



Article

Investigation of Viscoplastic Regularization for Simulating Dynamic Impact Soil-Structure Interaction Problems

Tewodros Y. Yosef ^{1,2}

¹ Department of Civil and Environmental Engineering, University of Nebraska-Lincoln, Lincoln, NE 68508, USA; tyosef2@unl.edu; Tel.: +1-402-472-0965

² Midwest Roadside Safety Facility, University of Nebraska-Lincoln, 130 Prem S. Paul Research Center at Whittier School, 2200 Vine St., Lincoln, NE 68583, USA

Abstract

Lateral impact of a soil-embedded pile mobilizes pressure-dependent yielding, large deformation, and post-peak degradation of the surrounding granular soil. In a rate-independent softening formulation, the governing equations lose hyperbolicity once the tangent stiffness becomes negative, and deformation may concentrate in a mesh-controlled zone. This paper examines whether the Duvaut–Lions-type rate update of a pressure-dependent damage formulation changes this behavior in full-scale pile–soil impact. Three-dimensional explicit models were analyzed in pairs that differed only in the rate parameters: a rate-independent elastoplastic-damage variant (Model EP) and its rate-dependent extension (Model EVP). Two tests were analyzed: a lower-energy (51 kJ), rotation-dominated impact used to back-analyze the soil parameters, and a higher-energy (252 kJ) impact with combined pile rotation and structural bending used to assess parameter transfer. Model EP developed deformation trapping after the initial transient, producing a localized pile bend that was not observed in the lower-energy test and excessive sustained resistance in the higher-energy test. In the lower-energy case, this occurred even though the cumulative work approached the measured value. With the rate update active, cumulative force–displacement work at the last common displacement was within about 5% of the measurements in both cases, the rotation-dominated mechanism was reproduced, and the plastic-hinge location in the higher-energy case was within 5 mm of the measured position. Cumulative work alone did not identify the deformation mechanism; force history, integrated resistance, and permanent deformation must be assessed together. Within the investigated formulation, discretizations, and configurations, the improvement reflects the combined action of physical rate dependence and the numerical effect of the rate-dependent stress update.

Keywords: dynamic pile–soil interaction; viscoplastic regularization; strain localization; pressure-dependent plasticity; granular soil; lateral impact; large deformation; mesh sensitivity



Received: 16 September 2026

Revised: 9 October 2026

Accepted: 12 October 2026

Published: 14 October 2026

Copyright: © 2026 by the author.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

1. Introduction

Lateral impact on a soil-embedded pile transfers momentum to the pile and the surrounding ground over a few milliseconds. The pile accelerates before the soil mobilizes pressure-dependent resistance. The near-field material then shears, dilates, heaves, and flows around the pile. A boundary-value model must therefore reproduce the transmitted force, cumulative force–displacement work, and permanent deformation over displacements large enough to include the post-peak response [1–4]. Figure 1 shows the measured descending resistance and the associated soil flow.

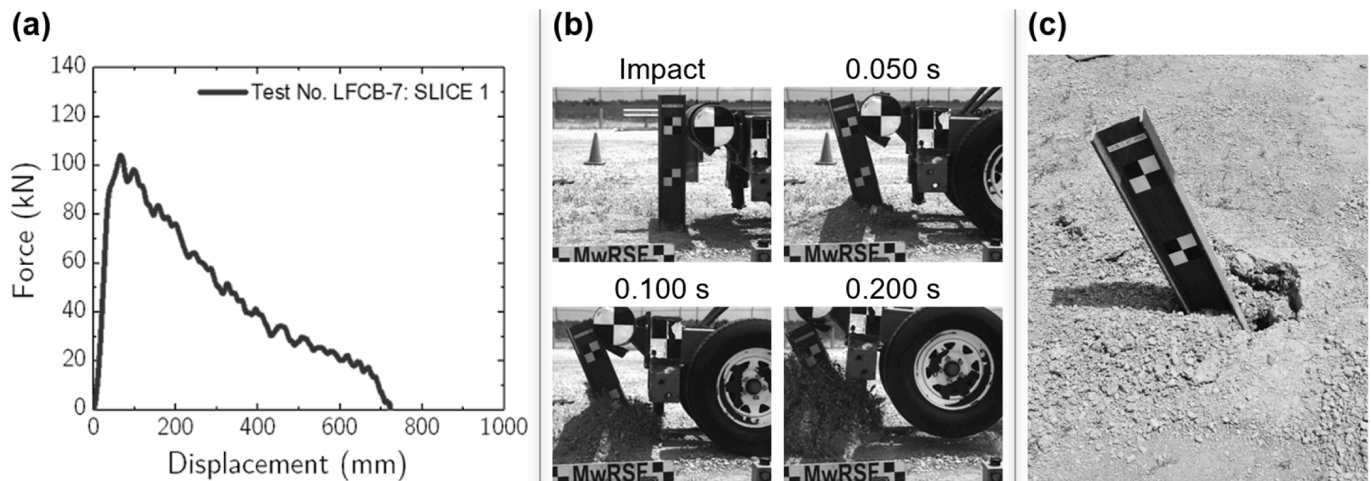


Figure 1. Measured force–displacement response of a pile in granular soil and the associated granular-soil flow during impact [3,5]. (a) Measured force–displacement response for Test No. LFCB-7 (SLICE 1); (b) sequential photographs of the pile at initial impact and 0.050, 0.100, and 0.200 s after impact; (c) condition of the pile and surrounding soil after impact.

Pressure-dependent elastoplastic formulations based on Mohr–Coulomb or Drucker–Prager criteria are widely used for this class of problem. Their parameters have direct geotechnical interpretations, and their local stress updates are efficient in explicit analysis. Pressure-sensitive yielding alone does not describe post-peak degradation or the transition from a confined granular skeleton to rapid flow. Physical rate strengthening requires a rate-dependent constitutive relation [2,6–9].

Strain softening introduces a mathematical difficulty. A negative constitutive tangent can cause loss of hyperbolicity under dynamic loading and loss of well-posedness of the initial-value problem. Deformation can then concentrate in one layer of finite elements, with localization width, dissipated energy, and structural post-peak response governed by the discretization [10–16].

Local energy scaling and continuum regularization address this difficulty differently. Fracture-energy scaling adjusts the softening law to the element dimension but introduces no interaction between material points. Nonlocal, gradient, and Cosserat formulations introduce spatial interaction and an internal length. Viscoplasticity introduces time dependence whose regularizing effect depends on the rheological arrangement and rate equation [17–27].

A dashpot in series with an elastoplastic branch can represent creep and wave attenuation without regularizing strain softening. In the one-dimensional rheological models analyzed in [20], a dashpot in parallel with the plastic dissipative element changes the highest-order terms of the dynamic field equation and permits real wave speeds in the softening range. These properties do not follow from rate dependence alone; localization and mesh convergence must be assessed for the constitutive law and boundary-value problem considered [20–22,28,29].

Large soil deformation in pile–soil impact analyses has been treated through element erosion, multi-material arbitrary Lagrangian–Eulerian (ALE) methods, adaptive finite element (FE)–smoothed particle hydrodynamics (SPH) coupling, and rate-dependent constitutive laws. These treatments require assessment beyond peak force: force and cumulative work histories must be considered alongside soil flow and permanent pile deformation [1–5,30–36].

The question is whether a Duvaut–Lions-type rate update suppresses the post-peak numerical pathologies of a local pressure-dependent damage formulation while reproducing

measured pile–soil impact response. A one-dimensional characteristic analysis identifies the difficulty associated with a negative tangent. Paired three-dimensional simulations of two full-scale tests then compare force, cumulative work, and permanent pile response with only the rate parameters changed [37]. The comparison assesses the combined effects of physical rate dependence and numerical regularization; the experiments do not identify those effects separately.

The contribution of this work is a mechanics-based evaluation of rate-dependent regularization in full-scale pile–soil impact. Here, regularization means that the rate update suppresses the post-peak pathology of the rate-independent model for the discretizations and configurations examined; it does not imply mesh objectivity or a finite localization width. The rate-independent and rate-dependent variants of one formulation are analyzed in pairs in which the rate parameters constitute the only change, so that the effect of the update can be assessed directly from the paired responses. The modeling procedures and the calibration against the lower-energy test come from the preceding computational study [37], and related large-deformation treatments of the same tests have been reported with multi-material ALE and coupled particle methods [5,34]. A characteristic analysis of the one-dimensional softening problem states the mathematical motivation. A lower-energy, rotation-dominated calibration case and a substantially higher-energy transfer case with combined rotation and pile bending provide the physical evidence. Force–displacement response, cumulative work, permanent deformation, and deformation mechanism are assessed together, so that agreement in an integrated measure is tested against the mechanism it represents.

In outline, the soil is represented by the pressure-dependent damage formulation of Section 3, with parameters drawn from field measurements, published correlations, laboratory tests on similar road-base soils, and back-analysis (Section 4.5). Pile, soil, and bogie are assembled with the discretization, contact, and boundary treatments of Sections 4.3–4.8; gravity is initialized by dynamic relaxation, and the bogie is assigned the measured impact speed. Each test is analyzed with the rate update disabled and enabled, and the computed bogie acceleration is reduced with the procedure applied to the test records before force history, cumulative work, and permanent deformation are compared with the measurements (Section 4.9).

2. Mathematical Background for Strain-Softening Continua

Strain softening denotes a reduction in constitutive resistance with increasing inelastic deformation. A specimen's descending load–displacement response can also reflect structural softening as deformation localizes. A local continuum supplies no intrinsic length to set the localization width [10,12–17,19]. The following one-dimensional analysis identifies the change in character of the rate-independent equations about a homogeneous loading state.

For a three-dimensional continuum occupying a volume V , linear momentum balance can be written as

$$\nabla \cdot \sigma + \rho g = \rho \ddot{u} \quad (1)$$

Equation (1) can be expressed in compact matrix-vector form as

$$L^T \sigma + p = R \ddot{u} \quad (2)$$

where σ denotes the Cauchy stress tensor in the balance law and its vector representation in the matrix form, u is displacement, and ρ is mass density. The gravitational acceleration is g ; p is the body-force vector per unit volume, and $R = \rho I$ is the density operator. The superscript T denotes transpose.

The small-strain kinematic relation is

$$\varepsilon = Lu \quad (3)$$

where ε is the strain vector and L is the differential-operator matrix, written as

$$L^T = \begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix} \quad (4)$$

Consider a homogeneous one-dimensional bar with constant density and no time-varying body force. Time differentiation of momentum balance gives Equation (5); the velocity–strain-rate relation is Equation (6) [12,20,28,29,38].

$$\frac{\partial \dot{\sigma}}{\partial x} = \rho \frac{\partial^2 v}{\partial t^2} \quad (5)$$

$$\dot{\varepsilon} = \frac{\partial v}{\partial x} \quad (6)$$

where v is particle velocity. During monotonic plastic loading, the strain decomposes as $\varepsilon = \varepsilon^{\text{el}} + \varepsilon^{\text{pl}}$, with $\sigma = E\varepsilon^{\text{el}}$. The stress on the loading branch depends on plastic strain through the function f in Equation (7).

$$\sigma = f(\varepsilon^{\text{pl}}) \quad (7)$$

Differentiation with respect to time gives

$$\dot{\sigma} = f' \dot{\varepsilon}^{\text{pl}} \quad (8)$$

The plastic modulus is $f' = d\sigma/(d\varepsilon^{\text{pl}})$. Combining elastic compliance with Equation (6) and Equation (8) gives the total tangent $E_t = Ef'/(E + f')$. With coefficients held at the homogeneous reference state, spatial differentiation yields

$$\frac{\partial \dot{\sigma}}{\partial x} = \frac{f'E}{E + f'} \frac{\partial^2 v}{\partial x^2} \quad (9)$$

Substitution of Equation (9) into Equation (5) gives the one-dimensional wave equation

$$\frac{E + f'}{c_e^2} \frac{\partial^2 v}{\partial t^2} - f' \frac{\partial^2 v}{\partial x^2} = 0 \quad (10)$$

where $c_e = \sqrt{E/\rho}$ is the elastic bar-wave speed. Equation (10) is exact for a constant tangent on the active loading branch. For an evolving tangent, it describes the principal part of the locally linearized problem; spatial variation of the tangent introduces additional terms.

The total differentials in Equation (11) and Equation (12) determine the characteristic directions [12,20,28].

$$d\left(\frac{\partial v}{\partial x}\right) = \frac{\partial^2 v}{\partial x \partial t} dt + \frac{\partial^2 v}{\partial x^2} dx \quad (11)$$

$$d\left(\frac{\partial v}{\partial t}\right) = \frac{\partial^2 v}{\partial t^2} dt + \frac{\partial^2 v}{\partial x \partial t} dx \quad (12)$$

Combining Equation (10), Equation (11), and Equation (12) produces the second-order system

$$\begin{cases} \frac{E+f'}{c_e^2} \frac{\partial^2 v}{\partial t^2} - f' \frac{\partial^2 v}{\partial x^2} = 0 \\ \frac{\partial^2 v}{\partial t^2} dt + \frac{\partial^2 v}{\partial x \partial t} dx = d \left(\frac{\partial v}{\partial t} \right) \\ \frac{\partial^2 v}{\partial x \partial t} dt + \frac{\partial^2 v}{\partial x^2} dx = d \left(\frac{\partial v}{\partial x} \right) \end{cases} \quad (13)$$

Equation (13) can be written in the standard characteristic form

$$M \begin{pmatrix} \frac{\partial^2 v}{\partial t^2} \\ \frac{\partial^2 v}{\partial x \partial t} \\ \frac{\partial^2 v}{\partial x^2} \end{pmatrix} = \begin{pmatrix} 0 \\ d \left(\frac{\partial v}{\partial t} \right) \\ d \left(\frac{\partial v}{\partial x} \right) \end{pmatrix} \quad (14)$$

with coefficients

$$M = \begin{pmatrix} \frac{(E+f')}{c_e^2} & 0 & -f' \\ \frac{dt}{dt} & dx & 0 \\ 0 & dt & dx \end{pmatrix} \quad (15)$$

The characteristic determinant is then

$$D = \det \begin{pmatrix} \frac{(E+f')}{c_e^2} & 0 & -f' \\ \frac{dt}{dt} & dx & 0 \\ 0 & dt & dx \end{pmatrix} = \frac{(E+f')}{c_e^2} dx^2 - f' dt^2 \quad (16)$$

Setting the determinant to zero gives the characteristic directions

$$\frac{dx}{dt} = \pm c_e \sqrt{\frac{f'}{E+f'}} \quad (17)$$

The squared characteristic speed is E_t/ρ . It is positive for a positive tangent and approaches c_e^2 in the elastic limit $f' \rightarrow \infty$. For $-E < f' < 0$, E_t is negative: the characteristic speeds are imaginary, the one-dimensional equation is elliptic in space and time, and the dynamic initial-value problem is ill posed [12,20,28]. At $f' = 0$, the equation is degenerate, whereas $f' = -E$ makes the tangent relation singular. The sign of f' alone is insufficient without specifying its range. Nonlocal, gradient, and Cosserat formulations add spatial terms that carry an internal length, whereas viscoplastic formulations add a rate term to the constitutive relation [18–27]. This result is exact for the one-dimensional rate-independent bar. The three-dimensional pressure-dependent formulation, its rate-dependent update, and the discretized problem lie outside the analysis, and the effect of the rate update on localization in the impact simulations is assessed from the computed responses.

3. Pressure-Dependent Elastoplastic and Elasto-Viscoplastic Formulations

The soil formulation combines isotropic elasticity, a smoothed modified Mohr–Coulomb yield surface, and associated plastic flow. Plastic volumetric work drives isotropic damage, with residual frictional strength and an element-volume adjustment of the softening energy. A Duvaut–Lions-type stress update supplies the rate-dependent extension. The LS-DYNA (version R13.1; Livermore Software Technology Corporation, Livermore, CA, USA) implementation is *MAT_147 [2,39–41]. The formulation was adopted because

it was developed for the road-base (granular) soils used in pile–soil impact testing and represents low-confinement strength, damage, and rate effects. Its rate update can also be disabled without altering any other input, so that paired analyses differ only in the rate parameters [37,40]. The relations below define the symbols and assumptions used in the comparison; their calibration is described in Section 4.5.

The two variants retain the same elastic, plastic, and damage parameters. Activating the rate update changes the constitutive response without changing the prescribed geometry, contact, mesh, steel response, or erosion thresholds [40–42].

3.1. Elastic Response

The isotropic elastic compliance relation is written in rate form as

$$\dot{\epsilon}_{ij}^{(el)} = \frac{\dot{s}_{ij}}{2G} + \frac{1-2\nu}{3E} \dot{\sigma}_{kk} \delta_{ij} \quad (18)$$

where $\dot{s}_{ij}$ is the deviatoric stress rate, $\dot{\sigma}_{kk}$ is the trace of the stress rate, G is the shear modulus, ν is Poisson's ratio, E is Young's modulus, and δ_{ij} is the Kronecker delta. Stress rates denote objective constitutive rates after rotational correction, and strain rates denote the rate-of-deformation tensor.

3.2. Yield Condition

The Mohr–Coulomb yield condition couples deviatoric stress to mean stress. Plastic yielding begins when

$$f = (\sigma_1 - \sigma_3) + (\sigma_1 + \sigma_3) \sin \phi - 2c \cos \phi = 0 \quad (19)$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the ordered principal stresses, compression is negative, ϕ is the friction angle, and c is cohesion.

For three-dimensional stress updates, Equation (19) is expressed in terms of stress invariants so that explicit calculation of the principal stresses is avoided:

$$\sigma_m = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) \quad (20)$$

$$\bar{\sigma} = \sqrt{J_2} = \sqrt{\frac{1}{2}(s_{11}^2 + s_{22}^2 + s_{33}^2) + \sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2} \quad (21)$$

$$\theta = \frac{1}{3} \sin^{-1} \left(-\frac{3\sqrt{3}}{2} \frac{J_3}{(J_2)^{\frac{3}{2}}} \right) \quad -30^\circ \leq \theta \leq 30^\circ \quad (22)$$

where σ_m is mean stress, J_2 is the second deviatoric stress invariant, and $\bar{\sigma}$ denotes its square root.

$$J_3 = s_{11}s_{22}s_{33} + 2\sigma_{12}\sigma_{23}\sigma_{13} - s_{11}\sigma_{23}^2 - s_{22}\sigma_{13}^2 - s_{33}\sigma_{12}^2 \quad (23)$$

The deviatoric normal stresses follow by subtracting the mean stress:

$$s_{11} = \sigma_{11} - \sigma_m, s_{22} = \sigma_{22} - \sigma_m, s_{33} = \sigma_{33} - \sigma_m \quad (24)$$

The principal stresses are then

$$\sigma_1 = \frac{2}{\sqrt{3}} \sqrt{J_2} \sin(\theta + 120^\circ) + \sigma_m \quad (25)$$

$$\sigma_2 = \frac{2}{\sqrt{3}} \sqrt{J_2} \sin(\theta) + \sigma_m \quad (26)$$

$$\sigma_3 = \frac{2}{\sqrt{3}} \sqrt{J_2} \sin(\theta - 120^\circ) + \sigma_m \quad (27)$$

Substitution of Equation (25) and Equation (27) into Equation (19) gives the invariant form of the yield function

$$f = \sigma_m \sin \phi + \sqrt{J_2} K(\theta) - c \cos \phi = 0 \quad (28)$$

where the Lode-angle function is

$$K(\theta) = \cos \theta - \frac{1}{\sqrt{3}} \sin \phi \sin \theta \quad (29)$$

The classical Mohr–Coulomb surface is nonsmooth at its corners and apex, where a unique flow direction is unavailable. The hyperbolic meridional approximation of Abbo and Sloan smooths the apex; the deviatoric section is modified separately below [39,40].

For a fixed Lode angle, the meridional section of the Mohr–Coulomb surface can be written as the straight line

$$\sqrt{J_2} = \frac{1}{K(\theta)} (c \cos \phi - \sigma_m \sin \phi) \quad (30)$$

Following Abbo and Sloan [39], the straight line is approximated by an asymptotic hyperbola:

$$\frac{(\sigma_m - d)^2}{a^2} - \frac{J_2}{b^2} = 1 \quad (31)$$

The hyperbola is constrained to have the same asymptotic slope and intercept as the Mohr–Coulomb line, which gives

$$\frac{b}{a} = \frac{\sin \phi}{K(\theta)}, \quad d = c \cot \phi \quad (32)$$

Substitution of Equation (32) into Equation (31) gives the smoothed yield function

$$f = \sigma_m \sin \phi + \sqrt{J_2 K^2(\theta) + a^2 \sin^2 \phi} - c \cos \phi = 0 \quad (33)$$

The parameter a has units of stress and controls meridional smoothing, with $0 \leq a < c \cot \phi$. As a tends to zero, the smoothed meridian recovers the unsmoothed relation for the same deviatoric function [40].

For low-confinement granular soil, the deviatoric section is modified through an eccentricity-dependent function [39,40]. The cosine-angle parameter is $\theta_c = \theta + \pi/6$, so $0 \leq \theta_c \leq \pi/3$ and $\cos(3\theta_c) = 3\sqrt{3}J_3/(2J_2^{3/2})$. In the following expression, K remains a function of the sine-defined angle θ :

$$K(\theta) = \frac{4(1 - e^2) \cos^2 \theta_c + (2e - 1)^2}{2(1 - e^2) \cos \theta_c + (2e - 1) \sqrt{4(1 - e^2) \cos^2 \theta_c + 5e^2 - 4e}} \quad (34)$$

For $0.5 < e \leq 1$, e is the triaxial-extension to triaxial-compression strength ratio at fixed mean stress [40]. Triaxial extension has $\theta = -\pi/6$ and $\theta_c = 0$, giving $K = 1/e$; triaxial compression has $\theta = \pi/6$ and $\theta_c = \pi/3$, giving $K = 1$. At $e = 1$ the deviatoric section is circular. A fixed e selects a fixed shape; this expression does not itself prescribe evolution of eccentricity with confinement. At $J_2 = 0$, the Lode angle is undefined, but the yield value follows from the zero-deviatoric-stress limit.

3.3. Flow Rule

The constitutive rate description decomposes the strain rate into elastic and plastic contributions. The plastic contribution is

$$\dot{\varepsilon}_{ij}^{(pl)} = \dot{\lambda} \frac{\partial g}{\partial \sigma_{ij}} \quad (35)$$

where $\dot{\lambda}$ is the plastic multiplier rate, g is the plastic potential, and σ is the undamaged Cauchy stress tensor used in the plastic return. The rate-independent loading conditions are

$$\begin{aligned} \dot{\lambda} &\geq 0 \\ f(\sigma_{ij}) &\leq 0 \\ \dot{\lambda} f(\sigma_{ij}) &= 0 \end{aligned} \quad (36)$$

together with the consistency condition

$$\dot{\lambda} f(\sigma_{ij}) = 0 \quad (37)$$

Associated flow sets $g = f$, so the yield-surface normal determines both shear flow and plastic dilation. Dilation cannot be adjusted independently through a separate angle. Its effect on confinement and localization is therefore constrained only indirectly, through calibration of the global response [14–16,40].

The plastic strain-rate tensor is therefore

$$\dot{\varepsilon}_{ij}^{(pl)} = \dot{\lambda} \frac{\partial f}{\partial \sigma_{ij}} \quad (38)$$

For the undamaged plastic return with fixed yield-surface parameters and no active friction hardening, the multiplier rate follows from consistency:

$$\dot{\lambda} = \frac{2G\dot{\varepsilon}_{ij} \frac{\partial f}{\partial \sigma_{ij}} + \left(K - \frac{2G}{3}\right) \dot{\varepsilon}_{kk} \frac{\partial f}{\partial \sigma_{ij}} \delta_{ij}}{2G \frac{\partial f}{\partial \sigma_{pq}} \frac{\partial f}{\partial \sigma_{pq}} + \left(K - \frac{2G}{3}\right) \frac{\partial f}{\partial \sigma_{pq}} \delta_{pq} \frac{\partial f}{\partial \sigma_{rs}} \delta_{rs}} \quad (39)$$

3.4. Strain Softening

Large-scale impact tests show soil heave, particle ejection, and a marked reduction in resistance after the peak load. The model represents this response through an isotropic continuum-damage law driven by plastic volumetric work [3–5,36,37,40,41].

The strain-energy-based damage measure is

$$\xi = -\frac{1}{K} \int \sigma_m d\varepsilon_v^{(pl)} \quad (40)$$

where K is the bulk modulus, σ_m is the mean undamaged stress, and $\varepsilon_v^{(pl)}$ is plastic volumetric strain. Compression is negative and dilation is positive. Thus $-\sigma_m d\varepsilon_v^{(pl)}$ is positive for plastic dilation under confinement, and ξ is a dimensionless normalized-work measure.

The damaged stress is obtained from the undamaged stress through

$$\sigma_{ij} = \bar{\sigma}_{ij}(1 - D) \quad (41)$$

where D is the scalar damage variable and $\bar{\sigma}_{ij}$ is the undamaged stress. Damage is bounded below by zero and above by its prescribed maximum.

Damage evolves irreversibly. With $D_0 = 0$, the bounded update is

$$D_{j+1} = \min \left(D_{\max}, \max \left(D_j, \frac{\xi_{j+1} - \xi_0}{\alpha - \xi_0} \right) \right) \quad (42)$$

where j denotes the previous step, ξ_0 is the dimensionless damage-onset threshold (D_{int}), and $\alpha > \xi_0$ sets the nominal end of the linear softening branch. The maximum operation prevents healing during unloading; the prescribed maximum damage retains residual strength. The equivalent history threshold is $r_{j+1} = \max(r_j, \xi_{j+1})$, with $r_0 = \xi_0$.

The maximum allowable damage is tied to the peak and residual friction angles so that the material retains a pressure-dependent residual strength:

$$D_{\max} = \frac{(\sin \phi_{\text{peak}} - \sin \phi_{\text{res}})}{\sin \phi_{\text{peak}}} \quad (43)$$

The void-formation parameter G_f is the positive area under the nominal linear pressure–volumetric-strain softening branch multiplied by the element length. With the pressure magnitude at onset defined as $K\xi_0$, the energy adjustment of [40] gives

$$G_f = \frac{1}{2} K \xi_0 (\alpha - \xi_0) V^{\frac{1}{3}} \quad (44)$$

where V is the element volume and $V^{\frac{1}{3}}$ is the associated element length. G_f has dimensions of energy per area (force per length). This nominal softening-area definition is distinct from the signed loading-path integral that drives ξ ; residual strength truncates the damage branch.

The softening parameter α is adjusted with element volume according to

$$\alpha = \frac{2G_f}{K\xi_0 V^{\frac{1}{3}}} + \xi_0 \quad (45)$$

This scaling controls the energy dissipated by the local softening relation for a selected mesh and deformation mode. It does not introduce spatial interaction between neighboring material points, and it does not by itself prove convergence to a finite localization width. Recent nonlocal and gradient formulations address that issue directly by adding an internal length or spatial averaging operator [23–27].

3.5. Elasto-Viscoplastic Extension

The rate-dependent extension uses a Duvaut–Lions-type stress update [40,42]. At each explicit time step, the updated stress is interpolated between the inviscid stress on the static yield surface and the elastic trial stress:

$$\bar{\sigma}_{ij}^{(vp)} = \bar{\sigma}_{ij}(1 - \zeta) + \zeta \bar{\sigma}_{ij}^{(trial)} \quad (46)$$

$$\zeta = \frac{\eta}{\Delta t + \eta} \quad (47)$$

$$\eta = \left(\frac{\gamma}{\dot{\epsilon}_e} \right)^{1 - \frac{1}{n}} \quad (48)$$

where ζ is the dimensionless interpolation weight, Δt is the time step, η is an effective relaxation time, γ is the rate coefficient, n is the rate exponent, and $\dot{\epsilon}_e$ is the effective strain rate. The sum $\Delta t + \eta$ requires η to have units of time. For $n > 1$, dimensional consistency of the rate equation gives $[\gamma] = T^{\frac{1}{n-1}}$, hence $[\gamma] = T$ for $n = 2$; γ is not dimensionless. The coefficient and the strain rate must use the same time unit. In the parameter studies of [37]

summarized in Section 4.5, γ is reported as a numerical value in the kg–mm–ms system for all values of n ; the unit ms applies only for $n = 2$.

$$\dot{\epsilon}_e = \sqrt{\frac{2}{3} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij}} \quad (49)$$

The deviatoric contribution used to calculate the effective strain rate is

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij} - \frac{1}{3} \dot{\epsilon}_{kk} \delta_{ij} \quad (50)$$

Setting $\gamma = 0$ disables rate enhancement and recovers Model EP. For $n > 1$ and nonzero effective strain rate, this gives $\eta = \zeta = 0$; at exactly zero rate, the disabled update is understood directly rather than by evaluating $0/0$. For $\gamma > 0$, the zero-rate limit is $\eta \rightarrow \infty$ and $\zeta \rightarrow 1$. For positive η and Δt , $0 < \zeta < 1$: the updated undamaged stress lies between the static return and the elastic trial state, retaining an overstress through a partial plastic correction. This relaxation supplies a temporal mechanism for limiting plastic evolution [20]. It does not by itself establish well-posedness, a finite localization width, or mesh objectivity [21,22]. In these simulations, physical rate strengthening and numerical regularization remain coupled.

Equations (46)–(48) can be read as follows. The overstress retained in a step is the fraction ζ of the difference between the trial stress and the static return. For $\eta \gg \Delta t$, ζ approaches 1 and most of the trial increment is retained; for $\eta \ll \Delta t$, ζ approaches 0 and the static return is recovered. For $n > 1$, η decreases as the effective strain rate increases (Equation (48)). Because ζ depends on $\Delta t/\eta$, the stress computed for given γ and n is not independent of the numerical time increment. The time-step controls therefore belong to the numerical model rather than to the constitutive description (Section 4.1).

3.6. Constitutive Comparison

Model EP and Model EVP were evaluated in pairs. Elasticity, yield surface, flow rule, damage evolution, residual strength, element-volume adjustment, and erosion thresholds were identical. Model EP used $\gamma = 0$. Model EVP used the calibrated coefficient and exponent listed in Table 1. Within each pair, only the rate update was changed.

Table 1. Constitutive settings used in the paired numerical analyses.

Constitutive Feature	Model EP	Model EVP
Elastic law	Isotropic, identical parameters	Isotropic, identical parameters
Yield surface	Smoothed modified Mohr–Coulomb	Smoothed modified Mohr–Coulomb
Flow rule	Associated	Associated
Post-peak law	Volumetric-work damage and residual friction	Volumetric-work damage and residual friction
Element-volume softening adjustment	Active	Active
Rate coefficient, γ (ms)	0	1.0×10^{-3}
Rate exponent, n (dimensionless)	Inactive	2.0
Erosion thresholds	Identical damage and principal-strain criteria	Identical damage and principal-strain criteria

4. Experimental Basis and Numerical Model

Full-scale three-dimensional models were prepared for tests P3G-7 and P3G-20. The updated Lagrangian formulation, erosion treatment, element formulations, contact, boundary conditions, gravity initialization, and response-extraction procedures were developed in the preceding computational study [37]. The P3G-7 geometry, discretization, and baseline parameter set were also reported in [37]. The P3G-20 model and the paired analyses of

both tests are those of the present study. Each test was analyzed with Model EP and Model EVP. Only the rate coefficient γ and the rate exponent n differed within a pair (Section 4.9).

4.1. Solver and Large-Deformation Strategy

The models were solved with the nonlinear explicit hydrocode LS-DYNA. The pile and soil were described in an updated Lagrangian frame, and the transient response was integrated with the explicit central-difference scheme [37,43,44]. The stress measure is Cauchy stress. The standard LS-DYNA hypoelastic update accounts for material spin through a Jaumann rotational correction before adding the constitutive stress increment; the constitutive strain rate is the symmetric spatial velocity gradient. Soil material points remained attached to eight-node hexahedral elements until severe distortion and constitutive failure activated element erosion. The explicit time increment Δt is a numerical quantity governed by the element stability limit and the solver time-step controls; it is not a material parameter. Its role in the rate update is described in Section 3.5.

Element erosion was used as a numerical treatment for highly distorted soil elements, not as a separate physical fracture law. An element was removed only after the prescribed damage level was reached and the maximum-principal-strain threshold was exceeded. In the procedure of [37], both thresholds were set equal to the maximum damage $D_{\max}$ of Equation (43). Nodal mass and momentum were retained after deletion, while the deleted element no longer stored internal energy. Total energy is therefore only approximately conserved once erosion begins [37]. Model EP and Model EVP used identical erosion criteria; their deformation histories could nevertheless produce different deletion times and deleted volumes [36,37].

4.2. Impact Tests, Instrumentation, and Data Processing

The physical program used a rigid-frame bogie with a detachable steel-pipe impact head wrapped with 19 mm of neoprene. Two SLICE 6DX (Diversified Technical Systems, Inc., Seal Beach, CA, USA) systems recorded longitudinal acceleration near the bogie center of gravity. The sensors had a range of ± 500 g, a sampling rate of 10,000 Hz, and a 1650 Hz anti-aliasing filter. The acceleration histories were processed with an SAE Class 60 Butterworth filter in accordance with SAE J211/1 [37,45,46].

Pile resistance was calculated from filtered longitudinal acceleration multiplied by bogie mass. Velocity and impact-point displacement were obtained from the measured initial speed and successive integration of acceleration. For P3G-7, the displacement obtained in this way was checked against high-speed video motion analysis [37]. Cumulative force–displacement work was calculated as $W(u) = \int_0^u F(\tilde{u})d\tilde{u}$; average resistance at a prescribed displacement was the corresponding work divided by displacement [37,45]. High-speed video identified the end of the test at the first occurrence of bogie override or loss of contact. Cumulative work is thus the work done by the bogie on the pile, an integrated measure of the force history. It does not separate the contributions of pile deformation, soil dissipation, and contact, and it differs from an energy balance of the simulations. Comparisons use the common displacement interval of the measured and computed histories.

The test soil was a well-graded crushed-limestone road-base material meeting AASHTO M 147 Grade B requirements. The material was placed in 203 mm lifts and compacted with a pneumatic hand tamper. The adopted bulk density, specific gravity, and gravimetric moisture content were 1900 kg/m³, 2.65, and 3.4%, respectively [37,45,47]. Their basis is given in Section 4.5.

P3G-7 used a 152 mm by 203 mm rectangular steel tube with a 4.76 mm wall, 2134 mm total length, and 1219 mm embedment. An 851 kg bogie struck the pile approximately

632 mm above grade at 10.95 m/s. P3G-20 used the same outer section with a 9.53 mm wall, 3353 mm total length, and 2438 mm embedment. A 2270 kg bogie struck the pile at the same height and at 14.89 m/s. Table 2 summarizes the two configurations [37,45]. The P3G-7 data are reported in [37,45] and the P3G-20 data in [45]. P3G-20 differs from P3G-7 in wall thickness, embedment, bogie mass, and impact speed. The tests are used as two boundary-value cases, not as a controlled one-variable comparison. The back-analysis of the rate and damage parameters in [37] used P3G-7; P3G-20, which was not part of that back-analysis, is used to assess transfer of the same parameter set to a higher-severity configuration (Section 4.5).

Table 2. Physical impact configurations and initial severity.

Parameter	P3G-7	P3G-20
Tube cross section	152 mm × 203 mm	152 mm × 203 mm
Wall thickness	4.76 mm	9.53 mm
Total pile length	2134 mm	3353 mm
Embedment depth	1219 mm	2438 mm
Impact height above grade	632 mm	632 mm
Bogie mass	851 kg	2270 kg
Impact speed	10.95 m/s	14.89 m/s
Impact momentum	9.32 kN·s	33.8 kN·s
Initial kinetic energy	51.0 kJ	251.6 kJ
Assessment role	Baseline calibration and rate-on/rate-off comparison	Higher-severity transfer assessment

The time-sequential photographs in Figure 2 show distinct deformation mechanisms. P3G-7 was characterized primarily by progressive pile rotation through the soil, followed by bogie override, with only minor permanent tube deformation. P3G-20 exhibited rotation and structural bending and developed a plastic hinge about 533 mm below grade [45].

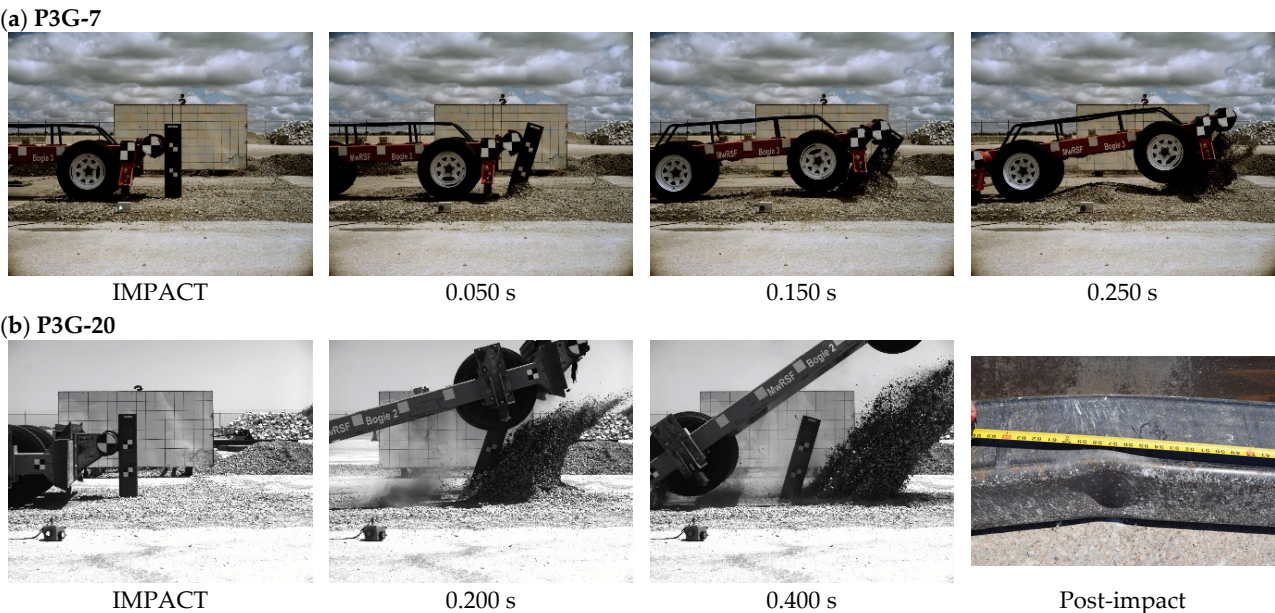


Figure 2. Time-sequential and post-impact physical response: (a) P3G-7, governed primarily by pile rotation through the soil; (b) P3G-20, with combined rotation and structural bending. The final P3G-20 panel shows the post-impact plastic hinge, reported approximately 533 mm below grade.

4.3. Computational Geometry and Initial Conditions

The ground surface was traction free, and the pile was embedded within the refined near-field zone described in Section 4.4. Figure 3 shows the P3G-7 computational geometry

and local discretization developed in the preceding computational study [37]. The test-specific pile dimensions and impact conditions are listed in Table 2.

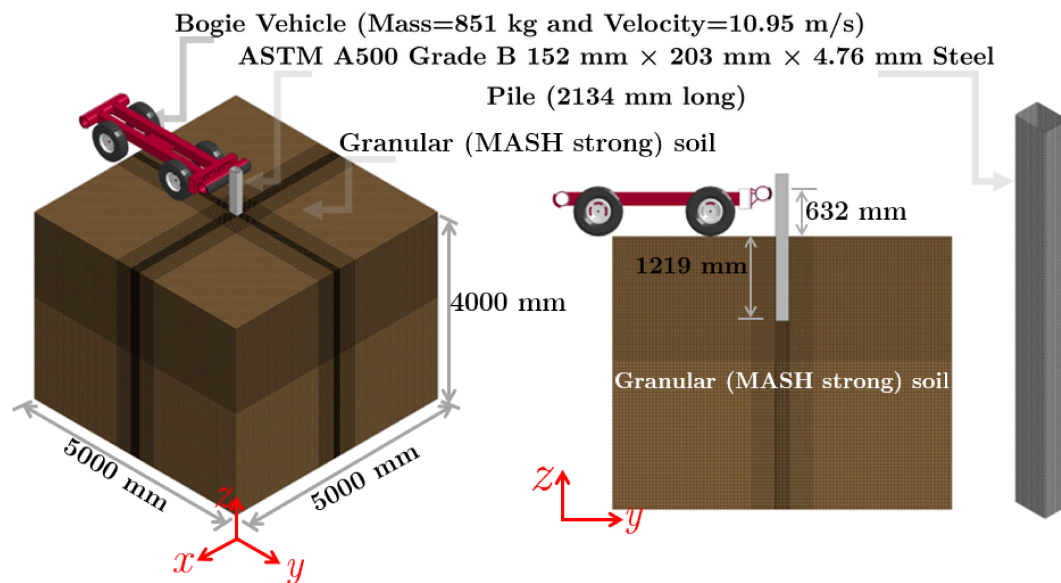


Figure 3. P3G-7 computational geometry, soil domain, pile discretization, and initial impact configuration.

4.4. Spatial Discretization and Hourglass Control

The soil was discretized with eight-node hexahedral elements and one-point integration. A 914 mm × 914 mm near-field zone was refined around the pile, where large deformation and plastic flow concentrate, and the elements coarsen gradually toward the boundaries; abrupt size changes between adjacent elements were avoided because they introduce spurious numerical errors in explicit transient analysis [37].

One-point integration limited the cost of the three-dimensional calculation. The soil elements used Flanagan–Belytschko stiffness-form hourglass control, LS-DYNA Type IV, with a coefficient of 0.01 [37,48]. The P3G-7 tube was represented by four-node, fully integrated shell elements with a nominal plan dimension of 10 mm. Within a test, Model EP and Model EVP used identical soil and pile meshes. Table 3 lists the adopted discretization and controls.

Table 3. Spatial discretization and numerical controls for the paired analyses.

Model Quantity	Specification	Purpose
Soil element formulation	Eight-node hexahedra	Represent the soil as a three-dimensional continuum
Soil integration	One-point quadrature	Limit the cost of the explicit large-deformation analysis
Near-field plan extent	914 mm × 914 mm	Resolve shearing, heave, and flow around the pile
Soil mesh grading	Hexahedral elements, finest in the near-field zone and coarsening gradually outward [37]; domain boundaries outside the large-deformation zone	Resolve the deforming zone while limiting model size
Soil hourglass treatment	Flanagan–Belytschko stiffness form, Type IV, coefficient 0.01	Suppress zero-energy modes
Pile discretization	P3G-7: fully integrated shell elements, nominal 10 mm; P3G-20: fully integrated shell elements of the same formulation	Resolve pile curvature without shell hourglass modes
Paired-model control	Identical pile and soil meshes for Model EP and Model EVP	Isolate the effect of the rate update

4.5. Soil State, Constitutive Parameters, and Calibration

The compacted road-base material was represented as a low-moisture continuum. Table 4 lists the parameters used in both variants and the basis of each value. State variables were based on field measurements and published values [37]. The bulk density range of $1.8\text{--}2.2 \times 10^{-6} \text{ kg/mm}^3$ combines published values for dense granular soils with field density tests at the Midwest Roadside Safety Facility (MwRSF) and the Texas A&M Transportation Institute. The moisture content of approximately 1–6% corresponds to measurements from dynamic bogie tests at MwRSF, and the specific gravity was taken as a typical value for granular soil. The elastic constants were obtained from correlations: Young's modulus from correlations with in situ tests and published values for dense crushed limestone (25.0–46.7 MPa), and Poisson's ratio from published values for gravel (0.20–0.30) [37]; the adopted shear and bulk moduli correspond to $E = 30 \text{ MPa}$ and $\nu = 0.25$. The peak friction angle and cohesion come from triaxial compression tests on crushed-limestone road-base material at confining pressures of 48 and 83 kPa and from large-scale direct shear tests on NCHRP 350 strong soil at normal stresses of 11–29 kPa. These tests gave peak friction angles of $40\text{--}48^\circ$ and cohesion of 5–6.2 kPa, from which 45° and 5 kPa were adopted [37]. The eccentricity of 0.7 follows from triaxial tests on road-base material at the U.S. Army Corps of Engineers Waterways Experiment Station, and the hyperbolic smoothing parameter retains the baseline value of [37], below $c \cot \phi$, so that the stress-free state lies inside the smoothed yield surface (Section 3.2). The travel of the MwRSF shear device was too short to reach a residual plateau, so the residual friction angle of 15° was adopted from the range of $15\text{--}20^\circ$ recommended for granular soils under high-strain conditions, well below the quasi-static values of about $26\text{--}36^\circ$ [37]. The rate and damage parameters were investigated through sensitivity studies and full-scale back-analysis [2,37,40,41]. The computational unit system is kg–mm–ms, with force in kN and stress in GPa. Rate and softening-energy inputs are stated in this system; the other parameters are reported in conventional engineering units.

The back-analysis in [37] used test P3G-7. With the state, elastic, and strength inputs fixed, the computed force–displacement and cumulative work–displacement histories were compared graphically with both SLICE records while the rate or damage parameters were varied. The rate exponent was varied from $n = 0.25$ to 10 with γ fixed at 1.0×10^{-3} , and the rate coefficient from $\gamma = 1.0 \times 10^{-3}$ to $1.0 \times 10^2 \text{ ms}$ with $n = 2$. A grid of $n = 2, 5$, and 10 with $\gamma = 1.0 \times 10^{-3}$, 1.0×10^{-2} , and 1.0×10^{-1} was then examined. Values of γ are numerical inputs in the kg–mm–ms system; the unit ms applies only for $n = 2$ (Section 3.5). Exponents below 2 and coefficients of 1 or larger produced greater resistance and cumulative work than measured. Combinations with $n = 2\text{--}10$ and $\gamma = 1.0 \times 10^{-3}\text{--}1.0 \times 10^{-1}$ gave broadly similar histories and were judged acceptable [37]. The adopted pair, $\gamma = 1.0 \times 10^{-3} \text{ ms}$ and $n = 2$, lies within this range. The comparisons do not identify a unique optimum: other pairs within the range gave similar global responses for P3G-7, and the adopted pair is one acceptable solution selected by graphical comparison. For the baseline parameter set, [37] reports the difference in average resistance at 125, 250, 375, and 500 mm as a quantitative check. The damage parameters were examined in the same way (Section 4.9). Agreement for P3G-7 is therefore calibration agreement; P3G-20 provides one assessment of parameter transfer.

The formulation uses an associated flow rule. The flow rule has no separate dilation angle: dilation follows from the pressure-dependent yield surface and normality. Calibration of the global response constrains its combined effect with stiffness, strength, damage, and residual resistance, but does not independently validate plastic dilation [14–16,40]. Because the damage measure in Equation (40) is driven by plastic volumetric work, the dilation implied by normality also affects the onset and progression of softening in both variants.

Table 4. Granular-soil parameters and determination basis.

Parameter	Value	Constitutive Role	Determination Basis
Bulk density, ρ	1900 kg/m ³	Mass and inertia	Published values and field density tests (1.8–2.2 $\times 10^{-6}$ kg/mm ³) [37]
Specific gravity, G_s	2.65 (dimensionless)	Index state	Typical value for granular soil [37]
Moisture content, w	3.4%	Test-state descriptor	Measured in dynamic bogie tests (approximately 1–6%) [37]
Shear modulus, G	12 MPa	Deviatoric elasticity	Correlations for E and ν ($E = 25.0$ – 46.7 MPa, $\nu = 0.20$ – 0.30); adopted $E = 30$ MPa, $\nu = 0.25$ [37]
Bulk modulus, K	20 MPa	Volumetric elasticity	Correlations for E and ν
Peak friction angle, φ_{peak}	45°	Peak pressure-dependent strength	Triaxial and large-scale direct shear tests (40–48°) [37]
Cohesion, c	5 kPa	Low-confinement intercept	Triaxial and large-scale direct shear tests (5–6.2 kPa) [37]
Hyperbolic smoothing, a	3.7 kPa	Meridional smoothing	Baseline value of [37]; below $c \cot \varphi$ (Section 3.2)
Deviatoric eccentricity, e	0.7 (dimensionless)	Lode-angle dependence	Triaxial tests on road-base material [40]
Rate coefficient, γ	1.0×10^{-3} ms ($n = 2$)	Rate-dependent stress update	Back-analysis of P3G-7; acceptable range 1.0×10^{-3} – 1.0×10^{-1} (Section 4.5) [37]
Rate exponent, n	2.0 (dimensionless)	Strain-rate dependence	Back-analysis of P3G-7; acceptable range 2–10 (Section 4.5) [37]
Damage-onset threshold, ξ_0	1.0×10^{-5} (dimensionless)	Damage initiation	Initial value from published inputs, refined by simulation trials; sensitivity in Section 4.9 [37]
Void-formation energy, G_f	6.0×10^{-8} kN/mm	Post-peak energy scaling	Initial value from published inputs, refined by simulation trials; sensitivity in Section 4.9 [37]
Residual friction angle, φ_{res}	15°	Residual pressure-dependent strength	Literature range for high-strain conditions (15–20°) [37]

4.6. Steel Constitutive Model

The steel tubes were modeled with a piecewise-linear isotropic elastoplastic law using the von Mises yield criterion, associated flow, and isotropic hardening. Density was 7860 kg/m³, Young's modulus was 200 GPa, and Poisson's ratio was 0.30. The effective stress–plastic strain input curves are listed in Table 5. Steel rate sensitivity used the Cowper-Symonds relation with $C = 40.4 \text{ s}^{-1}$ and $p = 5$ [37,45,49,50]. These are values commonly used for mild steel and were not calibrated for each tube batch. The tabulated curves control the baseline hardening response. The curves were derived from tensile tests on ASTM A500 Grade B tube samples reported in [45] and used in [37]; the 9.53 mm curve was obtained from samples of the P3G-20 tube.

Table 5. Effective plastic-strain and stress data used for the two steel tube piles.

Point	P3G-7 ε_p	P3G-7 Stress (MPa)	P3G-20 ε_p	P3G-20 Stress (MPa)
1	0.0000	389.1	0.0000	413.0
2	0.0324	397.5	0.0255	496.9
3	0.0506	431.2	0.0516	527.8
4	0.0754	461.5	0.0772	550.5
5	0.0982	482.1	0.1023	568.1
6	0.1239	499.7	0.1325	585.3
7	0.1439	511.3	0.1433	590.4
8	1.0426	807.4	1.0255	975.6

4.7. Contact Treatment

The physical impact head included the 19 mm neoprene wrap, but the neoprene was not represented as a separate deformable part in the numerical model. The impact head was idealized as rigid. This idealization omits the compliance of the neoprene layer and may affect the initial force transient. Bogie-pile contact used an automatic node-to-surface penalty formulation with soft option 1 and equal static and dynamic friction coefficients of 0.10, based on friction tests between rubber and steel. The equal coefficients avoided an abrupt friction transition during the short contact event [37,51].

Pile–soil interaction used a two-way eroding surface-to-surface penalty contact. Compressive and tangential tractions transferred in either direction, and contact faces were regenerated when failed soil elements were removed. The interface coefficient was defined as one-half of the peak friction angle expressed in radians. For a peak friction angle of 45 degrees, the static and dynamic coefficients were both 0.393 [37,52,53]. This relation was adopted before the paired rate comparison and was not fitted separately to the P3G-7 or P3G-20 force histories.

4.8. Boundary and Initial Conditions

The ground surface was traction free. Non-reflecting boundary conditions were applied to the four vertical faces and the bottom of the soil blocks described in Section 4.3, allowing outgoing stress waves to leave the computational domain [37,43,44]. Because these boundaries are not intended for regions of large plastic deformation, the domain boundaries were placed outside the large-deformation zone; the effect of domain size and boundary type is summarized in Table 6 [37].

Table 6. Prior sensitivity studies used to select the numerical model [37]. Percentages are absolute differences between each simulation and the P3G-7 experimental reference values tabulated in [37]. Average resistance is cumulative work divided by displacement.

Check	Variables	Metric	Result
Mesh pattern	Separate graded and uniform near-field configurations; vertical refinement also changed	Average resistance at 125, 250, 375, and 500 mm; final cumulative work	Both meshes reproduced the main response; local refinement changed force and cumulative work differently. Graded: 9.2, 2.1, 4.6, 6.2%; uniform: 1.2, 10.2, 4.8, 1.2%; final work 6.6% (graded) and 8.2% (uniform). The uniform mesh gave a higher initial peak and greater work to about 400 mm.
Domain size	Six domains from $1.5 d \times 1.5 d \times 1.5 d$ to $4 d \times 4 d \times 3 d$	Reported average-force differences at 125, 250, 375, and 500 mm	Larger domains were less sensitive to the boundary treatment. With BNR boundaries, differences at 250–500 mm decreased from 9.2–11.8% ($1.5 d \times 1.5 d \times 1.5 d$) to 2.3–4.6% ($4 d \times 4 d \times 3 d$); at 125 mm, they were 1.5% and 9.6%, respectively.
Boundary treatment	Non-reflecting (BNR) and fixed single-point-constraint (SPC) boundaries	Force and cumulative work histories	Non-reflecting boundaries reduced the effect of reflected waves in smaller domains; boundary effects decreased as the domain increased. With SPC boundaries, differences at 250–500 mm were 11.3–17.2% for the smallest domain and 2.3–4.6% for the largest; for the largest domain, both treatments gave the same rounded values.

Table 6. Cont.

Check	Variables	Metric	Result
Rate parameters	Rate coefficient and exponent	Force and cumulative work histories	Investigated: $n = 0.25$ – 10 ($\gamma = 0.001$), $\gamma = 0.001$ – 100 ms ($n = 2$), and a 3×3 grid; γ values are numerical inputs in kg–mm–ms units (Section 3.5). Ranges judged acceptable: $\gamma = 0.001$ – 0.1 ms at $n = 2$; $n = 2$ – 10 . Several combinations gave similar histories; comparisons were graphical. Parameters were assessed through the global impact response. Damage initiation and softening-energy inputs affected post-peak resistance and deformation. Investigated:
Damage and erosion settings	Damage onset, softening energy, damage and strain thresholds	Post-peak force continuity and deformation	$Gf = 10^{-10}$ – 10^3 kN/mm ($\xi_0 = 10^{-5}$) and $\xi_0 = 10^{-5}$ – 10^{-1} ($Gf = 6 \times 10^{-8}$ kN/mm), plus a grid; little change for $Gf = 10^{-9}$ – 10^{-7} ; larger Gf values produced a pronounced secondary force peak; comparisons were graphical.

Initialization used two stages. Gravity was first applied through explicit dynamic relaxation to establish the self-weight stress field. The procedure of [37] reduces nodal velocities at each step by the default dynamic-relaxation factor of 0.995 until the distortional kinetic energy satisfies the convergence criterion. The stabilized state was then transferred to the transient analysis, and the measured bogie velocity was assigned in the impact direction. The pile and soil began the impact stage with zero prescribed translational velocity [37,44].

4.9. Numerical Verification, Sensitivity, and Comparison Protocol

Prior sensitivity studies examined mesh pattern, soil-domain size, boundary treatment, rate parameters, and damage parameters [37]. These studies used test P3G-7 and the rate-dependent baseline parameters only; their scope is considered in Section 6.5. Table 6 summarizes the response over those configurations. The P3G-7 mesh study compared graded and uniform near-field discretizations in a $5 \text{ m} \times 5 \text{ m} \times 4 \text{ m}$ domain. The graded mesh used 20 mm vertical elements to a depth of 2000 mm; the uniform mesh used $10 \text{ mm} \times 10 \text{ mm}$ plan elements near the pile and 10 mm vertical elements to a depth of 1500 mm; vertical elements below these depths were 50 mm [37]. Because plan and vertical dimensions changed together, the study compares two discretizations rather than a systematic refinement sequence. The domain study used six sizes from $1.5 d \times 1.5 d \times 1.5 d$ to $4 d \times 4 d \times 3 d$, where d is embedment depth, and compared non-reflecting and fixed boundaries. These comparisons characterize sensitivity over the meshes and domains examined.

The physical and numerical responses were reduced by the same acceleration-based procedure. The computed acceleration was taken from a node at the bogie center of gravity [37]. Model assessment used the force–displacement history as the primary response measure, cumulative force–displacement work over the common displacement interval as an integrated measure of sustained resistance, and permanent pile deformation as an independent structural measure. Peak force was not used as a quantitative measure (Section 6.2). High-speed image sequences provided an independent record of pile rotation, soil displacement (including heave and flow), bogie override, and permanent pile deformation. For P3G-7 and P3G-20, the cumulative-work values reported later were evaluated

at approximately 850 mm and 950 mm, respectively, corresponding to the last common displacement the measured and computed histories. Within each test, Model EP and Model EVP used identical solver controls, geometry, soil mesh, pile mesh, contact, boundary conditions, gravity state, steel curve, damage parameters, and erosion thresholds. Model EP set γ to zero. Model EVP used $\gamma = 1.0 \times 10^{-3}$ ms and $n = 2.0$.

5. Response of Model EP

5.1. P3G-7 Response

The first analyses used Model EP. The rate coefficient was set to zero, while the pressure-dependent plasticity, damage, residual strength, erosion, geometry, and all numerical controls remained unchanged. The stress update is that of Equations (46)–(48) with $\gamma = 0$.

With $\gamma = 0$, the stress returns to the static yield surface, and the constitutive response is rate independent. The resulting P3G-7 sequence, force–displacement history, cumulative work–displacement history, and final pile shape are shown in Figures 4–6.

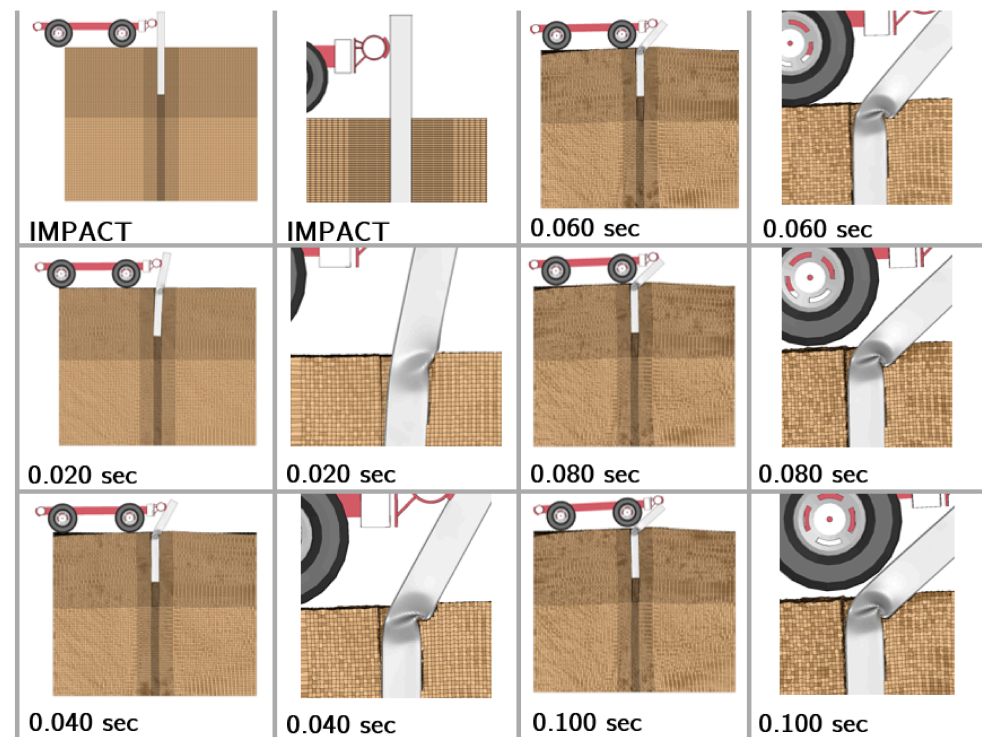


Figure 4. Sequential P3G-7 response with Model EP.

The computed and measured force histories were initially similar over approximately the first 50 mm of pile displacement. The computed force then dropped abruptly, recovered to a pronounced secondary peak, and later fell below the measured sustained resistance. Cumulative force–displacement work initially exceeded the measurements but approached them near the end of the common displacement interval. Despite that final cumulative-work agreement, deformation concentrated near the pile and produced a localized bend inconsistent with the progressive pile rotation through the soil shown by the experimental sequence in Figure 2a.

Deformation trapping and the nonphysical pile shape in Figures 4–6 are consistent with localization in a rate-independent softening calculation. They are numerical evidence for this boundary-value problem, rather than a proof of loss of hyperbolicity in the full

three-dimensional system. The one-dimensional argument in Section 2 identifies a possible mathematical origin [13,20–22,28].

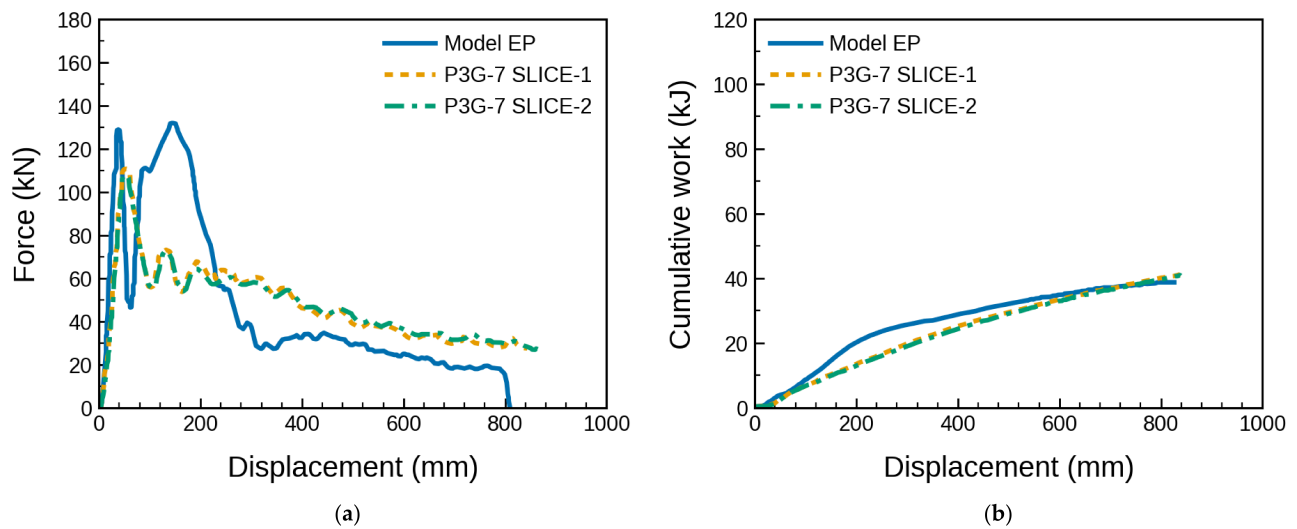


Figure 5. Measured and Model EP response: (a) force–displacement and (b) cumulative work–displacement responses for P3G-7.

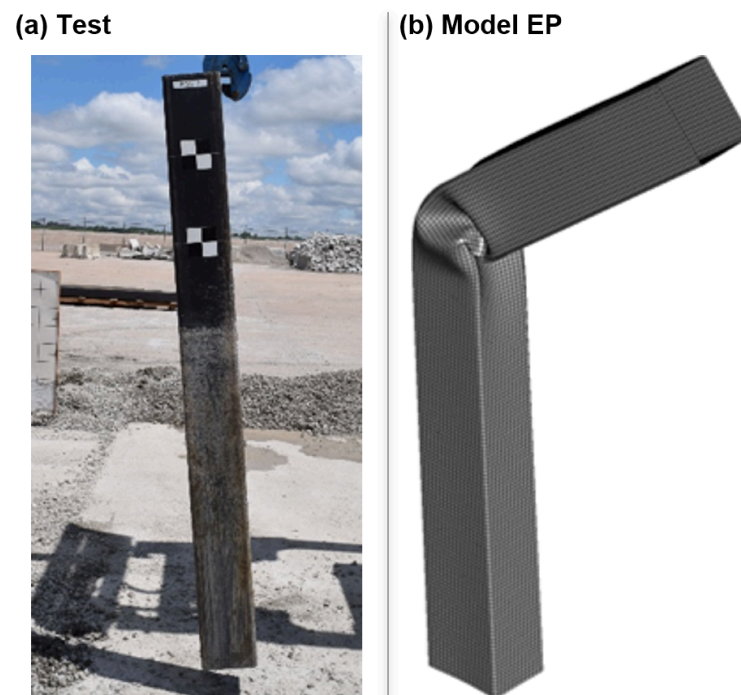


Figure 6. Post-impact P3G-7 pile: (a) test and (b) Model EP.

5.2. P3G-20 Response

P3G-20 subjects the same soil formulation to a thicker pile wall, greater embedment, larger bogie mass, and higher impact speed. These changes define a second boundary-value case rather than isolating any one severity parameter. Model EP retained all settings of the corresponding Model EVP analysis except the rate parameters. The physical sequence in Figure 2b shows pile rotation and structural bending, with the post-impact photograph documenting the permanent hinge. The computed Model EP sequence is shown in Figure 7, and both the force response and permanent deformation were compared with the test.

After the early force transient, Model EP sustained greater resistance over much of the displacement history and accumulated substantially more force–displacement work than the test (Figure 8). The computed force became relatively smooth at later displacement; the discrepancy was predominantly sustained resistance and cumulative work rather than progressively increasing oscillation. The P3G-20 sequence developed pronounced pile curvature (Figure 7); the permanent hinge is compared with the test in Section 6.2.

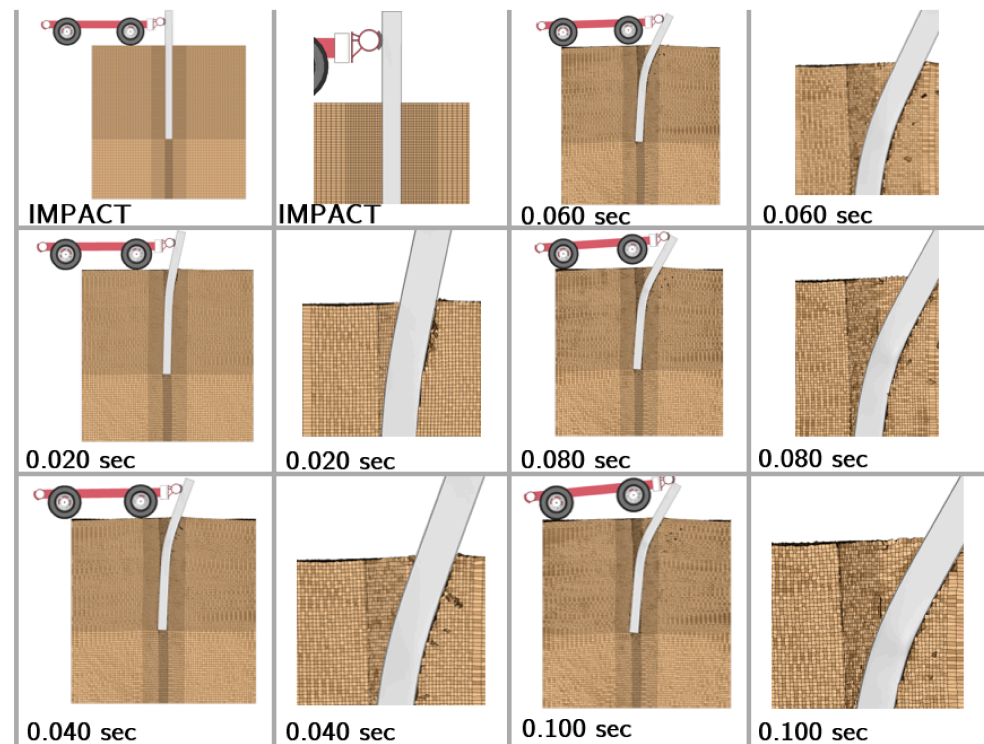


Figure 7. Sequential P3G-20 response with Model EP.

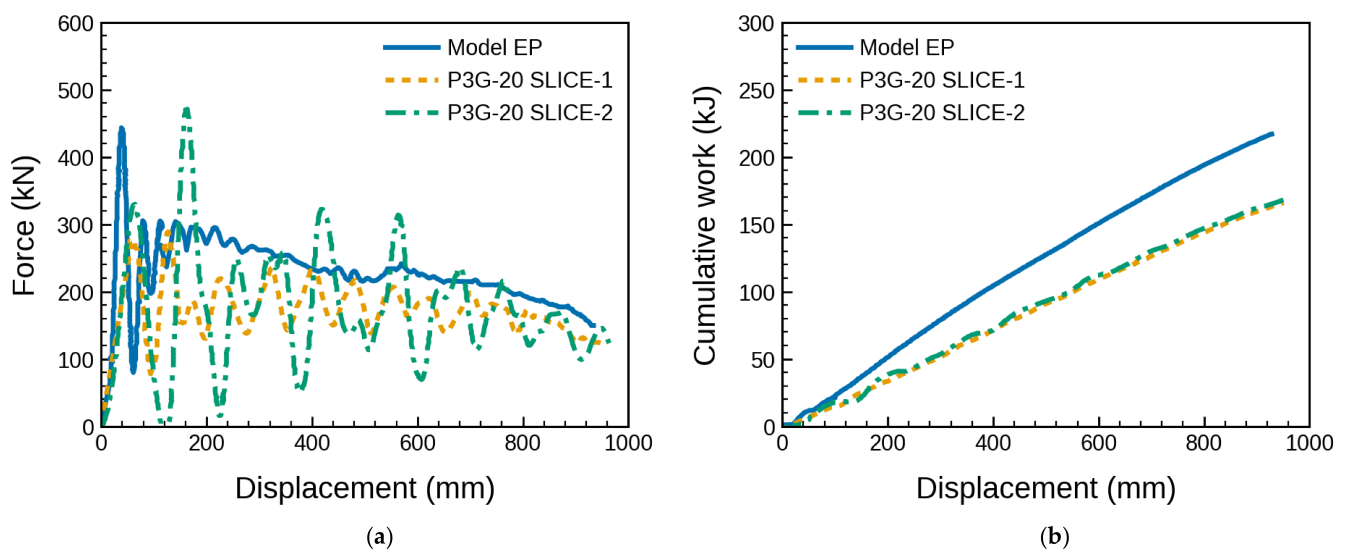


Figure 8. Measured and Model EP P3G-20 responses: (a) force–displacement and (b) cumulative work–displacement.

5.3. Interpretation of the Rate-Independent Response

Element-volume adjustment of the damage law did not prevent the rate-independent pathologies in either impact simulation. The adjustment controls the local post-peak energy relation but supplies neither spatial averaging nor a rate limit on plastic evolution.

Local energy scaling can therefore reduce one source of element-size sensitivity while leaving deformation trapping unresolved. The Model EVP calculations examine the effect of the rate update on force and deformation.

6. Response of Model EVP

6.1. Constitutive Action and Global Response

A reduction in local strength can attract further plastic deformation after softening begins. In a local rate-independent calculation, that feedback can concentrate deformation into a mesh-controlled band [13,16,17,19,24,25,54].

The elasto-viscoplastic update retains part of the elastic trial stress instead of applying the complete inviscid correction within each step. In the computed sequences, the rate update is accompanied by less concentrated deformation and, after the secondary peak, a less abrupt reduction in resistance; the initial transient is discussed separately below.

By construction, the update applies only a partial plastic correction in each step (Equation (46)); its effect on localization in the present boundary-value problems is assessed from the computed responses [20,28]. The mathematical properties still depend on the specific rheology: some local rate-dependent formulations retain localization and mesh sensitivity [21,22,55].

The calibrated rate coefficient and exponent were activated for both pile-impact models. Every other model quantity remained unchanged.

Model EVP reduced the pronounced secondary force peak in P3G-7 and reproduced sustained resistance more closely over much of the subsequent displacement (Figure 9). In P3G-20, the rate update reduced cumulative work toward the measured history, although force differences remained during the initial transient and at later displacement (Figure 10). Physical rate strengthening is a possible constitutive contribution [6–9], but the system response also depends on the redistribution of deformation.

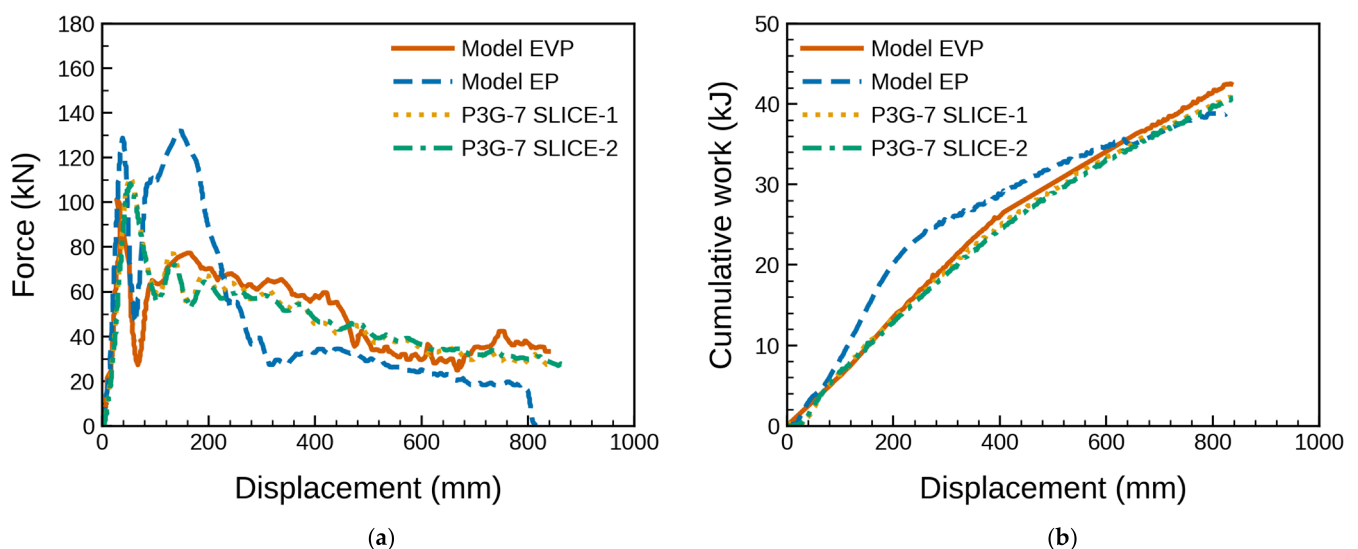


Figure 9. P3G-7 response for Model EP, Model EVP, and the two experimental SLICE records: (a) force–displacement and (b) cumulative work–displacement.

Identical erosion thresholds and eroding-contact settings exclude a change in the prescribed deletion rule as the cause of the improvement. Deletion timing, deleted volume, and contact evolution can nevertheless change with the deformation history. In P3G-7, Model EVP also shows a sharp force decrease immediately after the initial peak, to a lower force than Model EP at a similar displacement, followed by recovery (Figure 9). The decrease is a transient of the computed response, distinct from the later secondary peak, the sustained resistance, and the permanent deformation.

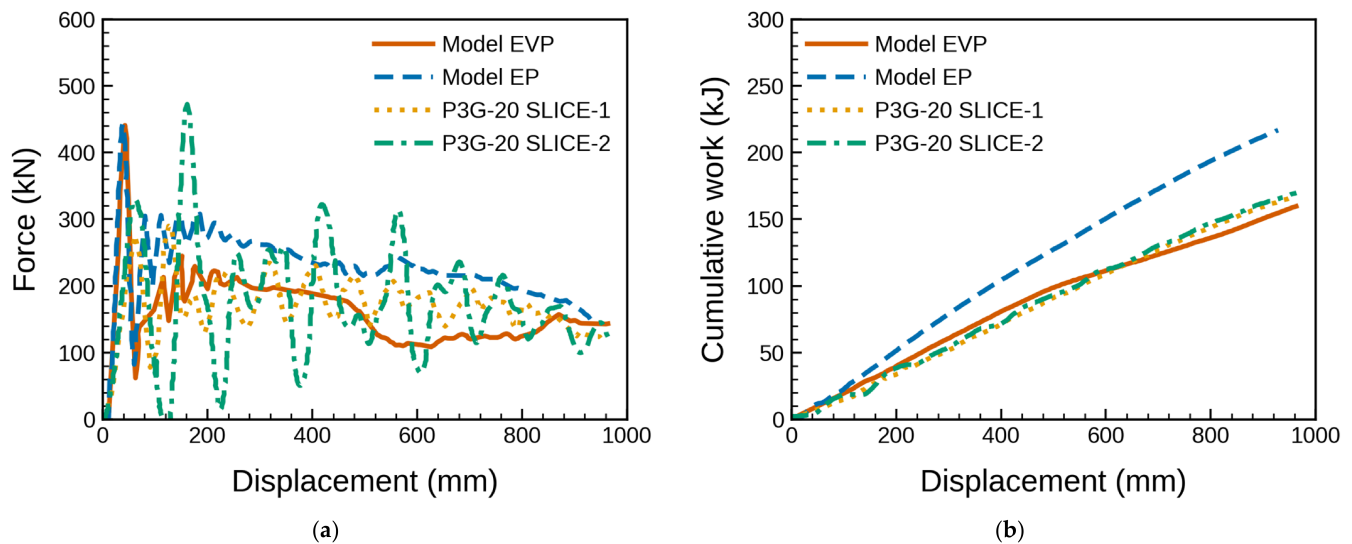


Figure 10. P3G-20 response for Model EP, Model EVP, and the two experimental SLICE records: (a) force–displacement and (b) cumulative work–displacement.

6.2. Quantitative Model Assessment

Table 7 summarizes the quantitative measures used to assess Model EVP. P3G-7 is the baseline calibration case, and P3G-20 is the higher-severity transfer assessment. Figures 9 and 10 show both experimental SLICE histories. The force–displacement history provides the primary response comparison. The measured P3G-7 initial peak was 25.0 kip (approximately 111 kN). The computed peak is not compared quantitatively, and the assessment rests on cumulative work and permanent deformation. For P3G-20, SLICE-1 recorded 65.3 kip (approximately 291 kN), whereas SLICE-2 recorded 106.5 kip (approximately 474 kN). The test report attributes the latter response to likely technical difficulties and identifies SLICE-1 as the credible measurement. SLICE-1 supplies the quantitative experimental comparison record for P3G-20 [45]. The computed P3G-20 response contains a pronounced initial transient. Its assessment emphasizes the complete force–displacement response, cumulative work over the common displacement interval, and permanent hinge location.

Table 7. Quantitative comparison of Model EVP with the experimental response.

Case	Response Measure	Experiment	Model EVP	Difference (Model EVP—Experiment)
P3G-7	Cumulative work at approximately 850 mm	41 kJ	43 kJ	+4.9%
P3G-20	Cumulative work at approximately 950 mm	157 kJ	149 kJ	−5.1%
P3G-20	Plastic-hinge depth below grade	533 mm	528 mm	−5 mm

The cumulative-work differences were approximately 4.9% for P3G-7 and 5.1% for P3G-20. These percentages use the rounded tabulated values. Differences are relative discrepancies, $\delta = (X_{\text{num}} - X_{\text{exp}})/X_{\text{exp}} \times 100\%$; magnitudes are quoted in the text, and the

P3G-20 cumulative-work discrepancy is negative (Model EVP below the measurement). The hinge-location difference is reported as a length. The nominal 5 mm difference should not be read as an accuracy estimate (Section 6.5). Because agreement in cumulative work does not imply agreement in mechanism, Table 8 compares the permanent response. Model EVP reproduced the rotation-dominated P3G-7 response without a governing groundline buckle. For P3G-20, the measured and computed hinge locations were 533 mm and 528 mm below grade, a difference of 5 mm.

Table 8. Permanent-response assessment for Model EVP.

Case	Physical Observation	Model EVP Response	Assessment
P3G-7	Rotation-dominated response; no governing groundline buckle	Rotation through soil; no governing groundline buckle	Dominant mechanism reproduced
P3G-20	Plastic-hinge region 533 mm below grade	Plastic-hinge region 528 mm below grade	Hinge location differed by 5 mm

6.3. Deformation Mode and Post-Impact Response

Soil deformation spread over a broader region around the pile in the P3G-7 Model EVP sequence (Figure 11). The pile rotated through the soil without the localized nonphysical curvature obtained with Model EP. Pile–soil contact and resistance persisted into the later impact stage. The experimental sequence in Figure 2a shows the same rotation-dominated response obtained with Model EVP. The comparison rests on deformed configurations and post-impact shapes.

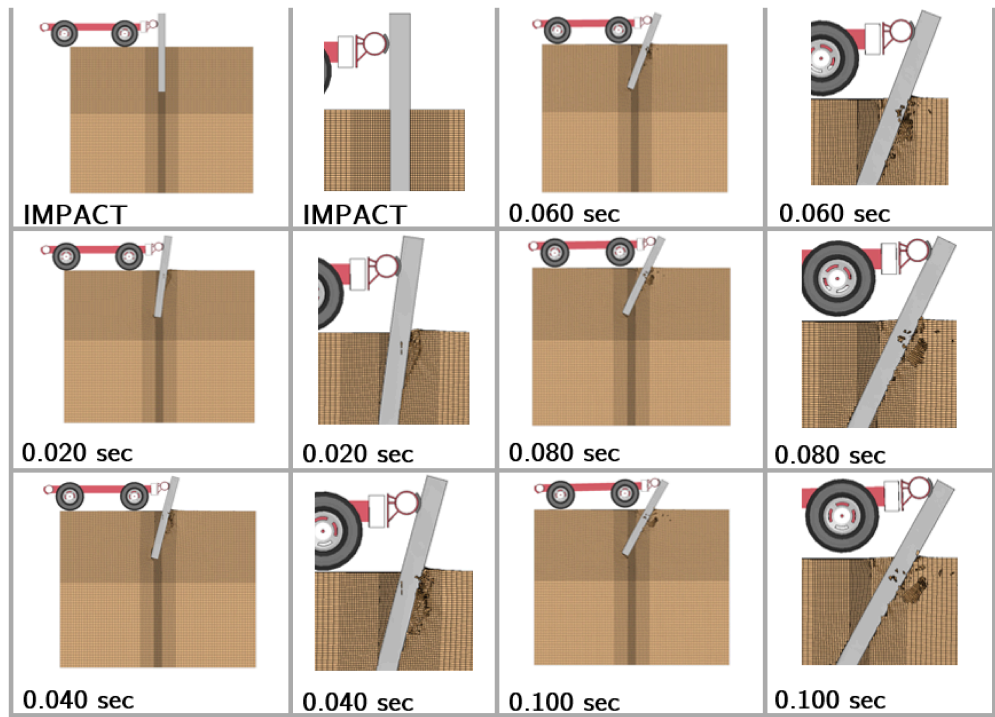


Figure 11. Sequential P3G-7 response with Model EVP.

The measured post-impact pile shape provides an additional comparison of the deformation mechanism (Figure 12). Model EVP reproduced the observed rotation and curvature more closely than Model EP, which developed an unrealistic localized bend. This comparison tests permanent deformation in addition to agreement in force and cumulative work.

6.4. Engineering Implications and Coupled Modeling Features

Force histories and deformation mechanisms must be assessed together. In the lower-energy test Model EP approached the measured cumulative work near the end of the common interval while producing a localized pile bend that did not occur (Section 5.1). An assessment based on cumulative work or average resistance alone, the quantities used in the design of soil-embedded posts [37], would have accepted the wrong mechanism. The force history by itself does not show whether resistance arises from rotation through the soil or from bending of the pile, so the force history, an integrated measure, and the permanent deformation are compared jointly.

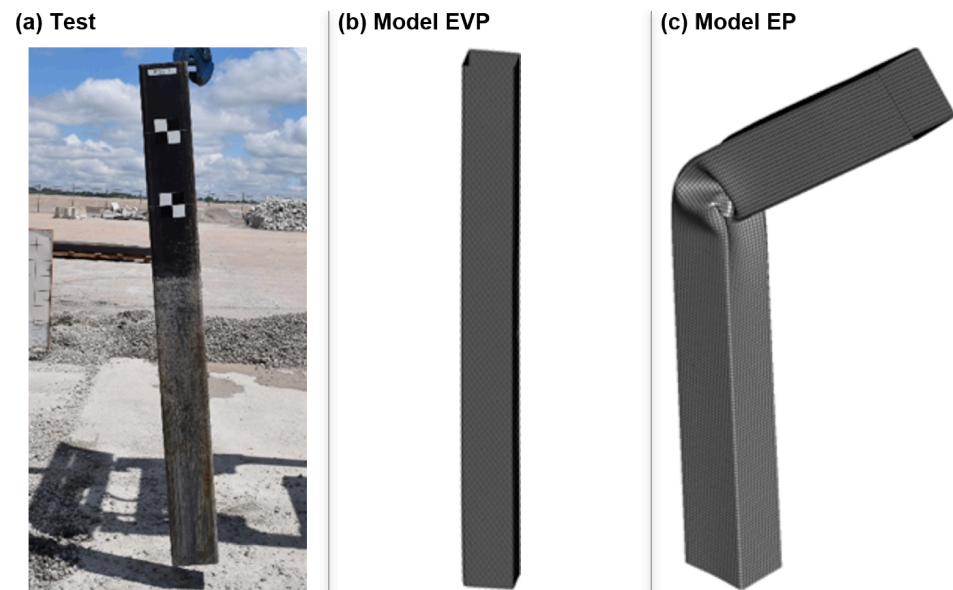


Figure 12. Post-impact P3G-7 pile: (a) test, (b) Model EVP, and (c) Model EP.

With the rate update disabled, Model EP exhibited different post-peak deficiencies in the two impact regimes. In the lower-energy case, the deformation mechanism was incorrect while the final work was nearly correct; in the higher-energy case, both the sustained resistance and the work were too large. A local rate-independent softening model can therefore misrepresent the mechanism, the energy absorbed, or both, depending on the loading.

The two variants share associated flow, volumetric-work damage, the element-volume energy adjustment, element erosion, and the contact idealization, and these features are coupled. Plastic dilation drives the damage measure (Equation (40)); damage reduces stiffness and strength and, with the principal-strain threshold, controls erosion; erosion alters the contact surface and removes internal energy; and the rate update changes the plastic increment in each step and hence the dilation and damage history. The paired comparison changes only the rate parameters, but the response difference develops through all of these features, so the separate contributions of rate strengthening, numerical regularization, damage evolution, and erosion are not isolated. The parameter sweep of [37] shows that larger rate coefficients increase sustained resistance and cumulative work in the lower-energy case, which is consistent with rate strengthening, whereas the reduction in work from Model EP to Model EVP in the higher-energy case is not produced by added strength alone.

The same soil formulation, including its rate option, has been used with multi-material ALE [5], coupled SPH-FE and EFG-FE discretizations [34], and element erosion, ALE, and adaptive FEM-SPH [37] for laterally impacted posts. Those studies treat large deformation and mesh distortion; their smoothing lengths (SPH) and support-domain radii (EFG) [34]

are parameters of the numerical approximation, not constitutive regularization lengths, and do not by themselves make a strain-softening solution mesh-objective. The present study keeps the discretization fixed and changes only the rate update.

Constitutive regularization is a separate matter. Nonlocal, gradient, and Cosserat formulations prescribe an internal length [13,16]. For the one-dimensional dynamic softening bar analyzed by de Borst and Duretz [20], a linear viscous element in parallel with the plastic dissipative element gives real wave speeds under softening and a nonzero internal length proportional to the viscosity, whereas a viscous element in series with the slider gives neither. That result is rigorous for the one-dimensional dynamic problem; their two-dimensional quasi-static calculations support the regularizing behavior, but the one-dimensional internal-length expressions do not transfer directly to general multidimensional quasi-static problems. The present Duvaut–Lions-type update also relaxes the stress toward the rate-independent state, but its mathematical equivalence to the analyzed arrangement has not been established, its relaxation time depends on the strain rate (Equation (48)), and softening is governed by pressure-dependent damage. The analysis of [20] identifies the mechanism by which the present update may act; it does not prove an internal length, a finite localization width, or mesh objectivity for the three-dimensional model.

Duretz et al. [56] compared viscoplastic, gradient, and Cosserat regularization for frictional shear banding in rock, a different material and loading regime. De Borst et al. [57] showed that non-associated frictional plasticity can lose ellipticity and become mesh dependent even without explicit strain softening, and they discuss the tendency of associated flow to overpredict plastic volumetric expansion in granular materials. The present formulation is associated, so the flow rule acts on dilation, confinement, volumetric-work damage, resistance, and localization together (Section 4.5). Safari Honar et al. [33] developed and validated an impact-oriented elastic–viscoplastic soil model for barrier-pile simulation with element-scale and full-scale evidence. The present analysis retains *MAT_147 and changes only the rate update between paired calculations, which allows the influence of that update on the pile–soil response to be examined within the adopted formulation; its parameters come from full-scale back-analysis (Section 4.5) rather than from independent high-strain-rate element tests. The practical advantage of the present approach is that the rate option is available in the explicit solver and adds no field variables, which keeps the paired comparison direct.

6.5. Limitations

The agreement obtained for the lower-energy test is calibration agreement. Its records were used to back-analyze the rate and damage parameters, which were not measured and are not unique within the acceptable range (Section 4.5), and the higher-energy test provides one assessment of parameter transfer. Within each pair, the physical strain-rate strengthening of the soil and the numerical action of the stress relaxation act together, and the two cases do not separate them.

The mesh, domain, and boundary studies of Table 6 quantify the sensitivity of the rate-dependent model over the configurations examined and do not form a refinement sequence. The paired analyses were designed to isolate the rate update at a fixed discretization, and a refinement sequence was not part of that design. Because the rate-independent problem loses hyperbolicity (Section 2), refining Model EP is expected to sharpen the localization rather than converge; the informative refinement is that of Model EVP, which is left for future work. The two meshes therefore support a limited conclusion: for the discretizations examined, the rate-dependent update suppressed the post-peak pathology of Model EP. They do not establish mesh-objective regularization. The deformation comparisons rest on

deformed configurations and post-impact shapes rather than on measured field quantities. Mesh objectivity and a finite localization width therefore remain to be established.

The comparison with the tests rests on the force–displacement response, the cumulative work, and the permanent deformation. The energy, hourglass, contact, erosion, and time-step histories of the paired simulations were not part of this assessment, so neither an independent energy balance nor time-step convergence is claimed. For the erosion-based modeling framework, these quantities are discussed in [36].

Several modeling features act together and were not isolated: the associated flow rule, which couples dilation to the volumetric-work damage; the rigid impact head without the neoprene layer; element erosion, which removes energy and alters the contact surface; the pile–soil friction coefficient; and the solver time-step controls. The two experimental channels differ substantially during the initial transient of the higher-energy test, and the uncertainty of the measured hinge location is not quantified. One soil, two pile configurations, and one constitutive framework were examined, and the conclusions apply to these.

7. Summary and Conclusions

Lateral impact of a soil-embedded pile involves pressure-dependent yielding, large deformation, post-peak degradation, and rapid redistribution of soil resistance. For a local strain-softening formulation, this response also raises the possibility of deformation localization governed by the spatial discretization. The one-dimensional characteristic analysis showed that a negative constitutive tangent can lead to loss of hyperbolicity in the rate-independent problem. The three-dimensional calculations examined the corresponding numerical response in two full-scale pile–soil impact tests using rate-independent and rate-dependent variants of the same pressure-dependent damage formulation. Geometry, mesh, contact, steel response, damage parameters, erosion criteria, boundary conditions, and solver controls remained unchanged within each pair.

Model EP reproduced the early impact response but developed post-peak behavior inconsistent with the measured force histories and deformation mechanisms. In P3G-7, resistance dropped abruptly and subsequently recovered to a pronounced secondary peak, while deformation concentrated into a localized pile bend that was absent from the physical test. The final cumulative work nevertheless approached the measured value, showing that agreement in an integrated scalar quantity does not ensure reproduction of the governing deformation mechanism. In P3G-20, Model EP maintained excessive resistance over much of the displacement history and consequently accumulated substantially greater cumulative work than measured. With Model EVP, deformation was distributed over a broader region, the secondary resistance in P3G-7 was reduced, and the excessive sustained resistance in P3G-20 diminished.

Element-volume scaling of the softening energy alone did not prevent these deficiencies. With the rate update active, the force histories followed the measured records more closely after the initial peak, the rotation-dominated response of the lower-energy test was recovered, and the combined rotation and bending of the higher-energy test was reproduced with a plastic hinge at the measured position.

Cumulative work at the last common displacement was within about 5% of the measurements in both cases, +4.9% for the lower-energy calibration case and –5.1% for the higher-energy transfer case, and the computed hinge location of 528 mm below grade compares with 533 mm measured. The permanent pile deformation agreed more closely with the tests than with the rate-independent variant for both configurations.

For the assessment of pile–soil impact models, force history, integrated resistance, and permanent deformation should be evaluated together. The rate-dependent option,

which is available in the explicit solver and adds no field variables, suppressed the post-peak pathologies observed in the rate-independent calculations for the discretizations examined and brought both the integrated resistance and the permanent deformation close to the measurements.

These conclusions hold for the formulation, parameter set, discretizations, soil, and pile configurations examined, and the improvement reflects the combined action of physical rate dependence and the numerical effect of the rate-dependent stress update. The regularizing effect observed here is the suppression of the post-peak pathology for the discretizations investigated; whether the formulation is mesh-objective and yields a finite localization width remains open. Further paired refinement studies, field measures of localization, and energy and erosion diagnostics are needed to separate these contributions and to extend the result to other soils, pile sections, and impact conditions.

Funding: This research received no external funding.

Data Availability Statement: The data presented in this study are available from the corresponding author upon reasonable request.

Acknowledgments: The author thanks MwRSF personnel for conducting the dynamic bogie tests and the Holland Computing Center at the University of Nebraska–Lincoln for computational support.

Conflicts of Interest: The author declares no conflict of interest.

Nomenclature

Principal symbols and abbreviations. Where a symbol has more than one meaning, the context is given.

Symbol	Meaning	Context
a	Hyperbolic smoothing parameter of the yield surface	Equations (31)–(33); Table 4
c	Cohesion	Equation (19); Table 4
C, p	Cowper–Symonds coefficient and exponent (steel)	Section 4.6; p also denotes body force in Equation (2)
c_e	Elastic bar-wave speed	Equations (10) and (17)
d	Embedment depth (soil-domain dimensions)	Section 4.9; Table 6
$D, D_{\max}$	Scalar damage variable; maximum damage	Equations (41)–(43)
e	Deviatoric eccentricity (extension/compression strength ratio)	Equation (34); Table 4
e_{ij}	Deviatoric strain rate	Equations (49)–(50)
E, E_t	Young’s modulus; total tangent modulus	Sections 2–4
f	Stress–plastic-strain function on the loading branch (Section 2); yield function (Section 3)	Equation (7); Equations (19), (28) and (33)
f'	Plastic modulus, $df/d\varepsilon^{\text{pl}}$	Equations (8)–(17)
F	Pile resistance force at the impact point	Section 4.2
g	Gravitational acceleration (Section 2); plastic potential (Section 3.3)	Equation (1); Equation (35)
G, K	Shear and bulk moduli	Equations (18) and (39)–(45); Table 4
$K(\theta)$	Lode-angle function (distinct from the bulk modulus K)	Equations (28)–(34)
G_f	Void-formation (softening) energy	Equations (44)–(45); Table 4
J_2, J_3	Second and third deviatoric stress invariants	Equations (21)–(23)
L, R	Differential-operator matrix; density operator	Equations (2)–(4)
n	Rate exponent (dimensionless)	Equation (48); Tables 1, 4 and 6

s_{ij}	Deviatoric stress	Equations (18) and (21)–(24)
u	Displacement field (Section 2); pile displacement at the impact point (Sections 4–6)	Equations (1)–(3); Section 4.2
v	Particle velocity	Equations (5)–(13)
V	Volume of the continuum (Section 2); element volume (Section 3.4)	Section 2; Equations (44) and (45)
$W(u)$	Cumulative force–displacement work	Section 4.2; Table 7
$X_{\text{num}}, X_{\text{exp}}$	Computed and measured response quantities	Section 6.2
α	Nominal end of the linear softening branch	Equations (42), (44) and (45)
γ	Rate coefficient; dimension $T^{1/(n-1)}$ from Equation (48), i.e., ms for $n = 2$	Equation (48); Tables 1, 4 and 6
δ	Signed relative discrepancy	Section 6.2
δ_{ij}	Kronecker delta	Equations (18), (39) and (50)
Δt	Explicit time increment (numerical quantity)	Equation (47); Sections 3.5 and 4.1
$\varepsilon, \varepsilon^{\text{el}}, \varepsilon^{\text{pl}}, \varepsilon_v^{\text{pl}}$	Strain; elastic, plastic, and plastic volumetric parts	Sections 2 and 3; Equation (40)
$\dot{\varepsilon}_e$	Effective strain rate	Equations (48) and (49)
ζ	Interpolation weight of the rate update	Equations (46) and (47)
η	Effective relaxation time	Equations (47) and (48)
θ, θ_c	Lode angle; cosine-angle parameter	Equations (22) and (34)
λ	Plastic multiplier	Equations (35)–(39)
ν, ρ	Poisson's ratio; mass density	Equations (1) and (18); Table 4
ξ, ξ_0	Damage measure (normalized plastic volumetric work); damage-onset threshold	Equations (40)–(45)
$\sigma, \sigma_{ij}; \bar{\sigma}_{ij}$	Cauchy (damaged) stress; undamaged stress	Equations (1)–(2), (41) and (46)
$\bar{\sigma}$ (scalar)	Square root of J_2	Equation (21)
$\sigma_1, \sigma_2, \sigma_3; \sigma_m$	Ordered principal stresses; mean stress	Equations (19), (20) and (25)–(27)
$\varphi, \varphi_{\text{peak}}, \varphi_{\text{res}}$	Friction angle; peak and residual values	Equations (19) and (43); Table 4
ALE, MM-ALE	(Multi-material) arbitrary Lagrangian–Eulerian	Sections 1 and 6.4
BNR, SPC	Non-reflecting boundary; single-point-constraint (fixed) boundary	Section 4.8; Table 6
EP, EVP	Rate-independent and rate-dependent model variants	Section 3.6
EFG, SPH, FE	Element-free Galerkin; smoothed particle hydrodynamics; finite element	Sections 1 and 6.4
SLICE-1, SLICE-2	Bogie-mounted SLICE 6DX data-recorder channels	Section 4.2
*MAT_147	LS-DYNA FHWA soil material model	Section 3

References

- Yosef, T.Y.; Faller, R.K.; Fang, C.; Kim, S. A state-of-the-art review on computational modeling of dynamic soil-structure interaction in crash test simulations. *Geotechnics* **2024**, *4*, 127–157. [[CrossRef](#)]
- Yosef, T.Y.; Fang, C.; Faller, R.K.; Kim, S.; Alomari, Q.A.; Atash Bahar, M.; Kumar, G.S. A review of soil constitutive models for simulating dynamic soil-structure interaction processes under impact loading. *Geotechnics* **2025**, *5*, 40. [[CrossRef](#)]
- Yosef, T.Y.; Fang, C.; Kim, S.; Faller, R.K. Dynamic response of piles embedded in granular soil to lateral impacts. *Soil Dyn. Earthq. Eng.* **2024**, *180*, 108593. [[CrossRef](#)]
- Yosef, T.Y.; Fang, C.; Faller, R.K.; Kim, S. Experimental and numerical investigations on the dynamic response of steel piles embedded in crushed limestone material under impact loading. *Geotech. Geol. Eng.* **2024**, *42*, 6253–6277. [[CrossRef](#)]
- Yosef, T.Y.; Fang, C.; Faller, R.K.; Kim, S. A multi-material ALE model for investigating impact dynamics of pile-soil systems. *Soil Dyn. Earthq. Eng.* **2023**, *164*, 107648. [[CrossRef](#)]

6. Zambrano-Cruzatty, L.E.; Yerro, A.; Macedo, J. Constitutive modelling of non-cohesive soils under high-strain rates: A consistency approach. *Géotechnique* **2024**, *74*, 743–760. [[CrossRef](#)]
7. Omidvar, M.; Iskander, M.; Bless, S. Stress-strain behavior of sand at high strain rates. *Int. J. Impact Eng.* **2012**, *49*, 192–213. [[CrossRef](#)]
8. Suescun-Florez, E.; Omidvar, M.; Iskander, M.; Bless, S. Review of high strain rate testing of granular soils. *Geotech. Test. J.* **2015**, *38*, 511–536. [[CrossRef](#)]
9. Su, Y.; Cai, G.; Yin, F.; Shan, Y.; Zhou, A. Elastic-viscoplastic model for coarse-grained soil considering particle breakage. *Comput. Geotech.* **2024**, *165*, 105962. [[CrossRef](#)]
10. Pietruszczak, S.; Mróz, Z. Finite element analysis of deformation of strain-softening materials. *Int. J. Numer. Methods Eng.* **1981**, *17*, 327–334. [[CrossRef](#)]
11. Belytschko, T.; Bažant, Z.P.; Hyun, Y.-W.; Chang, T.-P. Strain-softening materials and finite-element solutions. *Comput. Struct.* **1986**, *23*, 163–180. [[CrossRef](#)]
12. Bažant, Z.P.; Belytschko, T.B.; Chang, T.-P. Continuum theory for strain-softening. *J. Eng. Mech.* **1984**, *110*, 1666–1692. [[CrossRef](#)]
13. de Borst, R.; Sluys, L.J.; Mühlhaus, H.B.; Pamin, J. Fundamental issues in finite element analysis of localization of deformation. *Eng. Comput.* **1993**, *10*, 99–122. [[CrossRef](#)]
14. Rudnicki, J.W.; Rice, J.R. Conditions for the localization of deformation in pressure-sensitive dilatant materials. *J. Mech. Phys. Solids* **1975**, *23*, 371–394. [[CrossRef](#)]
15. Vermeer, P.A.; de Borst, R. Non-associated plasticity for soils, concrete and rock. *Heron* **1984**, *29*, 3–64.
16. Sabet, S.A.; de Borst, R. Structural softening, mesh dependence, and regularisation in non-associated plastic flow. *Int. J. Numer. Anal. Methods Geomech.* **2019**, *43*, 2170–2183. [[CrossRef](#)]
17. Pijaudier-Cabot, G.; Bažant, Z.P. Nonlocal damage theory. *J. Eng. Mech.* **1987**, *113*, 1512–1533. [[CrossRef](#)]
18. Mühlhaus, H.B.; Vardoulakis, I. The thickness of shear bands in granular materials. *Géotechnique* **1987**, *37*, 271–283. [[CrossRef](#)]
19. Bažant, Z.P.; Jirásek, M. Nonlocal integral formulations of plasticity and damage: Survey of progress. *J. Eng. Mech.* **2002**, *128*, 1119–1149. [[CrossRef](#)]
20. de Borst, R.; Duretz, T. On viscoplastic regularisation of strain-softening rocks and soils. *Int. J. Numer. Anal. Methods Geomech.* **2020**, *44*, 890–903. [[CrossRef](#)]
21. Stathas, A.; Stefanou, I. The role of viscous regularization in dynamical problems, strain localization and mesh dependency. *Comput. Methods Appl. Mech. Eng.* **2022**, *388*, 114185. [[CrossRef](#)]
22. Xue, D.; Shahin, G.; Lü, X.; Buscarnera, G. Strain localization criteria for viscoplastic geomaterials. *Int. J. Numer. Anal. Methods Geomech.* **2022**, *46*, 717–738. [[CrossRef](#)]
23. Shen, J.; Arruda, M.R.T.; Pagani, A. State of art in regularization methods for numerical analysis of structures with softening. *Int. J. Damage Mech.* **2026**, *35*, 72–118. [[CrossRef](#)]
24. Chen, J.; Hawlader, B.; Roy, K.; Pike, K. A nonlocal Eulerian-based finite-element approach for strain-softening materials. *Comput. Geotech.* **2023**, *154*, 105114. [[CrossRef](#)]
25. Chen, J.; Hawlader, B.; Roy, K.; Pike, K. Comparison of local and nonlocal regularization approaches in Eulerian-based finite element analyses. *Comput. Geotech.* **2023**, *162*, 105617. [[CrossRef](#)]
26. Cui, W.; Wu, X.; Potts, D.M.; Zdravković, L. Nonlocal strain regularisation for critical state models with volumetric hardening. *Comput. Geotech.* **2023**, *157*, 105350. [[CrossRef](#)]
27. Wu, X.; Yao, Y.; Dong, L.; Potts, D.M.; Zdravković, L.; Cui, W. Nonlocal strain regularisation for models with multiple state variables using an example of the UH model. *Comput. Geotech.* **2025**, *184*, 107296. [[CrossRef](#)]
28. Sluys, L.J.; de Borst, R. Wave propagation and localization in a rate-dependent cracked medium: Model formulation and one-dimensional examples. *Int. J. Solids Struct.* **1992**, *29*, 2945–2958. [[CrossRef](#)]
29. Wang, W.M.; Sluys, L.J.; de Borst, R. Viscoplasticity for instabilities due to strain softening and strain-rate softening. *Int. J. Numer. Methods Eng.* **1997**, *40*, 3839–3864. [[CrossRef](#)]
30. Yosef, T.Y.; Fang, C.; Faller, R.K.; Kim, S.; Bielenberg, R.W.; Stolle, C.S.; Pajouh, M.A. Adaptive coupling of FEM and SPH method for simulating dynamic post-soil interaction under impact loading. *Adv. Eng. Softw.* **2024**, *195*, 103707. [[CrossRef](#)]
31. Safari Honar, F.; Yousefpour, N.; Lam, N.T.K.; Perera, J.S. Towards a new soil constitutive model for simulation of road barrier piles. In *Proceedings of the 5th International Conference on Transportation Geotechnics, Sydney, Australia, 20–22 November 2024*; Lecture Notes in Civil Engineering; Springer: Singapore, 2025; Volume 407, pp. 157–167. [[CrossRef](#)]
32. Safari Honar, F.; Yousefpour, N.; Lam, N.T.K.; Perera, J.S. Investigating the nonlinear soil behaviour in piled barrier performance. *Transp. Infrastruct. Geotechnol.* **2025**, *12*, 154. [[CrossRef](#)]
33. Safari Honar, F.; Yousefpour, N.; Lam, N.T.K.; Perera, J.S.; Pouragha, M. Elastic-viscoplastic soil constitutive model for simulation of barrier piles under vehicle crash impact. *J. Geotech. Geoenviron. Eng.* **2025**, *151*, 102423. [[CrossRef](#)]

34. Yosef, T.Y.; Fang, C.; Kim, S.; Faller, R.K.; Alomari, Q.A.; Atash Bahar, M.; Kumar, G.S. Large deformation pile-soil interaction under lateral vehicle impact using hybrid SPH+FEM and EFG+FEM models. *Int. J. Numer. Anal. Methods Geomech.* **2026**, *50*, 2648–2669. [CrossRef]
35. Pajouh, M.A.; Yosef, T.Y.; Bielenberg, R.W.; Faller, R.K. Soil-embedded guardrail post modeling under vehicle impacts. In *Geo-Congress 2023*; American Society of Civil Engineers: Reston, VA, USA, 2023. [CrossRef]
36. Yosef, T.Y.; Fang, C.; Kim, S. Modeling dynamics of laterally impacted piles in gravel using an erosion method. *Geotechnics* **2023**, *3*, 1251–1278. [CrossRef]
37. Yosef, T.Y. Development of Advanced Computational Methodologies and Guidelines for Modeling Impact Dynamics of Pile-Granular Soil Systems. Ph.D. Dissertation, University of Nebraska-Lincoln, Lincoln, NE, USA, 2024.
38. Abellan, M.-A.; de Borst, R. Wave propagation and localisation in a softening two-phase medium. *Comput. Methods Appl. Mech. Eng.* **2006**, *195*, 5011–5019. [CrossRef]
39. Abbo, A.J.; Sloan, S.W. A smooth hyperbolic approximation to the Mohr-Coulomb yield criterion. *Comput. Struct.* **1995**, *54*, 427–441. [CrossRef]
40. Lewis, B.A. *Manual for LS-DYNA Soil Material Model 147*; FHWA-HRT-04-095; Federal Highway Administration: McLean, VA, USA, 2004. Available online: <https://www.fhwa.dot.gov/publications/research/safety/04095/04095.pdf> (accessed on 15 March 2023).
41. Reid, J.D.; Coon, B.A.; Lewis, B.A.; Sutherland, S.H.; Murray, Y.D. *Evaluation of LS-DYNA Soil Material Model 147*; FHWA-HRT-04-094; Federal Highway Administration: McLean, VA, USA, 2004. Available online: <https://www.fhwa.dot.gov/publications/research/safety/04094/index.cfm> (accessed on 15 March 2023).
42. Duvaut, G.; Lions, J.-L. *Inequalities in Mechanics and Physics*; Springer: Berlin/Heidelberg, Germany, 1976.
43. Hallquist, J.O. *LS-DYNA Theory Manual*; Livermore Software Technology Corporation: Livermore, CA, USA, 2014.
44. Livermore Software Technology Corporation. *LS-DYNA Keyword User's Manual, Volume II: Material Models*; LSTC: Livermore, CA, USA, 2020.
45. Meyer, D.T.; Stolle, C.S.; Bielenberg, R.W.; Holloway, J.C.; Faller, R.K. *Component Testing of Steel Posts for Use in a Non-Proprietary ASTM F2656-15 M30 P2 Barrier*; MwRSF Report No. TRP-03-350-17; Midwest Roadside Safety Facility, University of Nebraska-Lincoln: Lincoln, NE, USA, 2017.
46. SAE J211/1 MAR95; Instrumentation for Impact Test, Part 1, Electronic Instrumentation. SAE International: Warrendale, PA, USA, 2007.
47. *AASHTO M 147-17*; Standard Specification for Materials for Aggregate and Soil-Aggregate Subbase, Base, and Surface Courses. American Association of State Highway and Transportation Officials: Washington, DC, USA, 2021.
48. Flanagan, D.P.; Belytschko, T. A uniform strain hexahedron and quadrilateral with orthogonal hourglass control. *Int. J. Numer. Methods Eng.* **1981**, *17*, 679–706. [CrossRef]
49. Symonds, P.S. *Survey of Methods of Analysis for Plastic Deformation of Structures under Dynamic Loading*; Division of Engineering Report BU/NSRDC/1-67; Brown University: Providence, RI, USA, 1967.
50. Cowper, G.R.; Symonds, P.S. *Strain-Hardening and Strain-Rate Effects in the Impact Loading of Cantilever Beams*; Report No. 28; Brown University, Division of Applied Mathematics: Providence, RI, USA, 1957.
51. Deladi, E.L. Static Friction in Rubber-Metal Contacts with Application to Rubber Pad Forming Processes. Ph.D. Dissertation, University of Twente, Enschede, The Netherlands, 2006.
52. Yoshimi, Y.; Kishida, T. A ring torsion apparatus for evaluating friction between soil and metal surfaces. *Geotech. Test. J.* **1981**, *4*, 145–152. [CrossRef]
53. Uesugi, M.; Kishida, H. Frictional resistance at yield between dry sand and mild steel. *Soils Found.* **1986**, *26*, 139–149. [CrossRef]
54. Niazi, M.S.; Wisselink, H.; Meinders, T. Viscoplastic regularization of local damage models: Revisited. *Comput. Mech.* **2013**, *51*, 203–216. [CrossRef]
55. Duretz, T.; de Borst, R.; Le Pourhiet, L. On finite thickness of shear bands in frictional viscoplasticity, and implications for lithosphere dynamics. *Geochem. Geophys. Geosyst.* **2019**, *20*, 5598–5616. [CrossRef]
56. Duretz, T.; Räss, L.; de Borst, R.; Hageman, T. A comparison of plasticity regularization approaches for geodynamic modeling. *Geochem. Geophys. Geosyst.* **2023**, *24*, e2022GC010675. [CrossRef]
57. de Borst, R.; Sabet, S.A.; Hageman, T. Non-associated Cosserat plasticity. *Int. J. Mech. Sci.* **2022**, *230*, 107535. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.