



Article

Effect of Tooth Count and Rim Thickness on the Operational Durability of Cylindrical Involute Gears

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Abstract

This paper presents a numerical assessment of bending-fatigue durability in the tooth root region of cylindrical involute gears. Multiple gear pairs were modelled with different numbers of teeth and varying gear rim thicknesses. The generated geometry was implemented in the ANSYS 2025 R2 software suite, where the maximum normal stresses at critical locations in the tooth root region were determined through numerical simulation. A deformation-based method derived from Socie's models was applied to estimate the duration of the phase leading up to fatigue crack formation in terms of load cycle accumulation. The gear geometry, together with the generated finite element mesh, was transferred to the FRANC2D/L version 4 software suite, where fatigue crack propagation was numerically simulated. Numerical analysis provided effective stress intensity factors, which then enabled an estimation of the number of load cycles required for an initiated crack to grow to the critical length associated with tooth failure. The total fatigue life in the tooth root region was evaluated as the sum of load cycles in the crack initiation phase and the crack propagation phase up to the critical crack length. The results show that all analysed factors exhibit very high resistance to fatigue fractures in the tooth root region. Furthermore, for gears with a rim thickness ratio greater than 0.7, the fatigue crack propagates through the tooth and reaches the fracture toughness limit of the material (K_{Ic}), whereas for lower rim thickness ratios, crack propagation occurs through the gear rim itself.

Keywords: stress intensity factor; fracture mechanics; critical plane methods; crack propagation; fatigue life

1. Introduction

In most gear pairs, failure primarily occurs when the tooth loses structural integrity due to material fatigue developing at its root. Gear deterioration associated with this mechanism develops under cyclic loading, where a fatigue crack forms in the tooth root region; with increasing stresses at the crack front, the defect gradually propagates until it reaches a critical size leading to final tooth failure [1,2]. Therefore, the gears, as fundamental components of power and motion transmissions, must be designed in such a way that they fulfil the very high reliability and performance requirements [3]. Gear designers and manufacturers are often confronted with contradictory requirements that cannot be fulfilled simultaneously. Thus, gears are designed to transmit very high loads at high speeds and temperatures, while the transmission housings with all their work components must be as compact as possible, with a lesser mass in order to reduce the manufacturing



Received: 31 March 2026

Revised: 11 May 2026

Accepted: 19 May 2026

Published: 21 May 2026

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costs (materials and energy) and achieve environmentally sustainable manufacturing. Such requirements are largely met by thin-rim gears, which are essential elements of power transmission systems in aviation, space applications, and alternative energy technologies such as wind turbines, while in the automotive sector they are particularly used in hybrid and electric drivetrains [4,5]. These gears are capable of transmitting high loads while maintaining satisfactory contact stresses on the working tooth flanks, where, due to material fatigue, pitting may occur. However, under variable bending stresses a more critical failure mechanism is the initiation of fatigue damage in the tooth root region. Once initiated, the crack may propagate through the tooth body or into the gear rim, eventually leading to tooth fracture and, in severe cases, catastrophic failure. Therefore, determining the fatigue durability of the gear tooth root due to flexing is often the focus of professional and scientific interest of drive engineers, designers and scientists [6,7].

This problem is approached in three ways. The first involves the experimental determination of gear service life, which is very costly and demanding, and is thus used in calculations for complex constructions, such as aircraft and spacecraft. The second approach is characteristic of practical applications, and is based on conventional calculations of the gear service life according to ISO, DIN and AGMA standards. A common feature of these standards is that the tooth is treated as a console with a varying cross-section, while the loading is taken as uniformly distributed across its width. In addition, ISO and AGMA standards evaluate fatigue durability through material strength, whereas the DIN approach treats it solely as an inherent material property. Gear life, expressed in terms of load cycle endurance, is determined from a diagram that shows how the durability factor (Y_{NT}) varies with the cycle count required for the tooth root crack to reach the point of final failure. In practical terms, the durability factor describes the relationship between the normal stress acting at the critical tooth root region and the corresponding material strength parameters. The relevant strength characteristic in the tooth root region is the fatigue limit obtained from tests on model gears, representing the stress level in the tooth root at which 99% of the tested gears reach $3 \cdot 10^6$ load cycles. Standard procedures for calculating gear durability are relatively coarse and consider only the final fracture of the gear [8]. The third approach to determining gear service life is linked to the development of computer science and technology and has recently attracted considerable attention from scientists. Studies of crack growth under cyclic loading and the dependable estimation of gear service life are carried out using numerical modelling founded on finite element principles, making use of contemporary findings from fracture mechanics [9]. The first scientific publications in this field researched how gear rim thickness influences the stress state in the tooth root region, and so, in [10], a partial two-dimensional model of a one-toothed gear with an assumed planar stress state was considered. At the tooth root where the operating load acts, a tensile zone with positive normal stress was observed, while on the opposite side of the flank of the same tooth, a pressure zone with negative normal stress is found. The results of the study show that, with a reduction in the gear rim thickness (to a value of $1.5 \cdot m_n$), the greatest normal stress in the tensile zone exhibits a slight decreasing trend, and that, with further reduction in gear rim thickness, the observed stress rapidly increases. At the same time, a trend of continuous growth of the negative normal stress is noticeable in the pressure zone. This conclusion is supported by the findings reported in [11], which include experimental testing of a statically loaded lightweight gear and a numerical analysis performed using a 2D partial model, assuming a predominantly planar stress state within the root region of the gear tooth. In addition, the sign of the stress acting at the tooth root region was experimentally varied, and gears with thinner rims exhibited a shorter service life compared with full-rim gears. Similar findings are reported in [12], although in their numerical study the authors worked with a 3D segment of a gear featuring a reduced rim thickness.

According to [13], when loading along the tooth flank is symmetrically distributed in the tooth root region, the stresses obtained from two-dimensional and three-dimensional gear models are nearly identical. At the same time, significantly less time is required for the creation of the model itself and the obtaining of results with the two-dimensional gear model. Furthermore, according to the same author, for a two-dimensional gear model whose width-to-module ratio is greater or equal to six, it can be assumed that in the area of the root of the analysed tooth, a state of planar deformation exists, while in the opposite case a state of planar stress is present.

Further research on the stress state in the tooth root region of gears with reduced rim thickness analysed the influence of the thickness and the position of a supporting full disc linking the gear rim to the gear hub. The position of the disc across the gear width, together with its thickness, significantly affects both the level and the position of the highest normal stresses that occur in the root region of the tooth. Simultaneously, elastic deformation of the gear rim and the disc occurs [14], which leads to a non-uniform stress distribution in the root region along the tooth width. Reference [15] examines the effect of centrifugal force on the stress distribution in the tooth root region of a gear with a reduced rim thickness and a disc positioned asymmetrically along the gear width. In this research, it has been observed that the gear rim elastically deforms with an increase in the rotation rate and the centrifugal force, which influences the engagement of the gear, because the contact stresses along the tooth flank increase. Under such a loading condition, the maximum normal stress in the tooth root region remains approximately unchanged up to a certain rotational speed threshold. After reaching the threshold speed of rotation, an increase in stress in the tooth root region begins, and research has shown that said increase is more pronounced as the gear rim thickness decreases.

The mechanism of fatigue fracture in the root of the teeth of involute gears is examined through four phases of fatigue crack inception. The initial phase involves the appearance of microcracks in the tooth root region, while the point at which they begin to extend marks the start of the second phase, where the cracks remain relatively short but continue to advance. In the third phase, the crack enters its long-crack regime and continues to advance until it reaches a critical size, after which the process shifts into the fourth phase, leading to final failure in the tooth. The onset of material degradation near the root corresponds to the first two phases and marks the stage at which a fatigue crack initiates. As the process advances, the crack enters its long-crack phase and continues to grow toward a critical dimension, at which point the tooth fails; this entire stage is regarded as the crack propagation period. Accordingly, gear life expressed in load cycles (N) is determined by combining the cycles accumulated during the crack initiation stage (N_i) with those required for the crack to advance to its critical length, at which point the gear fails (N_p) [16–18]. More relevant fatigue crack research papers follow this approach. However, they consider only the crack expansion period, while the initiation period is ignored or is determined experimentally. Such an approach was used in [19], where the growth of a fatigue fracture with different gear rim thicknesses was researched for the practical application in power transmission in helicopters and turboprop aircraft. In that study, the author accounted for where the crack first appears and for the effect of centrifugal force, while the moment at which the crack initiates was established experimentally. In the research paper, gears without gear rim supports (discs) have been considered, and it was determined that a fatigue crack propagates through the gear rim in cases where the gear rim thickness value is below the threshold thickness and at rotation speeds which are greater than the threshold value.

Many papers on fatigue crack propagation in thin-rim gears can be found in the literature. Thus, in [20], the effect of different stress distributions along the tooth flank on crack propagation was experimentally researched, while in [21] a numerical model

for predicting the fatigue service life of cylindrical thin-rim gears was proposed, which includes the initiation phase and the fatigue crack expansion phase. In the expansion phase, the paths of fatigue cracks (which propagated through the tooth or the gear rim) and the remaining service life until the final fracture of the gear were researched. The influence of the crack length on the rigidity of the tooth during engagement has been researched in [22], while in [23] tribological properties of the gear tooth with a crack at its root have been considered. A study presented in [24] examined how centrifugal loading influences the service life of gears with thin rims, and it showed that rim thickness plays a more dominant role in determining fatigue resistance than the centrifugal force itself. Study [25] explored how friction during gear meshing affects where a fatigue crack first appears at the tooth root, while study [26] examined how variations in load direction, magnitude and point of application during meshing influence both the onset of cracking and the subsequent development of the crack in that region. In [26], the onset of cracking at the critical point in the tooth root region was estimated using the critical plane approach, while the period needed for the crack to grow to its critical dimension was assessed through fracture mechanics. The crack propagation stage was determined by accounting for variations in stress intensity values, since cyclic contact between mating surfaces during gear engagement causes partial closure of the crack near its tip. It was demonstrated that this approach influences the path followed by the fatigue crack. In this study, this path differed from the one obtained when the load was simplified to an applied force acting at the top point of engagement, which corresponds to a pulsating gear fatigue cycle.

Study [27] examined fatigue crack growth for various rim thicknesses by combining numerical modelling with principles of linear elastic fracture behaviour, and showed that the crack will pass through the rim when its thickness (m_b) is below 0.38. According to [28], where a series of simulation models of the expansion of a crack in the root of a tooth was conducted using the expanded finite element method (XFEM), it was shown that the expansion of the crack through the gear rim begins with a gear rim thickness (m_b) of 0.4, and that a significant contribution of it comes from the centrifugal stress which affects the position and direction of expansion of the initiated crack. In their research, the authors took into consideration the gear rim thickness, gear rim support (disc) thickness, and the effect of centrifugal load of the gear.

More recent research papers, refs. [29,30], compare the fatigue crack expansion in a symmetric and an asymmetric involute tooth profile for different gear rim thicknesses and engagement angles of the gear (α_w). The results show that flexural stresses in the tooth root region decrease with increasing engagement angle, and that the crack propagation rate in asymmetric profiles is significantly lower than in symmetric profiles. Furthermore, the fatigue strength of asymmetric cylindrical gear profiles is higher.

Recent studies have increasingly focused on the influence of rim thickness and gear geometry on fatigue crack propagation behaviour and bending durability of thin-rim gears [29,30]. At the same time, numerical approaches combining crack initiation and crack propagation analyses for fatigue life assessment have attracted growing attention [16–18]. However, the combined influence of tooth count and rim thickness on the total fatigue life and crack path behaviour of cylindrical involute gears without rim supports has not yet been sufficiently clarified.

At present, no model provides a reliable prediction of the operational life of thin-rim gears, as the available studies—and the numerical approaches they employ—differ considerably. The differences are primarily contingent on the stress which has a randomly varying amplitude, the geometry of the researched gears (number of teeth, step, profile displacement, degree of overlay, profile modification, etc.), and the form, arrangement and number of supports connecting the gear rim to the hub. A more precise and reliable

determination of gear service life also implies a more accurate determination of said input parameters [31,32]. Therefore, the greatest novel contribution of this paper compared to those previously published lies in the identified regularity of the shared influence of the number of teeth (z) and the gear rim thickness (m_b) on the fatigue durability of cylindrical involute gears made without gear rim supports (a disc). The study was carried out numerically under the assumption of a plane-strain condition in the region at the base of the tooth. The onset point (N_i) was obtained using the critical plane approach, identifying the orientation on which the damage parameter reaches its maximum. The shear and tensile damage parameters are linked to the amplitudes of normal and shear deformation, and these values are derived from the stress-response history at the base of the tooth, obtained through numerical modelling. The propagation of the crack up to its critical length was analysed using linear elastic fracture mechanics, because this phase is controlled by the stress intensity factor evaluated at the crack tip for every load cycle. In this context, the stress intensity value reflects both the opening and sliding components of crack displacement in the region at the base of the tooth. The interval in which the crack grows to its critical length (N_p) is obtained through numerical integration of Paris's law, which links the growth duration to the effective SIF evaluated at the crack front. When the effective SIF is used, it is taken into account that the actual intensity near the crack front decreases; a short distance behind that point the crack becomes partially closed, which offers some shielding from cyclic loading. The total gear service life (N) is the sum of load cycles in the initiation phase and in the fatigue crack expansion phase in the tooth root region. To obtain the most realistic load cycle possible, a quasi-static numeric simulation of a partial 2D gear model with three teeth was conducted. Therein, all cases of loads into which the gear engagement is divided were individually analysed.

2. Numerical Models

The overall numerical procedure adopted in the present study is schematically illustrated in Figure 1.

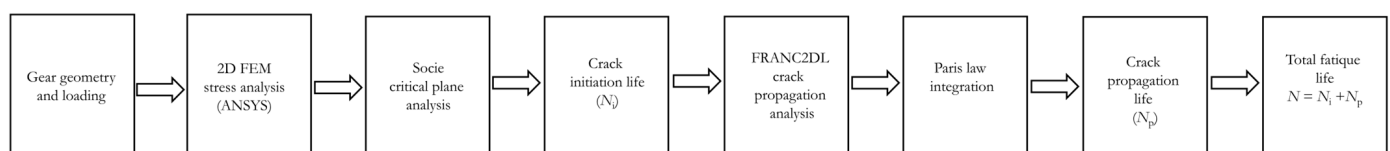


Figure 1. Schematic representation of the numerical methodology adopted in the study.

The methodology combines the crack initiation model and crack propagation analysis in order to estimate the total fatigue life of thin-rim gears. Detailed descriptions of the individual stages of the numerical procedure are presented in the following subsections.

2.1. Initial Phase of Crack Appearance in the Gear Tooth Root

The load cycle count associated with the initial phase of cracking in the tooth root region was obtained through numerical modelling for four Nul gear pairs with external flat gearing and five different rim thickness values. All gears have an ideal surface roughness and are loaded with a temporally variable cyclic load. Table 1 presents the loading conditions together with the geometry of the investigated gear pairs.

Table 1. Geometry, material, and load data for the researched gears.

Quantity	Notation	First Pair	Second Pair	Third Pair	Fourth Pair
Tooth count of gear 1	z_1	18	20	22	24
Tooth count of gear 2	z_2	32	30	28	26
Normal cross-section gear pair modulus	m_n , mm	4	4	4	4
Gear width	b , mm	25	25	25	25
Gear pair axial spacing	a , mm	100	100	100	100
Gear rim thickness	$m_b = s_R/h_t$	1.2; 1; 0.7; 0.5; 0.3	1.2; 1; 0.7; 0.5; 0.3	1.2; 1; 0.7; 0.5; 0.3	1.2; 1; 0.7; 0.5; 0.3
Normal cross-section gear engagement angle	α_n	20°	20°	20°	20°
Gear engagement radial clearance factor	c^*	0.25	0.25	0.25	0.25
Addendum associated with the cutting tool	h_a^*	1	1	1	1
Dedendum defined by the cutting tool	h_f^*	1.25	1.25	1.25	1.25
Diameter of the circle atop the gear hub	d_a	Standard engagement clearance	Standard engagement clearance	Standard engagement clearance	Standard engagement clearance
Gear material	-	42CrMo4	42CrMo4	42CrMo4	42CrMo4
Gear linear load	F_b/b , N/mm	300	300	300	300

The tooth profile and 2D geometric gear model of the analysed gear pairs were created in Auto CAD. As noted in the introduction, two- and three-dimensional gear models produce nearly identical stresses at the gear tooth root when loading along the tooth flank is symmetrically distributed; under these conditions, constructing a 2D model and obtaining the results requires less time [13]. In this research, a partial model of a gear with three teeth is analysed, because, according to [26], the stress difference in the tooth root between this model and a full gear model amounts to just 0.4%. Files representing eight partial three-tooth gear models were imported into the ANSYS environment, where a two-dimensional finite element approach—based on the assumption of planar deformation, justified by the width-to-module ratio exceeding six [13]—was applied to determine stresses in the tooth root region for all analysed gears. Before the stress analysis, an interlinking of the model with rectangular linearly elastic finite elements with eight nodes was conducted, the material properties, borderline conditions and load were determined, and the convergence of the solution was controlled with the aim of obtaining the optimal number of finite elements in the model. A locally refined finite element mesh was applied in the tooth root region in order to improve the accuracy of the stress evaluation in the critical area and to minimise distorted and skewed elements. Depending on the analysed geometry, the FE models contained approximately 800 finite elements. The displacements at the left and right boundary nodes of the three-tooth partial model were constrained, which is illustrated in Figure 2.

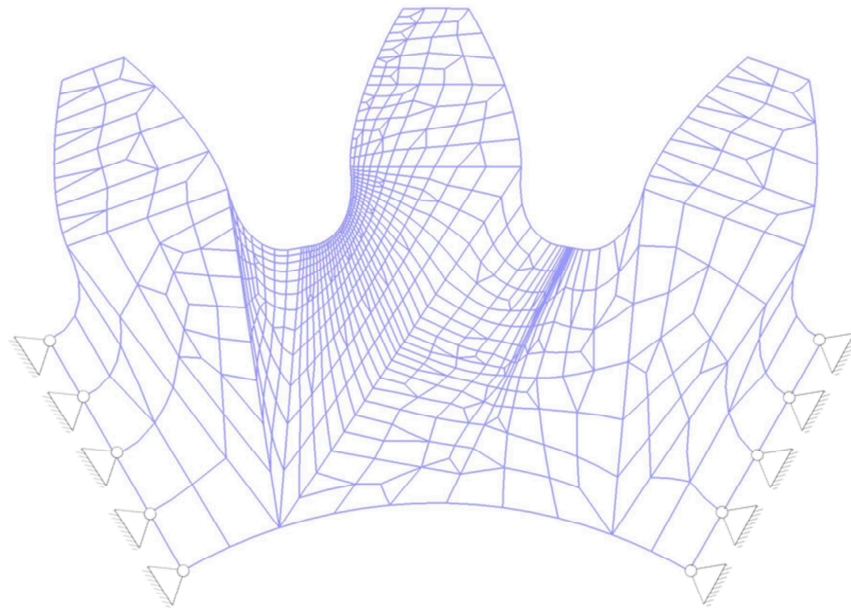


Figure 2. Boundary conditions applied to the partial gear geometry.

For every analysed configuration, the shifting position of the load along the flank was examined, since during meshing the load's magnitude, point of action and direction change continuously across the flank. Accordingly, this phenomenon was assessed by considering 16 individual meshing cases, through which the load cycle of each tooth was characterised. This includes six distinct load positions within the single-tooth meshing interval, during which the entire load is carried by the analysed tooth. Two positions correspond to the region of double-tooth contact when the load is distributed between two pairs of gear teeth in engagement, and four positions correspond to cases where the load is applied to the tooth in front and to the adjacent preceding tooth relative to the analysed tooth. The described load positions, shown on Figure 3, define the load cycle for all teeth of the considered gear pairs.

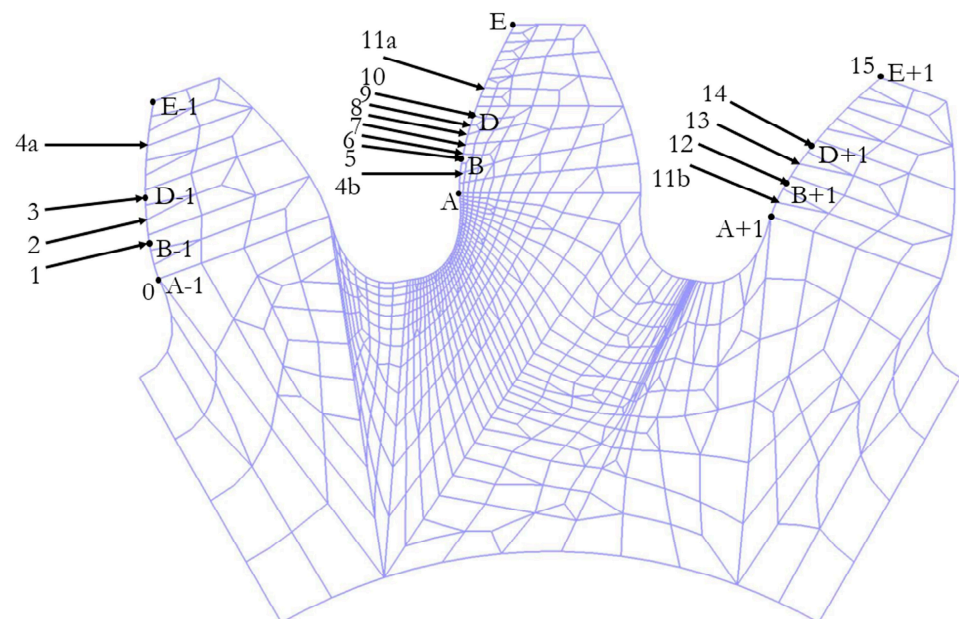


Figure 3. Engagement points along the tooth flank defining the tooth load cycle of the considered gears.

The locally based deformation model presented in [26] makes it possible to estimate the load cycle count associated with the stage in which a crack first forms at the tooth root. Within this framework, the first appearance of a crack is expected at the point in the tooth root region where deformation is most concentrated, arising from repeatedly imposed cycles of local plastic strain. Therefore, the analysis of the stress cycles has been conducted when the gear teeth are located in the external point of single engagement, because it is a case of the most unfavourable load of the analysed tooth. Because a multiaxial stress state prevails at the surface of the analysed tooth root, the crack initiation period was evaluated using Socie's critical plane models [33]. The models link the damage parameter to the number of load cycles until crack initiation so that it is necessary to find the plane (critical plane) where said parameter has the greatest value. Critical plane methods provide for two models of fatigue crack initiation. In the first model, crack propagation follows the orientation of maximum shear deformation, whereas in the second model it forms along a line approximately orthogonal to the direction associated with the peak normal stress.

Expression (1) represents the functional relation between time to fatigue crack initiation and the shear damage parameter:

$$d_s = \gamma_a \left(1 + k \frac{\sigma_{\max}}{R_t} \right) = \frac{\tau_f'}{G} (2N_i)^{b_{0i}} + \gamma_f' (2N_i)^{c_{0i}} \quad (1)$$

where

γ_a is the shear deformation amplitude;

k is the gear material constant;

$\sigma_{\max}$ is the peak normal stress acting on the orientation where shear deformation is greatest;

R_t is the gear material yield strength;

τ_f' is the parameter describing shear-fatigue strength;

G is the under shear load;

N_i is the load cycle count associated with the moment when a crack first appears;

γ_f' is a parameter describing cyclic shear deformation;

b_{0i} is the exponent characterising shear-fatigue strength;

c_{0i} is the exponent of cyclical shear deformation.

Expression (2) represents the functional relation between the time to fatigue crack initiation and the tensile damage parameter:

$$d_t = \sigma_{\max} \varepsilon_a = \sigma_f' \varepsilon_f' (2N_i)^{b_i + c_i} + \frac{(\sigma_f')^2}{E} (2N_i)^{2b_i} \quad (2)$$

where:

ε_a is a measure of the normal deformation amplitude;

$\sigma_{\max}$ is the maximum normal stress;

σ_f' is a parameter representing fatigue strength;

E is the material's elastic property describing stiffness;

N_i is the load cycle count corresponding to the moment when a crack first forms;

ε_f' is a parameter characterising cyclic deformation behaviour;

b_i is the exponent describing fatigue-strength behaviour;

c_i is an exponent characterising cyclic deformation behaviour.

From Expressions (1) and (2) it follows that the shear-related damage parameter depends on the local amplitude of shear deformation, while the tensile counterpart is governed by the local amplitude of normal deformation; in addition, both parameters are influenced by the maximum normal stress evaluated at the critical location along the transitional curve of the tooth under critical loading conditions. Under the assumption of

a planar deformation state, the elastic components of principal stresses and strains (with the superscript “e” indicating elastic quantities) are determined from the stress tensor in a plane normal to the transitional curve surface at the critical location in the tooth root region, using standard relations from the strength of the materials, as follows:

$$\sigma_1^e = \frac{\sigma_y^e + \sigma_z^e}{2} + \sqrt{\left(\frac{\sigma_y^e - \sigma_z^e}{2}\right)^2 + (\tau_{yz})^2} \quad (3)$$

$$\sigma_2^e = \nu^e (\sigma_y^e + \sigma_z^e) \quad (4)$$

$$\sigma_3^e = \frac{\sigma_y^e + \sigma_z^e}{2} - \sqrt{\left(\frac{\sigma_y^e - \sigma_z^e}{2}\right)^2 + (\tau_{yz})^2} \quad (5)$$

On the same plane, the deformation components are then readily determined according to the following formulas:

$$\varepsilon_1^e = \frac{1}{E} [\sigma_1^e - \nu^e (\sigma_2^e + \sigma_3^e)] \quad (6)$$

$$\varepsilon_2^e = \frac{1}{E} [\sigma_2^e - \nu^e (\sigma_1^e + \sigma_3^e)] = 0 \quad (7)$$

$$\varepsilon_3^e = \frac{1}{E} [\sigma_3^e - \nu^e (\sigma_1^e + \sigma_2^e)] \quad (8)$$

Then the normal stress on a plane perpendicular to the transitional curve surface is:

$$\sigma_n^e = \max\{|\sigma_1^e|, |\sigma_3^e|\} \quad (9)$$

The remaining two values of the stress components on the same plane in the tangential and axial direction are:

$$\sigma_t^e = \min\{|\sigma_1^e|, |\sigma_3^e|\} \quad (10)$$

$$\sigma_2^e = \nu^e \sigma_n^e \quad (11)$$

The deformation components in the normal and tangential directions on the same plane are obtained by substituting Expressions (9)–(11) in (6)–(8) and they are:

$$\varepsilon_n^e = \frac{1 - (\nu^e)^2}{E} \sigma_n^e \quad (12)$$

$$\varepsilon_2^e = 0 \quad (13)$$

$$\varepsilon_t^e = -\frac{\nu^e (1 + \nu^e)}{E} \sigma_n^e \quad (14)$$

The biaxiality ratio of the stress components is determined from Expressions (9) and (11) as follows:

$$a^e = \frac{\sigma_2^e}{\sigma_n^e} = \nu^e \quad (15)$$

So, from (15), it follows that the stress components ratio on a plane perpendicular to the transitional curve of the tooth root in the axial and normal directions is constant, and since the maximal normal stress is located in the same plane, the angle which its vector closes with the local horizontal axis is constant as well. It follows, then, that the stress field at the critical region near the tooth root region—as well as in the plane normal to the transitional curve surface—is multiaxial and corresponds to a constant-amplitude loading pattern. Within this plane, the tensile-related damage parameter reaches its highest level, making it the critical orientation for assessing tensile damage. For the stress cycle given in

(12), the corresponding normal deformation is obtained, and the tensile damage parameter is then evaluated using Expression (2), which relates directly to the moment when a fatigue crack first forms, as follows:

$$d_t = \frac{1 - (\nu^e)^2}{2E} (\sigma_n^e)^2 = \sigma_f' \epsilon_f' (2N_i)^{b_i + c_i} + \frac{(\sigma_f')^2}{E} (2N_i)^{2b_i} \quad (16)$$

The peak level of the shear damage parameter (1) occurs at the critical location along the transitional surface at the tooth root, where the constant-amplitude stress field is proportional. This maximum lies in a plane inclined by 45° relative to the critical orientation used in the tensile-based model. This plane represents the critical orientation for the shear damage parameter, as the shear deformation amplitude reaches its maximum there; see Figure 4.

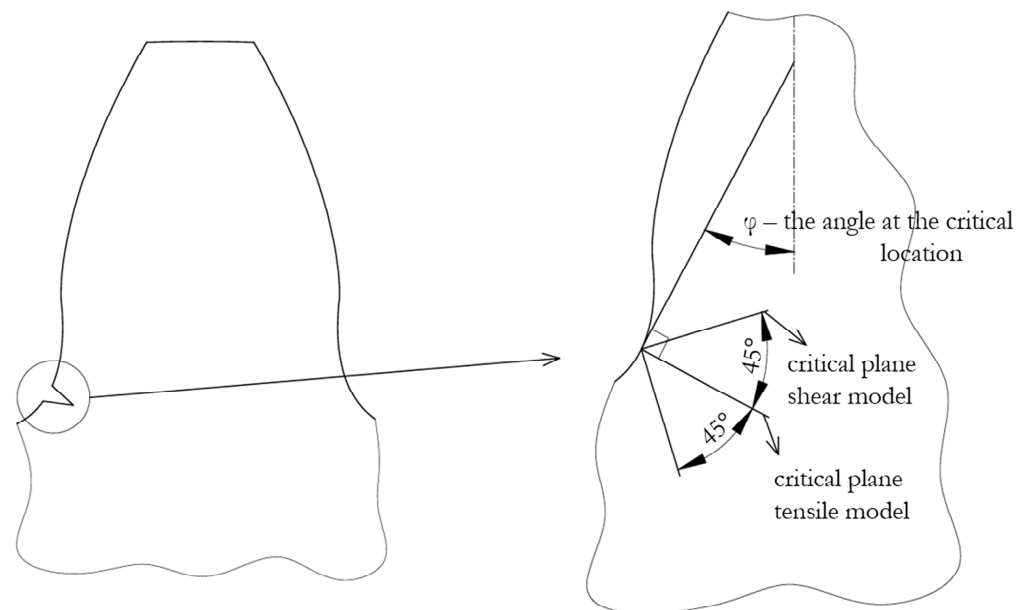


Figure 4. Critical planes in the transitional curve point, in which the tensile and shear damage parameters take on maximum values.

From the components of deformation in the normal and tangential directions (12) and (14), the shear deformation is determined in the critical plane, which is:

$$\gamma^e = \epsilon_n^e - \epsilon_t^e = \frac{1 + \nu^e}{E} \sigma_n^e \quad (17)$$

From (17) it is evident that the change in shear deformation depends on the normal stress and for a pulsating stress cycle the shear deformation amplitude is:

$$\gamma_a^e = \frac{1 + \nu^e}{2E} \sigma_n^e \quad (18)$$

By including (18) in (1), the shear damage parameter is:

$$d_s = \frac{1 + \nu^e}{2E} \sigma_n^e \left(k \frac{\sigma_n^e}{R_t} + 1 \right) = \gamma_f' (2N_i)^{c_{0i}} + \frac{\tau_f'}{G} (2N_i)^{b_{0i}} \quad (19)$$

Before estimating the moment at which a crack first forms in the analysed gears, an elasto-plastic correction of local stresses and strains was carried out, since plastic flow develops in the critical region near the base of the tooth. The calculation of local elasto-plastic stresses and deformations was conducted using the Hoffman–Seeger method, which

is applied with a multiaxial proportional stress state [34], where the multiaxial proportional stress is translated to an equivalent uniaxial stress state and the obtained stresses and deformations are substituted in Neuber's rule. It is then possible to calculate the main stresses and deformations in the elasto-plastic area using Hencky's flow rule. Firstly, it is necessary to calculate von Mises' equivalent elastic stress on the tooth root surface, where $\sigma_3^e = 0$ and the ratio of biaxiality for the proportional load is constant and equal to Poisson's coefficient (15), and so the equivalent stress is:

$$\sigma_q^e = \sigma_n^e \sqrt{1 - \nu^e + (\nu^e)^2} \quad (20)$$

Based on Hook's law, the equivalent deformation is:

$$\varepsilon_q^e = \frac{\sigma_q^e}{E} \quad (21)$$

Knowing the equivalent stresses and deformations in the elastic area, it is possible to determine equivalent deformations (ε_q) and stresses (σ_q) in the elasto-plastic area by solving two equations using Newton–Raphson's numerical process. The first Equation (22) represents an expression for a uniaxial cyclical curve of the dependence of equivalent stresses and deformations in the elasto-plastic area, and the second Equation (23) represents Neuber's rule.

$$\varepsilon_q = \frac{\sigma_q}{E} + \left(\frac{\sigma_q}{K'} \right)^{\frac{1}{n'}} \quad (22)$$

$$\varepsilon_q \sigma_q = \varepsilon_q^e \sigma_q^e \quad (23)$$

where

K' is the coefficient of material hardening under cyclical load;

n' is the exponent of material hardening under cyclical load.

When equivalent stresses and deformations in the elasto-plastic area are known, the effective Poisson coefficient (ν_{eff}) can be calculated according to the following expression:

$$\nu_{\text{eff}} = \frac{1}{2} - \left(\frac{1}{2} - \nu^e \right) \frac{\sigma_q}{E \varepsilon_q} \quad (24)$$

Assuming an equal ratio of the main deformation components for the elastic and elasto-plastic areas

$$\frac{\varepsilon_2^e}{\varepsilon_n^e} = \frac{\varepsilon_2}{\varepsilon_n} = 0 \quad (25)$$

we can also determine the elasto-plastic biaxial ratio as follows:

$$a = \frac{\frac{\varepsilon_2}{\varepsilon_1} + \nu_{\text{eff}}}{1 + \nu_{\text{eff}} \frac{\varepsilon_2}{\varepsilon_1}} = \frac{0 + \nu_{\text{eff}}}{1 + \nu_{\text{eff}} 0} = \nu_{\text{eff}} \quad (26)$$

Using Hencky's flow law, the local main stresses and the local main deformations are calculated as follows:

$$\sigma_n = \sigma_q \frac{1}{\sqrt{(\nu_{\text{eff}})^2 - \nu_{\text{eff}} + 1}} \quad (27)$$

$$\sigma_2 = \nu_{\text{eff}} \sigma_n \quad (28)$$

$$\sigma_t = 0 \quad (29)$$

$$\varepsilon_n = \varepsilon_q \frac{1 - (\nu_{\text{eff}})^2}{\sqrt{(\nu_{\text{eff}})^2 - \nu_{\text{eff}} + 1}} \quad (30)$$

$$\varepsilon_2 = 0 \quad (31)$$

$$\varepsilon_t = -\varepsilon_q \frac{(1 + \nu_{\text{eff}})\nu_{\text{eff}}}{\sqrt{(\nu_{\text{eff}})^2 - \nu_{\text{eff}} + 1}} \quad (32)$$

Based on Expressions (30) and (32), the normal and shear deformation amplitudes in the elasto-plastic area are obtained, which are then used to calculate the damage parameters. Using the obtained damage parameter values together with the cyclic material data from Table 2, and applying Expressions (16) and (19), the load cycle count corresponding to the stage when a crack first appears at the tooth root (N_i) can be derived. The load cycle count is evaluated using whichever damage parameter attains the higher value, which yields a more conservative estimate of the time required for crack initiation and growth.

Table 2. Static and cyclical gear material characteristics.

Notation	Numerical Value	Notation	Numerical Value	Notation	Numerical Value
R_m , GPa	1	σ_f' , GPa	1.82	b_i	−0.08
R_t , GPa	0.8	τ_f' , GPa	1.051	b_{0i}	−0.08
E , GPa	206	σ_{td} , GPa	0.7	c_i	−0.76
G , GPa	80	n'	0.14	c_{0i}	−0.76
σ_d , GPa	0.55	ν	0.3	γ_f'	1.13
K' , GPa	2.259	k	1	ε_f'	0.65

2.2. Fatigue Crack Propagation Period in the Tooth Root Region and Gear Service Life

The second stage of the numerical analysis concerns the evaluation of the parameter (N_p), representing the load cycle count required for an already-formed crack to advance to its terminal size, the point at which failure of the tooth takes place. This simulation phase was conducted using the FRANC2DL software suite, where it is necessary to determine the location, orientation, and length of the initiated crack. The critical plane approaches used to estimate when a crack first forms also indicate its likely position and orientation along the transitional curve surface in the root region of the tooth, as illustrated in Figure 4. In this way, the necessary basis was established for continuing the analysis of crack growth and, ultimately, for determining the overall load cycle count that defines the gear's fatigue life. According to the described fatigue fracture mechanism, the initiated crack expansion begins with an increase in long cracks, so for the length of the initiated crack, the value (a_2) was adopted, which represents the length threshold value when short cracks become long. Thus, this length marks the boundary between the second and third phase of the fracture initiation process in the region near the tooth root. According to [35], the initiated crack length (a_2) is defined by the following expression:

$$a_2 = \frac{\delta}{2} e^{2\left(\frac{4\sqrt{2}}{\pi} \frac{\sigma_{td}}{\sigma_d} - 1\right)} \quad (33)$$

where

δ is crystallite diameter;

σ_d is the laboratory sample fatigue limit;

σ_{td} is the gear material dynamic yield strength.

Once the crack length exceeds the value defined by Expression (33), its subsequent behaviour can be described according to the principles of linear elastic fracture mechanics. Based on the data given in Tables 2 and 3, which define the propagation characteristics

of the gear material, and in accordance with (33), the initial crack length was taken as 200 micrometres.

Table 3. Gear material propagation properties.

Notation	Numerical Value
K_{Ic} , MPa $\sqrt{\text{mm}}$	2620
ΔK_{th} , MPa $\sqrt{\text{mm}}$	269
δ , μm	32
C , $\frac{\text{mm}}{\text{cycle} \times (\text{MPa}\sqrt{\text{mm}})^m}$	$3.31 \cdot 10^{-17}$
m	4.16

As previously stated, the fatigue crack expansion was simulated in the Franc2D/L version 4 program suite, which semi-automatically propagates the crack, that is, the user defines the length of each lengthening of the crack. The analysis is conducted using the finite element method after each lengthening which gives the solution for the crack tip location, further expansion direction, and stress intensity factor (SIF) of the cleaving (K_I) and sliding type (K_{II}). The effect of the mode II stress intensity factor is accounted for through the equivalent stress intensity factor, as expressed by the following relation:

$$\Delta K_{eq} = \cos^2 \frac{\theta_0}{2} \left(\Delta K_I \cos \frac{\theta_0}{2} - 3 \Delta K_{II} \sin \frac{\theta_0}{2} \right) \quad (34)$$

where

θ_0 is the angle describing the direction in which the crack advances.

Expression (34) for the equivalent SIF does not fully capture the mechanisms responsible for crack closure arising from plastic deformation, corrosion effects or surface roughness, conditions that partly shield the crack front from cyclic loading. Accordingly, the effective SIF is defined for the part of the loading cycle in which the crack faces remain separated and the fracture is fully open. The factor is valid for the entire crack growth area, and is calculated according to the expression from [26]:

$$\Delta K_{ef} = K_{max} \left\{ 1 - \left[1 + g' \cdot \left(\frac{2}{\pi} - 1 \right) \right] \cdot \left(1 - 2\sqrt{1 - \zeta} \right) \right\} \quad (35)$$

where

g' is the form function, which is:

$$g' = e^{-\left(\frac{K_{max}}{K_{th}} - 1\right)} \quad (36)$$

and parameter ζ relates the $CTOD_{plastic \ strain}$ to the $CTOD_{max}$ value measured perpendicular to the opening direction, with its magnitude expressed through a fourth-degree polynomial in the load-ratio function (r), as presented in [34]:

$$\zeta = \frac{CTOD_{plastic \ strain}}{CTOD_{max}} = 0.8561 + 0.0205 \cdot r + 0.1438 \cdot r^2 + 0.2802 \cdot r^3 - 0.3007 \cdot r^4 \quad (37)$$

The crack growth phase concludes once the equivalent SIF from Expression (34) attains the toughness limit (K_{Ic}) of the gear material, given in Table 3. At the completion of the growth phase, both the crack path and the functional relation between the equivalent SIF and the progressing crack length are established for all gears in the analysed pairs.

An effective SIF (ΔK_{eff}), used to modify the standard Paris relation, is obtained from Expression (35) once the equivalent SIF values from (34) are inserted into it. By numerically integrating the corrected Paris relation and applying the material data from Table 3, the parameter (N_p)—which represents the load cycle span required for a crack to grow from its initial size (a_2) to the critical value (a_c), where failure near the tooth root takes place—is obtained according to the following expression:

$$\int_0^{N_p} dN = \frac{1}{C} \int_{a_2}^{a_c} \frac{da}{[\Delta K_{\text{ef}}(a)]^m} \quad (38)$$

The described process was conducted for all gears of the considered gear pairs and the paths of their fatigue cracks in the gear tooth root, critical crack length values, and the numbers of load cycles in the fatigue crack expansion period (N_p) were obtained.

The overall fatigue life of the analysed gear pairs is expressed through the parameter (N), obtained by combining the contribution from the stage in which a crack first forms (N_i) with the cycle span required for that crack to advance to its critical size, where failure at the tooth root occurs (N_p).

$$N = N_i + N_p \quad (39)$$

The obtained numerical trends regarding crack propagation behaviour and the influence of gear rim thickness are generally consistent with previously published experimental and benchmark numerical studies [11,19,27–30], particularly with respect to the transition of crack propagation from the tooth toward the gear rim for reduced rim thickness values, as will be discussed in the following section.

3. Results and Discussion

3.1. Initial Crack Formation Stage in the Tooth Root Region

Before evaluating the load cycle span associated with the stage in which a crack first forms, the normal stress history at the critical location along the transitional curve was established. The stress cycle response corresponds to the plane at the critical location near the tooth root, oriented perpendicular to the transitional curve, and for one of the analysed gear pairs it is illustrated in Figure 5. The trends of change in the stress cycles for other analysed pairs are similar to the diagrams in Figure 5. The similarity arises from the fact that the considered gear pairs have no profile shift and a constant axial offset, resulting in equal degrees of overlap.

The cycles relate to gears of different gear rim thicknesses (m_b), where an equal trend of pulsating stress change for the driving and driven gear is observable. The maximum normal stress is in the external points of single engagement, and the stress curve in the area of single engagement for the driving gear is placed to the left, while on the driven gear it is placed to the right side. Somewhat greater are the stresses in gears with a lesser number of teeth, but the gear rim thickness still has the dominant effect on the stress cycle. There is practically no difference in cycles up to a gear rim thickness of 0.7 for all gears, and a more significant stress increase occurs after further reduction in the gear rim thickness. The change is more clearly illustrated in Figure 6, which shows the variation in maximum normal stress (σ_n^e) with gear rim thickness (m_b) for all considered gears. Up to a gear rim thickness of 0.7, normal stresses behave in accordance with the research papers [10,11]. For thicknesses between 0.6 and 0.7, a smaller increase in stress is observed for all gears, whereas a more pronounced increase occurs with further reduction in the gear rim thickness and is most significant between 0.4 and 0.3.

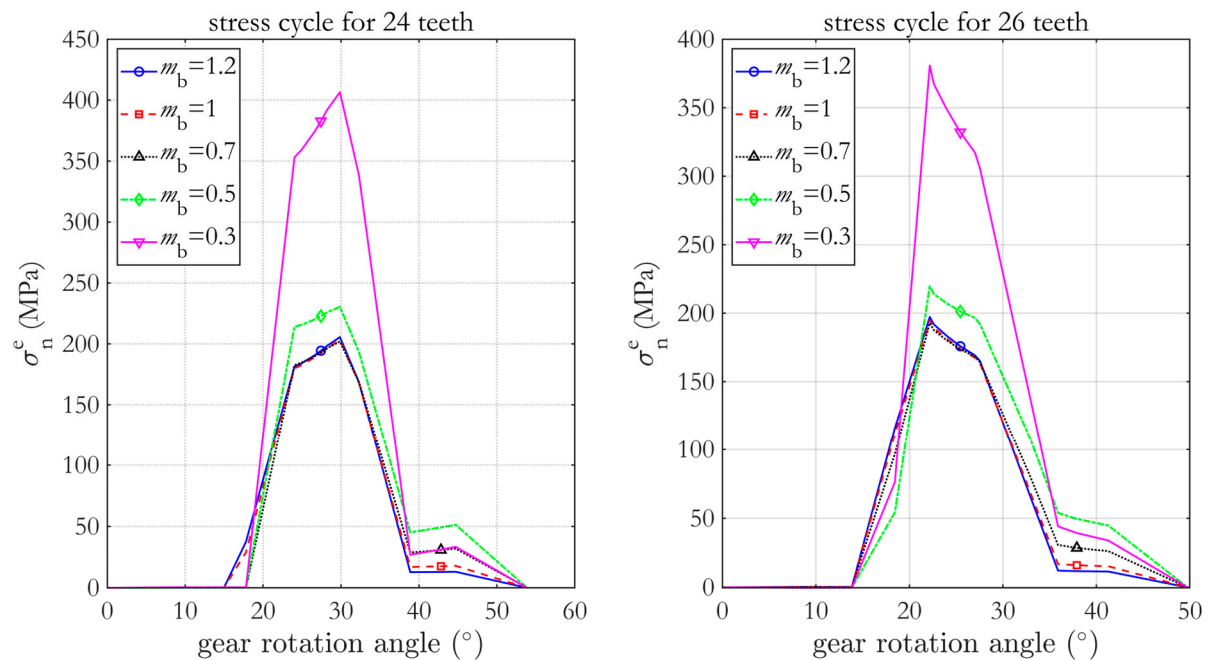


Figure 5. Stress cycles of the fourth gear pair.

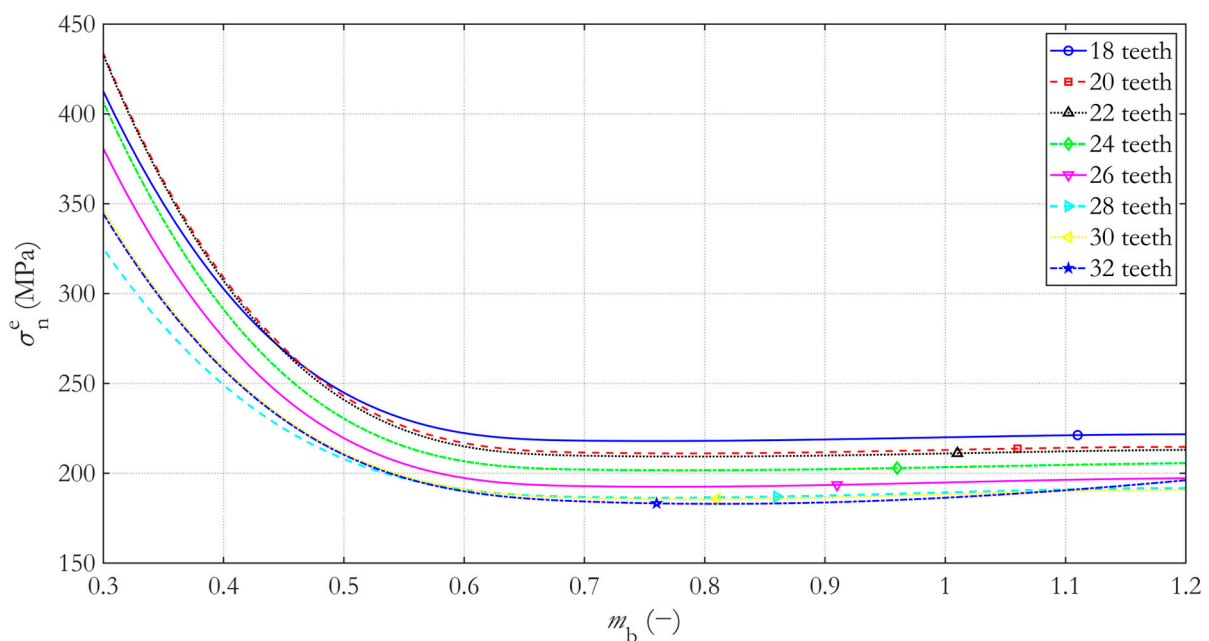


Figure 6. Stress change (σ_n^e) with the gear rim thickness (m_b) of all gears.

Load cycles in the fatigue fracture initiation period in the tooth root region are calculated based on the Expressions (16) and (19) after conducting elasto-plastic correction of stresses and deformations according to Expressions (20) to (32), and the obtained normal and shear deformation amplitudes. Finally, the value (N_i), obtained from the tensile model for all analysed gears, is presented in Figure 7 as a function of tooth count (z) and rim thickness (m_b). It is evident that the load cycle count decreases as the tooth count becomes smaller. However, a more significant reduction in the load cycles, for all numbers of gear teeth, occurs with a reduction in the gear rim thickness below 0.6. It is interesting that this trend is more pronounced with a greater number of teeth.

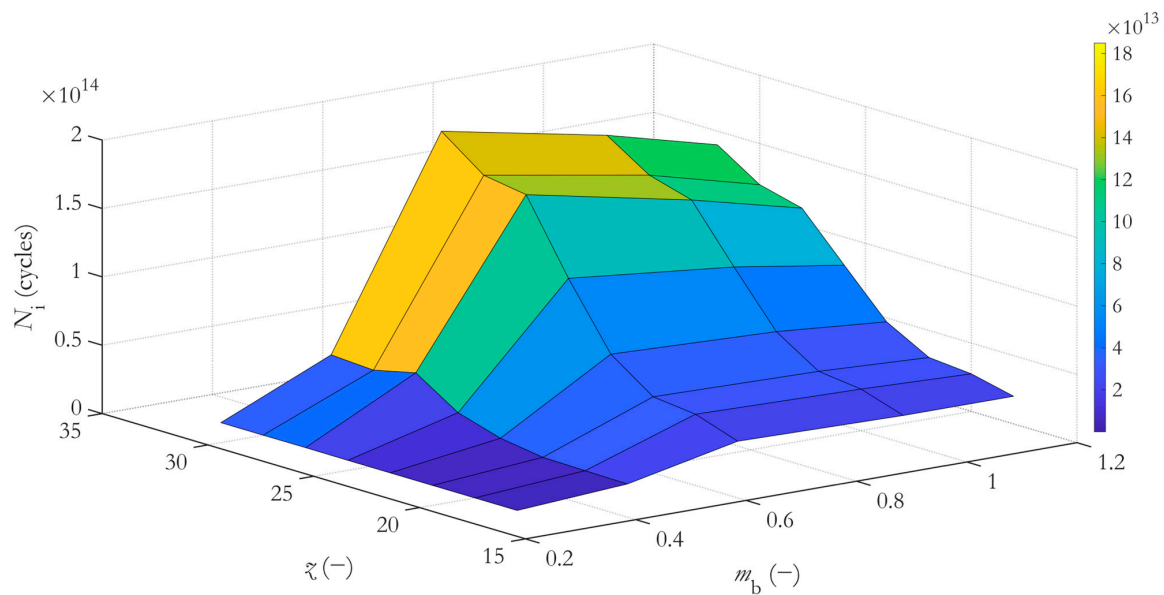


Figure 7. Dependence of the fatigue crack initiation time (N_i) on gear rim thickness (m_b) and number of teeth (z).

3.2. Tooth Root Area Fatigue Crack Expansion Period

The growth of the initiated crack from its initial length (a_2) to the critical size (a_c), marking the onset of the fourth phase of the fracture process near the tooth root region, is evaluated within the framework of LEFM. Therein, the equivalent stress intensity factors were obtained, which were corrected for the closing of the crack due to the plasticity, corrosion, and roughness of the gear material. The corrected equivalent SIF values serve as effective SIFs and, through Paris' law (38), are related to the cycle span required for crack growth in the analysed gears. Similarly to the evaluation of the fatigue crack initiation period, Figure 8 presents only one variation in the equivalent stress intensity factor (K_{eq}) with the crack length (a), since the behaviour for the remaining gear pairs is very similar; the example shown refers to the fourth gear pair.

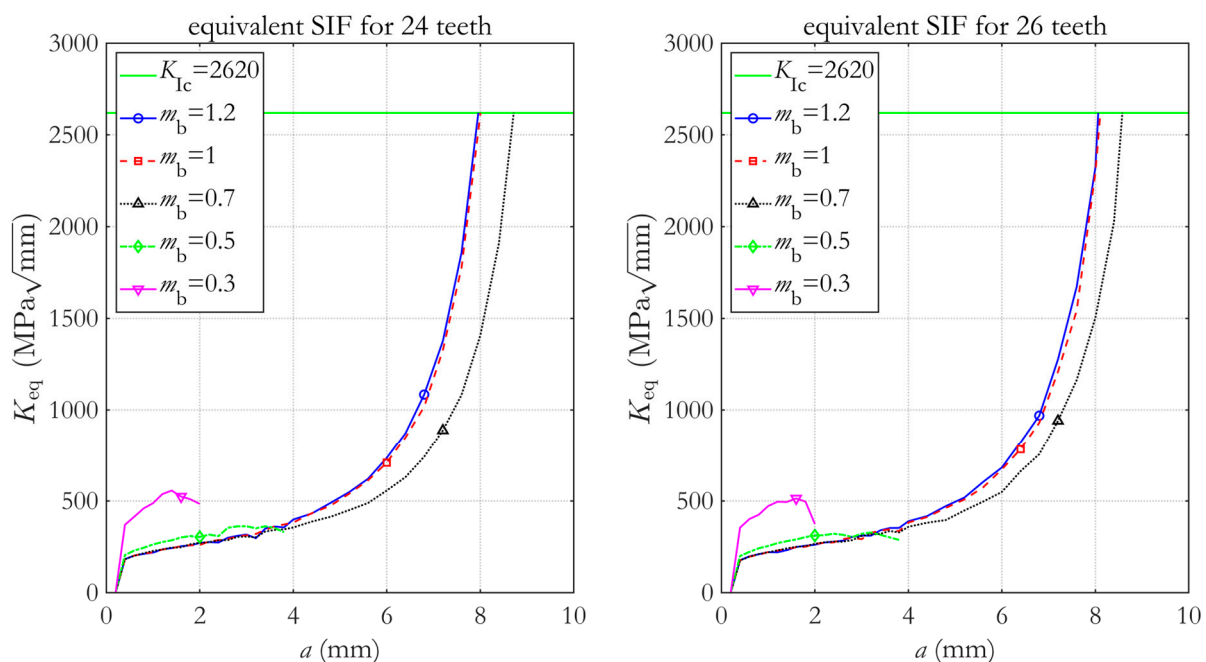


Figure 8. Dependence of the equivalent SIF (K_{eq}) on the fatigue crack length (a) for the fourth gear pair.

Crack growth comes to an end once the equivalent SIF (K_{eq}) reaches the fracture toughness limit (K_{Ic}) of the gear material. From Figure 8, it can be seen that only gears with a rim thickness above 0.7 reach the material's toughness limit, while Figure 9 illustrates that, in such cases, the crack continues through the tooth, illustrating the propagation paths for different rim thickness configurations of the fourth gear pair.

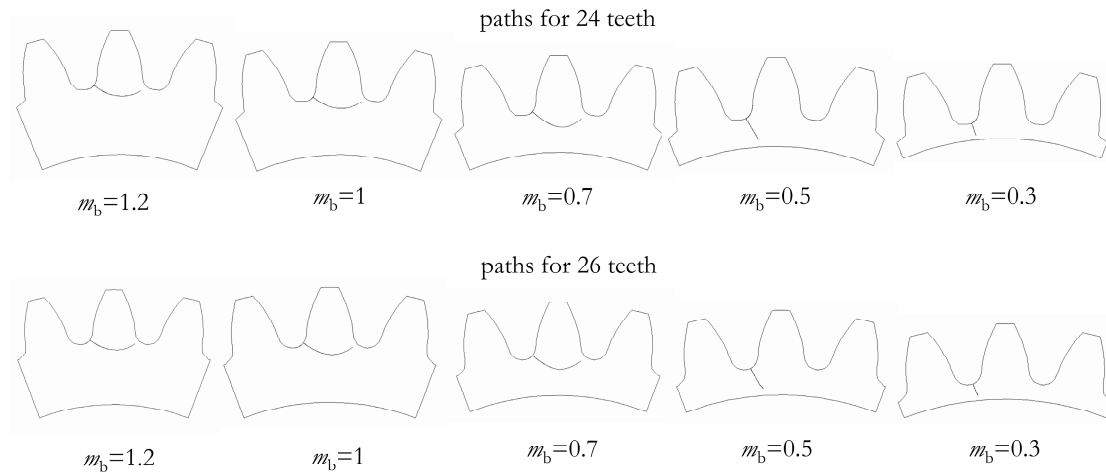


Figure 9. Paths of the fatigue cracks of the fourth gear pair.

As with the determining of the time to fatigue crack initiation, a trend of slight increase in crack length for lower values of gear rim thickness is noticeable here as well. For gear rim thickness values below 0.7, all considered gears resist fatigue fracture for a shorter time and have shorter fatigue crack lengths which then propagate through the gear rim. Figure 10 confirms this as well, presenting how the crack length (a) varies with the cycle span (N_p) during its growth for the fourth gear pair. The trend of the change in the crack length is the same as the trend of the change in the equivalent stress intensity factor. So, for gear rim thicknesses from 1.2 to 0.7, there is a slight increase in the crack length, and at lower gear rim thickness values the crack lengths are significantly shorter.

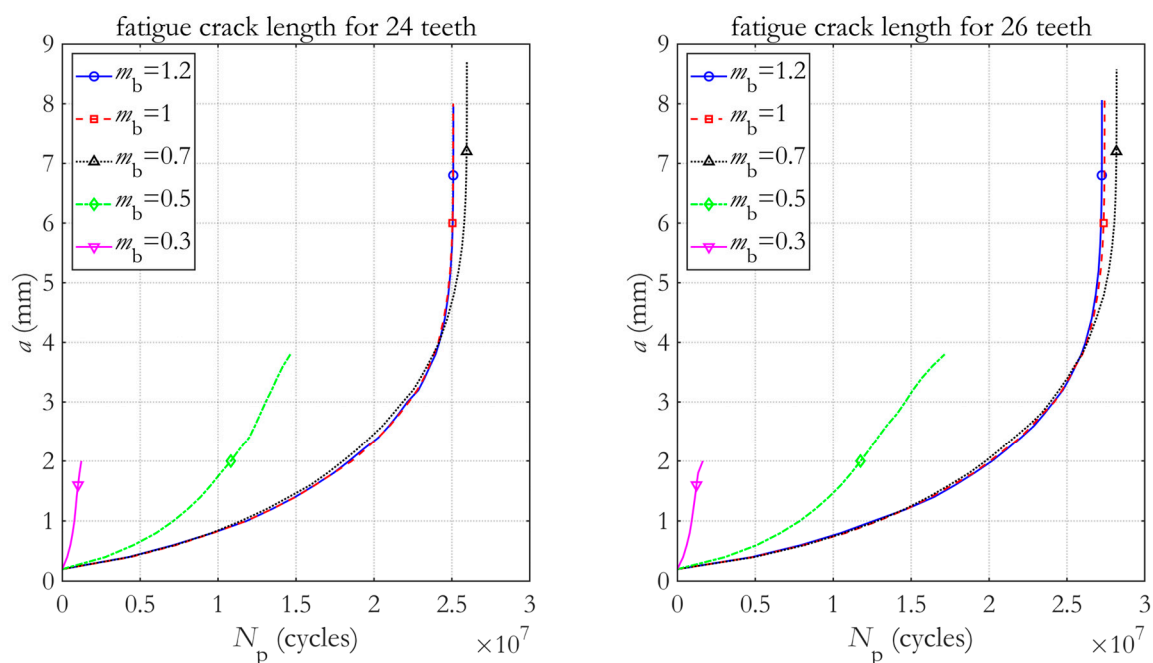


Figure 10. Variation in crack length (a) with the load cycle span (N_p) during propagation in the fourth gear pair.

Figure 11 presents how tooth count together with rim thickness affect the cycle span (N_p) required for crack growth, while Figure 12 shows how these same parameters influence the corresponding crack lengths. It is worth noting that in both figures the tooth count shows a much smaller effect on the duration of crack growth and on the resulting crack size than the rim thickness parameter. The number of fatigue crack expansion cycles for every number of gear teeth changes slightly if the gear rim thickness is reduced to a value of 0.7. Below that thickness, a more significant drop of fatigue crack expansion cycles occurs, and this trend is even more pronounced with the change in fatigue crack lengths shown in Figure 12.

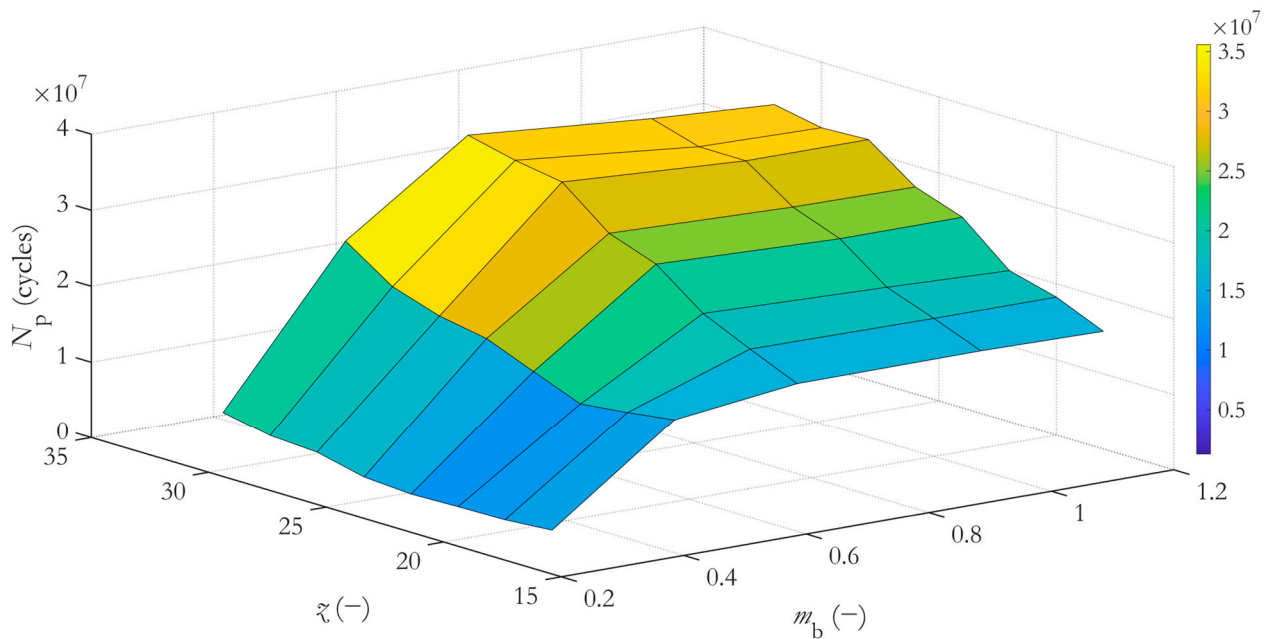


Figure 11. Dependence of the fatigue crack propagation time (N_p) on the gear rim thickness (m_b) and number of gear teeth (z).

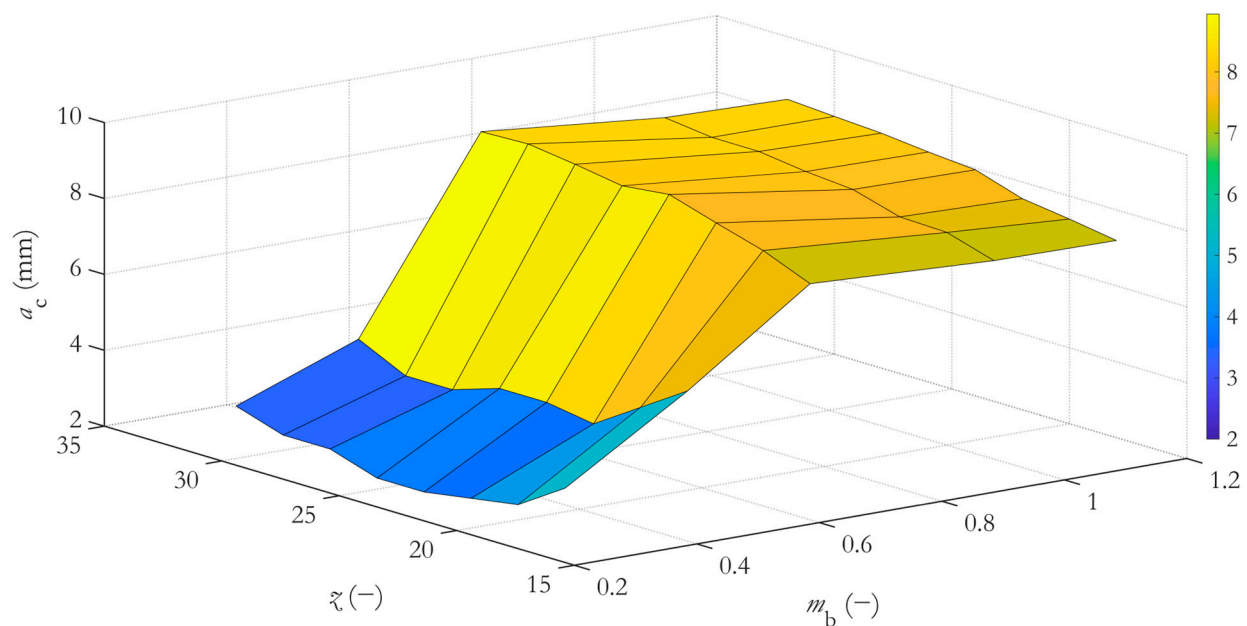


Figure 12. Dependence of the fatigue crack critical length (a_c) on the gear rim thickness (m_b) and number of gear teeth (z).

3.3. Gear Fatigue Durability Due to Fracture in the Tooth Root Area

Fatigue durability is expressed through the parameter (N), representing how many load cycles the analysed gears can sustain up to the moment when the tooth fails. Figure 13 illustrates how the load cycle trend varies with tooth count (z) and rim thickness value (m_b). A similar pattern was observed as in Figure 7, where the values associated with the initial crack formation stage (N_i) are presented. Such behaviour is expected, since these components operate under very high cycle counts, and most of their service life is consumed before crack initiation occurs.

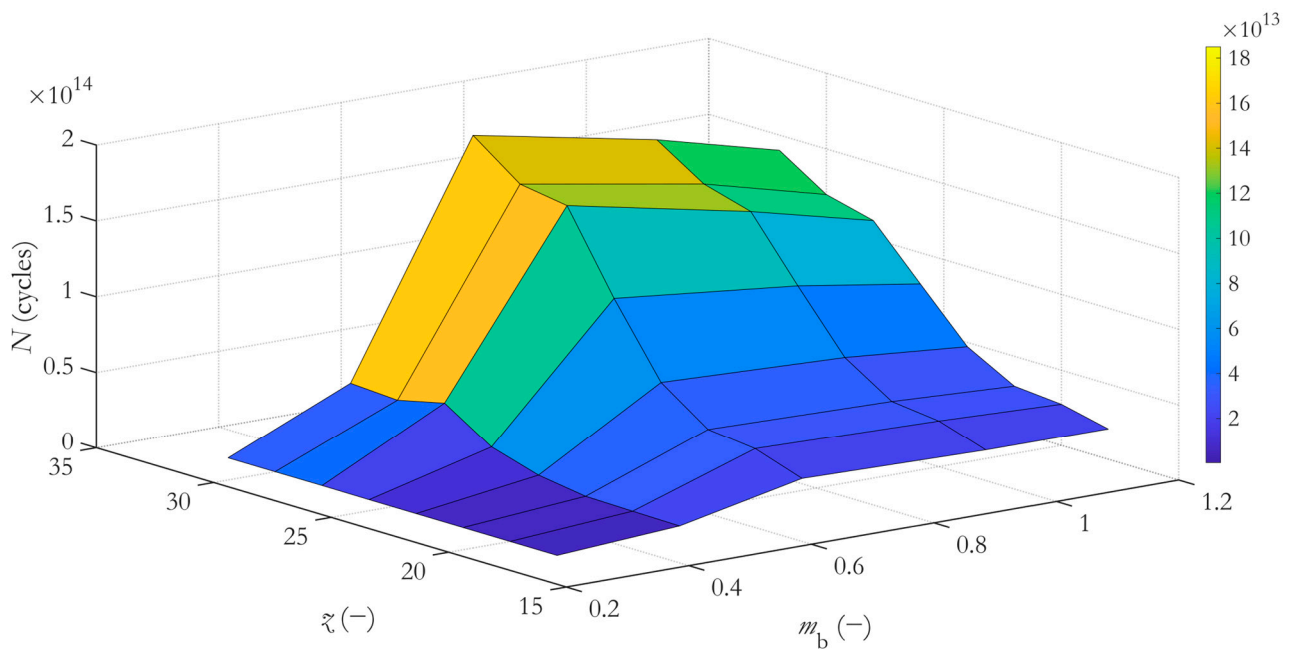


Figure 13. Variation in total crack growth cycles (N) with rim thickness (m_b) and tooth count (z).

According to Figure 13, the considered gears of all gear rim thicknesses, subjected to loads from Table 1, have an unlimited durability regarding fatigue fracture in the tooth root, which is not in accordance with ISO standard [36]. The standard provides for the correction of the nominal normal stress (σ_{F0}) at the critical point of the tooth root in gears with a gear rim thickness (m_b) greater than 0.5 and less than 1.2. The stress is corrected by the gear rim thickness factor (Y_B) according to the following expression:

$$Y_B = 1.6 \ln \left(2.242 \frac{h_t}{s_R} \right) \quad (40)$$

The ISO standard provides for the correction of the stress in the tooth root for all gear load amounts and it is necessary to avoid using gears whose gear rim thickness is below 0.5. Applying the standard to the gear sets listed in Table 1 yields the durability factor trend (Y_{NT}) shown in Figure 14, presented as a function of tooth count (z) and rim thickness (m_b). The red plane in Figure 14 represents the threshold between limited ($Y_{NT} > 1$) and unlimited gear durability ($Y_{NT} \leq 1$) and it is evident that only gears with all teeth numbers to a gear rim thickness 0.8 have unlimited durability. After this threshold gear rim thickness, nominal normal stress in the tooth root increases more significantly due to an increase in the gear rim thickness (40), which leads to a fall in gear durability. This is characteristic of all gear teeth numbers of the considered gears.

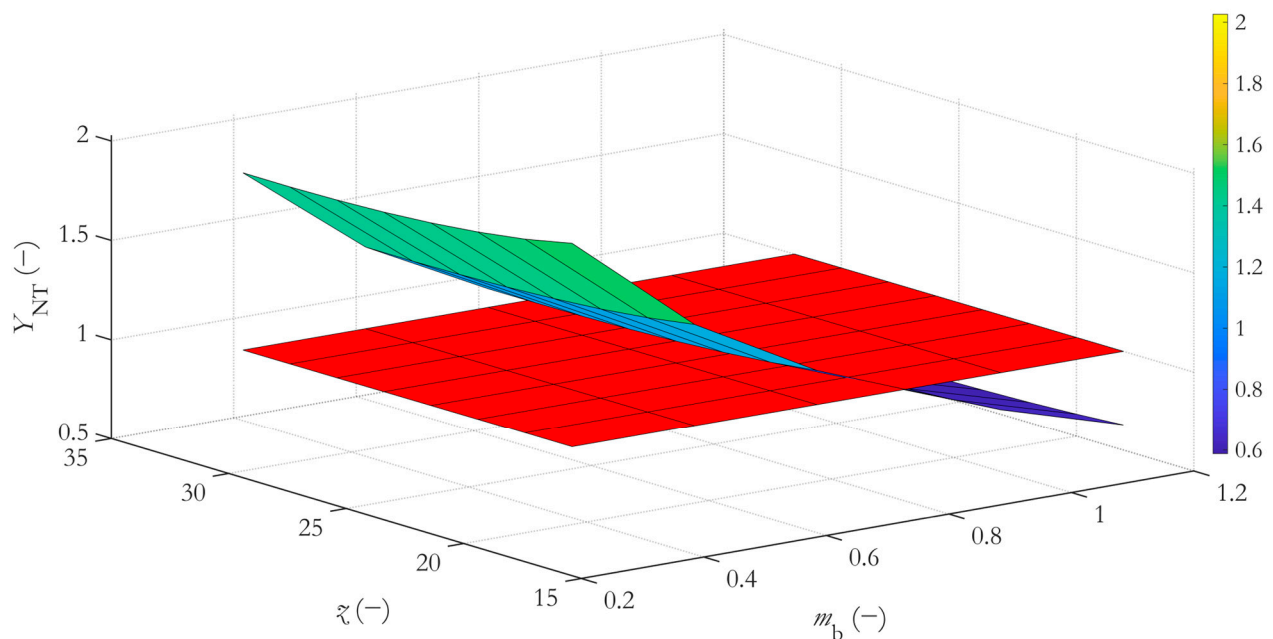


Figure 14. Change in the durability factor (Y_{NT}) of the considered gears according to the ISO standard [36].

4. Conclusions

The presented numerical procedure for determining the durability of gears with different numbers of teeth and varying gear rim thicknesses was conducted according to current scientific trends in fracture mechanics and drive strength. In this way, the stress cycle is described more realistically, providing the basis for applying critical plane approaches to estimate the moment at which a crack first appears in the region near the tooth root. The second part of the durability assessment relies on LEFM, where Paris' relation connects the effective SIF during crack growth with the load cycle span (N_p) required for the crack to extend to its critical size, the point at which integrity is lost and fracture of the gear occurs. This part of the numerical model includes the influence of both opening- and sliding-mode SIF components on the time of crack propagation, and also considers crack closure caused by plastic deformation, corrosion and surface roughness, which partly protect the crack tip from cyclic loading. The indicated mechanisms within the numerical model aim to describe the gear load conditions as realistically as possible. Through such an approach, a more detailed assessment of gear durability is obtained in comparison with conventional standard-based procedures, such as ISO standard [36]. Based on the conducted numerical process of determining fatigue durability of involute gears, the following conclusions emerged:

- The normal stress cycle at the critical location of the gear tooth root is predominantly governed by the thickness of the gear rim (m_b). The results indicate that the analysed gears exhibit a similar trend in flexural stress at the key location near the tooth root. It follows that the cycle count remains unchanged even when the rim thickness is reduced to 0.7. A lesser stress increase occurs with a reduction in the thickness to a value of 0.6, and a more significant increase occurs with further reduction in the gear rim thickness, and is greatest for thicknesses between 0.4 and 0.3.
- For all analysed gears, the load cycle span associated with the stage when a crack first appears (N_i) is derived from the tensile damage parameter (d_t), and the resulting values decrease as the tooth count is reduced. However, a more significant fall of the time to initiation, for all numbers of gear teeth, occurs with a reduction in the gear rim thickness below 0.6, and the trend is more pronounced with a greater number of teeth.

- According to the numerical results, the material fracture strength (K_{Ic}) is reached for gears with a gear rim thickness greater than 0.7, while for lesser rim thickness values the fatigue crack tends to propagate through the gear rim. For gear rim thickness values below 0.7, all considered gears resist fatigue fracture for a shorter time and have shorter fatigue crack lengths (a) which then propagate through the gear rim.
- At a reduction in the gear rim thickness from 1.2 to 0.7, a slight increase in crack length (a), is observed, whereas at lower rim thickness values the crack lengths are significantly shorter.
- The numerical results indicate that the number of teeth has a smaller influence on the fatigue crack expansion period (N_p) and crack length (a) than the gear rim thickness (m_b) in the analysed gears. From an engineering perspective, the obtained results indicate that gear rim thickness represents one of the key parameters governing fatigue durability and crack propagation behaviour in thin-rim involute gears operating under high-cycle loading conditions. The number of fatigue crack expansion cycles (N_p) for every number of gear teeth changes slightly until a gear rim thickness of 0.7, and below that thickness a more significant fall of fatigue crack expansion cycles occurs. This effect becomes even more evident when observing how crack length varies in relation to tooth count and rim thickness values in the analysed gears.
- A similar pattern appears in the total load cycle count, leading to tooth root failure (N) for all analysed involute gears, matching the trend observed in the cycle values for the early crack formation stage (N_i). Such behaviour is expected under the analysed loading conditions, since most of the operational lifetime is spent in the crack initiation phase before noticeable crack propagation occurs.
- Maximum normal stresses in the elastic region obtained through the numerical model based on the finite element approach differ from the analytical values reported in [36], both in magnitude and in the position of the critical cross-section near the tooth root. According to the ISO standard procedure [36], the increase in normal stress with decreasing gear rim thickness is more pronounced than that obtained through the present numerical analysis.
- The finite element approach applied in the present study enables a more detailed evaluation of bending stresses in the tooth root region for actual gear geometry, whereas the ISO standard procedure [36] includes several simplifications related to gear geometry and loading conditions. Therefore, particular attention should be devoted to stress-state analysis in gears with reduced gear rim thickness.

Author Contributions: Conceptualisation, M.P. and S.P.; methodology, V.T.; software, S.P.; validation, M.P., V.T. and S.P.; formal analysis, S.P.; investigation, K.D.; resources, K.D.; data curation, S.P.; writing—original draft preparation, M.P.; writing—review and editing, V.T.; visualisation, S.P.; supervision, M.P. All authors have read and agreed to the published version of the manuscript.

Funding: This research was supported by the European Union—NextGenerationEU through the University of Split (Project IP-UNIST-06).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data is contained within the article.

Acknowledgments: Artificial intelligence tools were used only for linguistic refinement and improvement of English expression. The scientific content, analysis, and conclusions were developed solely by the authors.

Conflicts of Interest: The authors declare no conflicts of interest.

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