

Article

Comparison of Direct and Telescopic Grid Nesting in a High-Resolution Operational Ocean Model

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Abstract

High-resolution regional ocean models are typically nested within coarser parent models, with resolution ratios limited to avoid spurious artefacts. Consequently, a telescopic grid-nesting strategy is commonly adopted for very high-resolution applications by introducing one or more models of intermediate resolution. However, such an approach is complex and computationally expensive. The purpose of this study is to assess the effectiveness of the recently developed Nesting with Data Assimilation (NDA) method in reducing the need for multi-stage telescopic nesting. The NDA method assimilates observations into the child model indirectly via the 3D outputs from the parent model, which in turn assimilates observations directly. Direct nesting of a $1/60^\circ$ resolution child into a $1/12^\circ$ parent model is compared with telescopic nesting via an intermediate $1/20^\circ$ model in the Southeast Arabian Sea for the year 2016. Validation against observations demonstrates that, when combined with the NDA method, the directly nested model achieves accuracy comparable to, or better than, that of the double-nested configuration. Direct nesting eliminates the intermediate model entirely, reducing the complexity and cost of the modelling system while maintaining at least equivalent, or often better, performance. The findings suggest that an efficient assimilation technique can effectively replace intermediate dynamical downscaling steps in operational ocean forecasting, where reducing complexity is particularly valuable.

Keywords: ocean modelling; Indian Ocean; data assimilation; operational forecast

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1. Introduction

High-resolution ocean models are frequently employed to simulate mesoscale and sub-mesoscale ocean processes, enabling the assessment of the influence of horizontal currents on vertical motion in the upper ocean and the transport of pollutants [1]. Several approaches exist for refining the results from a coarse ocean model in regional and coastal zones [2], one of which is grid nesting, also referred to as dynamic downscaling [3]. In dynamic downscaling, a high-resolution (child) model is embedded within a coarser (parent) model, from which it derives its initial and boundary conditions. This technique is common practice not only in ocean modelling but also in atmospheric and climate models [4–6]. The study by [5] demonstrated that grid nesting offers major advantages over other

refinement techniques, such as stretched or unstructured grids. Dynamic downscaling has been shown to be an effective method for operational regional oceanographic models. Nevertheless, the nesting process introduces certain unavoidable errors, particularly near the open boundary, arising from discrepancies in grid resolutions and updating intervals [3]. The errors near boundaries are exacerbated when telescopic nesting is used.

In common with the published literature [4–6], we use the terms ‘parent’ and ‘child’ models for the original coarse and nested high-resolution models, respectively. The sensitivity of child model performance to variations in the nesting ratio r within the range $r = 3:1$ to $7:1$ was examined by Spall and Holland [7], who found that larger ratios lead to a deterioration in model quality. High nesting ratios could lead to instabilities and create artefacts at the nesting boundary [8]. The recommendations for strict limits between $2:1$ and $5:1$ to ensure computational stability were formalised by [9]. They proved that a $3:1$ ratio became the convention because it allows a child grid point to perfectly align with a parent cell centre, keeping the discrete spatial filters symmetric and preventing artificial wave reflection. Similarly, the study in [3] indicated that the optimal spatial resolution ratio between a parent model and its nested child is likely to be less than $r = 3:1$. Consistent with these findings, many regional ocean modelling studies adopt a ratio of $r = 3:1$, as seen in [10,11] and related works.

To attain very high resolution in the child domain, a double- or multi-nesting strategy is often employed, whereby one or more intermediate grids are introduced. For example, high-quality results for a small Mediterranean domain were achieved using a telescoping nesting strategy between resolutions of 10 km and 225 m when two intermediate grids were introduced [12]. Another example was presented by Barth et al. [13], who downscaled from a coarse ($1/4^\circ$) to a fine ($1/60^\circ$) grid, incorporating an intermediate grid at $1/20^\circ$ resolution in their Mediterranean Sea study. In order to avoid high nesting ratios an operational setup used in [14] included six levels of nesting when moving from the resolution of the Baltic Sea to that of the port of Riga. This setup ran several times slower than the base setup, which forced the authors to decrease the child model size [8]. Similar multi-stage nesting strategies are also employed in atmospheric and climate modelling contexts [15–17].

The study in [18] demonstrated that the assimilation of observational data, which is a standard practice in operational oceanography, imposes additional constraints on the permissible nesting ratio. A high-resolution model (2 km) nested directly into a global model ($1/12.5^\circ$, approximately 7–9 km), with a corresponding nesting ratio of $r = 3.5:1$, yielded unacceptably large errors when combined with a three-dimensional variational (3DVar) data assimilation (DA) scheme. These findings suggest that the assimilation of low-resolution observational data into a nested high-resolution model may degrade model performance. To mitigate these errors, ref. [18] proposed two alternative approaches: the introduction of an intermediate model with a reduced nesting ratio of $r = 2:1$, or the application of a four-dimensional variational (4DVar) data assimilation scheme in a direct nesting framework. This study demonstrated that a change in the DA technique (in this case, from 3DVar to 4DVar) can remove the necessity for an intermediate dynamical model. In a data-assimilating model, special attention must be paid to the treatment of the open boundary conditions to prevent assimilation increments from decoupling the nested model from the parent model [19]. A telescopic nesting strategy requires such alignment to be performed a number of times for each intermediate model. Therefore, both strategies, the use of 4DVar or an intermediate model, entail an increase in the complexity of the modelling system and its computational cost.

An alternative approach for improving the quality of the child model involves the use of two-way nesting rather than a two-step downscaling strategy. In two-way nesting, the parent model solution is periodically updated using the child model output in regions where the two grids overlap. This facilitates a degree of interaction and blending between

the parent and child solutions within the nested domain [7]. However, this approach requires the parent and child models to be run simultaneously, substantially increasing the computational cost, which remains a major limitation for operational forecasting and other resource-intensive applications despite ongoing improvements in computing hardware.

A further development of the idea of two-way nesting was the introduction of the Nesting with Data Assimilation (NDA) method [20,21]. According to this method, the outputs from the parent model (e.g., temperature, salinity, velocity) are assimilated into the child model, enabling a consistent blending of coarse- and fine-resolution information not only at the lateral boundaries but throughout the entire three-dimensional domain of the nested model. By assimilating dense parent model data instead of sparse observations, the NDA method reduces the degradation of model performance caused by the assimilation of low-resolution observational data. The NDA method applies a combination of stochastic and dynamic downscaling from the coarse to the fine (or intermediate) grid [20,22,23]. The two components, stochastic and dynamic, are combined using a Kalman filter. Although model-to-model data assimilation shares conceptual similarities with the assimilation of observational data, it differs in practice because, in contrast to observations, errors in model outputs are spatially correlated. The NDA method does not require the simultaneous execution of both coarse- and fine-grid models. The output from the parent coarse model can be sourced off-line from publicly accessible datasets, thereby reducing the complexity and cost of high-resolution regional modelling.

Another advantage of assimilating model-derived fields, rather than direct observations, lies in their availability on a regular grid, which enhances the computational efficiency of the NDA approach [20]. An additional advantage is that the velocity fields derived from the parent model are dynamically consistent with the temperature, salinity, and density fields. As a result, the NDA technique does not require commonly used geostrophic balancing between density and velocity fields (see, e.g., ref. [24]), since these fields are already physically balanced within the parent model. The NDA method has also proven effective at mitigating errors in meteorological forcing [25].

The present study takes a further step in the assessment of the capabilities of the NDA method. It evaluates whether the NDA approach can achieve comparable or better forecast quality using direct rather than telescopic nesting. The downscaling is carried out from a $1/12^\circ$ global ocean model to a $1/60^\circ$ operational model without the intermediate nesting step traditionally required. The performance of this configuration is compared with that of a conventional two-stage telescoping approach comprising successive nesting ratios of 1.7:1 and 3:1. To our knowledge, this is the first assessment of the ability of NDA to reduce the complexity of telescoping nesting in a high-resolution operational ocean modelling system.

2. Materials and Methods

The modelling was conducted in the Lakshadweep Sea (Indian Ocean), also known as the South East Arabian Sea. The study area is shown in Figure 1. The 3D outputs of temperature, salinity, U, and V components of current velocities are taken from the parent model CMEMS [26]. Three model configurations named LD20_COPDA, LD60small_COPDA, and LD60small_LD20DA were developed and executed within the operational Rose-CYLC computing framework on a high-performance computing (HPC) cluster using between 96 and 140 computing cores [23].

The details of the LD model setup and its validation against observations are presented in [23]. The calibration and validation datasets were independent. Here, we briefly summarise the main characteristics of the models and the data assimilation (DA) technique employed. All configurations are based on the NEMO 3.6 codebase [27] and are configured as consistently as possible. Each model includes tidal forcing and employs 50

geopotential vertical levels spanning depths from 0.5 m to 4600 m. The vertical level distribution for all models is the same as in the parent model. The configurations differ only in their horizontal resolution and time steps to keep identical Courant numbers. Bathymetry is derived from the GEBCO database [28] at 1 arc-minute resolution and is linearly interpolated onto the respective model grids.

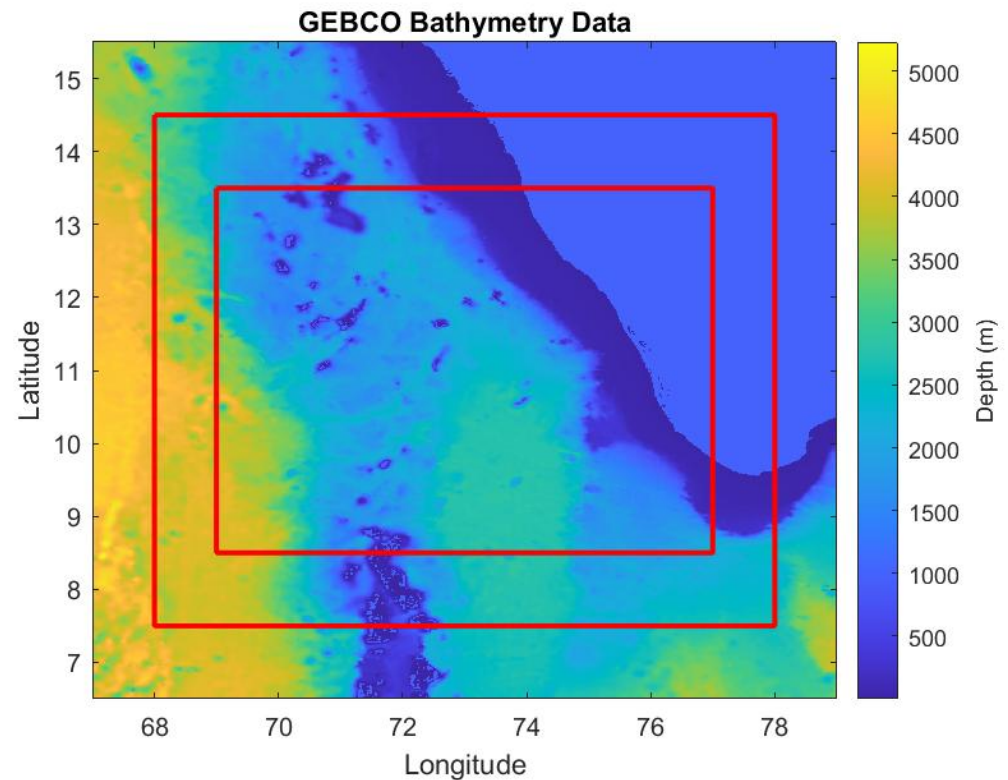


Figure 1. The bathymetry map and the model domains: the full box delineates the domain from which parent CMEMS model outputs are extracted for use in this study; the outer red box indicates the domain of LD20_COPDA; the inner red box shows the domain used for both LD60small_COPDA and LD60small_LD20DA models.

The horizontal viscosity and diffusion coefficients are calculated using the Smagorinsky algorithm, with identical Smagorinsky coefficients adopted from [29]. The bottom friction in CMEMS and all LD models is prescribed using a quadratic friction formulation in accordance with the NEMO guide [27].

The models are forced by hourly wind velocities and three-hourly air temperature, total downward shortwave radiation, longwave radiation, precipitation, and relative humidity [30]. The flow relaxation scheme (FRS) is applied at the open lateral boundaries using an unstructured NEMO BDY algorithm [31] on a narrow sponge rim for temperature, salinity, and baroclinic velocity, while barotropic velocities are treated using Flather radiation conditions [32]. The baroclinic time steps are 120 s, 16 s, and 16 s for LD20_COPDA, LD60small_COPDA, and LD60small_LD20DA, respectively. The ratio between the baroclinic and barotropic time steps is 20:1 for all LD models. The three models are nested within a parent global model, either directly (LD20_COPDA and LD60small_COPDA) or through a two-step downscaling approach (LD60small_LD20DA). Simulations are performed over a two-year period from 1 January 2015 to 31 December 2016. All configurations follow the Big/Little Brother concept [3], whereby the nested models share the same codebase, consistent parameterisations of physical processes, namelist parameters, compilation keys, and the same NDA method

for data assimilation. This design isolates the effects of the nesting type (direct versus telescopic) while minimising the influence of other sources of discrepancy.

All three LD models employ the Nesting with Data Assimilation methodology, which is briefly described below in relation to the purpose of this study. Full details are given in [20]. Within this framework, three-dimensional fields of temperature (T), salinity (S), and velocity components (U, V) generated by the parent (CMEMS) or intermediate (LD20_COPDA) model are assimilated into a higher-resolution child model. Because the parent CMEMS model itself incorporates assimilation of observations, it serves as an intermediary that propagates real-world observations to the higher-resolution model.

A principal distinction between the NDA methodology and traditional observation-based assimilation schemes lies in the formulation of the computations and the structure of the error covariance matrices (ECMs). These matrices are used to minimise the cost function that quantifies the discrepancy between the estimated and true ocean states [33,34]. In conventional data assimilation schemes which use observations, measurement errors are assumed to be spatially uncorrelated [35], resulting in a diagonal structure of the observational error covariance matrix [36]. However, this assumption is not valid when model outputs are treated as pseudo-observations, as such data inherently contain spatial correlations. Consequently, an alternative formulation for estimating ECMs is required in model-to-model assimilation contexts. Detailed procedures for deriving both background and observational ECMs under this framework are provided in [20].

2.1. The Parent Model CMEMS

The parent model, hereafter referred to as CMEMS, is a global ocean reanalysis with a horizontal resolution of $1/12^\circ$ and 50 geopotential vertical levels, based on the NEMO 3.1 codebase [26]. Within the study region, the reanalysis uses the NEMOVAR system to assimilate in situ temperature and salinity (T/S) profiles together with satellite-derived sea surface temperature (SST) and sea level anomaly (SLA). Further details of the model configuration are provided in [26]. For the present study, model outputs are taken from the product known as GLOBAL_REANALYSIS_PHY_001_030 [37]. The dataset provides daily mean fields of potential temperature (T), salinity (S), and the zonal (U) and meridional (V) velocity components, together with a range of additional oceanographic variables.

2.2. The LD20_COPDA Model

The LD20_COPDA model serves as the intermediate model in the telescopic nesting configuration. It is configured on a 200×140 (longitude $\times$ latitude) grid covering the region between 7.5 and 14.5° N and between 68 and 78° E, with a horizontal resolution of $1/20^\circ$. Initial conditions are taken from the CMEMS reanalysis on 1 January 2015, while lateral boundary conditions for temperature, salinity, and the velocity components (U and V) are derived from the same parent model. Because the parent CMEMS reanalysis does not include tidal forcing, nine tidal constituents (M2, S2, K1, O1, Q1, P1, N2, K2, and M4) are extracted from the TPXO version 7.1 global tidal model [38] and superimposed on the barotropic boundary velocities under the assumption of linear superposition. The LD20_COPDA model assimilates three-dimensional temperature, salinity, and velocity fields from the parent CMEMS reanalysis every five days at 00:00 GMT using the NDA method. The choice of assimilation cycle length was the subject of the studies presented in [20,23]. Further details of the LD20_COPDA configuration are provided in [23].

2.3. The LD60small_COPDA Model

The LD60small_COPDA model is a high-resolution configuration established as a direct nest within the global parent model. The LD60small_COPDA model is discretized on a 480 (longitude) $\times$ 300 (latitude) mesh, covering the domain 8.5–13.5° N and 69–77° E (see the inner box in Figure 1). It shares the same structural design as LD20_COPDA, differing only in its higher resolution (1/60° compared to 1/20°), along with proportionally reduced barotropic and baroclinic time steps. Boundary conditions are derived from the CMEMS and TPXO models, following the same approach used for LD20_COPDA, and the model employs identical atmospheric forcing. This model assimilates 3D temperature, salinity, and velocity components from the CMEMS model every five days, also at midnight.

2.4. The LD60small_LD20DA Model

The LD60small_LD20DA model represents the high-resolution model configuration implemented as the final nest within the multi-nesting framework. It employs the same three-dimensional grid with a horizontal resolution of 1/60° as the LD60small_COPDA. However, unlike LD60small_COPDA, its open boundary conditions and three-dimensional fields (T, S, U, V) used for model-to-model data assimilation are derived from the intermediate model LD20_COPDA rather than from the global CMEMS reanalysis. As the intermediate configuration includes tides, no additional tidal currents are imposed at the open boundaries. The width of the flow relaxation scheme (FRS) sponge layer is identical to that used in LD60small_COPDA. Data assimilation of temperature, salinity, and velocity fields from LD20_COPDA is also performed at five-day intervals.

2.5. Data for Model Validation

Sea surface temperature (SST) data used for model validation are obtained from the Operational Sea Surface Temperature and Ice Analysis (OSTIA), available through the UK Met Office at a spatial resolution of 1/20° [39]. OSTIA provides a high-resolution SST analysis derived from a combination of satellite and in situ observations. In this study, the daily foundation SST (i.e., the temperature unaffected by the diurnal warming cycle) is used. Vertical profiles of temperature and salinity used for model validation are obtained from Argo float observations [40,41].

2.6. Data Assimilation

The NDA process used in this study comprises two sequential stages. The first stage, referred to as Stochastic–Deterministic Downscaling (SDD), involves projecting the outputs from the coarse-resolution parent model (either CMEMS or LD20_COPDA) onto the finer grid of the child model using data-driven stochastic methods [22]. The second stage consists of dynamic downscaling, whereby the child model (either LD20_COPDA, LD60small_COPDA, or LD60small_LD20DA) is run in the forecasting mode, and the stochastically downscaled three-dimensional fields from the parent (or intermediate, in the case of telescopic nesting) model are assimilated into the child model forecast using a Kalman filter to generate a high-resolution analysis. Since the CMEMS model provides only daily averaged outputs rather than instantaneous fields, an additional temporal synchronisation step is required to ensure consistency between the stochastically downscaled data and the child model forecast for the direct nesting and the first nest in the case of telescopic nesting.

A flowchart of the model-to-model data assimilation is shown in Figure 2

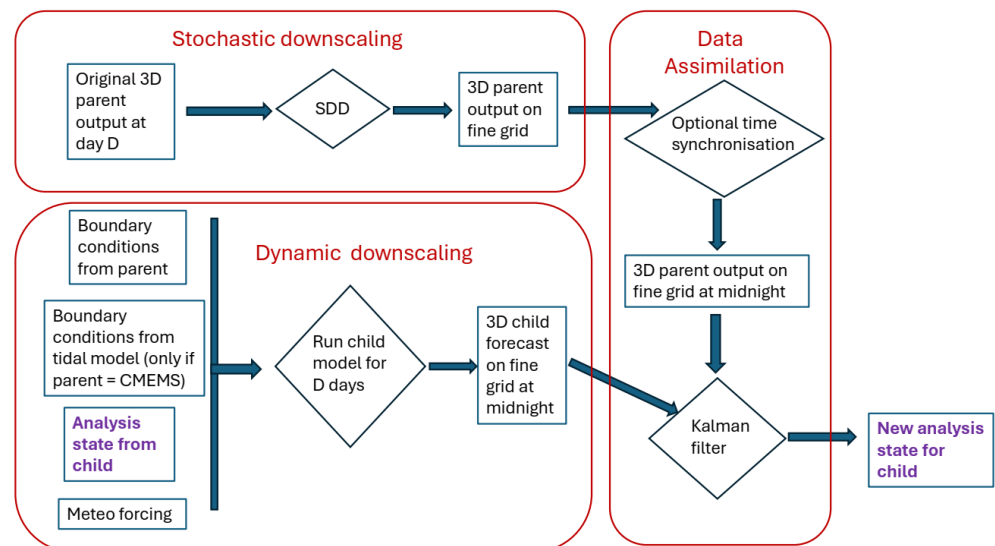


Figure 2. Flowchart of the model-to-model data assimilation process. Parallelograms show data/inputs/outputs, diamonds show actions. The violet colour denotes the analysis states before and after assimilation.

2.7. Stochastic Downscaling

The SDD procedure broadly follows the methodology of objective analysis and can be classed as a best linear unbiased estimator (BLUE), see [42]. The implementation of SDD involves a sequence of steps that are performed once at the start of the simulation period, as follows:

- (i) Correlation functions for each of the four state variables (temperature, salinity, and the zonal and meridional velocity components, T , S , U , and V) are computed at every three-dimensional grid point of the parent model within the model domain shown in Figure 1 using a one-year dataset of model outputs from the year 2015. Data from the preceding year are used for two reasons. First, a one-year record provides 365 data points, which is sufficient for robust curve fitting. Second, it is short enough to minimise the influence of long-term climate trends. Error covariance functions in data assimilation are physically anisotropic and flow-dependent in reality, but they are frequently assumed to be isotropic and static in practice for simplicity and computational efficiency [33].
- (ii) Approximation of the correlation function using a two-length-scale correlation curve [43], under the assumptions that the correlation structure is isotropic and time-invariant, while remaining dependent on the spatial location of the central node [44]. Approximation is carried out by a standard curve-fitting technique [43].
- (iii) Analysis of the spatial distribution of the short correlation length scale, L_s , obtained in the previous step. It is observed that for the period of study, the distributions of L_s are highly consistent across all four state variables within the upper 1000 m of the water column. Consequently, the surface-derived L_s for temperature is adopted for all three LD models, for all variables, and across all depth levels (see Figure 3). The long correlation scale is 440 km for all variables, similar to [43].

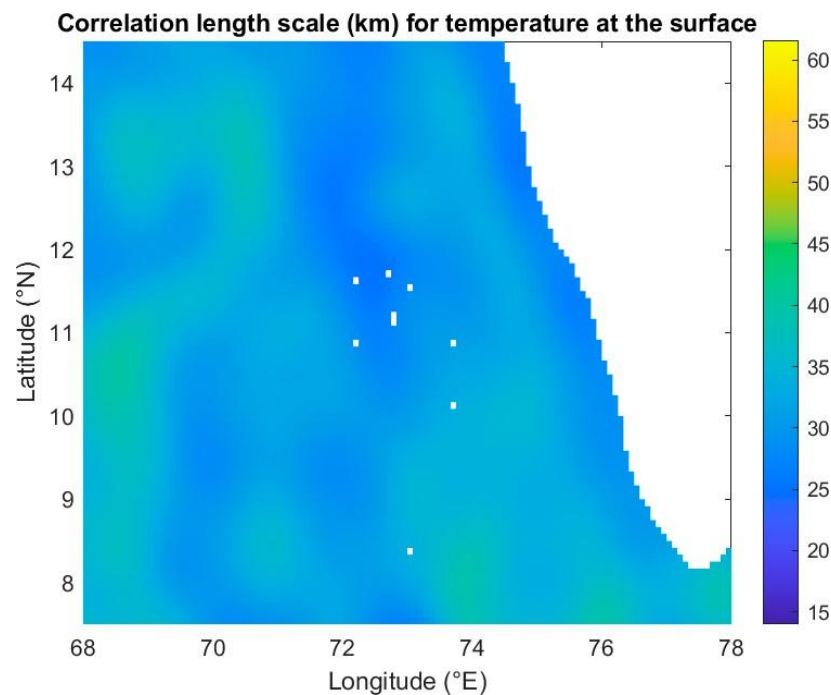


Figure 3. Horizontal distribution of the short correlation length scale for temperature at the surface in the study area. Reproduced from [44].

- (iv) Linear interpolation of the short correlation length scale, L_s , from the parent (or intermediate) model grid onto the higher-resolution (intermediate or fine) child model grid. This interpolated field is then used to construct a single-scale Gaussian correlation function of the form

$$C(\mathbf{r}, \mathbf{r}_0) = \exp \left[- \left(\frac{r}{L_s(\mathbf{r}_0)} \right)^2 \right], \quad (1)$$

where $\mathbf{r}$ denotes the distance between two points within the area of influence (typically six short correlation lengths), and $\mathbf{r}_0$ denotes the ‘central’ node on the fine-resolution grid at which the corresponding value of the short correlation length scale, L_s , is obtained.

- (v) Computation of objective analysis (also referred to as optimal interpolation) weights $P_{n,m}$ using the algorithms described in [45–47]. These weights are determined individually for each of the three LD models, reflecting the distinct parent–child grid combinations. The procedure involves solving a large system of linear Equation (2) for every relevant grid node, $\mathbf{r}_m$, for each of the fine and intermediate resolution models.

$$\sum_{n \in CI} C(|\mathbf{r}_n - \mathbf{r}_k|, \mathbf{r}_m) P_{n,m} = C(|\mathbf{r}_k - \mathbf{r}_0|, \mathbf{r}_m). \quad (2)$$

Here, $\mathbf{r}_n$ and $\mathbf{r}_k$ denote the coordinates of nodes on the coarser grid (i.e., CMEMS for direct nesting or LD20_COPDA for the second step of multi-nesting), while $\mathbf{r}_m$ represents the coordinate of a node on the finer grid (LD60small_COPDA for direct nesting, LD60small_LD20DA for the second step of multi-nesting, or LD20_COPDA for the first step of multi-nesting). The quantities $P_{n,m}$ are the weights to be computed, and CI defines the area of influence around the central node $\mathbf{r}_0$ used in calculating the Gaussian correlation function in Equation (1). Solving the system of linear equations for each fine-grid node generates hundreds of millions of weights per LD model. This computationally intensive step is performed only once at the start of the modelling experiment and does not slow down the routine cycling of model forecasts. Further details on the methodology and calculations are provided in [33,44–47].

- (vi) A subsequent step of the SDD, performed at each data assimilation cycle, involves applying the previously computed weights to the three-dimensional fields obtained from either the global CMEMS model (for direct nesting into LD60small_COPDA or into the intermediate LD20_COPDA) or from the intermediate LD20_COPDA model (for the secondary nest in the double-nesting approach). This procedure is carried out according to Equation (3):

$$T'_m = \sum_{n \in CI} P_{n,m} T'_n, \quad (3)$$

where T'_m , T'_n are temperature anomalies on the finer and coarser grids, respectively.

Theoretically, temperature anomalies should be computed relative to a statistical “norm” or statistical mean, which is generally unknown. By invoking the ergodic hypothesis, this norm can be approximated as the spatial average within the area of influence, CI . Similar procedures are applied to the other state variables. In certain scenarios, such as when nesting ratios are low, the SDD method alone may suffice as the primary downscaling approach, eliminating the need to run a dynamic high-resolution model [22].

2.8. Time Synchronisation

In our experiments, the data assimilation (DA) cycle is set to five days, with updates occurring at midnight GMT on the fifth day. For SDD using LD20_COPDA outputs, the necessary data are obtained directly from the model’s restart files, which are recorded at midnight, making the final SDD step straightforward. In contrast, CMEMS outputs are only provided as daily averages. To synchronise CMEMS data with the LD models, a midnight correction is applied, calculated as the difference between the midnight value from the relevant LD model (prior to DA) and its corresponding daily average. Under the assumption that the CMEMS model exhibits the same daily cycle at the same grid nodes as the LD models, the stochastically downscaled CMEMS daily averages are adjusted by this correction to produce instantaneous fields at midnight. The final outcome of the SDD procedure is a three-dimensional field of T , S , U , and V for each LD model at midnight, prepared for subsequent blending with outputs from dynamic downscaling.

2.9. Combination of Dynamic and Stochastic Downscaling

Dynamic downscaling involves running the relevant LD model in forecasting mode for a five-day period, using the analysis fields from the previous DA cycle as initial conditions and usual boundary conditions and meteorological forcing. By the end of the fifth day, two competing datasets exist on the same LD grid: (i) the stochastically downscaled dataset described previously, and (ii) the dataset generated by the dynamic LD model. These two datasets have to be blended together, thus finalising the DA cycle. This stage is summarised briefly here, with detailed descriptions available in [20,23]. Assimilation is performed through minimisation of the cost function J_S , as defined in Equation (4).

$$J_S(\mathbf{x}') = (\mathbf{x}' - \mathbf{x}'^b)^T \mathbf{B}^{-1} (\mathbf{x}' - \mathbf{x}'^b) + (\mathbf{S}(\mathbf{y}') - \mathbf{x}')^T \mathbf{R}^{-1} (\mathbf{S}(\mathbf{y}') - \mathbf{x}'). \quad (4)$$

In Equation (4), the vectors $\mathbf{x}$, $\mathbf{x}^b$, and $\mathbf{y}$ represent the (unknown) true ocean state, the output from the child model, and the stochastically downscaled fields from the parent model at midnight, respectively. The minimisation is performed not on the absolute values but on their anomalies (denoted by primed variables) relative to a statistical “norm”, which is computed as a spatial average within a small search area surrounding the ‘central’ node.

The matrices $\mathbf{B}$ and $\mathbf{R}$ are the error covariance matrices (ECMs) associated with the fields from the child and stochastically downscaled parent models, respectively. The symbol $\mathbf{R}$ denotes error covariances of the parent model ECM, since the parent fields act as pseudo-observations within the NDA framework. The operator $\mathbf{S}$ denotes the projection

applied to the parent model anomalies ($\mathbf{y}'$); it represents the SDD stage of the data assimilation procedure.

The minimum of the cost function is obtained by differentiating J_S with respect to its argument. This minimum is reached when

$$2(\mathbf{x}'^a - \mathbf{x}'^b)^T \mathbf{B}^{-1} - 2(\mathbf{S}\mathbf{y}' - \mathbf{x}'^a)^T \mathbf{R}^{-1} = 0, \quad (5)$$

where $\mathbf{x}'^a$ denotes the anomaly of the best estimate of the true state (i.e., analysis) relative to the statistical “norm”.

The NDA method utilises row-normalised matrices $\mathbf{Q}_B$ and $\mathbf{Q}_R$ in the form

$$\mathbf{B} = \mathbf{V}_B \mathbf{Q}_B, \quad \mathbf{R} = \mathbf{V}_R \mathbf{Q}_R. \quad (6)$$

Here, $\mathbf{V}_B$, $\mathbf{V}_R$ are diagonal matrices containing the respective error variances (i.e., diagonal elements of $\mathbf{B}$ and $\mathbf{R}$) at each child model grid node, i.e.,

$$\mathbf{V}_B = \text{diag}(\text{diag}(\mathbf{B})), \quad \mathbf{V}_R = \text{diag}(\text{diag}(\mathbf{R})). \quad (7)$$

The row-normalised matrices $\mathbf{Q}_B$ and $\mathbf{Q}_R$ consist of elements obtained by dividing elements of each row of the respective ECM by their maximum value, i.e., the corresponding diagonal element. These matrices describe the spatial structure of error covariances for the child and stochastically downscaled parent models.

The difference in the structure of the matrix $\mathbf{Q}_R$ represents the principal distinction between conventional data assimilation approaches and the NDA methodology. The common assumption in conventional assimilation of real-world observations is that the observational errors are uncorrelated and therefore the matrix $\mathbf{R}$ is assumed to be diagonal [33,35,36], which is equivalent to $\mathbf{Q}_R$ being an identity matrix $\mathbf{I}$. This assumption does not hold for pseudo-observations obtained from the model output where errors are likely to be spatially correlated [35]. Therefore, $\mathbf{R}$ is a non-diagonal symmetric matrix and $\mathbf{Q}_R$ cannot be an identity matrix (see more details in [20,23]).

The outputs from the child model and the stochastically downscaled parent model are defined on the same grid, cover the same oceanic region, and are generated using the same numerical code (in this case, NEMO 3.6); hence, it is reasonable to assume that their spatial correlation structures represented by normalised matrices $\mathbf{Q}_B$ and $\mathbf{Q}_R$ are similar. The validity of this assumption has been assessed in [20,23] and will be further discussed in Sections 3.1 and 3.2. Consequently, the NDA approach adopts $\mathbf{Q}_R = \mathbf{Q}_B$, implying that the pseudo-observational errors share the same spatial correlation structure as those of the child model, rather than assuming that $\mathbf{Q}_R = \mathbf{I}$ (i.e., that the errors are spatially uncorrelated), see [20]. The validity of this assumption has been extensively evaluated through comparisons between the child model outputs and real-world observational data, as documented in [20,23]. From a practical point of view, the NDA technique enables a significant simplification of calculations by eliminating very large and generally unknown matrices $\mathbf{Q}_R$ and $\mathbf{Q}_B$ from the equations.

After performing the necessary algebraic manipulations, one can obtain the following expression for each element $x_i'^a$ of the anomaly of the analysis vector $\mathbf{x}'^a$:

$$x_i'^a = \frac{V_{Rii}}{V_{Rii} + V_{Bii}} x_i'^b + \frac{V_{Bii}}{V_{Rii} + V_{Bii}} S(\mathbf{y}')_i = KW_i x_i'^b + (1 - KW_i) S(\mathbf{y}')_i, \quad (8)$$

where

$$KW_i = \frac{V_{Rii}}{V_{Rii} + V_{Bii}}$$

is the Kalman gain coefficient [42] at each child model grid node. The full value of $x_i'^a$ is obtained by adding $x_i'^a$ to the statistical ‘norm’ $\langle S(\mathbf{y}) \rangle_i$ computed from the parent model output. Equation (8) indicates that the NDA technique effectively reduces the dimensionality of the Kalman filter to zero, as the filtering procedure is applied independently at

each grid point, without requiring data from the neighbouring nodes. More details of the NDA procedure are given in [20,23].

3. Results and Discussion

Sample SST outputs from both LD60 models, as well as from CMEMS and OSTIA, are presented in Figure 4 for 31 March 2016, approximately 15 months after the commencement of simulations on 1 January 2015. As anticipated, the higher-resolution LD60 models resolve finer-scale structures more effectively than the coarser-resolution global model.

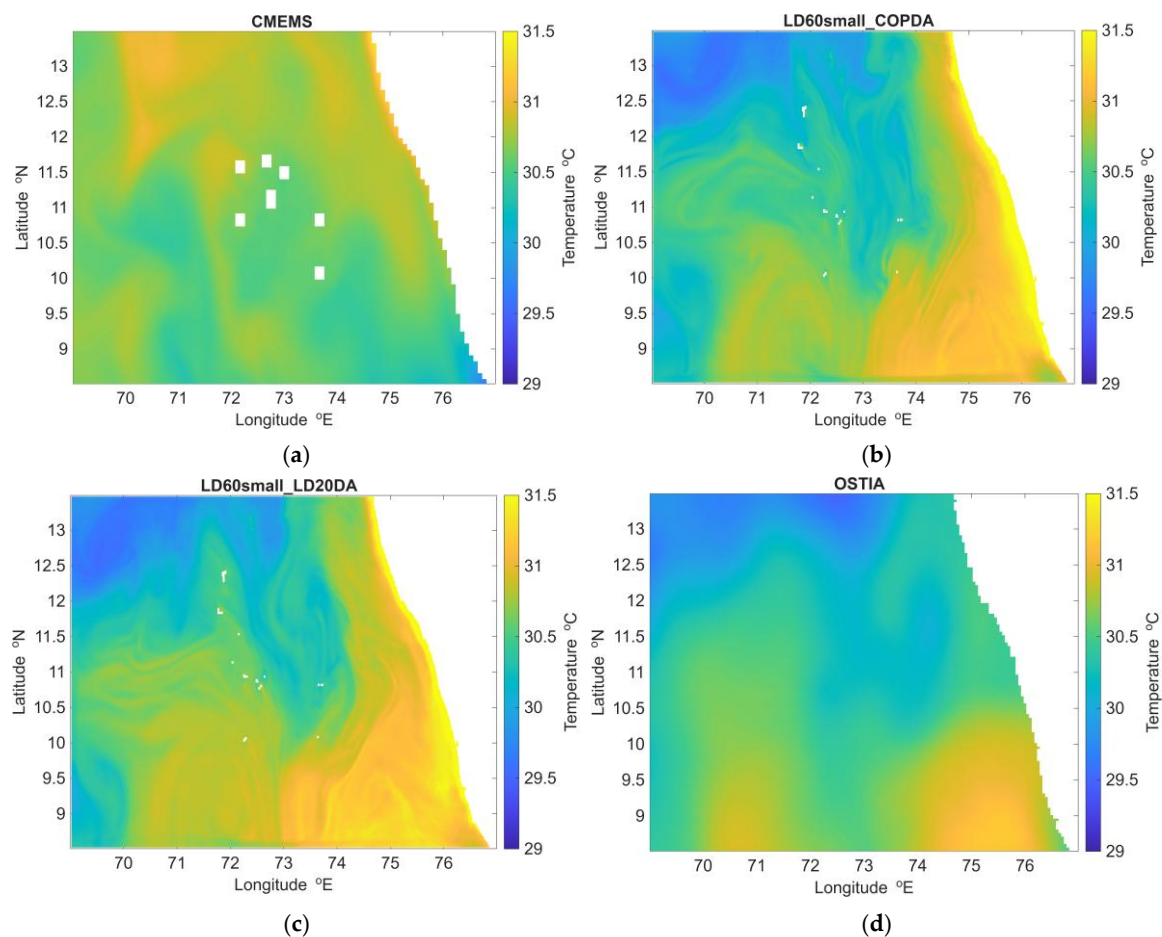


Figure 4. Sea surface temperature on 31 March 2016 from the (a) CMEMS model; (b) LD60small_COPDA; (c) LD60small_LD20DA; and (d) OSTIA.

The capacity of datasets from different sources (models and observations) to capture small-scale SST variability is further illustrated in Figure 5, which depicts maps of the squared SST gradient. The gradient is expressed in units of $^{\circ}\text{C}/^{\circ}\text{Lat}$, where $1^{\circ}\text{Lat} = 60$ nautical miles. Notably, despite its finer nominal resolution compared to CMEMS ($1/20^{\circ}$ versus $1/12^{\circ}$), OSTIA yields a comparatively smoother SST field (see Figure 4a,d).

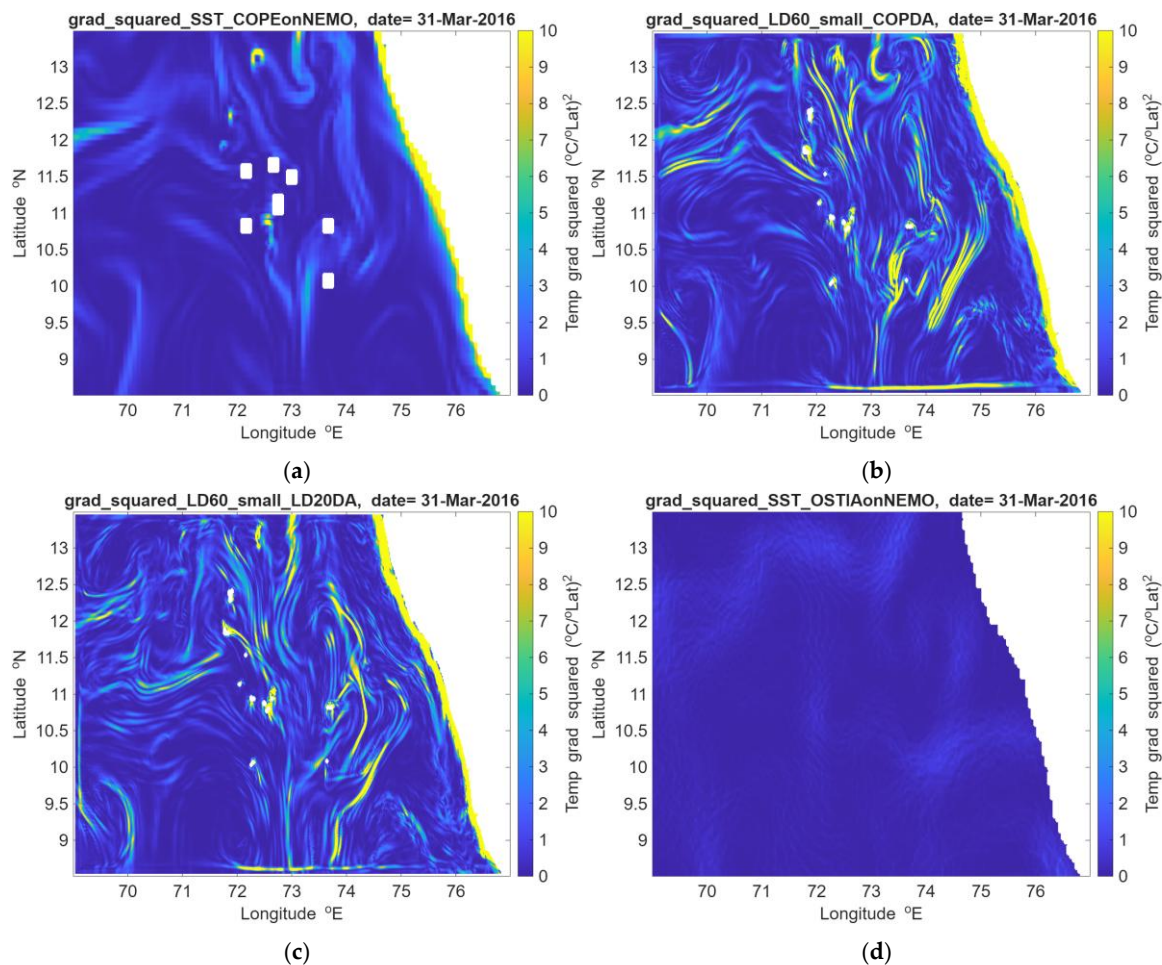


Figure 5. The square of the SST gradient computed from the same data sources as in Figure 4, (a) CMEMS model; (b) LD60small_COPDA; (c) LD60small_LD20DA; and (d) OSTIA. All data have been interpolated onto the LD60 grid.

A comparison of Figure 5b,c indicates that both the telescopically and directly nested models reveal broadly similar, though not identical, structures of temperature fronts. The most intense fronts are also captured by the coarser-resolution global model, whereas they are largely absent on the OSTIA-derived map. This discrepancy is likely attributable to the spatial interpolation procedures inherent in Level 4 products such as OSTIA [39], which tend to smooth the SST distribution during periods of extensive cloud cover.

3.1. Model Validation Against OSTIA

The outputs from the LD60small_COPDA and LD60_LD20DA models are validated against SST data from OSTIA [39] over the period from 1 January to 31 December 2016. The model-derived SST is extracted from the 5 m model layer to minimise the influence of the diurnal cycle and provide a closer match to the OSTIA foundation SST. The OSTIA dataset, originally provided at a spatial resolution of $1/20^\circ$ (approximately 6 km), is interpolated onto the LD60 model grid with a resolution of $1/60^\circ$ (approximately 1.8 km).

Figure 6 illustrates the seasonal cycle of area-averaged SST for the year 2016. The highest temperatures, approximately 31.5°C , occur in late spring, while the lowest values, around 27.5°C , are observed at the end of the southwest monsoon period in September. Given the relatively narrow range of temperature variability, achieving high accuracy in SST forecasts is essential for reliably representing and predicting the underlying dynamical processes in the study region.

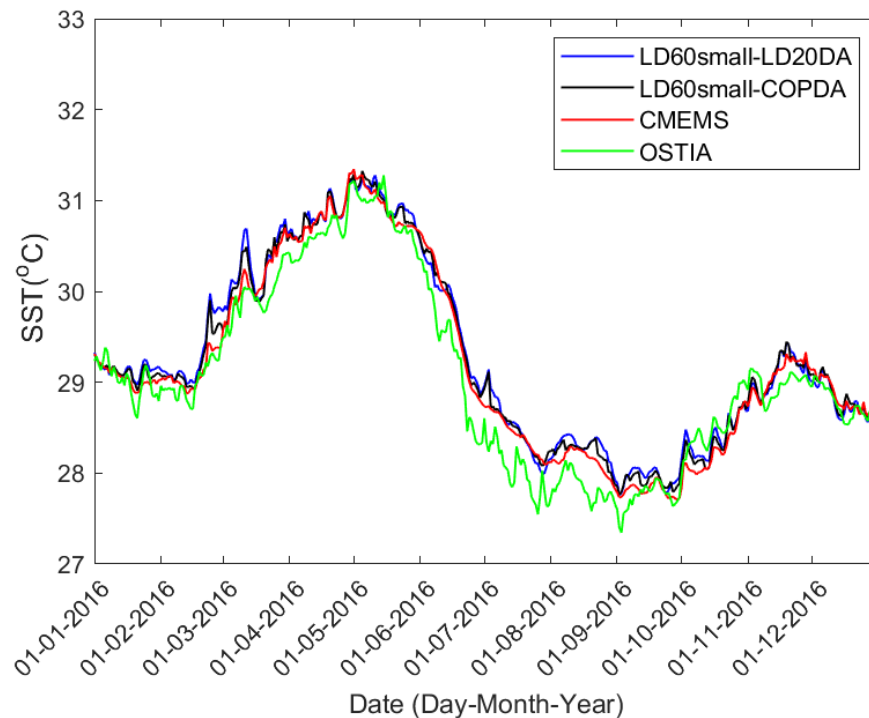


Figure 6. Time series of daily average sea surface temperature (SST) from the directly nested LD60small_COPDA model (black) and the double-nested LD60small_LD20DA model (blue). For comparison, corresponding SST time series from OSTIA observations (green) and the CMEMS global reanalysis (red) are also shown.

In line with similar studies assessing the model skill (e.g., ref. [3,5,48]), the area-averaged bias, root-mean-square error (RMSE), and the Pearson correlation coefficient are computed on a daily basis to evaluate the performance of both LD60 models in forecasting the SST using the following equations:

$$BIAS_SST(t) = \frac{1}{N} \sum_{i=1}^N (M_i(t) - O_i(t)), \quad (9)$$

$$RMSE_SST(t) = \left[\frac{1}{N} \sum_{i=1}^N (M_i(t) - O_i(t))^2 \right]^{1/2}, \quad (10)$$

$$COR_SST(t) = \frac{1}{N} \sum_{i=1}^N \frac{(M_i(t) - M(t))(O_i(t) - O(t))}{\sigma_M(t)\sigma_O(t)}. \quad (11)$$

Here, $M_i(t)$ and $O_i(t)$ denote the model and OSTIA SST values at the fine-grid node i on day t , $M(t)$ and $O(t)$ are the corresponding area-averaged values, and $\sigma_M(t)$ and $\sigma_O(t)$ are their standard deviations for the same day. N represents the total number of wet grid nodes at the sea surface.

Figures 7–9 present the time series of the model skill statistics, with corresponding metrics for the parent data-assimilating global model CMEMS included for comparison. Figure 7 shows that the bias from the directly nested LD60small_COPDA model is nearly identical to, and occasionally better than, that of the double-nested LD60small_LD20DA model. For both high-resolution models, the bias remains within the reported OSTIA accuracy of 0.5 °C. Slightly elevated biases are observed in April and during the July–September period, with differences between the fine-resolution LD60 models and the coarser CMEMS model also increasing slightly in these months. A similar pattern is evident in the time series of the RMSE, as illustrated in Figure 8.

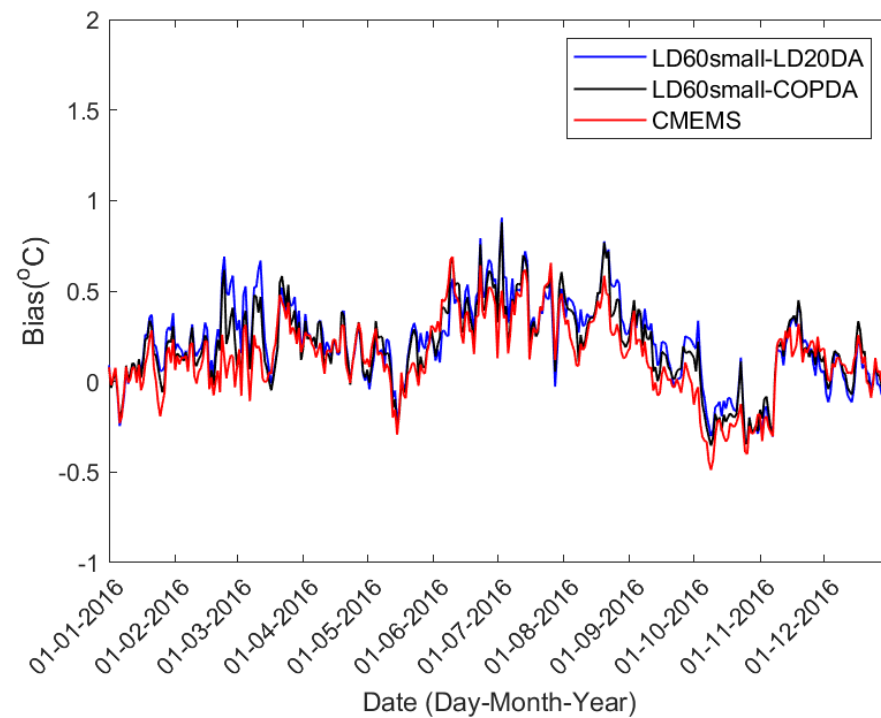


Figure 7. Time series of area-averaged bias for the same models as shown in Figure 6, with biases computed relative to the OSTIA reference dataset.

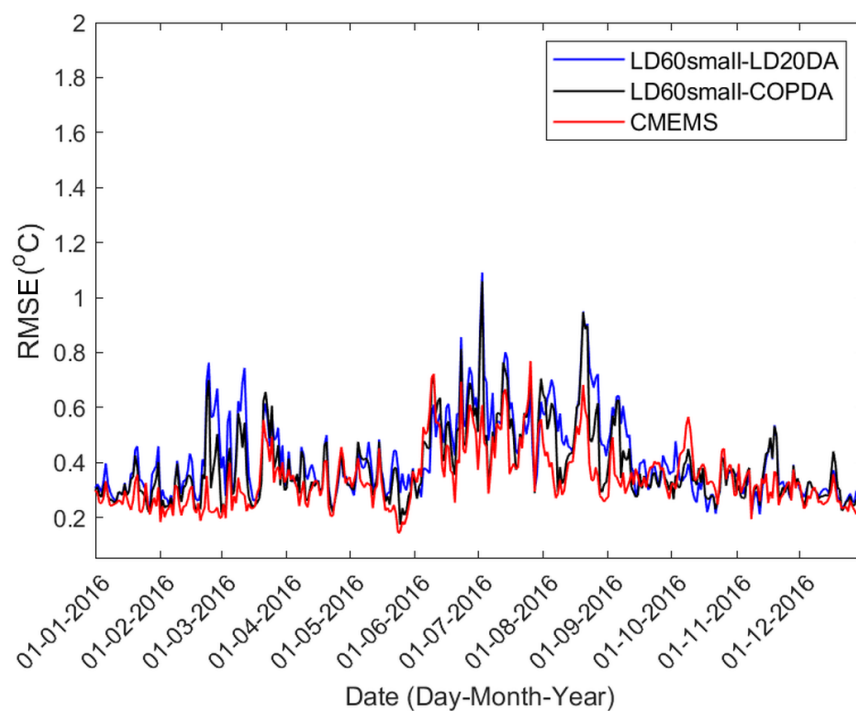


Figure 8. Time series of area-averaged RMSE for the same models as in Figure 6 relative to OSTIA.

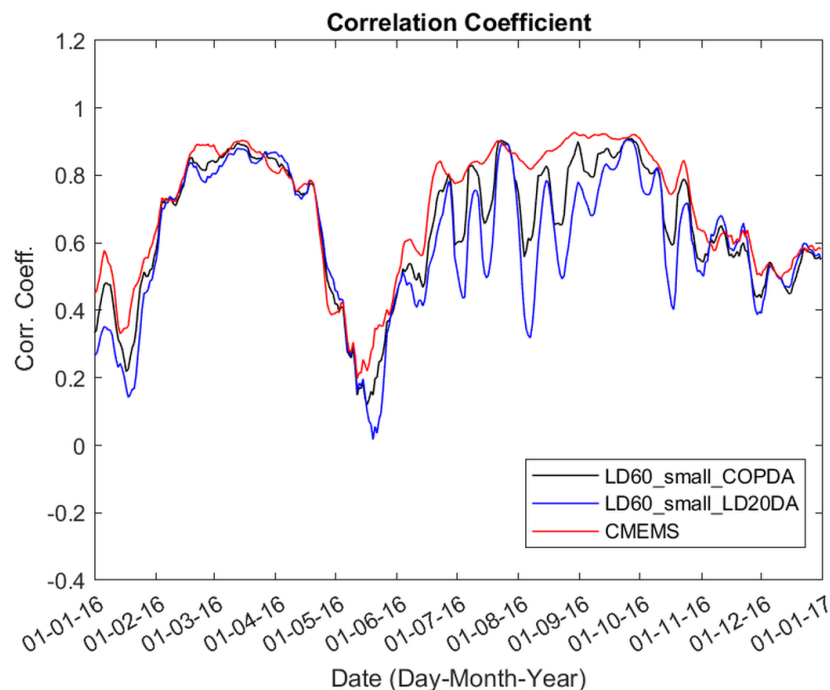


Figure 9. Time series of Pearson correlation coefficient COR_{SST} (see Equation (11)) for the same models as in Figure 5, relative to OSTIA.

Figure 8 demonstrates that the RMSE exhibits greater variability than the bias, particularly during spring (March–April) and the southwest monsoon period (July–August). For both high-resolution LD60 models, deviations from the OSTIA in terms of bias and RMSE are larger than those observed for the coarser CMEMS model. A partial explanation for the March–April differences is increased cloudiness, as indicated by a higher multi-year average of rainy days (15–22 days) in March–April compared to only 5 days in January–February [49]. During the southwest monsoon period, persistent cloud cover, fewer sunny days, and increased rainfall lead to extensive interpolation of satellite-derived SST data, which can obscure meso- and sub-mesoscale features that are captured by the models. The elevated bias and RMSE, along with a lower SST correlation (COR_{SST} , see Figure 9) during this period, may also reflect higher meso- and sub-mesoscale activity driven by stronger winds: the maximum gust in August 2016 reached 17.0 km/h, compared with 12.0 km/h in May. The wind data referenced above are the area average over the neighbouring state of Kerala (India) [50].

Periods of low correlation coefficients shown in Figure 9 correspond to intervals of low spatial SST variability, characterised by low standard deviations of 0.3 °C or less (see Figure 10). Such small variations fall within the uncertainty bounds of OSTIA (approximately 0.5 °C) [51] (Table IV.1) and are close to the accuracy limits of the models. The CMEMS model is reported to have an SST RMSE of approximately 0.6–0.7 °C for 2016 when compared with in situ measurements [52] (Figure 1.7.1c), while the LD60 models exhibit slightly better RMSE values (see Table 1).

Table 1. Statistics of SST uncertainty for nested models against OSTIA.

	RMSE_SST	COR_SST	BIAS_SST
LD60small_COPDA (direct nesting)	0.42	0.63	0.09
LD60small_LD20DA (double telescopic nesting)	0.40	0.58	0.11

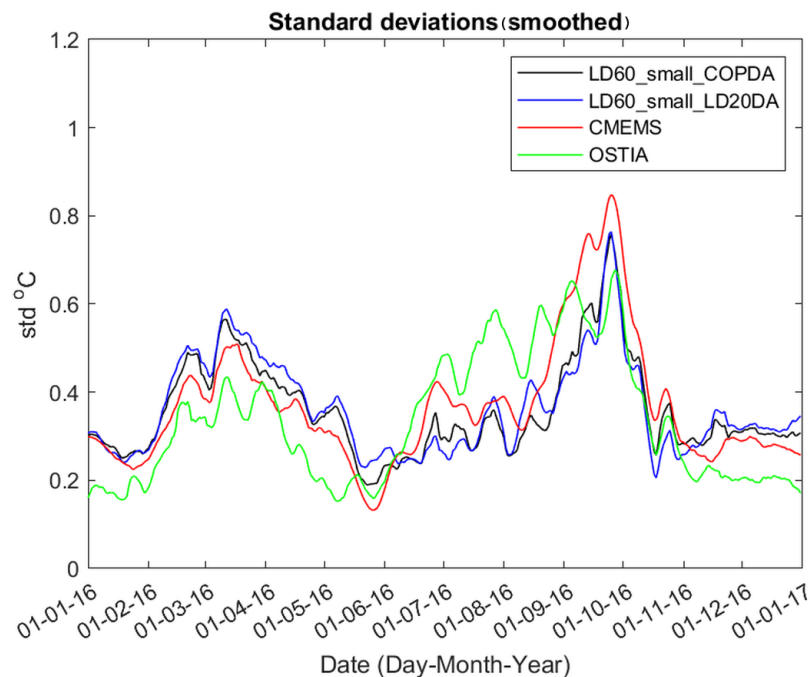


Figure 10. Time series of SST standard deviations across the study area for the three models and OSTIA. The daily time series is smoothed with a moving window of 7 days for clarity of presentation.

Therefore, SST variations within the standard deviation bounds of $0.3\text{ }^{\circ}\text{C}$ are likely dominated by random, uncorrelated errors, which act to reduce the correlation coefficient. In contrast, the correlation improves (with $\text{COR}_{\text{SST}} \geq 0.6$) when SST variability exceeds $0.5\text{ }^{\circ}\text{C}$, indicating more robust and physically meaningful signal representation from the models under conditions of stronger spatial gradients.

Table 1 presents the error statistics for the directly and telescopically nested models, averaged over the period from 1 January to 31 December 2016, using the same daily datasets as those shown in Figures 7–9. The results indicate that the directly nested model, LD60small_COPDA, achieves a level of accuracy comparable to that of the double-nested model.

Figure 11 presents bar charts of the time-averaged bias and RMSE for the LD60 models alongside those from the industry-leading data sources [39,48,51,52]. The averaging period for the LD60 models spans from 1 January to 31 December 2016. It should be noted that the CMEMS, AMM15, and AMM7 reanalysis products incorporate bias-correction algorithms [26,48,51,52], whereas no bias correction is applied to the LD60 models.

The results presented in Figures 7, 8 and 11, together with Table 1, indicate that the SST accuracy achieved by both directly and telescopically nested models is comparable to the best industry standards and lies within the uncertainty bounds of the OSTIA dataset itself. For both LD60 configurations, the SST bias relative to OSTIA is small, approximately $0.1\text{ }^{\circ}\text{C}$, and the RMSE is around $0.4\text{ }^{\circ}\text{C}$. The design specification for OSTIA indicates an RMSE of $0.8\text{ }^{\circ}\text{C}$; however, its actual performance is estimated to be closer to $0.5\text{ }^{\circ}\text{C}$ [53]. The most recent OSTIA version achieves a bias of $0.07\text{ }^{\circ}\text{C}$ and an RMSE of $0.45\text{ }^{\circ}\text{C}$ when validated against in situ observations in the Indian Ocean [51] (Table IV.1). By comparison, the CMEMS model exhibits an RMSE of approximately $0.65\text{ }^{\circ}\text{C}$ against observations over the same period as this study, see [52] (Figure 1.7.1c). The latest Met Office operational model, AMM15, achieves an RMSE of $0.34\text{ }^{\circ}\text{C}$ relative to OSTIA and $0.48\text{ }^{\circ}\text{C}$ against in situ data [48], which is comparable with the performance of the LD60 models ($\text{RMSE} \approx 0.40\text{--}0.42$; see Table 1).

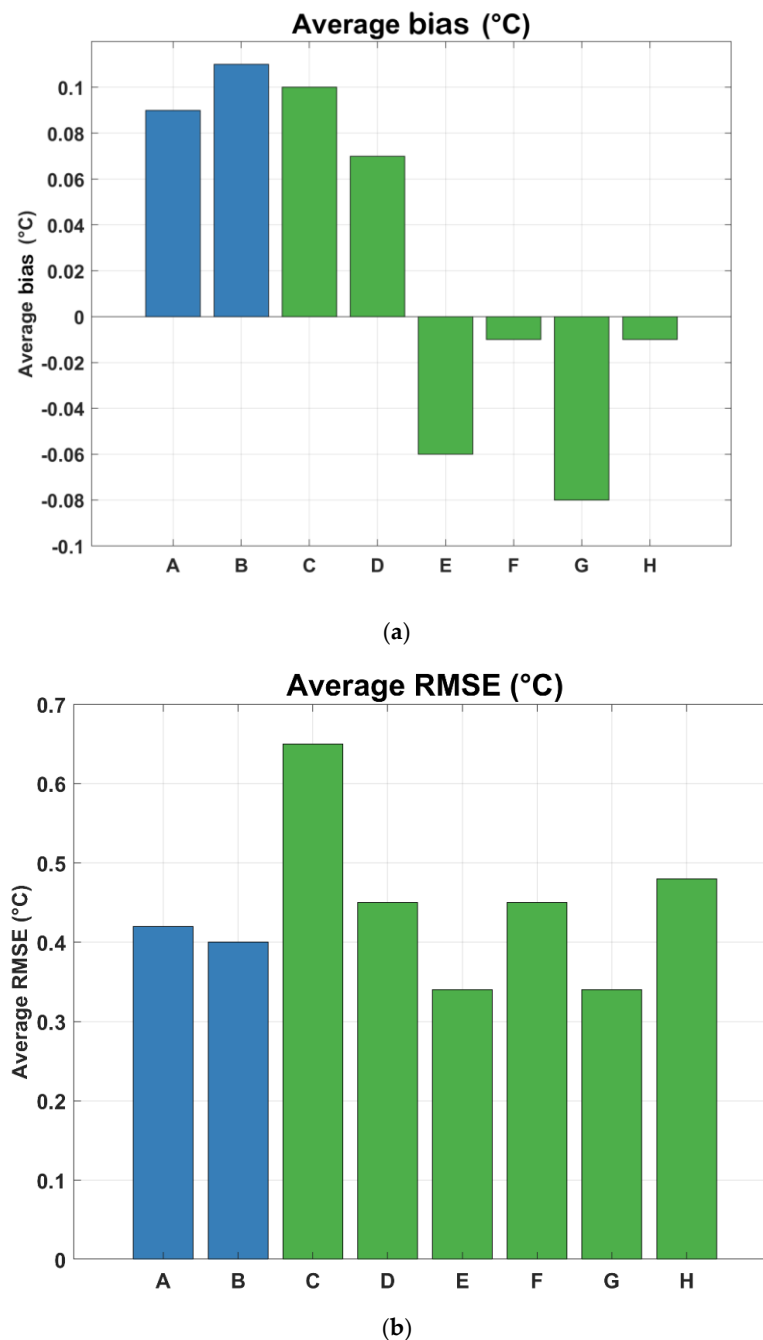


Figure 11. (a) Time-averaged bias and (b) RMSE for (A) LD60small_COPDA (direct nesting); (B) LD60small_LD20DA (telescopic nesting); (C) CMEMS reanalysis against in situ, (D) OSTIA; (E) AMM7 against OSTIA; (F) AMM7 against in situ; (G) AMM15 against OSTIA; and (H) AMM15 against in situ. LD60 data are shown in blue.

Higher-resolution models are often associated with an increased RMSE compared to their lower-resolution counterparts due to the so-called “double-penalty” effect [54]. However, the results presented in Table 1 and Figures 8 and 11 indicate that any potential deterioration in RMSE for the higher-resolution LD60 models, relative to the coarser CMEMS model, is negligible or absent.

3.2. Model Validation Against Argo Floats

A total of 43 Argo profiles are available within the study area for the year 2016. For the purpose of model skill assessment, temperature and salinity profiles from both LD60 models are co-located with the observations in space and time using bilinear interpolation.

Since the observational profiles have a higher vertical resolution than the model outputs, they are interpolated onto the model's computational temperature levels. The bias and RMSE are then calculated using the following equations:

$$BIAS_ARGO = \frac{1}{L} \sum_{i=1}^L (M_i(z) - A_i(z)), \quad (12)$$

$$RMSE_ARGO = \left[\frac{1}{L} \sum_{i=1}^L (M_i(z) - A_i(z))^2 \right]^{1/2}, \quad (13)$$

where $M_i(z)$ and $A_i(z)$ are the model and Argo data at model depth level z , and L is the total number of Argo profiles.

The results of calculations are presented in Figure 12. The accuracy of both models throughout the water column is broadly consistent with that observed at the surface, although slightly larger discrepancies are evident within the 50–250 m depth range. This layer is known to be particularly challenging to represent accurately in the Indian Ocean, and similar deviations have been reported in quality assessments of global reanalysis products [52]. A minor fictitious warm bias (often described as a “warm belly”) at approximately 200 m depth is apparent in Figure 12; this feature is common across all models known to us, including the global reanalysis [52]. Overall, the difference in performance between the directly and telescopically nested LD60 configurations is minimal.

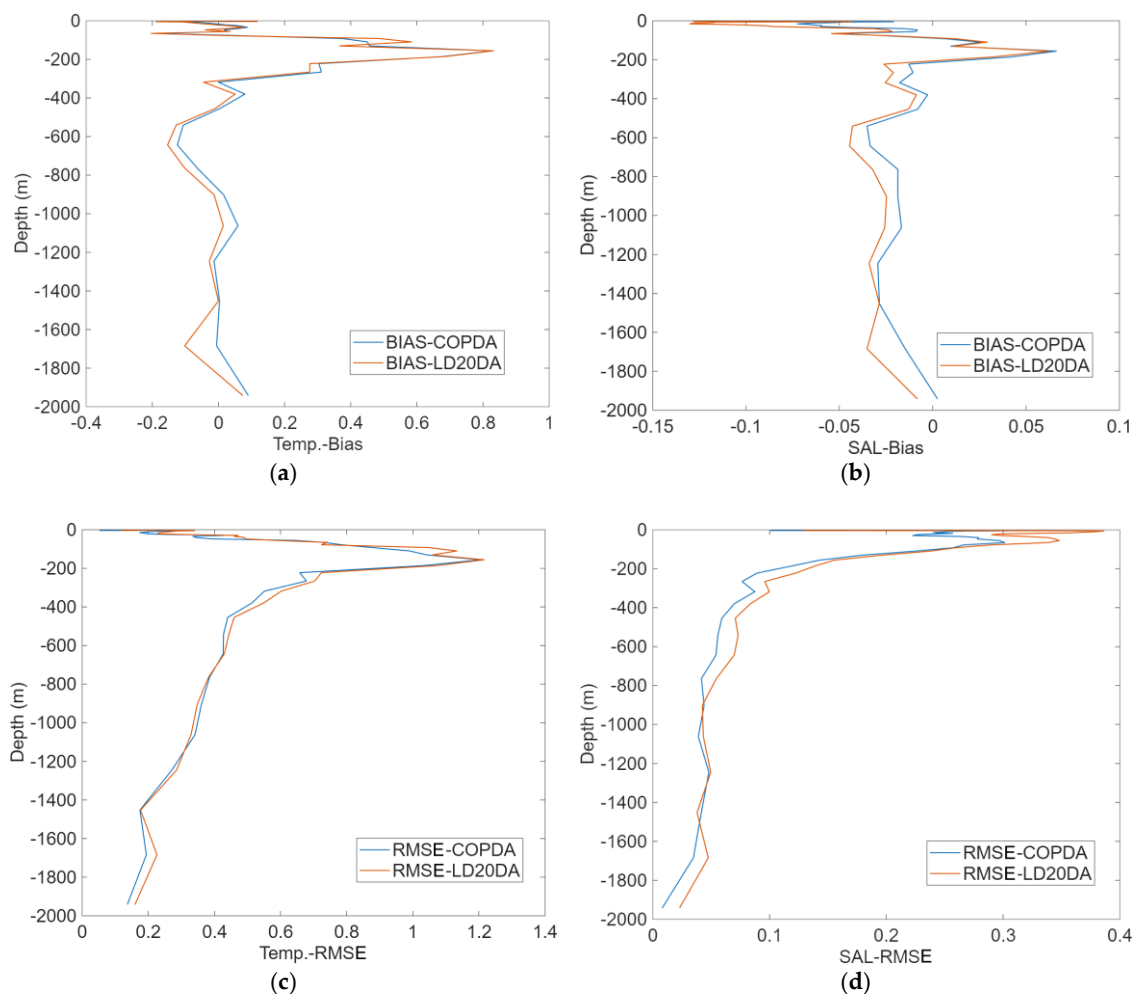


Figure 12. Validation of the directly and telescopically nested models against Argo observations: (a) temperature bias; (b) salinity bias; (c) temperature RMSE; (d) salinity RMSE.

3.3. Model Inter-Comparison

In this subsection, the directly and telescopically nested models are compared in order to assess the differences between them. Figure 13 presents time series of the correlation coefficient (CORR) between area-averaged potential temperatures produced by LD60small_COPDA and LD60small_LD20DA models at selected depth levels.

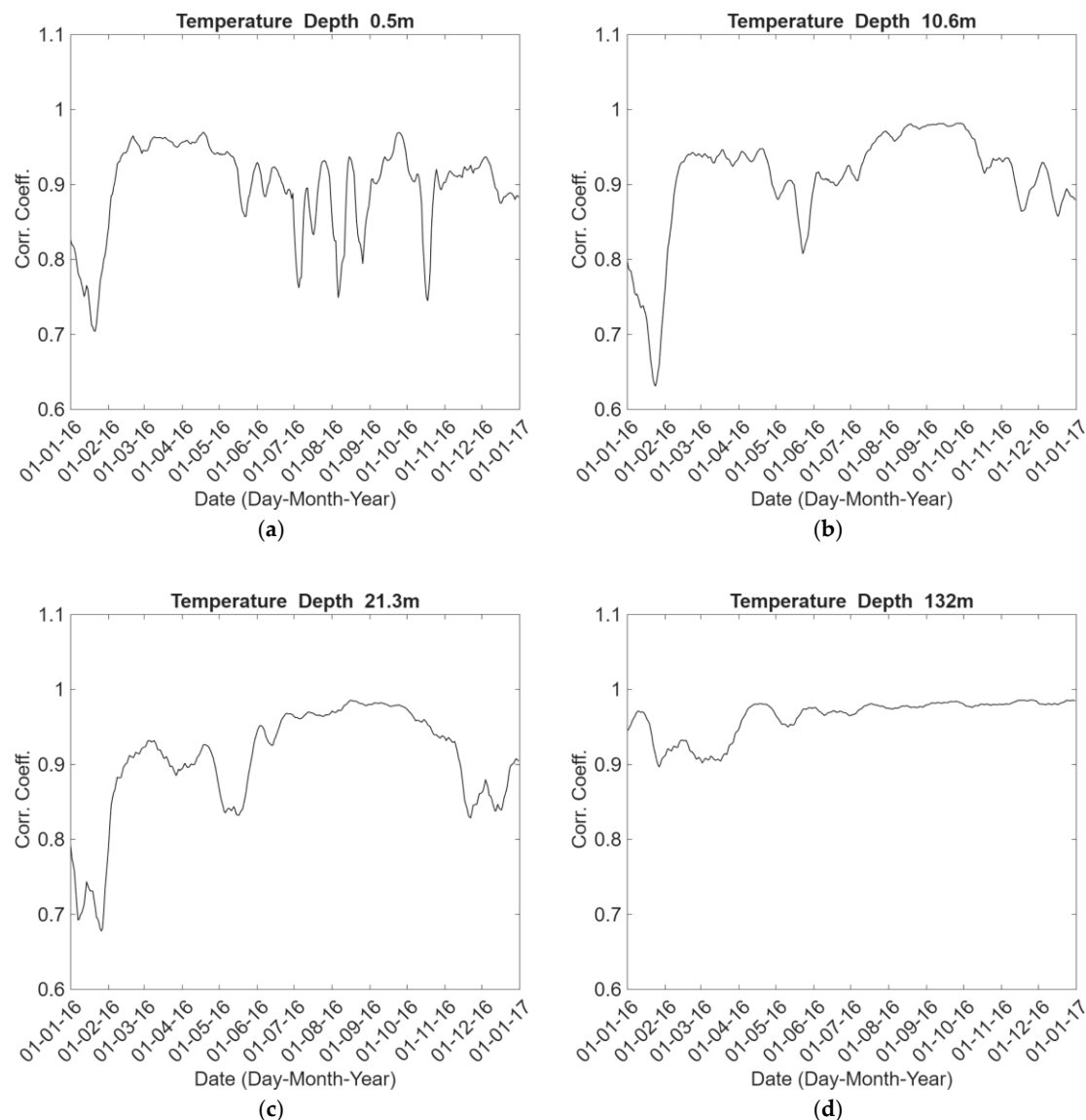


Figure 13. Pearson correlation coefficients of area-averaged temperature between LD60small_COPDA and LD60small_LD20DA models as a function of time at depths (a) 1.5 m; (b) 10.6 m; (c) 21.3 m; and (d) 132 m.

Figure 14 shows the corresponding correlations for horizontal salinity distributions within the study area. The correlation values are generally high, exceeding 0.9 for both temperature and salinity, with the exception of a slight reduction in temperature correlation observed in January 2016.

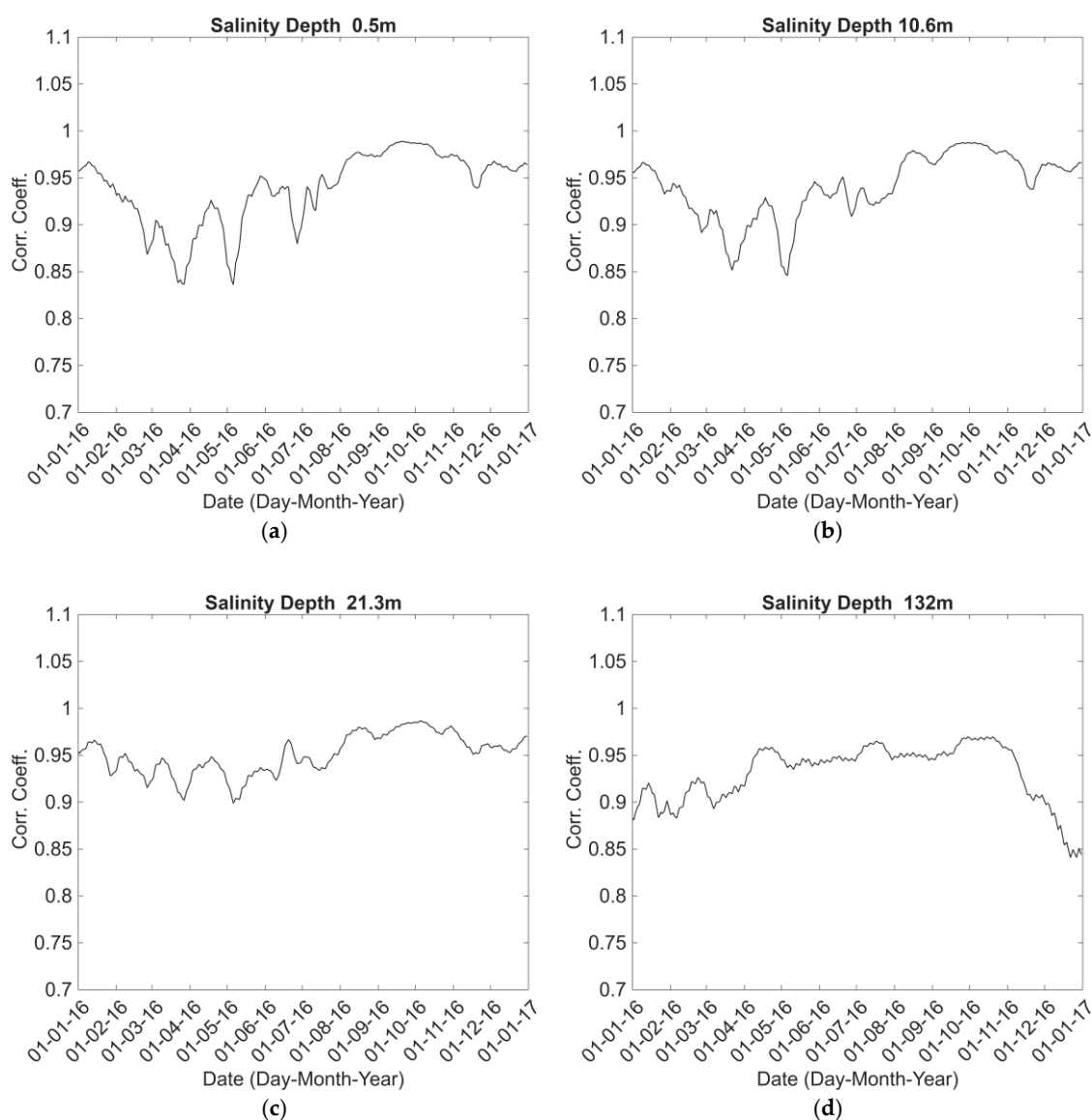


Figure 14. Pearson correlation coefficients of area-averaged salinity between LD60small_COPDA and LD60small_LD20DA models as a function of time at depths (a) 1.5 m; (b) 10.6 m; (c) 21.3 m; and (d) 132 m. In addition, the bias and RMSE between the two models are computed on a daily basis using formulations analogous to Equations (9)–(11). These statistics are subsequently averaged over the year 2016 and summarised in Table 2. When the differences between the two LD60 models (Table 2) are compared with the discrepancies between the models and observations (Table 1 and Figure 12), it is clear that the inter-model differences are contained well within the overall accuracy bounds of the models. Consequently, the LD60small_COPDA model, which is directly nested from the global model with a 5:1 nesting ratio, and the LD60small_LD20 model, which is telescopically nested via the intermediate LD20_DACOP model, yield broadly similar results. A closer examination indicates that the model featuring direct nesting exhibits slightly better performance, despite a higher nesting ratio.

Table 2. Differences between directly and telescopically nested models.

	Surface	Depth 10.6 m	Depth 21.3 m	Depth 132 m
CORR (temperature)	0.90	0.91	0.91	0.96
Bias (temperature, °C)	−0.023	−0.001	0.035	0.022
RMSE (temperature, °C)	0.17	0.18	0.21	0.35
CORR (salinity)	0.94	0.94	0.95	0.93

Bias (salinity)	0.026	0.010	0.004	0.005
RMSE (salinity)	0.20	0.16	0.16	0.06

A previous study by the authors [23] demonstrated that an operational regional model can achieve high-quality results using a nesting ratio as high as $r = 10:1$ when applying the NDA technique. However, telescopic nesting was not evaluated in that work. The present study extends this analysis by directly comparing outputs from directly nested and double-nested models. The results show that, when combined with the NDA method, direct nesting produces results that are consistent, or slightly better, than those from the double-nested approach.

From a computational perspective, the direct nesting approach is more efficient than its double-nesting counterpart. For the specific settings used in this study, a 5-day simulation cycle, including the nested data assimilation (NDA) procedure, requires, on average, 81 min of CPU time for the telescopic nesting approach, which comprises both the intermediate and fine-resolution models. By comparison, the directly nested configuration requires only 57 min, reducing the computational cost by approximately 42%. This estimate is based on the total CPU time required to perform one month of simulations, including all model components, the one-time cost of NDA weight computation, execution of the intermediate model, operation of the ROSE-CYLC workflow, and input/output operations. It does not account for the additional savings in staff time associated with setting up, calibrating, and validating only a single nested model.

4. Conclusions

High-resolution regional ocean models are often nested into coarser-resolution global models via one or more intermediate resolution models. Telescopic nesting reduces numerical artefacts caused by large differences in model resolution, but it also increases system complexity and computational cost. The present study evaluates whether the intermediate model can be eliminated by replacing traditional data assimilation techniques with a recently developed NDA method based on a model-to-model assimilation approach. In this approach, the role of an intermediate model is transferred to an intermediate layer in the data assimilation system. Real-world observations are assimilated into the parent model, which provides initial and boundary conditions for the child model. The child model then assimilates 3D data from the parent model. An important advantage of this approach is that the parent-model velocity fields are already dynamically consistent with density, thereby avoiding the need for additional geostrophic balancing.

The approach was evaluated by comparing direct nesting from a $1/12^\circ$ global into a $1/60^\circ$ regional model with telescopic nesting via an intermediate $1/20^\circ$ model in the South East Arabian Sea. Validation against OSTIA SST and Argo profiles demonstrates that direct nesting combined with model-to-model data assimilation produces accurate results, with performance comparable to that of the most advanced operational models.

By eliminating intermediate nesting levels, direct nesting with NDA simplifies the modelling system, reduces computational cost, and decreases the effort required to configure, calibrate, validate, and maintain multiple nested model configurations. These results demonstrate that the model-to-model NDA method provides an effective means for achieving accurate high-resolution simulations without the need for intermediate-resolution dynamical models. The approach is also potentially transferable to other ocean regions and modelling systems.

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