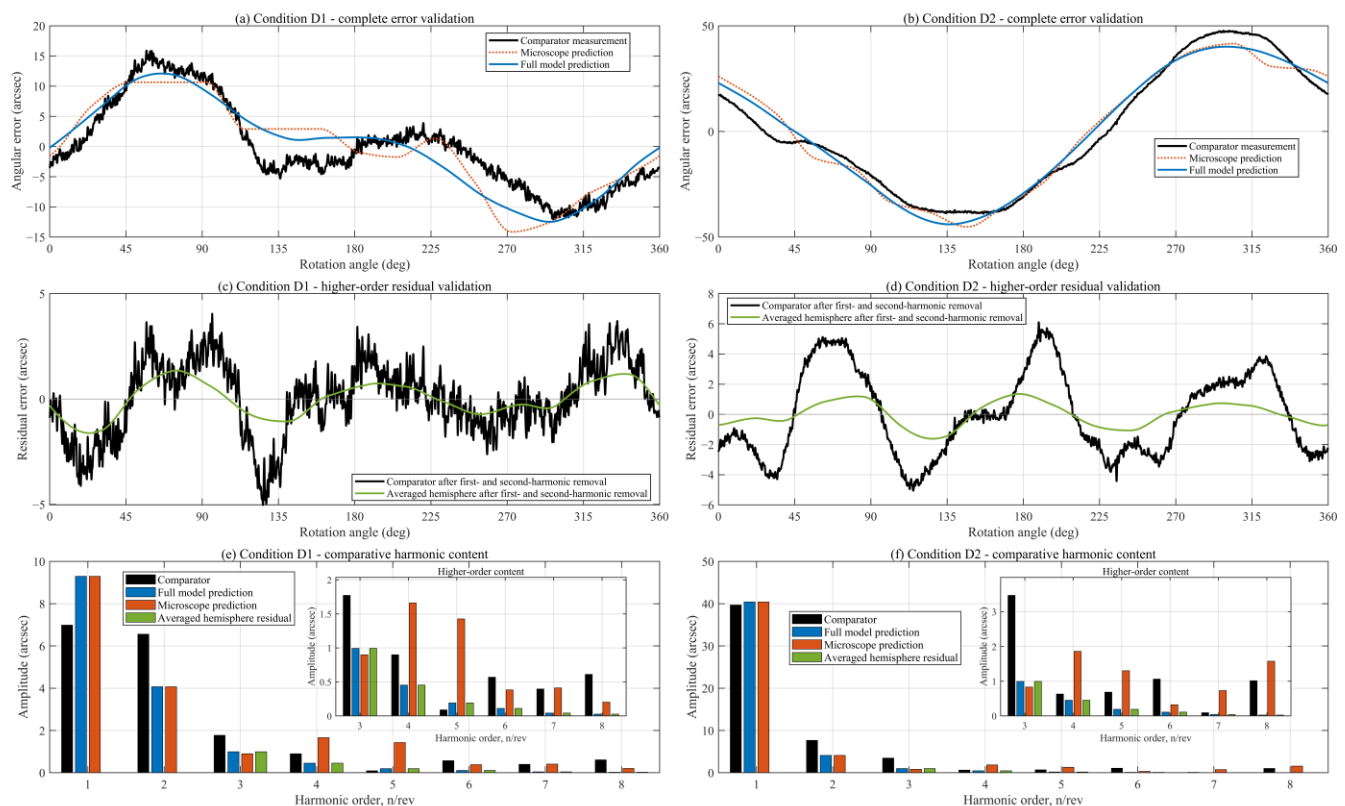


Figure 11. Experimental validation of the proposed mechanical error propagation model: (a,b) comparison of the measured comparator error with the microscope-based prediction and the combined model prediction for D1 and D2, respectively; (c,d) comparison of the comparator residual after removing the first and second harmonics with the averaged hemisphere residual; (e,f) comparison

of harmonic amplitudes for the measured comparator error, microscope prediction, combined model prediction, and averaged hemisphere residual.

The different prediction accuracy obtained for D1 and D2 is consistent with their harmonic composition. Condition D2 is strongly dominated by the first harmonic associated with grating disc eccentricity. Consequently, the principal shape of its angular error profile is already captured by the Stage 2 microscope measurement, resulting in a correlation of 0.977 before the rotor system contribution is incorporated. In the more centred D1 condition, the eccentricity-related component is substantially smaller, and the higher-order components represent a greater proportion of the total error. The prediction is therefore more sensitive to measurement uncertainty, harmonic reconstruction, and mechanical effects not represented by the simplified model. Nevertheless, incorporation of the Stage 1 rotor system residual improves both the correlation and RMSE for D1.

The relationship between the independently measured rotor system motion and the higher-order encoder error was examined separately in Figure 11c,d. After removal of the dominant low-order components from the comparator measurements, the remaining error profiles were compared with the averaged Stage 1 hemisphere residual. Pearson correlation coefficients of 0.820 for D1 and 0.817 for D2 were obtained. These values indicate a clear similarity in the angular distribution of the residual waveforms. The corresponding RMSE values of 1.16 arcsec for D1 and 2.18 arcsec for D2 confirm that amplitude differences remain, particularly for D2. The residual comparison therefore supports a relationship between the periodic rotor system motion and the higher-order encoder error, while indicating that the rotor system measurement alone does not fully reproduce the magnitude of the measured residual error.

The residual correlation coefficients should therefore be interpreted only as measures of similarity in waveform distribution. They do not demonstrate amplitude equivalence, establish the measured rotor system motion as the unique cause, or provide a complete explanation of all residual harmonics. Different superimposed mechanical and grating-related sources may produce components with similar harmonic orders and angular distributions.

The harmonic comparison in Figure 11e,f provides further information on this relationship. Several higher-order harmonics identified from the Stage 1 hemisphere measurements are also present in the comparator measurements; however, their amplitudes do not generally coincide. Therefore, the harmonic results should not be interpreted as demonstrating that the higher-order encoder error originates exclusively, or predominantly, from the measured rotor system motion. Instead, they show that periodic rotor system motion contributes to the higher-order error structure. The remaining differences may include higher-order grating imperfection effects, measurement uncertainty, and other mechanical contributions not represented by the simplified model. Importantly, despite these differences at individual harmonic orders, incorporation of the Stage 1 residual improves the prediction of the complete angular-error waveform for both centring conditions. The quantitative validation results are summarised in Table 3.

Table 3. Summary of the experimental validation results.

Parameter	Condition D1	Condition D2
Comparator peak-to-peak error (arcsec)	27.91	86.57
Comparator repeatability RMS (arcsec)	1.33	0.50
Microscope prediction r (95% CI)	0.871 (0.832–0.893)	0.977 (0.975–0.978)
Microscope prediction RMSE (arcsec)	3.68	6.21
Combined model r (95% CI)	0.897 (0.870–0.910)	0.984 (0.982–0.986)
Combined model RMSE (arcsec)	3.22	5.12

Higher-order residual r (95% CI) (averaged A and B hemisphere measurements)	0.820 (0.722–0.825)	0.817 (0.785–0.838)
Higher-order residual RMSE (arcsec) (averaged A and B hemisphere measurements)	1.16	2.18

Because the RMSE values in Table 3 were calculated from the averaged comparator profiles, an additional paired analysis was performed to determine whether the improvement obtained by incorporating the rotor system contribution was consistent across the five individual comparator repetitions. The results are summarised in Table 4.

Table 4. Paired per-repetition assessment of the RMSE reduction.

Parameter	Condition D1	Condition D2
Microscope prediction RMSE * (arcsec)	3.863	6.225
Combined model RMSE * (arcsec)	3.436	5.141
Mean reduction (arcsec)	0.427	1.084
Reduction (%)	11.1	17.4
95% Bootstrap CI (arcsec)	(0.405–0.453)	(1.073–1.094)
Paired t -test	$p = 6.12 \times 10^{-6}$	$p = 5.06 \times 10^{-9}$

NOTE: * Values are the means of RMSE values calculated separately for the five comparator repetitions. They differ slightly from the RMSE values calculated from the averaged profiles reported in Table 3. The confidence intervals were obtained from 5000 bootstrap resamples of the paired RMSE differences.

The paired analysis confirmed that incorporating rotor system contribution reduces RMSE in all five comparator repetitions under both conditions. The mean reductions were 0.427 arcsec (11.1%) for D1 and 1.084 arcsec (17.4%) for D2, and both were statistically significant according to the paired t -test. However, the D1 reduction was smaller than the comparator repeatability RMS of 1.33 arcsec and should therefore be interpreted as statistically consistent but practically modest. In contrast, the D2 reduction exceeded the corresponding repeatability RMS of 0.50 arcsec, indicating a more practically relevant improvement.

4.5. Measurement Uncertainty Estimation

Measurement uncertainty was estimated separately for each experimental stage. Type A uncertainty was determined from the repeated complete measurements and characterises the experimentally observed variability of the mean measurement profiles. The standard uncertainty of the mean was obtained by dividing the reported repeatability by the square root of the number of repetitions. Six complete repetitions were available for each Stage 1 condition, while five repetitions were available for each Stage 2 and Stage 3 condition.

Type A estimate reflects the combined short-term variability of the complete measurement procedure. Depending on the experimental stage, this includes the influence of sensor repeatability, instrument resolution, feature localisation, manual angular positioning, mounting stability and local measurement variation to the extent that these effects varied between repetitions. Type B contributions were evaluated from instrument specifications, resolution limits, reference artefact data and the previously characterised performance of the angular comparator.

Limits specified as $\pm a$ were represented by rectangular distributions. A resolution interval q was treated as a rounding interval of $\pm q/2$. Where several Type B contributions were identified for the same experimental stage, they were combined by the root-sum-of-squares method. Type A and Type B estimates were subsequently compared rather than

combined because the repeated complete measurements already incorporate part of the variability associated with the measurement equipment and procedure.

For Stages 1 and 2, displacement uncertainty was converted into angular uncertainty according to

$$u_{\theta} = u_d \frac{206,264.806}{R}, \quad (28)$$

where u_d is the displacement standard uncertainty in micrometres and $R = 53,250 \mu\text{m}$ is the grating track radius. The corresponding displacement-to-angle sensitivity coefficient was $3.8735 \text{ arcsec}/\mu\text{m}$. The resulting uncertainty estimates are presented in Table 5.

For Stage 1, the probe indication error limit was calculated as 1% of the complete $10 \mu\text{m}$ span of the selected $\pm 5 \mu\text{m}$ measuring range, giving an indication error limit of $\pm 0.10 \mu\text{m}$. The $0.1 \mu\text{m}$ probe resolution corresponds to rounding limits of $\pm 0.05 \mu\text{m}$. The reference hemisphere form deviation was represented as $\pm 0.03 \mu\text{m}$. After the conversion into angular units, the combined instrument-based standard uncertainty was 0.259 arcsec for both conditions.

The experimental Type A uncertainty was 0.217 arcsec for Condition A and 0.296 arcsec for Condition B. These values are comparable to the instrument-based estimate, indicating that the observed repeatability is consistent with the expected measurement capability of the probe and reference artefact arrangement. The slightly higher Type A value for Condition B reflects the greater repeat-to-repeat variability observed for the less centred hemisphere installation.

For Stage 2, the $1 \mu\text{m}$ visual grid interval corresponds to rounding limits of $\pm 0.5 \mu\text{m}$. With a rectangular distribution, this gives a displacement standard uncertainty of $0.289 \mu\text{m}$, equivalent to 1.118 arcsec . The contribution associated with the assumed microscope scale accuracy of $\pm 2\%$ depends on the measured displacement amplitude and was therefore evaluated separately for D1 and D2. The resulting contributions were 0.086 arcsec for D1 and 0.333 arcsec for D2. The combined instrument-based standard uncertainties were consequently 1.121 and 1.166 arcsec , respectively.

The experimental Type A uncertainties of the mean microscope profiles were 0.416 arcsec for D1 and 0.762 arcsec for D2. These values are lower than the corresponding instrument-based estimates. This indicates that repeated measurements produced stable mean waveform profiles, while the absolute localisation of the grating line position remained primarily limited by the microscope measurement scale. The visual grid resolution is therefore particularly relevant when interpreting the amplitudes of small higher-order harmonics, whereas the dominant low-order waveform remains clearly distinguishable from the measurement uncertainty.

For Stage 3, the Type A standard uncertainty of the mean comparator profile was 0.595 arcsec for D1 and 0.224 arcsec for D2. The comparator used in the present study was characterised independently in previous work [26]. After calibration and direction-aware compensation, its remaining unidirectional positioning error was 1.61 arcsec peak-to-peak. This residual error was represented by limits of $\pm 0.805 \text{ arcsec}$. Assuming a rectangular distribution, the corresponding Type B standard uncertainty was 0.465 arcsec .

The Stage 3 comparison shows that the D1 Type A uncertainty was slightly greater than the equipment-based estimate, indicating a stronger influence of repeat-to-repeat variation. For D2, the Type A uncertainty was lower than the comparator-based estimate. In both cases, the comparator-related uncertainty was substantially smaller than the complete measured peak-to-peak encoder error of 27.91 arcsec for D1 and 86.57 arcsec for D2.

Overall, the uncertainty estimates are consistent with the experimentally observed stability of the three measurement stages. For Stage 1, the Type A and Type B estimates are closely comparable. For Stage 2, the principal equipment-related limitation is the

microscope visual grid resolution, although averaging repeated measurements provides stable waveform estimates. For Stage 3, the experimental repeatability is comparable to the residual positioning capability of the compensated angular comparator. These results support the identification of the dominant angular error components and provide a quantitative basis for interpreting differences in the smaller residual harmonics.

Table 5. Stage-specific experimental and instrument-based standard uncertainty estimates.

Uncertainty Source	Input Quantity	Type/Distribution	Condition A or D1 (arcsec)	Condition B or D2 (arcsec)
Stage 1:				
Experimental repeatability of the mean profile	A: 0.137 μm /B: 0.187 μm	Type A/Normal	0.217	0.296
Probe resolution	$q = 0.1 \mu\text{m}$	Type B/Rectangular	0.112	0.112
Probe indication error	$\pm 0.10 \mu\text{m}$	Type B/Rectangular	0.224	0.224
Reference hemisphere form deviation	$\pm 0.03 \mu\text{m}$	Type B/Rectangular	0.067	0.067
Combined instrument-based uncertainty	-	Type B	0.259	0.259
Stage 2:				
Experimental repeatability of the mean profile	D1: 0.24 μm /D2: 0.44 μm	Type A/Normal	0.416	0.762
Microscope grid resolution	$q = 1 \mu\text{m}$	Type B/Rectangular	1.118	1.118
Microscope scale accuracy	2% of measured displacement amplitude	Type B/Rectangular	0.086	0.333
Combined instrument-based uncertainty	-	Type B	1.121	1.166
Stage 3:				
Experimental repeatability of the mean profile	D1: 1.33 arcsec/ D2: 0.5 arcsec	Type A/Normal	0.595	0.224
Compensated comparator residual positioning error	1.61 arcsec peak-to-peak [26]	Type B/Rectangular	0.465	0.465

4.6. Practical Implications, Limitations and Future Work

The proposed approach provides an engineering estimate of encoder accuracy from mechanical and optical measurements before demanding final system calibration. The strong agreement with the comparator measurements, with correlations reaching 0.984, shows that the method can identify the dominant mechanical contributions. It can therefore support decisions concerning disc centring, bearing quality, shaft and hub design, and assembly tolerances. This is particularly useful for open-type encoders and integrated axes, where the final error is influenced by the application mechanics and cannot be determined from the encoder components alone.

The measured error behaviour is qualitatively consistent with previous studies of circular grating encoders. The increase in peak-to-peak error from 27.91 arcsec for D1 to 86.57 arcsec for D2, together with the increasing dominance of the first harmonic, agrees with a coordinate-based model identifying grating eccentricity as a principal once-per-revolution error source [4,6,10,12]. The higher-order components and the remaining differences between the predicted and measured residuals are also consistent with reported effects of grating disc inclination, mounting-induced deformation and rotor system motion [17,18]. Unlike multi readhead self-calibration and output compensation approaches [21,22], the presented method used independently measured mechanical trajectories to predict error propagation and evaluate readhead placement before final calibration. Direct comparison of absolute error values is limited by differences in encoder geometry, mounting conditions, resolution and validation metrics among the reported values.

The readhead position analysis provides a potentially useful installation parameter. For the measured trajectories, the numerical projection produced an approximately 2.7-fold difference between the minimum and maximum predicted residual errors depending on circumferential readhead position. This result identifies calculated candidate positions at which the measured rotor trajectory produces a smaller tangential projection and indicates that favourable placement could improve the achievable accuracy of a single readhead encoder without the additional optical channels, electronics, space, and cost required for multi readhead self-compensation methods. However, this result represents a numerical sensitivity analysis for the specific investigated mechanical assembly rather than a universally applicable or fully experimentally validated design recommendation. The calculated favourable position depends on the measured mechanical trajectory and should be verified through physical readhead repositioning and recalibration.

Measurement-related variability was controlled through repeated complete measurements, fixed experimental arrangements within each stage and controlled environmental conditions. The stage-specific uncertainty estimates presented in Section 4.5 provide a quantitative basis for interpreting the agreement between the predicted and measured error profiles. Repeat-to-repeat variability was also propagated into the validation statistics by resampling complete experimental repetitions. Fixed installation conditions supported a consistent relative assessment of D1 and D2, while local grating pattern variations were retained as part of the measured disc motion. Together, the uncertainty estimates, repeatability results and bootstrap confidence intervals indicate that the observed differences between the two mounting conditions substantially exceed the measurement variability, whereas differences between small residual harmonic components should be interpreted more cautiously.

The main modelling limitation is that the method reproduces the high-order residual encoder error only partially. The remaining harmonics may arise from a combination of shaft-bearing motion, grating pattern and form imperfections, disc positioning and mounting-induced deformation, and readhead interaction. Because these contributions were not measured independently, their individual effects on the residual amplitude could not be separated quantitatively. Moreover, the axial runout and disc inclination were not measured or represented separately in the present planar model. Although the general modelling framework is transferable, the quantitative results are specific to the investigated encoder assembly. In particular, the calculated favourable readhead position, angular error magnitudes and relative improvements depend on the grating track radius, bearing assembly and shaft design, disc mounting quality, rotational speed and operating conditions. The present experiments were conducted under controlled quasi-static conditions, in which dynamic, vibration and thermal effects were not experimentally verified. Future work should therefore include multiaxial disc motion measurements, more detailed characterisation of the grating pattern, improved automated angular positioning and microscope image evaluation, and experimental validation of the predicted optimum readhead position. Measurements of additional assemblies under dynamic and thermal operating conditions would further extend the method from quasi-static design evaluation towards broader prediction of encoder performance in practical applications.

5. Conclusions

A combined theoretical and experimental methodology for analysing mechanical error propagation in optical rotary encoders has been developed and experimentally validated. Independent measurements of shaft centre motion, grating disc motion and encoder angular error enabled quantitative verification of the proposed model.

The principal findings of this study are as follows:

1. The proposed methodology enables the dominant encoder angular error to be predicted from independently measured kinematic error sources. The experimental validation under two different grating disc mounting conditions produced correlations between 0.897 and 0.984, demonstrating that the measured mechanical behaviour of the encoder assembly can reproduce the main shape of the resulting angular error profile.
2. The experimental results support the proposed superposition of individual mechanical error sources. Deterioration of grating disc centring increased the measured peak-to-peak angular error by more than three times, confirming the dominant influence of eccentricity. Incorporating the higher-order rotor system contribution reduced the prediction RMSE by up to 18%. However, the residual harmonics were reproduced only partially, indicating additional contributions not represented by the present planar model.
3. As the dominant eccentricity component is reduced, the relative importance of the remaining mechanical errors and readhead position increases. For the investigated optical encoder, the predicted residual error varied by a factor of approximately 2.7 with circumferential readhead position. This demonstrates the potential of readhead position selection as a practical means of improving the achievable accuracy of a single readhead encoder.

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