



Article

Seismic and Post-Seismic Ductility and Forces of Mid-Story Pin Moment-Resisting Frames Based on Scale of Local Deformation of I-Beam's Segments

Atsushi Suzuki, Ruiyu Che  and Yoshihiro Kimura * 

Graduate School of Engineering, Tohoku University, Sendai 980-8579, Japan;
atsushi.suzuki.c2@tohoku.ac.jp (A.S.); che.ruiyu.q4@dc.tohoku.ac.jp (R.C.)

* Correspondence: kimura@tohoku.ac.jp; Tel.: +81-22-795-7876

Abstract: A mid-story pin system to avoid moment-resisting frame column failure during seismic action was proposed recently. The solution consists of a reinforced concrete (RC) pier protruding from the foundations, the steel column connected with the superstructure, and plates and the anchor bolt working as a pinned connection in between. This paper utilizes shell finite element analysis (FEA) models to examine the demanded column-to-beam strength ratio to keep the column elastic and maximize the story drift at the moment of beam buckling of the frame. The method of calculating post-seismic residual strength based on maximal buckling deformation of the beam is also proposed.

Keywords: steel moment-resisting frame; soft-story mechanism; mid-story pin; local buckling deformation; residual strength; plastic story drift



Citation: Suzuki, A.; Che, R.; Kimura, Y. Seismic and Post-Seismic Ductility and Forces of Mid-Story Pin Moment-Resisting Frames Based on Scale of Local Deformation of I-Beam's Segments. *Appl. Mech.* **2024**, *5*, 938–963. <https://doi.org/10.3390/applmech5040052>

Received: 23 October 2024

Revised: 26 November 2024

Accepted: 2 December 2024

Published: 4 December 2024



Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Conventional frame systems for heavy-duty earthquake-resisting buildings are usually based upon the moment-resisting frame (MRF) concept. The target is to allow the structure to resist the earthquake load, utilizing the energy dissipation capacity relying on inelastic actions within frame members. One of the most probable mechanisms leading to failure is the concentration of the deformation within one story, called weak story (WS), which may lead to the soft-story mechanism (SSM) of this floor. During SSM, the top and bottom connections of the column of one floor turn into plastic hinges, and the story collapses, which is the ultimate state of the structure. To avoid that, MRF concepts introduce ways to uniformly distribute the story drift within the frame.

An easy and cost-efficient solution for typical steel frames is the concentrically braced frame (CBF) concept. This solution, however, does not perform well in the situation of a significant earthquake compared to the MRFs. One of the first attempts at CBF systematization was presented by Khatib et al. [1], where the authors analyzed CBF with various configurations of the braces. The results presented by Kimura and MacRae [2] have shown the effects of the stiffness of the continuous columns on the story drift distribution in CBF. This is important, as a high concentration of deformation within one story may lead to members' inelastic action, which is usually characterized by lowering the member's stiffness, potentially leading to SSM. In the abovementioned study [2], numerical analyses were employed to propose empirical equations showing the effects of column stiffness and to examine the drift concentration and column moment demand for frames. Ji et al. [3] presented the analyses of CBF experiencing WS in a study of models with varying stories. It was concluded that WS usually happens within the first floor but can be limited or avoided if stiff gravity columns are used. It was proved that the gravity columns with stiff, embedded bottom connections (hereinafter: FIX) perform better than those with exposed, elastic connections (hereinafter: PIN). Qu et al. [4] proposed a CBF rehabilitation method by implementing a stiff rocking system (RS). RS is a secondary solution providing

uniform distribution of the story drift between the floors, therefore limiting WS occurrence, which can be implemented for both CBF and MRF. Simpson and Mahin [5] checked the behavior of the retrofitted frames with RS. In numerical and experimental analyses, it was proved that when CBF fails, the structure relies on MRF action, with members and connections not designed for such loads. Therefore, the WS inevitably leads to a greater deformation concentration following SSM failure. In the presented research, the RS helps against SSM occurrence by mobilizing the floors around the WS at the stage of MRF action. Recently, Dong et al. [6] experimentally checked the low-rise CBF (characterized as a low-ductility class, and therefore designed for very limited earthquake resistance) and discovered that such structures collapse by SSM when the braces' capacity is depleted. Because the structural behavior of CBF with depleted brace capacity starts to rely on MRF action, it was proposed that the most endangered first story can be upgraded by strengthening the columns and column-to-beam connections to lower the collapse risk.

The CBFs are economic structures; however, in the situation of a significant earthquake, the capacity of the braces may be depleted, forcing the structure to rely on the stiffness of beams, columns, and connections in between. The structures designed to withstand the seismic load are MRFs. The dissipation energy is based on the plastic deformation of the structure. It is noticeable that MRF tends to be heavier than CBF, but the seismic performance and architectural benefits are better (see EC8 [7], and Yazdanpanah et al. [8]). In recent years, there has been noticeable research in this field. Yazdanpanah et al. [8] created numerical models of various MRFs with irregularities. They checked the influence of said irregularities on the failure probability, also from the point of view of strong column–weak beam (SC-WB) action disturbances. For the MRF, the achievement of the SC-WB action is the highest priority, as the assumption of the MRF system is not to allow for the plastic hinge formation within the columns. The formation of an excessive number of plastic hinges within the column leads to the failure of the structure via SSM of one or more floors. Therefore, the target for the MRF is to design the structure so that the plastic hinges are formed within the beams (and bottom of first story columns) until the system turns into a mechanism.

In another approach by Jia et al. [9], MRFs with various types of RS have been analyzed. The authors proposed self-centering RS (SCRS), and, in numerical analyses, they demonstrated its excellent performance. Zhu et al. [10] presented the numerical investigation of 4 to 16 story building MRF models utilizing self-centering viscous hysteretic devices (SC-VHDs) using force-based design and direct displacement-based design (DDBD). The authors proved the effectiveness of SC-VHDs in limiting the risk of SSM and allowing for SC-WB behavior up to the point of system failure. The demand for the column-to-beam plastic moment ratio was analyzed as well. In the case of frames below eight stories, it was discovered that the moment ratio was required to be 1.8 or greater. Shakeri and Dadkhah [11] defined the variant of DDBD just for MRF, as the general DDBD approach was found imprecise.

For all the solutions presented above, two typical solutions of the first story column support have been used—an exposed (PIN) and an embedded (FIX) type. In both cases, the structure is vulnerable to failure due to the formation of excessive hinges within columns, leading to an SSM (see [12–16]). This is because in both solutions, the stiffness of the top and bottom of the column is different, which causes the bending moments at the top and the bottom of the column to be different. Recently, a connection between an reinforced concrete (RC) beam and a steel column on the first floor was developed (see Figures 1 and 2). Kaneda et al. [17] analytically demonstrated the effectiveness of the proposed system.

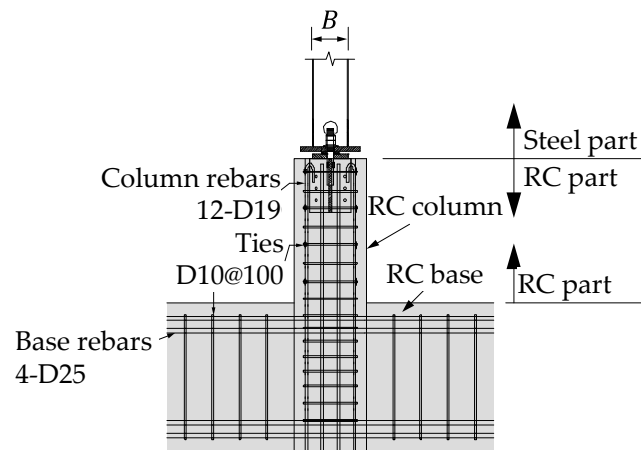


Figure 1. Details of the reinforced concrete (RC) column's support.



Figure 2. Industrial applications of MSP (mid-story pin) systems.

In this mid-story pin (MSP) solution, the first story column is divided into the bottom RC part and the upper steel column part connected to the balance of the superstructure. These parts are connected with a pin, ensuring that the ratio between the column's top and bottom moments will stay constant in the horizontal load situation. MSP effectively blocks the occurrence of abnormal concentrations of stresses in the column and, on a global scale, of the story drift concentration in the first story. Therefore, it makes obtaining a beam-yielding ultimate mechanism more straightforward (see Kaneda et al. [17]). Also, structural members can cause buckling in a huge deformation, as summarized in previous studies [18–44]. Moreover, the MSP structures experience higher deformations and lower base shear than traditional systems (see [17]). This allows for greater energy dissipation; however, the beams of MSP and MRF must be designed to withstand the excessive rotations and provide higher energy dissipation capacity without failing due to the elastoplastic local buckling (see [17]).

In previous research, the authors analyzed the behavior of the MRF models with MSP (as shown in Figures 1 and 2) using the static analyses of the frame loaded with forces/enforced deformations corresponding with the earthquake-originated motion of the frame (see [17]). The ultimate earthquake resistance evaluation was primarily focused on the rotational capacity and the distribution of forces between the upper and the lower part of the first story column (see Kimura et al. [45]). Previous research on the behavior of the frame with MSP used the simplified beam model approach utilizing the ideal pin in one node, whilst the ultimate seismic capacity has been considered as the point of attainment of the ultimate capacity of individual members. The trilinear curve has been used to describe the relationship between the bending moment and rotation of the standard I-beam (see

Kato et al. [46]). Experimental tests on the simplified frame consisting of the lowermost beams and column under the cycling horizontal loading have been conducted (see Kaneda et al. [47]); however, the influence of the vertical compressive force in the column was neglected. Therefore, the phenomena of the counter moment generated by the vertical force due to the shape of the bearing collar (as shown in Figure 3) were not taken into consideration. The existence of such phenomena is essential, as the changing location of the point of contact can either stabilize or destabilize the frame, significantly influencing the balance of the moment within the column. This adds to the lack of precision caused by previous research that did not consider the local stability loss of the beams and the ultimate seismic performance being determined simply by the collapse of the members. Because of the abovementioned simplifications, it cannot be stated that the seismic behavior of the frame at the point of collapse has been thoroughly scrutinized.

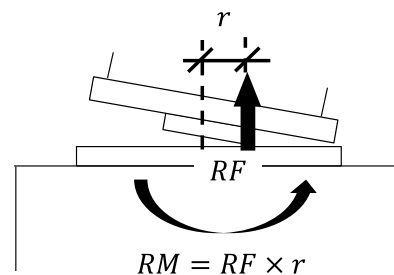


Figure 3. Moment-generating mechanism of the connection (RF : reaction force, RM : reaction moment).

Numerical analyses of the stress distribution within the I-beam cross-section of the MRF with MSP have not been provided in past research. This article grasps this kind of analysis by utilizing the shell and solid elements, showing the influence of the beam cross-section geometry, span-to-depth ratio, and the material properties of cross-sections. The performance evaluation of I-beams has been addressed in previous studies in terms of the reversed axial forces [48], restraint conditions [49,50], or steel frame subassembly [51,52]. In line with the earlier proposed method, this study applied a method of beam analysis based on the elastic local buckling load factor explained in Fujak's study [50]. For the connection part, the solid elements (C3D8) were used to reproduce the upper plate of the connection, collar (fillet) plate, anchor bolt, and lower plate of the joint, together with contact/friction between these elements. Therefore, the influence of the vertical force in the column and its impact on the behavior of the model was considered as well.

As an effect of previous research, the method for calculating the residual strength of the column after a disaster using a scale of local buckling deformation has been proposed (see Kishiki and Iwasaki [53]). In this research, a similar method is applied to beams. Faced with this need, structural health monitoring and damage detection methods have been emerging and have recently advanced [54–68]. The value of the deformation can be easily measured after a significant earthquake. The deformation value needs to be scaled using web thickness; however, estimating this value on-site is also possible by unscrewing one connection bolt or drilling the hole in the web. This makes the application of this method easy for estimating the level of damage and outstanding residual capacity to provide required repairing actions over the damaged members and allow for quick and efficient restoration work.

Based on the abovementioned description, this paper is focused on the following:

- (1) Describing the elastoplastic behavior considering the local loss of stability of the structural members using detailed numerical models of shell and solid elements.
- (2) Evaluating the relationship between the seismic performance and the scale of deformation of the individual members, i.e., I-beams.
- (3) Proposing equations to estimate the level of damage, residual stresses, and outstanding capacity in the disaster-caused damage evaluation.

To summarize, Figure 4 illustrates the scope of this research. The findings from this study enable a quick and accurate assessment of structural integrity following a major earthquake, thereby confirming the residual strength of the affected structures. Furthermore, engineers can quantify the safety margins for potential aftershocks, allowing them to make informed recommendations regarding evacuation. This contributes significantly to enhancing safety measures in the field of structural engineering.

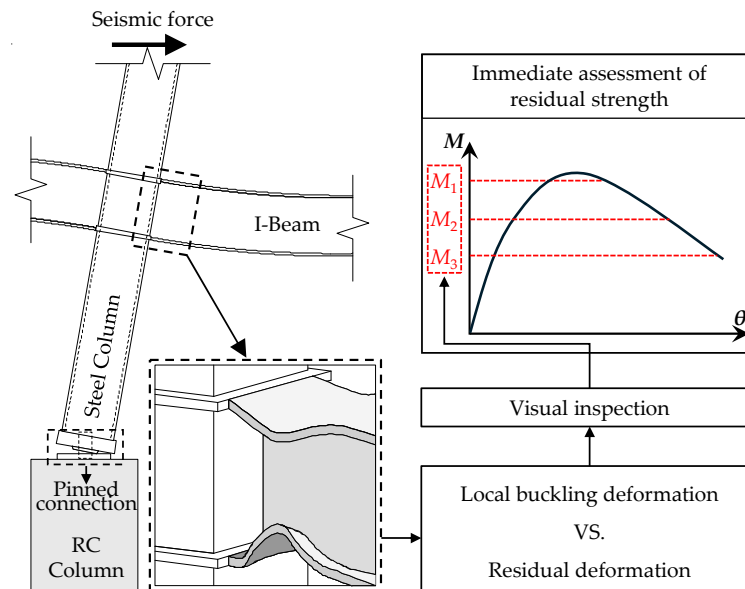


Figure 4. Scope of this research.

2. Numerical Model Definition and Validation for Square Hollow Section (SHS) Columns and I-Beams Considering Local Buckling

This section is focused on the definition of the shell-and-solid numerical model consisting of SHS columns and I-beams considering local buckling with the comparison of the results against previous experimental results. Figure 5 describes a numerical model of the components of the analyzed frame—steel SHS column and I-beam. The model has been defined using the Abaqus finite element analysis (FEA) program [69]. The experimental results from Kishiki and Iwasaki [70] and Kimura et al. [71] have been used for the model calibration. Because of the computation time and the different mesh size requirements across the model, the mesh becomes coarse towards the beam's free end and refined towards the supported edge, where the buckling will occur. For the model of the SHS member, the number of elements in the detailed area is 30 in the longitudinal direction and 12 (per wall) in the perpendicular direction. The refined part of the I-beam is divided into 15 elements longitudinally, with 8 elements across the flange width and 16 elements across the web height. This research conducted a mesh sensitivity analysis to ensure the accuracy and stability of the simulation results. The chosen mesh size was optimized to balance computational efficiency with solution accuracy, confirming convergence within acceptable limits. This approach to mesh sizing is consistently applied to all FEA models presented in this study.

The fillet radius of the SHS was determined to be $2.5 \times t$ (where t is wall thickness) in accordance with the general configuration of manufacturers. The initial imperfections were applied using the scaled value of the first buckling mode shape, as visible in the lower part of Figure 5. The shape was obtained from the Linear Buckling Analysis (LBA) assuming the loading with a single compressive force applied to the controlling node of the diaphragm at the free end of the member. It was scaled to be 1% of wall thickness at a given point of the model. The assumed material properties were based on the combined (kinematic and isotropic) hardening law. Boundary conditions for eigenvalue analyses were identical for

both models, with complete fixing at the detailed end and out-of-plane displacement, as well as rotation about the longitudinal axis of the member blocked at the free end.

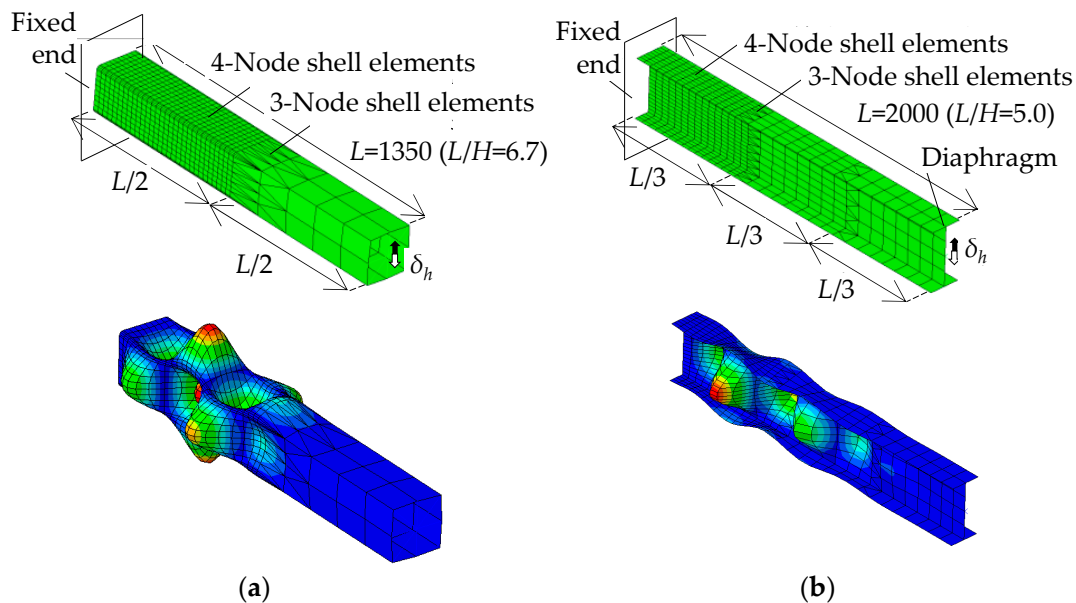


Figure 5. Numerical models of column and beam: (a) SHS; (b) I-beam.

For the SHS in Figure 5a, the width and height are 200 mm and the thickness is 6 mm. The yield and ultimate stresses are 330 N/mm² and 444 N/mm², respectively. In terms of the I-beam in Figure 5b, the height and width are 400 mm and 200 mm, respectively. The actual thickness of flange and web plates are 12.88 mm and 8.00 mm, respectively. The yield and ultimate stresses of flange plate are 352 N/mm² and 430 N/mm², respectively. The yield and ultimate stresses of web plate are 330 N/mm² and 444 N/mm², respectively.

In terms of the constitutive model, the simulation adopts the combined hardening law proposed by Lemaitre and Chaboche [72]. The parameters used in this study are as follows: initial yield stress (f_y), kinematic hardening modulus (C), the rate of hardening modulus decrease (γ), maximum change in the yield surface size (Q_∞), and the rate of initial yield stress change (b). Specifically, the parameters are set to $Cn = 4.0$ GPa, $\gamma = 30$, $Q_\infty = 240$ MPa, and $b = 8$. These values were derived based on stress–strain loops from both the initial and stabilized cyclic curves, following the methodology suggested by Chaboche [72].

In the case of the SHS column, the model with imperfections has been loaded twice with alternating positive and negative enforced deformation of the value $\theta = \arctan(\delta_h/L) = 0.02$ rad (where δ_h , L are displacement at free end and length of the beam, respectively) and later with altering positive and negative enforced deformation of $\theta = 0.025$ rad. In the case of the I-beam, the deformation was measured using the dimensionless value of θ/θ_{pl} , which is the rate of chord rotation θ versus the chord rotation achieving full plastic moment θ_{pl} . The value of θ/θ_{pl} has been set as 1, 2, 4, and 6, with one positive and one negative load for each value. However, in the frame model, there is also an axial force compressing the column generated by the self-weight of the building. Upon arising deformation, this force starts to be partially transferred into the bending moment and the shear force in the column. The initial force is 15% of the yielding axial capacity of the column's cross-section.

Since the model analyzed in this paper is not a model of the whole building but a fishbone model of one of the central columns and area nearby, the effect of fluctuation of the axial force is not considered, and the axial force is assumed to be constant.

Figure 6 presents the comparison between numerical analyses and experimental outputs (see [70,71]) using the moment-to-rotation hysteresis loops. For both members, the numerical analysis results follow the experimental output, and high convergence is also obtained for the post-yielding range. The strength deterioration caused by local buckling is also replicated in the FEA, validating the numerical models presented in Figure 6.

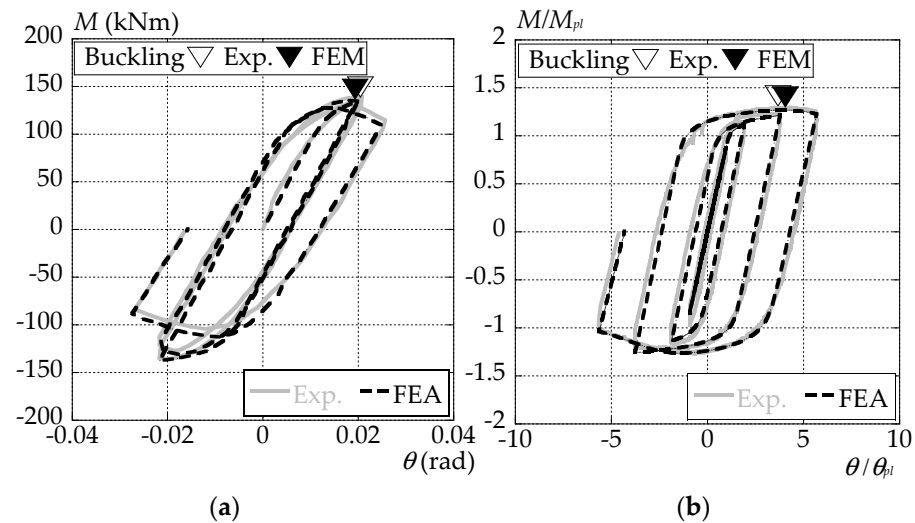


Figure 6. Hysteresis loops of column and beam: (a) SHS; (b) I-beam.

3. Simplified Residual Strength Indices in I-Beams After Local Buckling

This section presents the simple evaluation method of the strength in the member using the maximal local buckling deformation d , as proposed in the reference [53]. The I-beam residual stress index is proposed based on data from numerical analyses.

3.1. Estimation of Residual Stress Using Maximal Local Buckling Deformation

In this section, experimental results from the reference article [53] and numerical analyses of members shown in Section 2 are compared, and the validity of the evaluation index relating residual buckling stress with the local buckling deformation scale is verified.

Figure 7 compares the numerical and experimental results [53] using the skeleton curve of positive loading half-loops. The FEA model was newly constructed to match the exact specifications of the experimental setup, including the cross-sectional dimensions. This alignment ensures that the sections used in the analysis are identical to those in the experiments presented in reference [53]. The data presented on the graph show good convergence, and the validity of the numerical analysis model illustrating the behavior of the experimental model is confirmed.

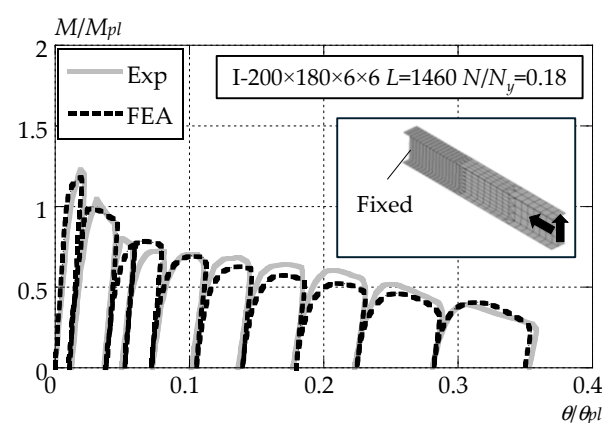


Figure 7. Skeleton curve derived from half-cycle loops.

The next step was to confirm if the numerical model can reproduce the local buckling, which occurs in the case of the experimental model. Figure 8 reveals the relation between the stress in the cross-section (in the form of M/M_{pl} , which is the moment of the beam versus the full plastic moment) and the value of the maximal deformation of the beam web d . The graph presents both the numerical and experimental results, and the continuous line

is obtained by connecting the maximal deformations per cycle of the model's loading. From the graph, it is possible to conclude that even in the case of very detailed results, such as the scale of the deformation caused by local buckling, the convergence of the experimental and numerical output is very high; therefore, the numerical approach may be used for further analysis.

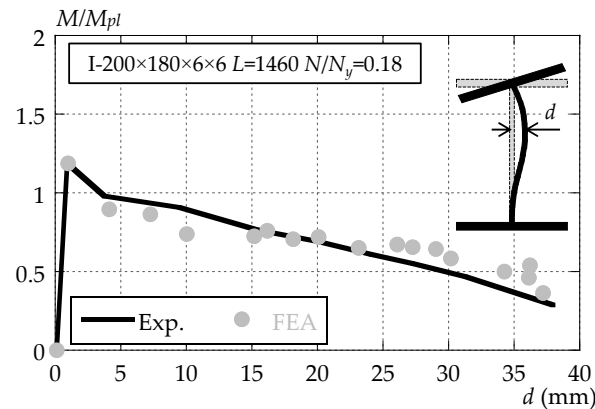


Figure 8. Simplified skeleton curve against buckling deformation.

3.2. Parametric Analysis Affecting Residual Strength

3.2.1. Overview

In previous research [53], the residual strength evaluation method was based on the most significant deformation of the flange of the beam that had undergone local buckling. Using the mechanical model, as shown in Figure 9, the method of calculating the bending moment related to the segment's local buckling deformation has been proposed. However, in the case of the majority of short-span I-beams, due to the shear-governed buckling, it is the size of the deformation of the web that governs. Therefore, in the case of this research, it is impossible to use an identical model as proposed in Figure 9 to estimate the post-buckling state of the member. Instead, web deformation (see Figure 8) is used as a base for the novel index of the post-buckling residual strength.

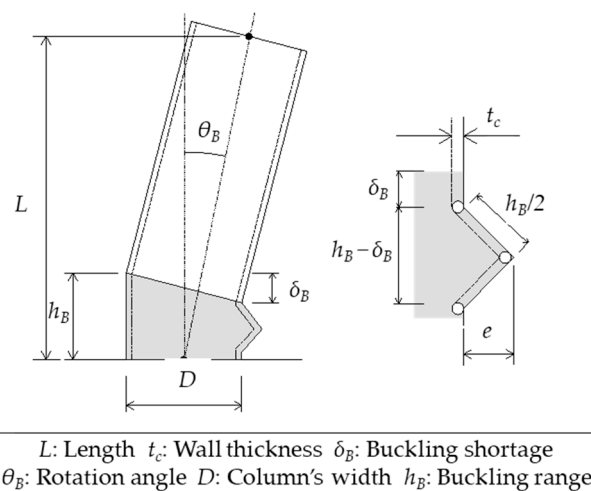


Figure 9. Buckling mechanism model.

This paper aims to express the relationship between the bending moment and the scale of the local buckling deformation of the member. Figure 10 illustrates four crucial indexes used in the proposed evaluation method. The first index, τ_1 , is the ratio of the maximum force that occurs in a member to the full plastic capacity of said member. The d_1 is the unitless deformation index, defined as the buckling deformation at the point of maximal

force occurring e_1 divided by the web thickness t_w . The d_2 is the unitless deformation index based on web deformation when the residual strength deteriorates to maximal plastic capacity value, and d_3 is the index based on the web's local buckling deformation when the residual force lowers to 50% of full plastic capacity. The equations to estimate those four values (using the cross-section properties of the I-beam as parameters) have been defined and checked against the results obtained from the numerical analyses. The parameters used were as follows: half-flange width–thickness ratio b_f/t_f , web height–thickness ratio h_w/t_w , beam span-to-depth ratio L/H , yield strain ε_y , and post-yielding secant modulus E_{st} . To consider the local buckling only, global out-of-plane deformation of the I-beam member has been restrained at the point laying $0.25 \times L$ from the fixed end of the beam, as reported by Suzuki et al. [48]. It was confirmed that the restrain at this point does not influence the maximal deformation of the web of the I-beam as it is sufficiently distant from the point where the maximal local buckling deformation occurs.

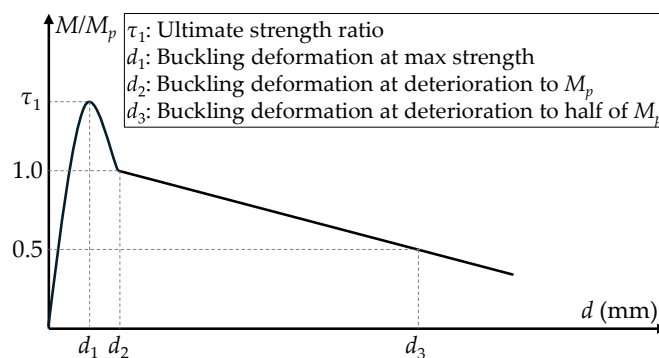


Figure 10. Residual strength against buckling deformation.

3.2.2. Numerical Analysis with Various Parameters

This section discusses the influence of the aforementioned five analysis parameters on the value of the four indexes. Table 1 presents the values for the numerical analysis used in this sub-section. Five types of cross-sections have been considered, with only the thickness of the web and flanges changing. Cross-section No. 5 was assumed as a reference case. Note that the class of the web and flange (as per Eurocode 3 (EC3) [73]) depends on the material used. Three values of span-to-depth ratio have been considered: 5.0, 6.0 (reference), and 7.0. Regarding the class of steel, Young's modulus E was assumed to be equal for all tests (205 kN/mm^2); however, the elongation at yielding ε_y varied between 1.14×10^{-3} , 1.58×10^{-3} (reference) and 1.95×10^{-3} . The respective yield strain corresponds to the yield strength f_y of 235, 325, and 400 N/mm^2 , respectively. Post-yielding secant modulus E_{st} has been assumed as $0.020 \times E$ (reference), $0.015 \times E$, or $0.005 \times E$. Note that the definition of the post-yielding secant modulus is a slope between ε_y and $\varepsilon = 0.020$. The analyses were performed using the monotonic loading within the range, which allowed us to obtain the defined points in Figure 10.

Table 1. Parameters of I-beam.

No.	H (mm)	h_w (mm)	B (mm)	b_f (mm)	t_w (mm)	t_f (mm)	Flange Class	Web Class
1	600	562	300	145.5	9.0	19.0	1, 2	1, 2
2	600	576	300	144.0	12.0	12.0	3, 4	1
3	600	562	300	140.5	19.0	19.0	1, 2	1
4	600	552	300	144.0	12.0	24.0	1	1
5	600	562	300	144.0	12.0	19.0	1, 2	1
Variables of all analyzed cross-sections			L/H	ε_y		E_{st} (GPa)		
			5.0	1.14×10^{-3}		$0.005 \times E$		
			6.0	1.58×10^{-3}		$0.015 \times E$		
			7.0	1.95×10^{-3}		$0.020 \times E$		

Figures 11–14 present the influence of these parameters on the values of the four aforementioned indexes. The L/H parameter has been changed between the shown series in all figures.

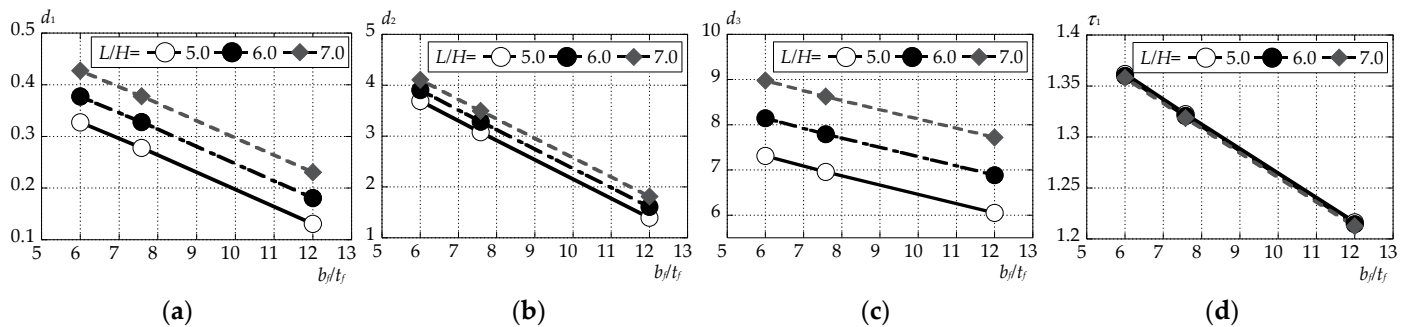


Figure 11. Influence of flange width–thickness ratio: (a) d_1 ; (b) d_2 ; (c) d_3 ; (d) τ_1 .

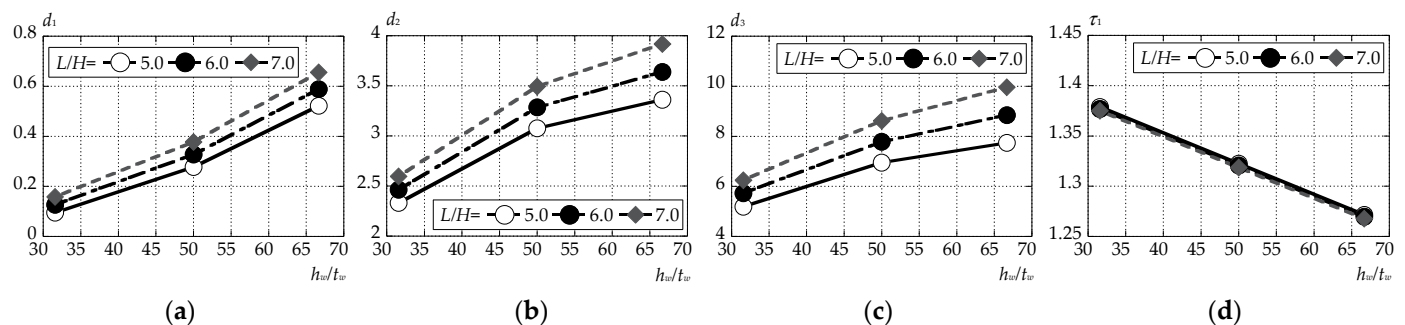


Figure 12. Influence of web height–thickness ratio: (a) d_1 ; (b) d_2 ; (c) d_3 ; (d) τ_1 .

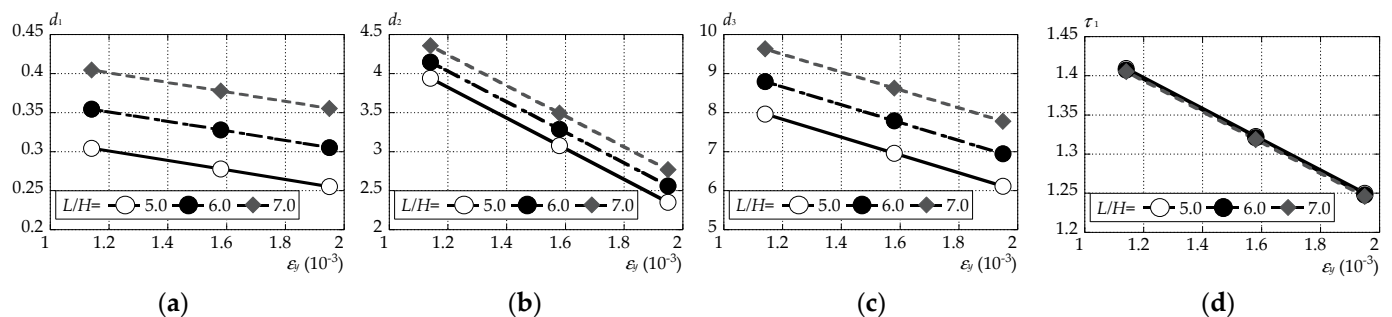


Figure 13. Influence of material yield strain: (a) d_1 ; (b) d_2 ; (c) d_3 ; (d) τ_1 .

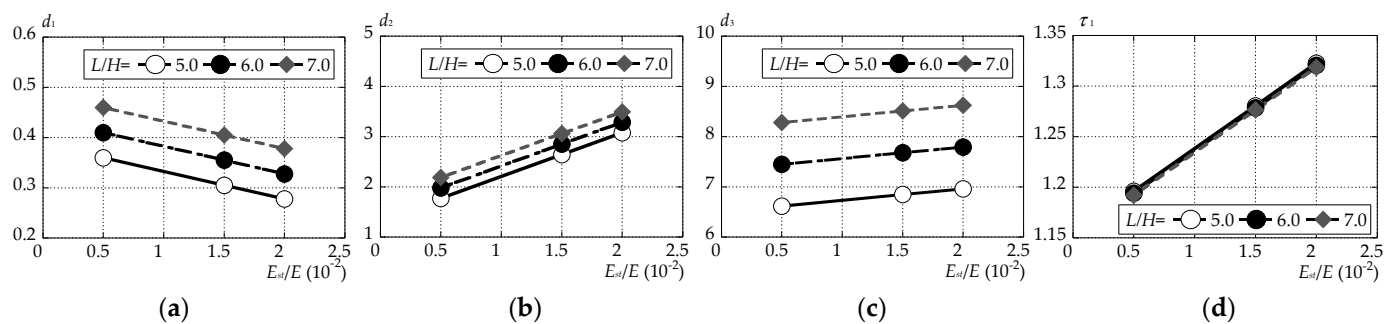


Figure 14. Influence of material secant modulus: (a) d_1 ; (b) d_2 ; (c) d_3 ; (d) τ_1 .

Based on Figure 11, it is possible to describe the influence of the flange width-to-thickness ratio on the behavior of the structure. d_1 decreases with the increasing slenderness of the beam's flange. This is an effect of the lower bending stiffness of the slender flange;

therefore, the slender flange does not drag the web together while buckling in the same scale as the bulky flange. Even after the point of maximum stress, this trend continued due to the low restraining effect between the flange and the web for cross-sections with slender flanges, as both the d_2 and d_3 indexes decreased with the flange's slenderness increasing. As described in reference [50], τ_1 tended to decrease as the flange width-to-thickness ratio became larger due to lower stiffness and ease of buckling in the case of slender flanges.

Figure 12 presents the influence of the web height-to-thickness ratio h_w/t_w . The d_1 becomes larger together with the increasing slenderness of the web. This is because the thinner the web, the lower its out-of-plane stiffness; therefore, the deformation increases, turning from compression to out-of-plane web bending. Upon achieving maximal stress, this trend continues, and the d_2 and d_3 indexes increase with the web's slenderness. As described in the literature (see [50]), τ_1 decreases as the web's height-to-thickness ratio increases.

The influence of yielding strain can be seen in Figure 13. As ε_y increases, the d_1 value decreases slightly. This and other results can be explained in a similar way to the influence shown in Figure 11. This confirms that changing the yield stress value effectively has the same effect as changing the thickness of the plate and the equivalent plate thickness. Models made of higher-class materials are plasticized on a lesser scale than those made of lower-class materials; therefore, the deformation at higher ε_y is smaller. Thus, similar results were obtained when reducing the flange width-to-thickness ratio by decreasing the yielding strain.

Figure 14 exhibits the influence of changing the post-yielding secant modulus. When the secant modulus is high, the stiffness after plasticizing becomes significant; therefore, the cross-section does not deform significantly, so d_1 tends to decrease with increasing E_{st} . Upon reaching the maximum residual stress, the stress starts to decline. However, the buckling zone within the cross-section propagates slower for the material with higher post-yielding stiffness, and the cross-section maintains a certain degree of strength. Therefore, the bending moment is greater for a given beam deflection, decreasing the strain to obtain the required stress level. Hence, both d_2 and d_3 rise together with the increase of E_{st} . Stiffer material allows the member to achieve a higher level of τ_1 .

In addition, to increase the span-to-depth ratio, L/H indexes d_1 , d_2 , and d_3 also rise. It is due to a longer unsupported span and, generally, higher slenderness of the beam itself. The value of τ_1 decreases slightly with increasing L/H .

3.3. Simplified Formula for Residual Stress Evaluation

In Section 3.2, the influence of changing the values of all the parameters on four evaluation indexes was described. Four equations describing the indexes were derived using the results from those analyses. Equations (1)–(4) were constructed based on the numerical analyses presented in this article using the least squares method. The base of all the equations is the elastic local buckling load factor of the beam summarized in [50], which, by definition, depends on cross-section geometrical parameters, material parameters, and beam length. The form of the equations describing elastoplastic behavior has been proposed by Fujak et al. [50]. Deformation indexes were adjusted using the web height-to-thickness ratio, which significantly influences the scale of deformation and the secant modulus of material used, as this parameter was not considered in the referred research. The powers used correspond to the influence of the parameters, as seen in Figures 12 and 14. For the ultimate strength ratio in Equation (4), the only correction compared with the original equation was applying the secant modulus factor (where the $\zeta_{cr,b}$ is the elastic local buckling load factor [50]).

$$d_1 = (0.4\zeta_{cr,b}^{0.5} + 0.3) \times (h_w/t_w)^2 \times (E_{st}/E)^{-0.03} \times 10^{-4} \quad (1)$$

$$d_2 = (2.6\zeta_{cr,b}^{0.5} - 2.0) \times (h_w/t_w)^{0.5} \times (E_{st}/E)^{0.03} \times 10^{-1} \quad (2)$$

$$d_3 = (2.2\zeta_{cr,b}^{0.5} + 6.2) \times (h_w/t_w)^{0.5} \times (E_{st}/E)^{0.03} \times 10^{-1} \quad (3)$$

$$\tau_1 = (-0.6\zeta_{cr,b}^{-0.5} + 1.6) \times (E_{st}/E)^{0.03} \quad (4)$$

Figure 15 shows a comparison between the results received from the proposed Equations (1)–(4) and the output obtained from the numerical analyses. As demonstrated, the equations estimated most of the results with $\pm 20\%$ accuracy, proving the effectiveness of the approach.

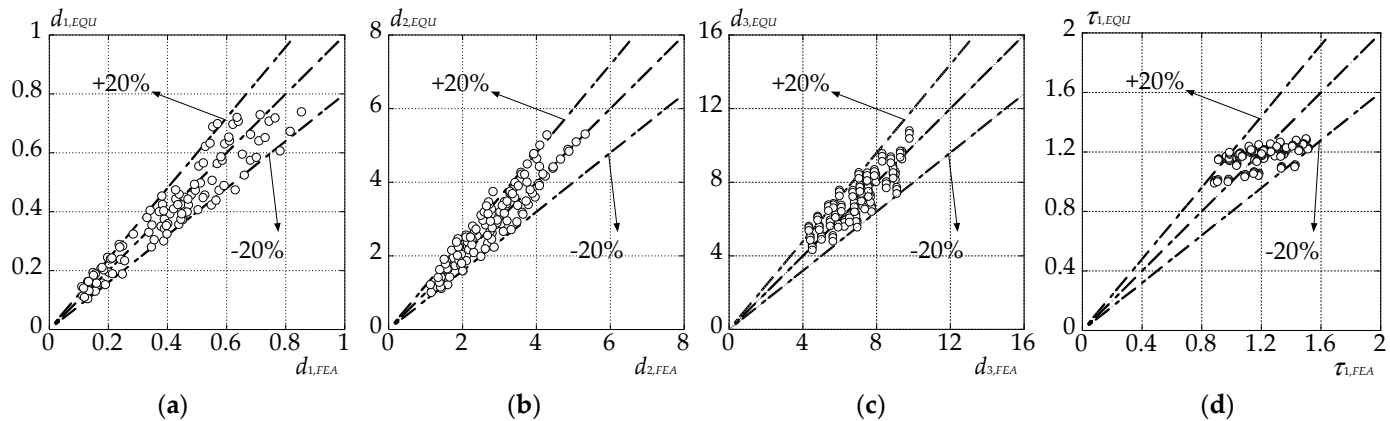


Figure 15. Comparison of results obtained from Equations (1)–(4) (EQU) with numerical analyses (FEA): (a) d_1 ; (b) d_2 ; (c) d_3 ; (d) τ_1 .

4. Static Pushover Analysis of a Three-Story Fishbone Frame with Mid-Story Pin Connections Considering Local Buckling

This section presents the static pushover analysis of the frame with an MSP connection accounting for the local buckling phenomenon of the beam. The ultimate seismic performance is determined by the collapse of the beam considering the shape of the beam's cross-section, the secant modulus of the material, and the influence of the compressive force in the column. Furthermore, the aim is to define the equation describing the relation between the local buckling of the beam and the angle of inter-story drift of the frame.

Generally, the concrete slab is placed on the top flange, and its influence varies depending on the stress transfer through the shear connectors. However, for the sake of simplicity, this research disregards the contribution of the concrete slab. This aspect will be addressed in future studies. Instead, to specifically investigate local buckling, the beam is constrained out-of-plane at a point 25% of the beam length from the column face. This setting aligns with the definition used in previous research [48].

4.1. Numerical Model for Single-Story Cruciform Frame with Pin Connection

Figure 16 shows the numerical model of the part of the building designed to scrutinize the influence of the mid-column hinge, as in reference [47]. The pin is built of a lower base plate, anchor bolt, collar plate, and upper base plate, as shown in Figure 16b. These parts have been modeled as 3D elements. As the stiffness of the lower RC part of the column is significantly higher than the steel frame, the RC deformation is neglected. Therefore, the bottom surface of the lower base plate is assumed to be fixed. The loading is an enforced deformation. The deformation index δ_T/H , obtained by dividing the value of the deformation divided by the total height of the frame (including the height of the RC pile), is set to achieve values of 0.0025, 0.005, 0.0075, 0.01, 0.015, 0.02, 0.025, 0.03, 0.04 and 0.05, both positive and negative, with three cycles per each value specified.

Figure 17 illustrates the comparison between experimental results and numerical analysis output. In Figure 17, P_H stands for the applied horizontal force and $P_{H,b,el}$ designates the horizontal force causing the beam yielding.

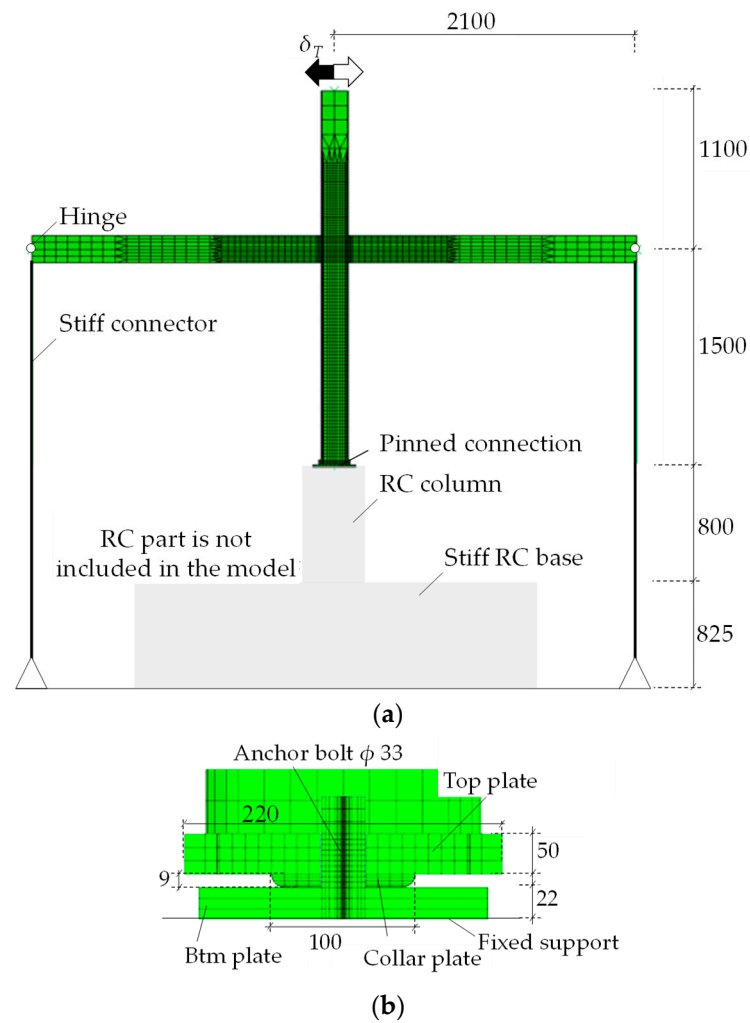


Figure 16. Numerical model of the frame with mid-story pin connections: (a) partial frame model; (b) detailed pinned connection model.

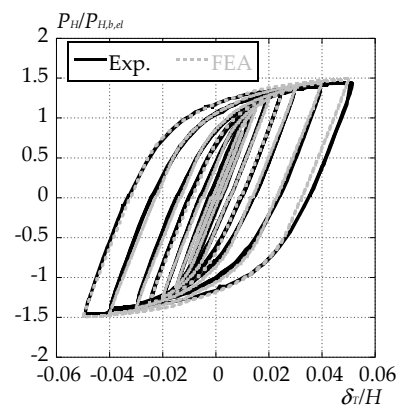


Figure 17. Hysteresis loop of the mid-story pin frame.

The numerical analysis result shows good convergence with the experiment results; therefore, it can be concluded that the analytical model of the pin connection shown in Figure 16 sufficiently reproduces the actual structure's behavior.

Figure 18 compares the analytical and numerical bending strain in the column ϵ_M against δ_T/H being 0.01, 0.02, and 0.05. It is proven that the pin model works as a hinge for both the experimental and numerical analyses.

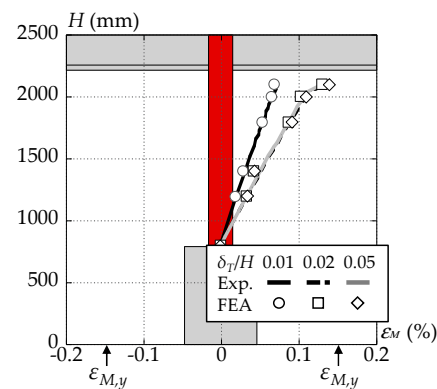


Figure 18. Bending strain in the column.

4.2. Overview of Three-Story Fishbone Frame Numerical Model with Mid-Story Pin Connection

Figure 19 shows a detailed three-story model of the frame with a pin connection in the middle of the first story column. Similar to Section 3, to include only the effects of local buckling, the out-of-plane deformation of the beams is restrained at the position of $0.25 \times L$. The anchor bolt material and parameters were chosen to keep it at the elastic stage until the level of enforced drift of 0.03, providing the failure does not occur before the beam buckling. The distribution of external force was assumed to be A_i distribution guided by AIJ provision [74].

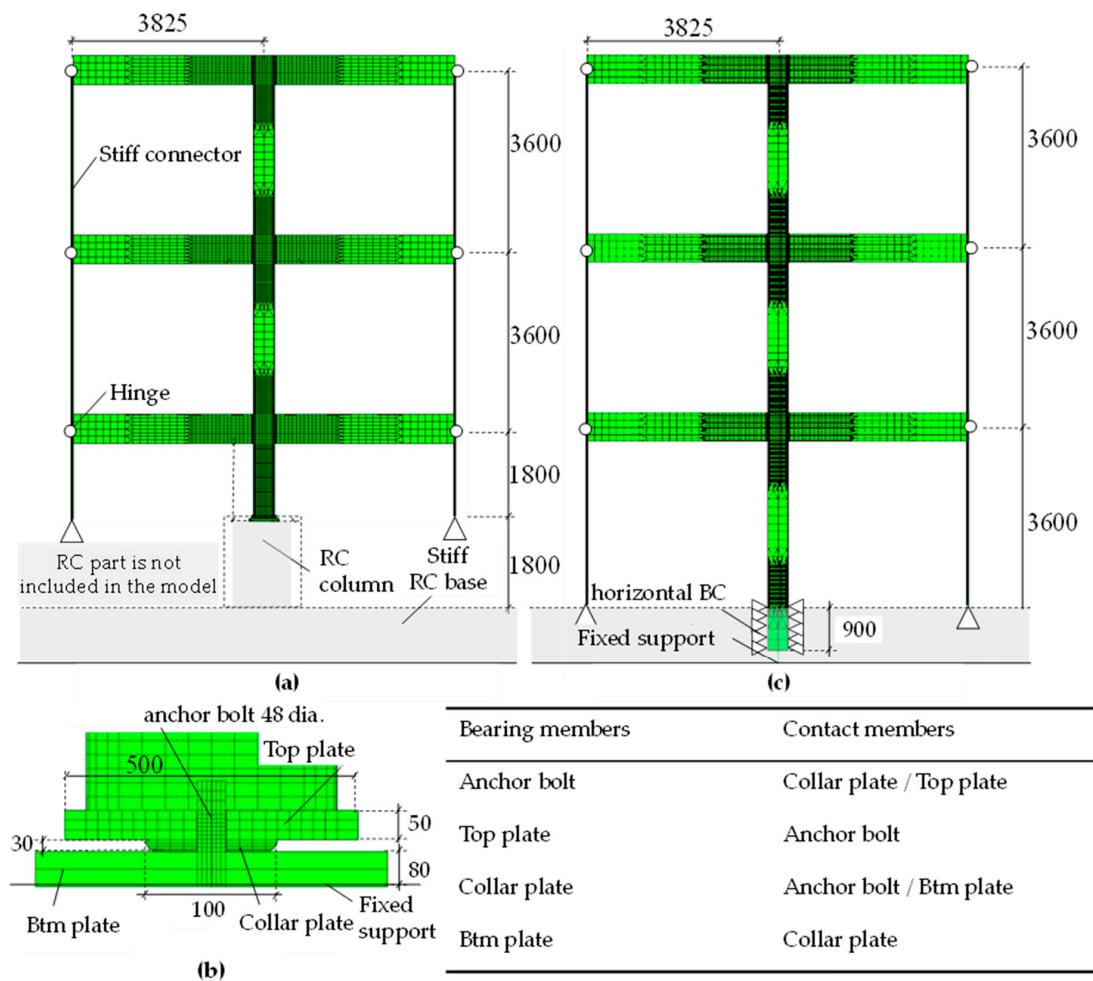


Figure 19. Three-story fishbone numerical models: (a) frame with mid-story pin support; (b) detailed pinned connection model; (c) frame with embedded support.

Tables 2–4 present the analysis parameters of the three-story fishbone frame with the pin connection in the middle of the first story, where the D and t_c are the side length and thickness of the column, respectively. The column was assumed to remain in the elastic stage to enable a straightforward investigation of the characteristics determined by the beam properties. The cross-section of the beam varied between 14 kinds, with a class of 1 and 2. The beam's material was set to have a full plastic moment, which differed by a maximum of 5% from the parameters taken for reference cross-section (No. 5). The cross-sections of the I-beams Nos. 1 to 11 had a height of 600 mm and a width of 300 mm. The cross-section performance was edited by changing the thickness of the flange and the web. For the cross-sections of beams Nos. 12 to 14, the width and the height were different compared with other cross-sections. As for the material properties of the beam, three types of secant modulus of $0.020 \times E$ (reference), $0.015 \times E$, and $0.005 \times E$ were used, the same as in Section No. 3. Note that, despite the column being an elastic element, for the section classification and the axial force utilization, the yield strength of 295 N/mm^2 was assumed. The referential axial force of the column was taken to be about 20% of the axial force column's cross-section capacity (615 kN per floor).

Table 2. SHS column parameters of 3-story fishbone frame.

No.	D (mm)	t_c (mm)	Wall Class	f_y (N/mm ²)
1	450	22	1	Elastic

Table 3. I-beam parameters of 3-story fishbone frame.

No.	H (mm)	h_w (mm)	B (mm)	b_f (mm)	t_w (mm)	t_f (mm)	Flange Class	Web Class	f_y (N/mm ²)
1	600	568	300	145.5	9.0	16.0	2	1	270
2	600	562	300	145.5	9.0	19.0	1	1	250
3	600	562	300	145.0	10.0	19.0	1	1	235
4	600	568	300	144.5	11.0	16.0	2	1	270
5	600	562	300	144.0	12.0	19.0	1	1	235
6	600	556	300	144.0	12.0	22.0	1	1	210
7	600	568	300	143.5	13.0	16.0	2	1	270
8	600	562	300	143.0	14.0	19.0	1	1	235
9	600	556	300	143.0	14.0	22.0	1	1	210
10	600	564	300	142.0	16.0	18.0	1	1	215
11	600	556	300	142.0	16.0	22.0	1	1	200
12	550	506	300	144.0	12.0	22.0	1	1	235
13	650	612	250	119.0	12.0	19.0	1	1	235
14	650	618	300	144.0	12.0	16.0	2	1	235

Table 4. Connection parameters and variables of three-story fishbone frame.

Connection Parameters		Variables of All Analyzed Frames	
Part	f_y (N/mm ²)	N (kN)	E_{st} (kN/mm ²)
Anchor bolt	835	$0.2 \times N_y$	$0.005 \times E$
Plate (Top, Collar, and Btm)	Elastic	$0.4 \times N_y$ ($N_y = 9200 \text{ kN}$)	$0.015 \times E$ $0.020 \times E$

4.3. Impact of Variable Parameters on the Ultimate Seismic Performance of the Frame

Figure 20 analyzes the cross-section of the beam of No. 5. The column-to-beam plastic strength ratio is 2.0 (note that in this diagram, the yield strength of the column is assumed to be 355 N/mm^2 in the case of $D/t_c = 20.5$ ($t_c = 22 \text{ mm}$) and 470 N/mm^2 in the case of $D/t_c = 28.1$ ($t_c = 16 \text{ mm}$)). The relation between story shear force and story drift for the mid-column hinge and embedded support is shown. The width-to-thickness ratio of 28.1 is about the lowest limit value of the 1st class. In the case of the MSP type of column's base,

since the local buckling does not occur in the column until the maximum story shear force is reached, the main parameter is the total plastic deformation capacity, so the influence of the width-to-thickness ratio of the column's wall is small. In the case of the embedded column base, when the width-to-thickness ratio for the column's cross-section is 20.5, it behaves in the same way as the MSP model does; however, in the case of the width-to-thickness ratio of 28.1, the model with embedded connection reaches the yielding stress early, causing the divergence in comparison with other models.

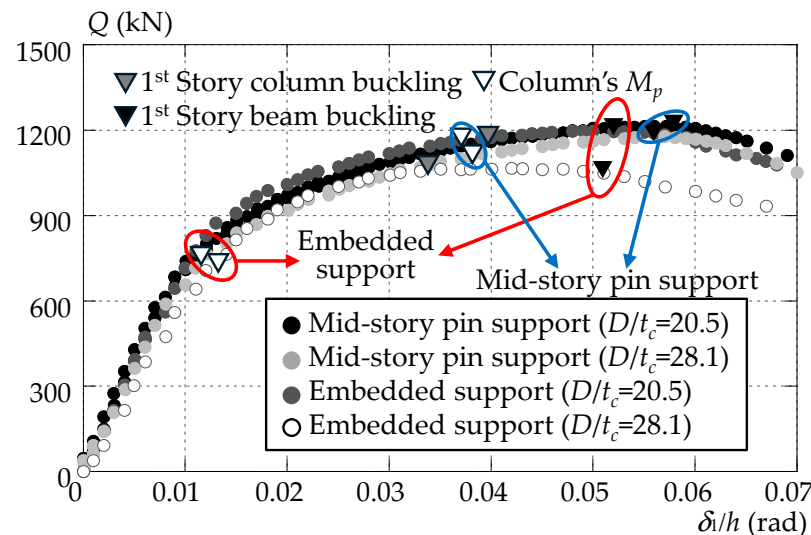


Figure 20. Shear force vs. story drift of the 1st story ($M_{c,pl}/M_{b,pl} = 2.0$).

Figure 21 presents the relationship between the story shear force and the story drift for the frames using various parameters. However, for each figure presented, only one parameter has been edited.

The result shown in Figure 21a,b is the outcome of decreasing the flange's and web's width-to-thickness ratios, showing the improvement of the performance of the beam by allowing for both the horizontal force and story drift to be greater at the point of maximal force. This is due to the rising capacity and stiffness of the beam with lower width-to-thickness ratios of the segments.

Figure 21c shows the influence of the cross-section shape variations. Cross-section No. 12 (550×300) has a smaller height-to-thickness ratio of the web than the reference, which was achieved by reducing the height of the beam. This edition makes the cross-section more compact and improves the performance of the beam significantly, allowing for greater force and enforced deformation to be taken. For cross-section No. 13 (650×250), the opposite edition was made, where the web is taller; hence, the web's height-to-thickness ratio is increased. At the same time, the flange's width-to-thickness ratio was decreased to maintain the same full plastic strength of the beam by making the flange narrower. For this type of cross-section, the outcome shown in Figure 21c does not change much compared with the reference one: the maximal force rises slightly, but the rotation at the point of maximum stays intact. The geometric properties of No. 14 (650×300) are identical to No. 13 for the height. However, the reduction in the flange's area was based on reducing the thickness while keeping the width unchanged (therefore increasing the width-to-thickness ratio). The effect visible in the skeleton curve is a deterioration of the story drift at the point of maximal strength occurrence, though the force does not change significantly. This behavior suggests that the buckling mode is flange buckling influenced by the restraint provided by web stiffness.

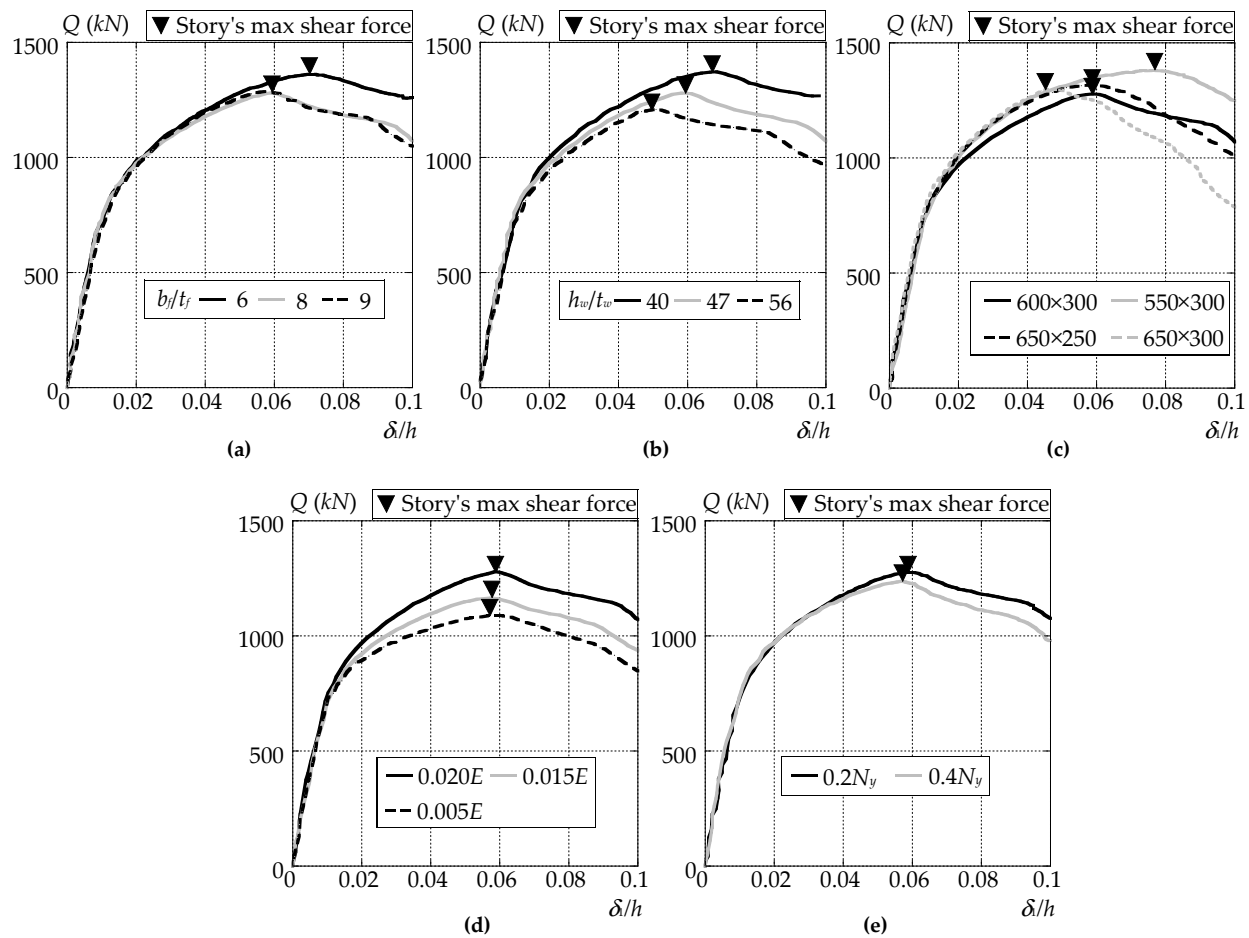


Figure 21. Influence of parameters over shear force vs. story drift ratio of the 1st floor: (a) flange's width–thickness ratio; (b) web's height–thickness ratio; (c) section's shape; (d) secant modulus; and (e) axial force.

Figure 21d presents the effect of the post-yielding secant modulus of the material. As the modulus increases, so does the maximal force. The value of the story drift registered during the peak point stays roughly the same, showing a minimal influence of the secant modulus.

In Figure 21e, the influence of the axial force value is in the form of the percentage of the maximum column's cross-section capacity (N_y). The axial force influences the value of the moment in the beam. The higher the column's utilization, the higher the I-beam's bending moment. Therefore, the higher the vertical force, the lower the inter-layer force and the smaller the corresponding enforced deformation. However, the difference between $0.2 \times N_y$ and $0.4 \times N_y$ is slight. This is strongly affected by the elastic model of the column. Interestingly, for greater deformations, the horizontal eccentricity between the top and bottom of the column creates the bending moment in the same way as the vertical force, therefore reducing the demand for the vertical force to reach the same column's rotation, as shown on the right side of the graph.

Figure 22 shows the relation between the column-to-beam full plastic strength ratio required to avoid column failure before beam buckling versus the normalized web's slenderness $\bar{\lambda}_w = h_w/t_w \sqrt{f_y/E}$ at the point of the maximal inter-layer force occurring for the 1st and 2nd floor. Additionally, specimens with different beam cross-sectional dimensions and those with the same $\bar{\lambda}_w$ value were excluded from the figure. The required capacity was calculated using the assumed elastic column, and calculated required strength based on the stress received from the analyses. The chart shows the influence of three analytical parameters. As the web slenderness increases, the demanded column-to-beam

strength ratio decreases for the first and second stories. The influence of the axial capacity utilization is visible when the axial force ratio μ_N is 0.4. The demand is about 1.15 times larger than that in the case of 0.2, yet the influence of the post-yielding secant modulus is even more significant; when the modulus is $0.020 \times E$, the demand is about 1.25 times larger than in the case of $0.005 \times E$. Furthermore, the required column-to-beam strength ratio for each parameter tends to be more significant for the 2nd story.

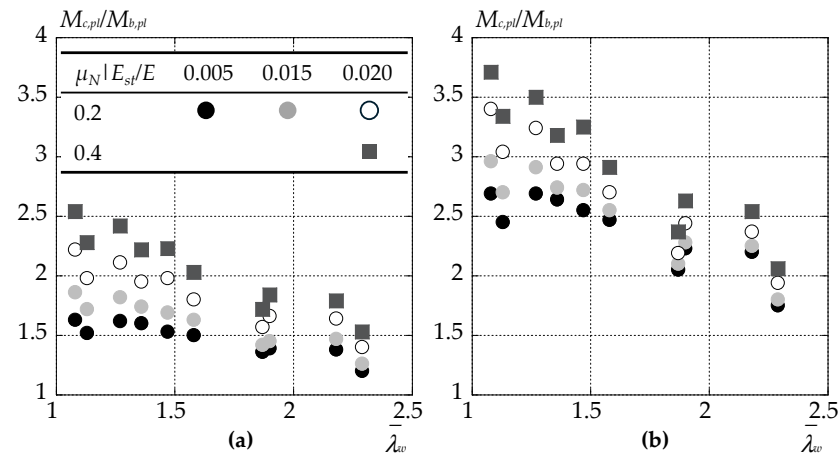


Figure 22. $M_{c,pl}/M_{b,pl}$ demand for 1st and 2nd story vs. web slenderness: (a) 1st story; and (b) 2nd story.

It is possible to compare this trend with the results shown in Figure 23. Here, H_I stands for the inflection point height within the story. This diagram shows the relationship between the 1st and 2nd stories' demanded column-to-beam strength and location of moment inflection points against the 1st story drift (the model used was the reference frame). The moment inflection point of the 1st floor is higher than the geometrical mid-point of the first story (at 0.5 value of the vertical axis) because of the negative moment occurring in the detailed connection due to the applied axial force in the column. This phenomenon continues up to the story drift of 0.01. After this point, the connection demonstrates stable hinge behavior, maintaining the inflection point in the middle of the first story. As for the 2nd story, the position of the inflection point gradually falls with the rising deformation of the structure. The reason for this behavior is that the stiffness of the beams on the 1st floor is gradually being reduced, therefore creating a smaller counter moment at the level of their connection with the column. As a result, the moment accumulates in the 2nd story column, causing greater stress and therefore demand for the moment capacity of this particular member, which is visible in Figure 23.

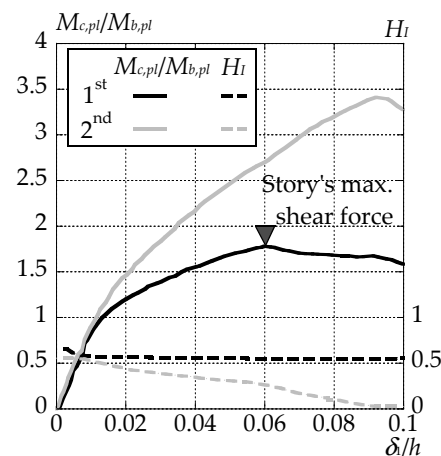


Figure 23. $M_{c,pl}/M_{b,pl}$ demand for 1st and 2nd story and inflection point location vs. 1st story drift.

Figure 24 shows the relation between the maximal inter-story drift (at the point of achieving maximal inter-story shear force) and the slenderness of the web. In Figure 21a,b, it is clear that the change in the slenderness of the beam segments changes the deformation at the point of maximal force—the slender the components, the lower the deformation. The influence of the secant modulus on the deformation at the maximal force point is negligible. The influence of the utilization of the beam is slightly greater. This output is reflected in Figure 24 as well. Noticeably, if the I-beam components fulfill the criterion of 1st class, the story drift at the point of maximal inter-story force is always over 0.04.

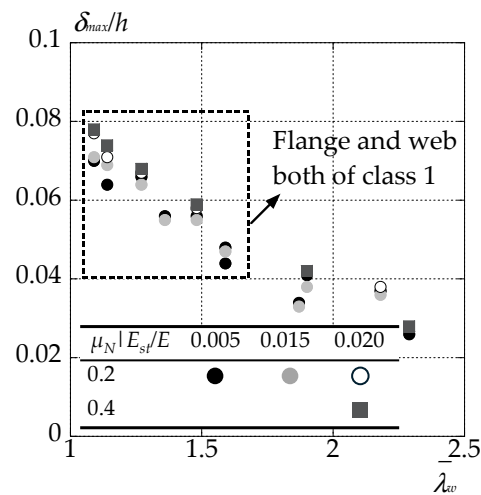


Figure 24. Maximal story drift at maximal shear force vs. web slenderness.

Figure 25 shows the necessary column-to-beam full plastic strength ratio against the value of the 2nd story drift for two groups of models: with the post-yielding secant modulus of $0.020 \times E$ and column utilization of 0.2 and with the post-yielding secant modulus of $0.005 \times E$ and column utilization of 0.2. The area between the line connecting maximal results and the line connecting minimal ones is filled with grey. The solid line in the center is the average value. The high convergence of the minimal and maximal demand for the 1st story finishes around the story drift value of 0.03, while for the 2nd story, it converges up to the drift of 0.05 (Figure 25a). For models shown in Figure 25b, the tendency for both floors is similar, with an increasing range starting around a story drift value of 0.01, slight divergence for the 1st story rising after story drift of 0.03, and an increase in divergence for the 2nd story around the drift of 0.04. The trajectories of the 1st and 2nd stories' demands correspond with the results shown in Figure 23 and the trend of lowering the moment inflection point (and, as a result, increasing the column capacity demand) for the 2nd-floor column along with rising story drift. Based on the results presented, it can be stated that the value of the required column-to-beam plastic strength to avoid column yielding is not strongly dependent on the parameters of the beam cross-section. The post-yielding secant modulus up to the level of story drift of 0.03 has a moderate influence; however, its importance (mainly for the 1st story column) starts to increase after the story drift rises over this value.

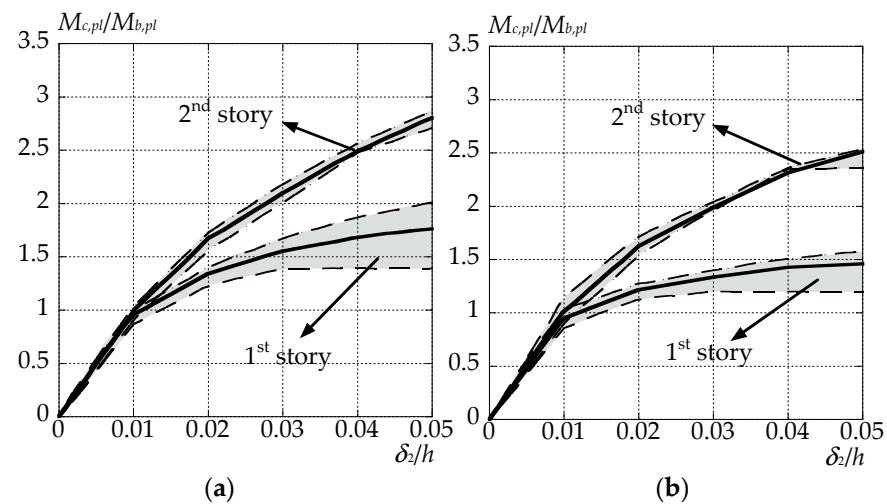


Figure 25. $M_{c,pl}/M_{b,pl}$ demand against story drift: (a) $N/N_y = 0.2$ and $E_{st}/E = 0.020$; (b) $N/N_y = 0.2$ and $E_{st}/E = 0.005$.

4.4. Method for Predicting Story's Cumulative Plastic Ductility Based on Local Buckling Deformation

This sub-section proposes the method of estimating the plastic ductility factor of post-earthquake state buildings by using the knowledge about its relationship with the scale of the local deformation.

Figure 26 presents the relationship between the value of the maximal plastic deformation index μ_f and the scale of local buckling deformation d (see Figures 9 and 10) concerning various parameters. In this paper, the maximal story drift has always been recorded within the 2nd floor; therefore, the maximal ductility analysis is focused on the 2nd story. The μ_f is a unitless value describing the scale of plastic deformation obtained by dividing the story drift at a certain point of time by the value of the story drift for the point when the 1st story beam reaches the full plastic strength (obtainable using the D-value method).

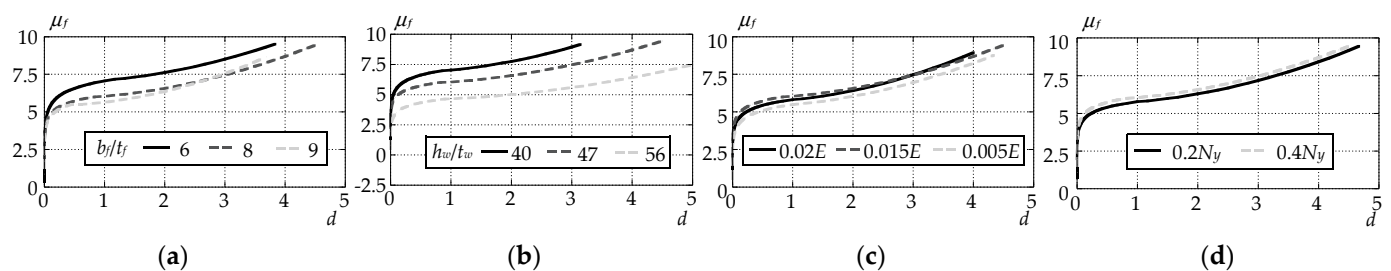


Figure 26. Plastic deformation index vs. the scale of local deformation: (a) b_f/t_f influence; (b) h_w/t_w influence; (c) E_{st}/E influence; and (d) N/N_y influence.

In Figure 26a,b, the influence of changing the slenderness of the cross-section components is shown. It is visible that the increase in both the web's and flange's slenderness causes the rise in the deformation for the same level of plastic ductility. It corresponds with Figure 21a,b, showing the relation between the beam's capacity and the aspect ratios of the cross-section's segments. Figure 26c,d explain the influence of the post-yielding secant modulus and axial force utilization of the column on the relation between ductility and buckling deformation. Those parameters are strongly related to the behavior of the column (the axial force is a column's parameter, and the post-yielding secant modulus has an influence on the growth of the bending moment in the column after the yielding of the 1st-floor

beams), and they do not have much of an influence over the μ_f to d relation. As the result of the analyses, Equation (5) has been proposed to describe the abovementioned relation.

$$\begin{cases} \mu_f = \alpha_1 \log_{10} d + \beta_1 & (0 < d < d_0) \\ \mu_f = \alpha_2 d^2 + \beta_2 & (d_0 < d) \end{cases} \quad (5)$$

Equation (5) intersects twice. In the equation above, d_0 is the intersection coordinate with a smaller value. Figure 27 compares the results from Equation (5) with those from numerical analyses of models Nos. 1, 4, 5, and 11. There are four parameters of the equations, α_1 , β_1 , α_2 , and β_2 , and the assumed values are given in the graphs. Correctly chosen parameters are a way to simplify the design process and achieve results even if the precise cross-section dimensions are not specified yet.

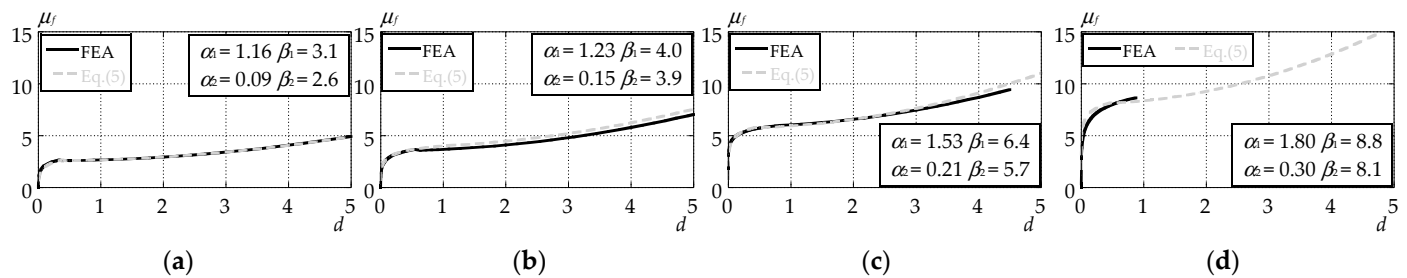


Figure 27. Comparison of Equation (5) and FEA output: (a) No. 1; (b) No. 4; (c) No. 5; and (d) No. 11.

Many models with different cross-sections, loading, and material characteristics have been considered during the numerical analyses, as shown in Tables 2–4. Those variables define the difference in the behavior of the structure, cumulative plastic ductility factor, and the scale of local buckling deformation. Figures 28–30 show the relationship between those variables and the values of four parameters of Equation (5) for the equations to correctly simulate the behavior of the numerical model. As was already proven in Figure 26c,d, the influence of post-yielding secant modulus and column's utilization ratio is negligible; therefore, the influence of those variables is not considered. However, to maintain a comparable level of the I-beams' full plastic cross-sectional capacities, the yielding strength of the material has been analyzed as well.

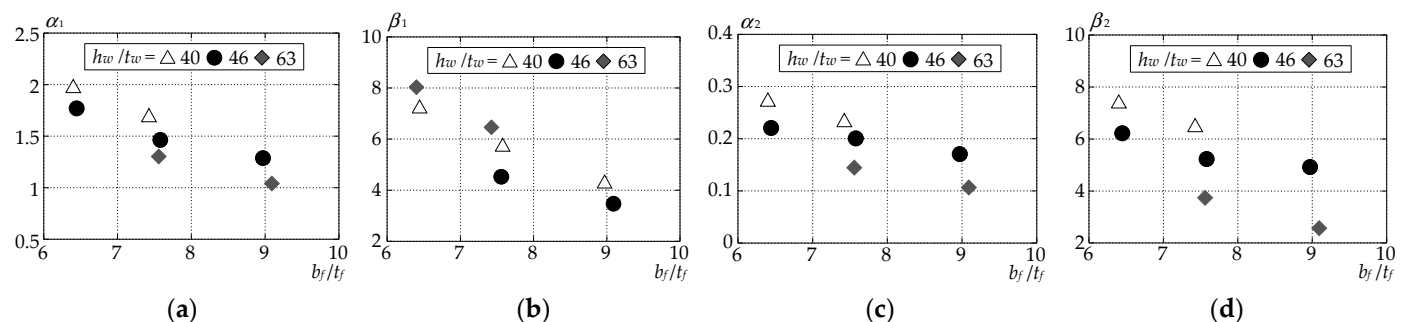


Figure 28. Influence of flange's width–thickness ratio on the following parameters: (a) α_1 ; (b) β_1 ; (c) α_2 ; (d) β_2 .

Figure 28 presents the influence of changing the width-to-thickness ratio of the flange. With the increasing b_f/t_f , all four parameters also decrease, making the final diagram flatter and giving lower results. This shows the effect of the decreasing performance of the member with the rising slenderness of the cross-section segments.

A similar situation is presented in Figure 29, as the rising h_w/t_w value causes a similar, yet more substantial, effect. This is related to the fact that in analyzed models, the web is generally a less stiff component of the cross-section; therefore, the influence is more visible. As shown in Figure 26, the higher the slenderness of the elements, the faster the

deterioration of the model's members; therefore, the demand for all four parameters is also reduced.

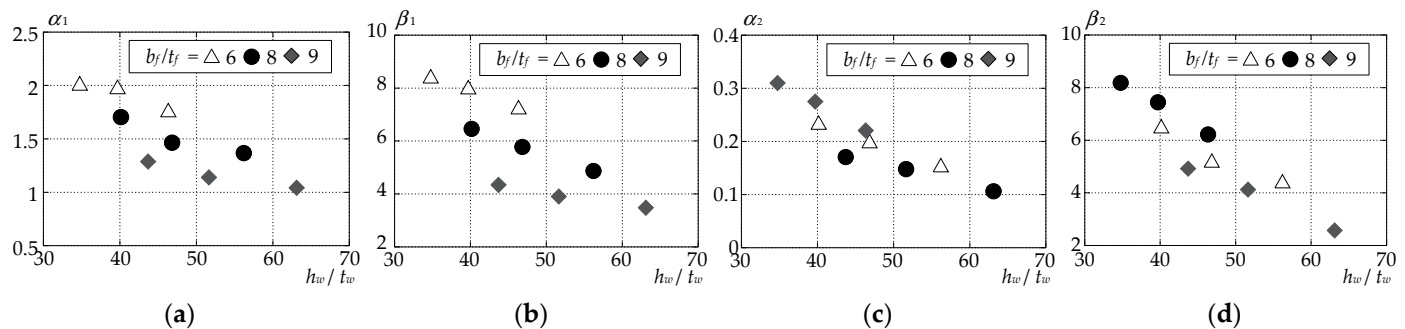


Figure 29. Influence of web height–thickness ratio on the following parameters: (a) α_1 ; (b) β_1 ; (c) α_2 ; (d) β_2 .

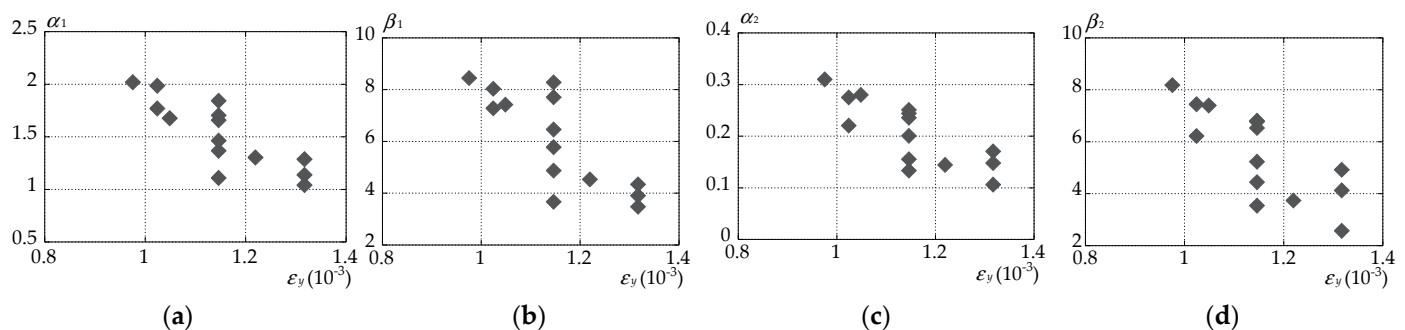


Figure 30. Influence of material yield strain on the following parameters: (a) α_1 ; (b) β_1 ; (c) α_2 ; (d) β_2 .

Figure 30 shows the difference in outcome from the models with respect to the ϵ_y of the beam. The yield strain ϵ_y is calculated by dividing the yield stress by Young's modulus, which is 205 GPa. For this graph, all the results are presented with one type of plot, as the ϵ_y varies between models, and a clear definition of groups (as in Figures 28 and 29) is impossible. Together with the increasing class of steel, four parameters are decreasing, showing deterioration in the model's capacity. This is not directly a function of the higher yielding strength but rather the aspect ratios of the components (mostly flange) of the cross-section, which, with the given outline of the beam (300×600 mm), requires the reduction in the segment's thickness. Moreover, it corresponds with the idea of the normalized slenderness index, where changing the geometrical slenderness and the steel class has a similar effect due to a similar impact on the M_{cr}/M_y ratio. Therefore, this parameter is not considered significant and is not taken into consideration in the equations proposed in the following sub-section.

4.5. Definition of the Simplified Residual Stress Evaluation Formula to Estimate Story's Cumulative Ductility

Section 4.4 presents the influence of changing the model's variables on the equations' parameters (see Figures 28–30). However, the influence of ϵ_y presented in Figure 30 is entangled with geometrical parameters of the cross-section. It was discovered that the equations for approximation of necessary values of α_1 , β_1 , α_2 , and β_2 can be defined based on the geometrical parameters only.

Based on those results the functions describing the values of parameters can be derived. They are presented below.

$$\alpha_1 = -0.005h_w/t_w - 0.19b_f/t_f + 3.2 \quad (6)$$

$$\beta_1 = -0.070h_w/t_w - 1.38b_f/t_f + 20.1 \quad (7)$$

$$\alpha_2 = -0.005h_w/t_w - 0.03bf/t_f + 0.675 \quad (8)$$

$$\beta_2 = -0.110h_w/t_w - 0.90bf/t_f + 17.7 \quad (9)$$

Figure 31 shows the comparison between the predicted values of the four parameters (using the abovementioned equations) and the approximated values describing curves from numerical analyses. As presented, the equations estimated the analysis results with around $\pm 10\%$ error, therefore proving the effectiveness of the formulas.

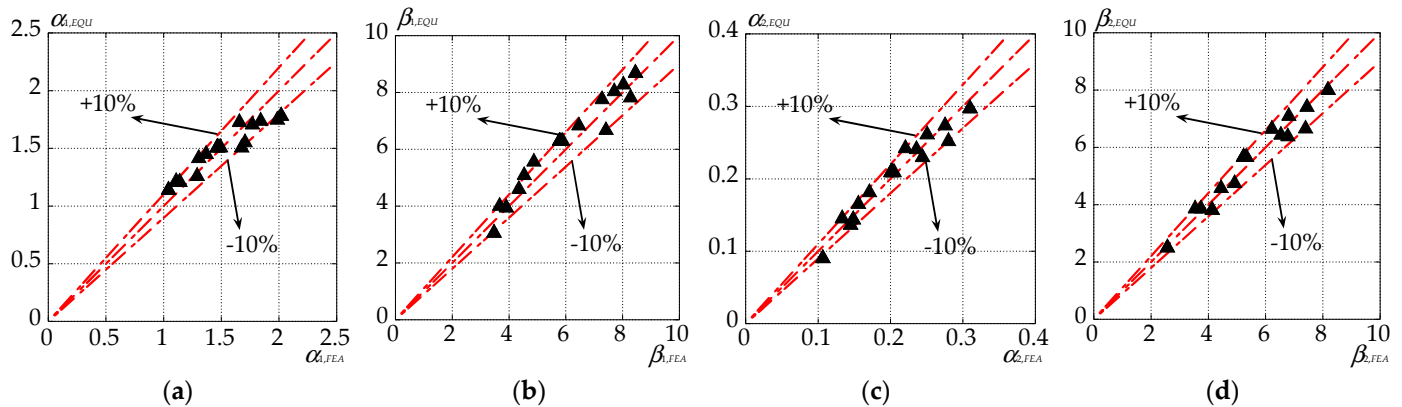


Figure 31. Comparison of numerical analyses (FEA) results with Equations (6)–(9) (EQU): (a) α_1 ; (b) β_1 ; (c) α_2 ; (d) β_2 .

5. Conclusions

This article aimed to check the feasibility and effectiveness of estimating the post-earthquake residual forces and capacity of the steel-and-concrete moment-resisting frames utilizing the mid-story pin connection located at the midpoint of the structure's first story. The main criterion of the ultimate state, in this case, was the local buckling of the members of the fishbone frame. Therefore, in the beginning, the precision of the numerical shell model was confirmed against the experimental results. Based on the elastic local buckling load factor, Equations (1)–(4) explaining the relation between the residual strength and the local buckling deformation scale were proposed for the I-beam-based model. The resultant convergence reached around $\pm 20\%$ compared with the numerical model outputs. The next section checked the requirements of the column-to-beam full plastic moment capacity for various parameters in the three-story model to avoid column yielding up to the point of beam buckling, therefore covering the range of possible seismic action of the frame. The models used in those analyses had the bottom connection built of solid elements, and their feasibility was confirmed by the results obtained from the previous experiment. Noticeably, the additional moment in the pin created by the behavior of the detailed connection and the axial force applied was visible only for the story drift below 0.01; upon reaching that point, the connection started working as a pin. Next, the equations to estimate the curve approximating the relationship between the total plastic ductility of the 2nd story beam were proposed depending on four geometrical parameters: H , b_f , t_w , and t_f . The resultant values reached the convergence of a minimum of $\pm 15\%$.

During the analyses, the effects of parameters such as the flange width-to-thickness ratio, web height-to-thickness ratio, yield strain, post-yielding secant modulus, and, in some cases, span-to-depth ratio, have been considered.

In this study, we thoroughly analyzed the static and seismic behavior of moment-resisting frames with the mid-story pin. The influence of the parameters is explained and proven, and based on the result, simplified equations of high accuracy are proposed. Therefore, the paper contributes to the following research and the industry.

Author Contributions: Conceptualization, Y.K.; methodology, A.S.; software, A.S. and R.C.; validation, A.S. and Y.K.; formal analysis, A.S.; investigation, A.S. and R.C.; resources, Y.K.; data curation, A.S. and R.C.; writing—original draft preparation, A.S. and R.C.; writing—review and editing, Y.K.; visualization, R.C.; supervision, Y.K.; project administration, A.S.; funding acquisition, Y.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by JSPS KAKENHI (Grant Number 24H00342) (Principal Investigator: Dr. Yoshihiro Kimura) and JSPS KAKENHI (Grant Number 23K13392) (Principal Investigator: Dr. Atsushi Suzuki). We express our deepest gratitude for their sincere support.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in the study are included in the article; further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Khatib, I.F.; Mahin, S.A.; Pister, K.S. *Seismic Behavior of Concentrically Braced Steel Frames*; UCB/EERC-88/01; University of California: Berkeley, CA, USA, 1988.
2. Kimura, Y.; MacRae, G.A. Effect of Column Flexural Stiffness and Strength on Story Drift Concentration Mechanism of 2 Story Braced Frames with Fixed Column Base. *J. Struct. Constr. Eng. (Trans. AIJ)* **2005**, *70*, 153–160. [\[CrossRef\]](#)
3. Ji, X.; Kato, M.; Wang, T.; Hitaka, T.; Nakashima, M. Effect of Gravity Columns on Mitigation of Drift Concentration for Braced Frames. *J. Constr. Steel Res.* **2009**, *65*, 2148–2156. [\[CrossRef\]](#)
4. Qu, B.; Sanchez-Zamora, F.; Pollino, M. Mitigation of Inter-Story Drift Concentration in Multi-Story Steel Concentrically Braced Frames through Implementation of Rocking Cores. *Eng. Struct.* **2014**, *70*, 208–217. [\[CrossRef\]](#)
5. Simpson, B.G.; Mahin, S.A. Experimental and Numerical Investigation of Strongback Braced Frame System to Mitigate Weak Story Behavior. *J. Struct. Eng.* **2018**, *144*, 04017211. [\[CrossRef\]](#)
6. Dong, Z.-Q.; Li, G.; Li, H.-N. Dynamic Tests of the Collapse-Prevention Performance of a Low-Ductility Low-Rise Steel Concentrically Braced Frame. *Eng. Struct.* **2021**, *240*, 112420. [\[CrossRef\]](#)
7. EN 1998-1:2004; European Committee for Standardization Eurocode 8: Design of Structures for Earthquake Resistance—Part 1: General Rules, Seismic Actions and Rules for Buildings. European Committee for Standardization: Brussels, Belgium, 2004.
8. Yazdanpanah, O.; Formisano, A.; Chang, M.; Mohebi, B. Fragility Curves for Seismic Damage Assessment in Regular and Irregular MRFs Using Improved Wavelet-Based Damage Index. *Measurement* **2021**, *182*, 109558. [\[CrossRef\]](#)
9. Jia, M.; Gao, S.; Fu, F.; Yang, N. Failure Mechanism and Seismic Performance Evaluation of Steel Frame Using Rocking Truss. *Structures* **2021**, *33*, 1180–1192. [\[CrossRef\]](#)
10. Zhu, R.; Guo, T.; Mwangilwa, F.; Han, D. Seismic Design of Self-Centering Viscous-Hysteretic Devices Used for Steel Moment-Resisting Frames. *Eng. Struct.* **2021**, *239*, 112369. [\[CrossRef\]](#)
11. Shakeri, K.; Dadkhah, H. Development of DDBD for Steel MRFs Using Inelastic Response-Based Seismic Lateral Force Distribution. *J. Build. Eng.* **2021**, *43*, 103063. [\[CrossRef\]](#)
12. Suita, K.; Matsuoka, Y.; Yamada, S.; Shimada, Y.; Tada, M.; Kasai, K. Experimental Procedure and Elastic Response Characteristics of Shaking Table Test—Complete Collapse Test of Full-Scale 4-Story Steel Building Part 1. *J. Struct. Constr. Eng. (Trans. AIJ)* **2009**, *635*, 157–166. [\[CrossRef\]](#)
13. Yamada, S.; Suita, K.; Matsuoka, Y.; Shimada, Y. Elasto-Plastic Responses and Process Leading to a Collapse Mechanism; Dynamic Collapse Test of Full-Scale 4-Story Steel Building; Part 2. *J. Struct. Constr. Eng. (Trans. AIJ)* **2009**, *74*, 1851–1859. [\[CrossRef\]](#)
14. Shimada, Y.; Suita, K.; Yamada, S.; Matsuoka, Y.; Tada, M.; Ohsaki, M.; Kasai, K. Collapse Behaviour on Shaking Table Test; Dynamic Collapse Test of Full-Scale 4-Story Steel Building; Part 3. *J. Struct. Constr. Eng. (Trans. AIJ)* **2010**, *75*, 1351–1360. [\[CrossRef\]](#)
15. Yamada, S.; Suita, K.; Tada, M.; Kasai, K.; Matsuoka, Y.; Shimada, Y. Collapse Experiment on 4-Story Steel Moment Frame: Part 1 Outline of Test Results. In Proceedings of the 14th World Conference on Earthquake Engineering, Beijing, China, 12–17 October 2008; Volume 4.
16. Suita, K.; Yamada, S.; Tada, M.; Kasai, K.; Matsuoka, Y.; Shimada, Y. Collapse Experiment on 4-Story Steel Moment Frame: Part 2 Detail of Collapse Behavior. In Proceedings of the 14th World Conference on Earthquake Engineering, Beijing, China, 12–17 October 2008; Volume 11.
17. Kaneda, K.; Kimura, Y.; Hamasaki, S.; Wada, A. Proposal of New Column Support System for Multi-Story Steel Moment Resisting Structures to Perform Beam Yielding Mechanism. *J. Struct. Constr. Eng. (Trans. AIJ)* **2010**, *75*, 1537–1546. [\[CrossRef\]](#)
18. Elamary, A.S. Ultimate Shear Resistance of Plate Girders Part 2—Höglund Theory. *Int. J. Civ. Environ. Eng.* **2013**, *7*, 918–926.
19. Alinia, M.M.; Shakiba, M.; Habashi, H.R. Shear Failure Characteristics of Steel Plate Girders. *Thin-Walled Struct.* **2009**, *47*, 1498–1506. [\[CrossRef\]](#)

20. Anastasiadis, A.; Mosoarca, M.; Gioncu, V. Prediction of Available Rotation Capacity and Ductility of Wide-Flange Beams: Part 2: Applications. *J. Constr. Steel Res.* **2012**, *68*, 176–191. [\[CrossRef\]](#)
21. Basler, K. Strength of Plate Girders in Shear. *J. Struct. Div.* **1961**, *87*, 151–180. [\[CrossRef\]](#)
22. Cheng, X.; Chen, Y.; Pan, L. Experimental Study on Steel Beam–Columns Composed of Slender H-Sections under Cyclic Bending. *J. Constr. Steel Res.* **2013**, *88*, 279–288. [\[CrossRef\]](#)
23. Elkady, A.; Lignos, D.G. Full-Scale Testing of Deep Wide-Flange Steel Columns under Multiaxis Cyclic Loading: Loading Sequence, Boundary Effects, and Lateral Stability Bracing Force Demands. *J. Struct. Eng.* **2018**, *144*. [\[CrossRef\]](#)
24. Gioncu, V.; Mosoarca, M.; Anastasiadis, A. Prediction of Available Rotation Capacity and Ductility of Wide-Flange Beams: Part 1: DUCTROT-M Computer Program. *J. Constr. Steel Res.* **2012**, *69*, 8–19. [\[CrossRef\]](#)
25. Glassman, J.D.; Moreyra Garlock, M.E. A Compression Model for Ultimate Postbuckling Shear Strength. *Thin-Walled Struct.* **2016**, *102*, 258–272. [\[CrossRef\]](#)
26. Ozkula, G.; Harris, J.; Uang, C.-M. Observations from Cyclic Tests on Deep, Wide-Flange Beam-Columns. *Eng. J.* **2017**, *54*, 45–60. [\[CrossRef\]](#)
27. Hanna, M.T. Failure Loads of Web Panels Loaded in Pure Shear. *J. Constr. Steel Res.* **2015**, *105*, 39–48. [\[CrossRef\]](#)
28. Ikarashi, K.; Suekuni, R.; Shinohara, T.; Wang, T. Evaluation of Plastic Deformation Capacity of H-Shaped Steel Beams with the Newly Proposed Limitation Value of Plate Slenderness. *J. Struct. Constr. Eng. (Trans. AIJ)* **2011**, *76*, 1865–1872. [\[CrossRef\]](#)
29. Kasai, K.; Popov, E.P. General Behavior of WF Steel Shear Link Beams. *J. Struct. Eng.* **1986**, *112*, 362–382. [\[CrossRef\]](#)
30. Kasai, K.; Popov, E.P. Cyclic Web Buckling Control for Shear Link Beams. *J. Struct. Eng.* **1986**, *112*, 505–523. [\[CrossRef\]](#)
31. Kato, B. Rotation Capacity of H-Section Members as Determined by Local Buckling. *J. Constr. Steel Res.* **1989**, *13*, 95–109. [\[CrossRef\]](#)
32. Lee, S.C.; Davidson, J.S.; Yoo, C.H. Shear Buckling Coefficients of Plate Girder Web Panels. *Comput. Struct.* **1996**, *59*, 789–795. [\[CrossRef\]](#)
33. Lee, S.C.; Lee, D.S.; Yoo, C.H. Flexure and Shear Interaction in Steel I-Girders. *J. Struct. Eng.* **2013**, *139*, 1882–1894. [\[CrossRef\]](#)
34. MacRae, G.A. *The Seismic Response of Steel Frames*; University of Canterbury: Christchurch, New Zealand, 1989.
35. Araújo, M.; Macedo, L.; Castro, J.M. Evaluation of the Rotation Capacity Limits of Steel Members Defined in EC8-3. *J. Constr. Steel Res.* **2017**, *135*, 11–29. [\[CrossRef\]](#)
36. Nakashima, M.; Takanashi, K.; Kato, H. Test of Steel Beam-Columns Subject to Sidesway. *J. Struct. Eng.* **1990**, *116*, 2516–2531. [\[CrossRef\]](#)
37. Newell, J.D.; Uang, C.-M. Cyclic Behavior of Steel Wide-Flange Columns Subjected to Large Drift. *J. Struct. Eng.* **2008**, *134*, 1334–1342. [\[CrossRef\]](#)
38. Torabian, S.; Schafer, B.W. Role of Local Slenderness in the Rotation Capacity of Structural Steel Members. *J. Constr. Steel Res.* **2014**, *95*, 32–43. [\[CrossRef\]](#)
39. Shokouhian, M.; Shi, Y. Classification of I-Section Flexural Members Based on Member Ductility. *J. Constr. Steel Res.* **2014**, *95*, 198–210. [\[CrossRef\]](#)
40. Suzuki, T.; Ikarashi, K.; Tsuneki, Y. A Study of Collapse Mode and Plastic Deformation Capacity of H-Shaped Steel Beams under Shear Bending. *J. Struct. Constr. Eng. (Trans. AIJ)* **2001**, *66*, 185–191. [\[CrossRef\]](#)
41. Höglund, T. Simply Supported Long Thin Plate I-Girders Without Web Stiffeners Subjected to Distributed Transverse Load. In *IABSE Colloquium London, Reports of the Working Comissions*; 1971; pp. 85–97. Available online: https://www.researchgate.net/publication/313444594_Simply_supported_long_thin_plate_I-girders_without_web_stiffeners_subjected_to_distributed_transverse_load (accessed on 23 October 2024).
42. Höglund, T. Shear Buckling Resistance of Steel and Aluminium Plate Girders. *Thin-Walled Struct.* **1997**, *29*, 13–30. [\[CrossRef\]](#)
43. Ragheb, W.F. Local Buckling of Welded Steel I-Beams Considering Flange–Web Interaction. *Thin-Walled Struct.* **2015**, *97*, 241–249. [\[CrossRef\]](#)
44. Chen, Y.; Cheng, X.; Nethercot, D.A. An Overview Study on Cross-Section Classification of Steel H-Sections. *J. Constr. Steel Res.* **2013**, *80*, 386–393. [\[CrossRef\]](#)
45. Kimura, Y.; Kaneda, K.; Wada, A. Estimation of Ultimate Seismic Capacity and Column Demand of Multi-Story Steel Moment Resisting Frames with New Column Support System to Fishbone-Shaped Frame. *J. Struct. Constr. Eng. (Trans. AIJ)* **2012**, *77*, 765–773. [\[CrossRef\]](#)
46. Kato, B.; Akiyama, H.; Obi, Y. Deformation of H-Shaped Cross-Section Member due to Local Buckling. *Trans. Archit. Inst. Jpn.* **1977**, *257*, 49–58. [\[CrossRef\]](#)
47. Kaneda, K.; Kimura, Y.; Mutsukura, K.; Kadoya, H.; Watanabe, T.; Takahashi, K. Cyclic Loading Test for Partial Frames with New Column Support System for Multi-Story Steel Moment Resisting Structures to Perform Beam Yielding Mechanism. *J. Struct. Constr. Eng. (Trans. AIJ)* **2011**, *76*, 1357–1366. [\[CrossRef\]](#)
48. Suzuki, A.; Kimura, Y.; Kasai, K. Rotation Capacity of I-Shaped Beams under Alternating Axial Forces Based on Buckling-Mode Transitions. *J. Struct. Eng.* **2020**, *146*. [\[CrossRef\]](#)
49. Kimura, Y.; Sato, Y.; Suzuki, A. Effect of Fork Restraint of Column on Lateral Buckling Behavior for H-Shaped Steel Beams with Continuous Braces under Flexural Moment Gradient. *J. Struct. Constr. Eng. (Trans. AIJ)* **2022**, *87*, 316–327. [\[CrossRef\]](#)
50. Fujak, S.M.; Kimura, Y.; Suzuki, A. Estimation of Elastoplastic Local Buckling Capacities and Novel Classification of I-Beams Based on Beam’s Elastic Local Buckling Strength. *Structures* **2022**, *39*, 765–781. [\[CrossRef\]](#)

51. Suzuki, A.; Kimura, Y. Rotation Capacity of I-Shaped Beam Failed by Local Buckling in Buckling-Restrained Braced Frames with Rigid Beam-Column Connections. *J. Struct. Eng.* **2023**, *149*. [[CrossRef](#)]
52. Fujak, S.M.; Suzuki, A.; Kimura, Y. Estimation of Ultimate Capacities of Moment-Resisting Frame's Subassemblies with Mid-Storey Pin Connection Based on Elastoplastic Local Buckling. *Structures* **2023**, *48*, 410–426. [[CrossRef](#)]
53. Kishiki, S.; Iwasaki, Y. Evaluation of Residual Strength Based on Local Buckling Deformation of Steel Column—Quick Inspection Method for Steel Structures Based on the Visible Damage Part 3. *J. Struct. Constr. Eng. (Trans. AIJ)* **2017**, *82*, 735–743. [[CrossRef](#)]
54. Chida, H.; Takahashi, N. Study on Image Diagnosis of Timber Houses Damaged by Earthquake Using Deep Learning. *Jpn. Archit. Rev.* **2021**, *4*, 420–430. [[CrossRef](#)]
55. Chang, P.C.; Flatau, A.; Liu, S.C. Review Paper: Health Monitoring of Civil Infrastructure. *Struct. Health Monit.* **2003**, *2*, 257–267. [[CrossRef](#)]
56. Carden, E.P.; Fanning, P. Vibration Based Condition Monitoring: A Review. *Struct. Health Monit.* **2004**, *3*, 355–377. [[CrossRef](#)]
57. Ji, X.; Fenves, G.L.; Kajiwara, K.; Nakashima, M. Seismic Damage Detection of a Full-Scale Shaking Table Test Structure. *J. Struct. Eng.* **2011**, *137*, 14–21. [[CrossRef](#)]
58. Okada, K.; Shiroishi, R.; Morii, Y.; Sagawa, R. Method of Structural Health Monitoring after Earthquake Using Limited Accelerometer—Case Study of Large-Scale Shaking Table Test on Six Story RC Building. *Concr. J.* **2017**, *55*, 138–145. [[CrossRef](#)] [[PubMed](#)]
59. Gislason, G.P.; Mei, Q.; Gül, M. Rapid and Automated Damage Detection in Buildings Through ARMAX Analysis of Wind Induced Vibrations. *Front. Built Environ.* **2019**, *5*, 63–73. [[CrossRef](#)]
60. Alampalli, S.; Fu, G.; Dillon, E.W. Signal versus Noise in Damage Detection by Experimental Modal Analysis. *J. Struct. Eng.* **1997**, *123*, 237–245. [[CrossRef](#)]
61. Ju, M.; Dou, Z.; Li, J.-W.; Qiu, X.; Shen, B.; Zhang, D.; Yao, F.-Z.; Gong, W.; Wang, K. Piezoelectric Materials and Sensors for Structural Health Monitoring: Fundamental Aspects, Current Status, and Future Perspectives. *Sensors* **2023**, *23*, 543. [[CrossRef](#)]
62. Gong, F.; Maekawa, K. Multi-Scale Simulation of Freeze-Thaw Damage to RC Column and Its Restoring Force Characteristics. *Eng. Struct.* **2018**, *156*, 522–536. [[CrossRef](#)]
63. Gong, F.; Wang, Z.; Xia, J.; Maekawa, K. Coupled Thermo-hydro-mechanical Analysis of Reinforced Concrete Beams under the Effect of Frost Damage and Sustained Load. *Struct. Concr.* **2021**, *22*, 3430–3445. [[CrossRef](#)]
64. Nobuhiro, S.; Tetsuya, N.; Akihiko, O.; Kazuhisa, N. Cuadra Carlos Comparison of Displacement Measurements in an Exposed Type Column Base Using Piezoelectric Vibration Sensors and Piezoelectric Limit Sensors. *Am. J. Remote Sens.* **2019**, *6*, 27–36.
65. Suzuki, A.; Fujita, T.; Kimura, Y. Identifying damage mechanisms of gymnasium structure damaged by the 2011 Tohoku earthquake based on biaxial excitation. *Structures* **2022**, *35*, 1321–1338. [[CrossRef](#)]
66. Aabid, A.; Parveez, B.; Raheman, M.A.; Ibrahim, Y.E.; Anjum, A.; Hrairi, M.; Parveen, N.; Mohammed Zayan, J. A Review of Piezoelectric Material-Based Structural Control and Health Monitoring Techniques for Engineering Structures: Challenges and Opportunities. *Actuators* **2021**, *10*, 101. [[CrossRef](#)]
67. Harada, T.; Yokoyama, K.; Tanabe, K. Study of Crack Detection of Civil Infrastructure Using PVDF Film Sensor. *J. Struct. Eng.* **2013**, *59*, 47–55.
68. Shimoi, N.; Cuadra, C.; Madokoro, H.; Nakasho, K. Comparison of Displacement Measurements and Simulation on Fillet Weld of Steel Column Base. *Int. J. Mech. Eng. Appl.* **2020**, *8*, 111. [[CrossRef](#)]
69. Smith, M. *ABAQUS/Standard User's Manual*, Version 6.9; Dassault Systèmes Simulia Corp.: Providence, RI, USA, 2009.
70. Kishiki, S.; Iwasaki, Y. Seismic Repair to the Damaged RHS Column with Strength Deterioration Due to the Local Buckling. *J. Struct. Constr. Eng. (Trans. AIJ)* **2018**, *83*, 179–189. [[CrossRef](#)]
71. Kimura, Y.; Yamanashi, T.; Kasai, K. Cyclic Hysteresis Behavior and Plastic Deformation Capacity for H-Shaped Beams on Local Buckling Under Compressive and Tensile Forces. *J. Struct. Constr. Eng. (Trans. AIJ)* **2013**, *78*, 1307–1316. [[CrossRef](#)]
72. Lemaitre, J.; Chaboche, J. *Mechanics of Solid Materials*; Cambridge University Press: Cambridge, UK, 1990.
73. *EN 1993-1-1:2005*; European Committee for Standardization Design of Steel Structures: Eurocode 3: Design of Steel Structures, Part 1-1: General Rules and Rules for Buildings. European Committee for Standardization: Brussels, Belgium, 2005.
74. Architectural Institute of Japan (AIJ). *Recommendations for Loads on Buildings 2015*; Maruzen Publishing Co., Ltd.: Tokyo, Japan, 2015.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.