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Möbius Quantum Computation: A New Topological Resource for Quantum Error Cancellation

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Abstract

We formulate a Möbius chain mechanism for passive cancellation of coherent quantum errors under specified symmetry and evolution conditions. Möbius Quantum Computation (MQC) is a new topological boundary framework for quantum information processing, in which non-orientable boundary conditions enter explicitly into the effective quantum-state evolution. Here, we investigate a concrete consequence within this framework: the suppression of selected coherent error contributions through Möbius chain boundary conditions. While conventional quantum error correction uses redundant logical encoding, syndrome extraction, and active recovery operations, the proposed mechanism exploits a global non-orientable boundary identification, together with a compatible transformation of the error dynamics. The Möbius identification relates propagation along the chain to a twisted counterpart. The resulting cancellation depends on how the boundary transformation acts on the relevant error generators, rather than on the identification alone. We show that coherent phase errors accumulated along the ordinary branch can act against their topologically reflected contribution and they cancel themselves. The resulting mechanism is boundary-driven and passive. We formulate the boundary condition, derive the basic cancellation relation, discuss residual errors due to imperfect symmetry, and compare the proposal with dynamical decoupling, decoherence-free subspaces, stabilizer codes, and topological quantum computation. The present results identify Möbius boundary conditions as a promising and efficient resource that may complement and optimize conventional quantum error mitigation and quantum error correction. Our quantum approach is theoretical and conceptual. Its engineering and experimental architecture deserve further investigation.

Keywords: quantum computation; Möbius chain; geometric error cancellation; topological boundary conditions; quantum information processing; coherent errors

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1. Introduction

Quantum computation has emerged as one of the most promising developments in contemporary science and technology. Since Feynman's seminal proposal that quantum systems could be efficiently simulated by computers governed by quantum-mechanical principles [1], the field has developed into a general framework for information processing [2] based on superposition, entanglement, and quantum interference, as systematically formulated by Nielsen and Chuang [3].

Potential applications range from the simulation of complex quantum systems and the discovery of new materials to cryptography, optimization, and problems that remain

intractable for classical computers [4]. Despite remarkable theoretical and experimental progress, however, the realization of large-scale and reliable quantum processors continues to face a central challenge: the extreme sensitivity of quantum states to noise, decoherence, and operational imperfections [5].

Quantum information processing is fundamentally limited by the unavoidable interaction of quantum systems with their environment, which produces decoherence, phase instability, and the accumulation of computational errors [6,7]. Conventional quantum-error-correction strategies, beginning with the pioneering schemes of Shor and Steane [8,9] and the general conditions formulated by Knill and Laflamme [10], address these effects through logical redundancy, ancillary qubits, and repeated syndrome measurements, often at the cost of considerable physical and operational resources. Topological quantum computation offers an alternative route in which information is protected through nonlocal geometric properties [11,12].

In this paper, within our new research on Möbius quantum computation [13], we investigate a new and complementary approach on quantum-error cancellation based on the global topology of the quantum system. We show here that Möbius chain boundary conditions can efficiently correlate apparently distinct error pathways and produce their destructive interference, thereby providing a *geometric mechanism for intrinsic quantum-error cancellation*.

Very recently, Google Quantum AI and Google DeepMind introduced an artificial-intelligence-based strategy for the continuous control of quantum error correction [14] using error-detection signals to compensate for processor drift computation. Our Möbius approach is conceptually different: rather than relying primarily on external adaptive control, we exploit non-orientable boundary conditions to correlate error pathways and promote their destructive quantum interference. In addition, our intrinsic geometrical mechanism could operate independently or be incorporated into adaptive schemes such as that of Google Ref. [14], potentially reducing the residual error burden and providing the learning agent with a more stable and structured error landscape.

The current interest and developments in the subject provides a timely experimental context for exploring different and complementary approaches—such as the Möbius boundary mechanism proposed here—in which error suppression arises intrinsically from topology, geometry and quantum interference.

Our quantum approach here is theoretical and conceptual, rather than engineering and experimental; its practical and quantitative implementation and combination with the new Google AI error control [14] and other controls deserve investigation.

Quantum computation is limited not only by stochastic decoherence but also by coherent and systematic errors. Such errors include phase drifts, calibration imperfections, systematic over-rotations, residual couplings, and slowly varying control errors. Because coherent errors can increase constructively over many operations, they may be especially damaging in long quantum circuits.

Standard quantum error correction suppresses errors by encoding logical qubits into larger Hilbert spaces, measuring stabilizers, and applying active recovery operations. This strategy is essential for fault-tolerant architectures, but it is resource-intensive. It is therefore useful to investigate complementary passive mechanisms capable of suppressing specific classes of errors before, or in parallel with, active correction.

In Ref. [13], we proposed Möbius Quantum Computation (MQC) as a topological boundary framework for quantum information processing, based on the idea that non-orientable boundary conditions can act as active constraints on quantum evolution rather than as merely geometrical illustrations.

The Möbius strip, owing to its single-sided non-orientable structure, provides a simple mathematical setting in which global boundary identification, geometric phase, and quantum-state evolution can be connected.

In this paper, we focus on one concrete new mechanism: coherent error cancellation by using a Möbius quantum chain, which was not considered before for such purpose.

The essential idea of this paper is that a quantum state propagating on a closed chain with a Möbius boundary identification does not return to itself trivially: instead, the return amplitude is related to the initial amplitude by a twist. This global constraint can force opposite coherent phase contributions to interfere destructively. Therefore, the cancellation is not produced by measurement or redundancy, but by the global topology of the boundary condition itself.

Twisted boundary conditions and Möbius geometries have been investigated in quantum systems, including proposals for protected qubits in frustrated Josephson-junction ladders [15] and studies of topological states on non-orientable bands [16]. Synthetic dimensions provide a framework for engineering internal-state connectivity [17,18]. Separately, dynamical decoupling and group symmetrization establish how unwanted Hamiltonian contributions can be suppressed through their transformation under unitary operations [19–32].

The present work investigates a boundary-implemented realization of selective coherent-error cancellation, specifying the twist-odd error channels, paired-traversal conditions, and sensitivity to imperfect symmetry. Its contribution concerns this implementation and quantitative analysis. In addition, this new mechanism could be economic with respect to the physical and operational resources, and it could also be considered to complete and optimize the other known conventional approaches.

The results of this paper are as follows:

(i) We formulate a quantum Möbius chain model;
(ii) We derive the cancellation of symmetric coherent phase perturbations;
(iii) We identify the residual error for imperfect symmetry;
(iv) We discuss the physical meaning of Möbius boundary conditions as a topological resource for passive quantum error suppression.

Further developments, including topological gate implementations and quantum algorithms based on Möbius topology, will be investigated in future work.

This paper is organized as follows: Section 1 describes the contextual background, current interest, purposes and results of this paper. In Section 2 we implement the Möbius boundary conditions as a quantum resource and describe the Möbius quantum chain. In Section 3 we describe the coherent phase error: the ordinary chain versus the Möbius error cancellation. Section 4 shows the Möbius boundary cancellation and the quantum gates, and an explicit finite-chain model. In Sections 5 and 6 we discuss a comparison with other error-suppression mechanisms, physical interpretation and possible implementations. Section 7 summarizes the conclusions.

2. Möbius Boundary Conditions as a Quantum Resource and Möbius Quantum Chain

The Möbius strip is the simplest non-orientable surface with a single boundary component. Its standard identification may be represented as

$$(x, 0) \equiv (L - x, 1), \tag{1}$$

where $x \in [0, L]$ labels the coordinate along the strip.

The important physical point is not the literal construction of a mechanical strip, but the existence of an effective boundary rule that identifies the end of a quantum trajectory with a reversed or twisted copy of its beginning.

In quantum information language, such a rule may be represented by a Möbius boundary operator $\mathcal{B}_M$. Acting on a state $|\psi\rangle$, this operator imposes a global constraint on the admissible evolution,

$$|\psi(L)\rangle = e^{i\phi_M}\mathcal{T}\,|\psi(0)\rangle, \tag{2}$$

where ϕ_M is a twist phase and $\mathcal{T}$ is an internal twist operation.

(i) Depending on the physical realization, $\mathcal{T}$ may act on spin, pseudospin, qubit orientation, path degree of freedom, or another internal two-level structure.
(ii) Such a boundary condition is the basic resource considered here.
(iii) It enriches the usual Hilbert-space evolution by imposing a non-orientable global identification.
(iv) In this sense, the Möbius boundary condition is not an additional local interaction. It is a topological rule selecting how amplitudes are compared after a complete traversal.

Let us now describe the Möbius quantum chain.

Consider a one-dimensional quantum chain of length L, parametrized by

$$x \in [0, L]. \tag{3}$$

For an ordinary periodic chain, the wavefunction satisfies

$$\psi(L) = \psi(0). \tag{4}$$

For a Möbius quantum chain, we impose instead

$$\psi(L) = e^{i\phi_M}\mathcal{T}\,\psi(0). \tag{5}$$

The simplest idealization is obtained when the twist acts as a sign or orientation inversion,

$$\mathcal{T}^2 = 1, \qquad \mathcal{T}\,\psi = \pm\psi. \tag{6}$$

A full traversal of the chain therefore combines dynamical propagation with a topological return rule. The state does not simply come back to the same local orientation. It comes back as a twisted copy of itself.

The effective propagation around the chain may be written schematically as

$$U_M = \mathcal{B}_M U_0, \tag{7}$$

where U_0 denotes the ordinary dynamical evolution and $\mathcal{B}_M$ encodes the Möbius boundary identification.

In Equation (7), the boundary operator $\mathcal{B}_M$ implements the Möbius identification on the relevant state space. This is formally reminiscent of transformations associated with braiding in topological quantum computation. However, a nontrivial unitary boundary operator alone does not by itself establish non-Abelian anyonic statistics: these require a specified fusion space and a noncommuting representation of braid operations.

We therefore regard this interesting analogy as a possible direction for further investigation, rather than an identification of the present mechanism with non-Abelian anyonic computation.

In regimes in which the topological contribution can be represented by a global phase, one may write

$$U_M = e^{i\gamma_M} U_0, \tag{8}$$

where γ_M denotes a geometric or topological phase associated with the Möbius traversal.

In the present paper, however, we emphasize not only the phase itself but the cancellation of coherent error contributions induced by the twisted return.

Figures 1 and 2 illustrate these properties.

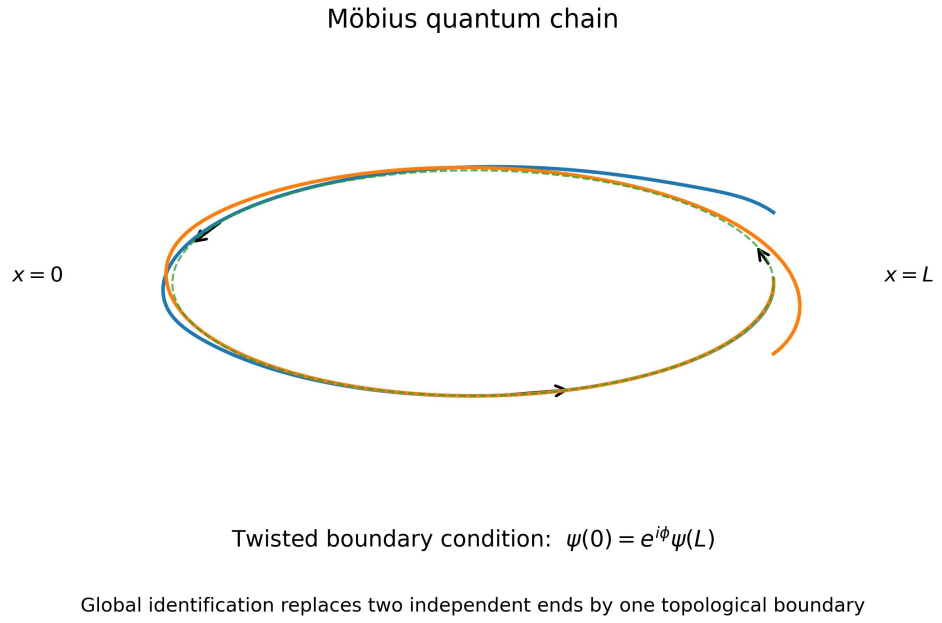


Figure 1. Möbius quantum chain with a twisted boundary condition. The two ends of the chain are not identified trivially, as in an ordinary periodic chain, but through a Möbius rule $\psi(L) = e^{i\phi_M}\mathcal{T}\,\psi(0)$. This global identification acts as a topological boundary resource.

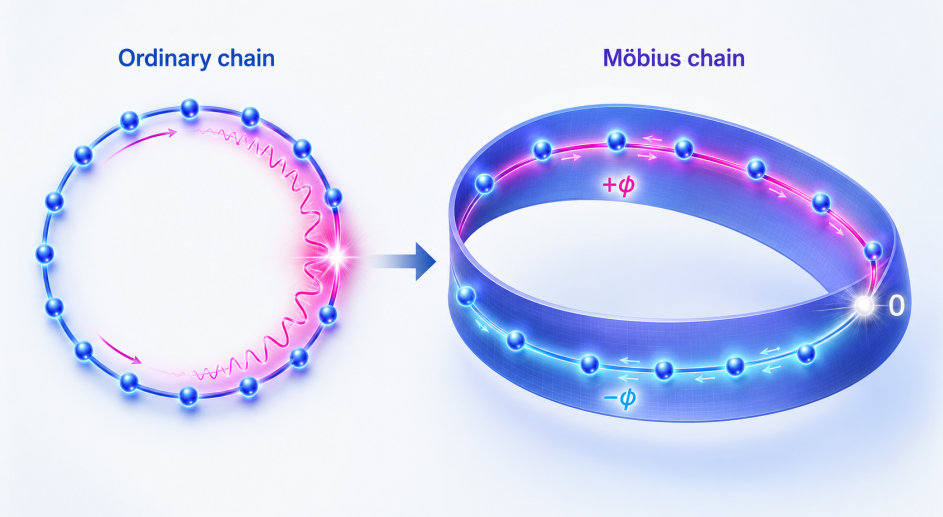


Figure 2. Error accumulation in ordinary topology versus passive cancellation in Möbius topology. Ordinary versus Möbius quantum topologies. In the ordinary geometry, coherent phase errors accumulate during propagation. The Möbius boundary identification generates opposite contributions, $+\phi$ and $-\phi$, producing passive cancellation after a complete traversal.

3. Coherent Phase Error. Ordinary Chain vs. Möbius Error Cancellation

Let a coherent phase error be described by a slowly varying perturbation $\epsilon(x)$ so that the wavefunction acquires the local phase

$$\psi(x) \longrightarrow e^{i\epsilon(x)}\psi(x). \tag{9}$$

On an ordinary chain, the accumulated coherent phase after one traversal is

$$\Delta_{\mathrm{ord}} = \int_0^L \epsilon(x)\, dx. \tag{10}$$

If the same systematic perturbation is repeated over many cycles, the accumulated phase may grow coherently,

$$\Delta_{\mathrm{ord}}^{(N)} = N\Delta_{\mathrm{ord}}. \tag{11}$$

This constructive accumulation is one of the reasons coherent errors can be more dangerous than purely random errors.

Random errors may partially average out, while systematic coherent errors can reinforce themselves.

On the contrary, as we show here, the presence of a Möbius strip topology (Möbius boundary conditions) is naturally produced in a clean (geometric) annihilation of one error by each other, instead of the error construction which operates in the trivial topology.

The Möbius boundary condition changes the global phase balance. Because the state returns through a twisted identification, the second contribution to the accumulated phase is effectively compared with a topologically reversed branch. We write the total Möbius phase mismatch as

$$\Delta_M = \frac{1}{2}\left[\int_0^L \epsilon(x)\, dx - \int_0^L \epsilon_M(x)\, dx\right], \tag{12}$$

where $\epsilon_M(x)$ denotes the error field seen on the twisted branch after the Möbius identification.

For a symmetric coherent perturbation,

$$\epsilon_M(x) = \epsilon(x), \tag{13}$$

Equation (12) gives

$$\Delta_M = 0. \tag{14}$$

Thus, the coherent error cancels at the level of the global boundary condition. The cancellation is not local. It is a result of comparing the forward branch with its topologically related Möbius return, as a consequence of the non-global orientation of the Möbius manifold.

More generally, if the perturbation is not perfectly symmetric, we may write

$$\epsilon_M(x) = \epsilon(x) + \delta\epsilon(x), \tag{15}$$

and the residual Möbius phase becomes

$$\Delta_M = -\frac{1}{2}\int_0^L \delta\epsilon(x)\, dx. \tag{16}$$

The remaining error is therefore controlled by the asymmetry of the perturbation rather than by its full magnitude. This is conceptually a key feature of the suppression mechanism. Figures 3 and 4 illustrate these features.

Let us explicitly show the connection of the boundary-operator formulation to the error-cancellation expression in Equation (12). The equality of the perturbation amplitudes on the two branches does not, by itself, establish cancellation. The relative minus sign must follow from the transformation of the relevant error operator under the implemented twist.

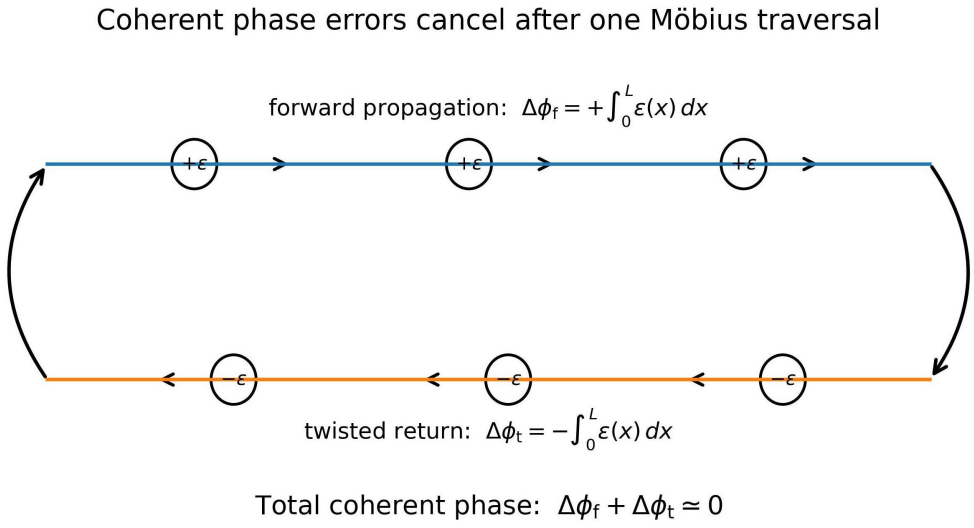


Figure 3. Coherent phase-error cancellation in a Möbius quantum chain. A phase accumulated along the ordinary branch is compared with the contribution accumulated along the twisted return branch. For symmetric coherent perturbations, the two contributions cancel in the global Möbius boundary condition.

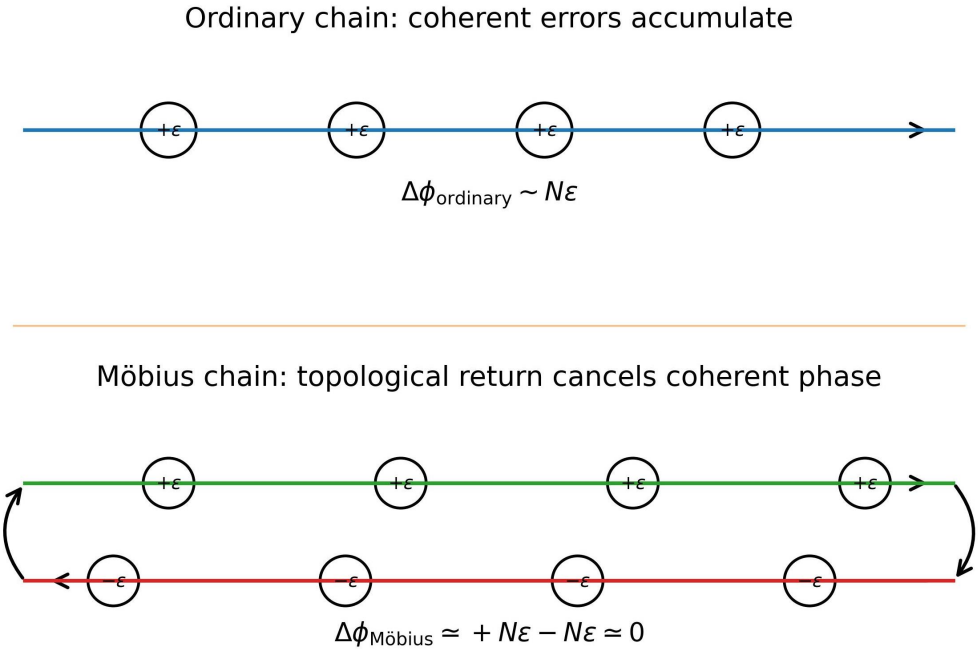


Figure 4. Quantum Errors: Comparison between an ordinary chain and a Möbius chain. In an ordinary chain, coherent errors can accumulate constructively. In a Möbius chain, the twisted return branch can cancel the leading coherent contribution, leaving only the asymmetry between the two branches.

Consider the boundary identification

$$|\psi(L)\rangle = e^{i\phi_M} T|\psi(0)\rangle, \tag{17}$$

where T is a unitary transformation acting on a specified internal degree of freedom. When the evolution on the twisted segment is expressed in the same internal reference frame as that on the ordinary segment, its error Hamiltonian becomes

$$\widetilde{H}_{\text{err},M}(t) = T^\dagger H_{\text{err},M}(t) T. \tag{18}$$

The scalar boundary phase $e^{i\phi_M}$ cancels in this conjugation and therefore does not generate the required sign reversal.

For a coherent-error channel described by a dimensionless Hermitian generator A, let

$$H_{\mathrm{err},o}(t) = \epsilon_o(t)A, \qquad H_{\mathrm{err},M}(t) = \epsilon_M(t)A, \tag{19}$$

where the coefficients have units of energy. If the physical twist satisfies

$$T^\dagger A T = -A, \tag{20}$$

the error generator on the twisted segment, expressed in the common frame, is

$$\widetilde{H}_{\mathrm{err},M}(t) = -\epsilon_M(t)A. \tag{21}$$

Thus, the negative sign follows from the transformation of the error operator, rather than from a definition of the phase mismatch.

For two successive segments of duration τ, with evolution generated by this single error operator in the interaction frame of the intended dynamics, the residual error propagator is

$$U_{\mathrm{err}} = \exp\left[+\frac{iA}{\hbar}\int_0^\tau \epsilon_M(t)\,dt\right]\exp\left[-\frac{iA}{\hbar}\int_0^\tau \epsilon_o(t)\,dt\right] \tag{22}$$

$$= e^{-iA\Delta_M}, \tag{23}$$

where

$$\Delta_M = \frac{1}{\hbar}\int_0^\tau [\epsilon_o(t) - \epsilon_M(t)]\,dt. \tag{24}$$

This gives the operator-level origin of the difference appearing in Equation (12). For propagation at a common constant speed v, the corresponding spatial expression is

$$\Delta_M = \frac{1}{\hbar v}\int_0^L [\epsilon_o(x) - \epsilon_M(x)]\,dx. \tag{25}$$

If the manuscript's spatial error coefficients are defined as phase accumulated per unit length, the factor $1/(\hbar v)$ is already included in those coefficients.

Under matched integrated perturbations,

$$\int_0^\tau \epsilon_o(t)\,dt = \int_0^\tau \epsilon_M(t)\,dt, \tag{26}$$

one obtains $\Delta_M = 0$ and $U_{\mathrm{err}} = I$ within this commuting, single-generator model. Pointwise equality of the two perturbations is sufficient but stronger than necessary.

An explicit example is a two-channel encoding in which the Möbius identification exchanges the channels:

$$T = \sigma_x. \tag{27}$$

For a differential phase error,

$$H_{\mathrm{err}}(t) = \epsilon(t)\sigma_z, \tag{28}$$

the relation

$$\sigma_x\sigma_z\sigma_x = -\sigma_z \tag{29}$$

provides the required sign reversal. A channel exchange therefore reverses a differential detuning error, whereas an error proportional to σ_x remains unchanged:

$$\sigma_x\sigma_x\sigma_x = \sigma_x. \tag{30}$$

These two error channels consequently have different suppression properties.

More generally, when $T^2 = e^{i\chi} I$, the error Hamiltonian can be decomposed into components that are even and odd under conjugation by T:

$$H_{\mathrm{err}}^{(\pm)} = \frac{1}{2}\left(H_{\mathrm{err}} \pm T^{\dagger} H_{\mathrm{err}} T\right), \tag{31}$$

with

$$T^{\dagger} H_{\mathrm{err}}^{(\pm)} T = \pm H_{\mathrm{err}}^{(\pm)}. \tag{32}$$

For matched exposures, the odd component cancels in the first-order integrated error generator, whereas the even component survives. If different error components do not commute, higher-order contributions generally remain; exact cancellation then requires additional dynamical conditions. In particular, the sign-reversal condition must hold in the interaction frame of the intended evolution, not merely for the laboratory-frame perturbation.

Accordingly, the proposed mechanism is selective: it suppresses matched coherent-error contributions that are odd under the implemented twist. It does not establish universal cancellation of arbitrary coherent errors, uncorrelated fluctuations, or irreversible noise. Furthermore, a boundary identification alone does not specify the required paired evolution: the physical implementation must realize the internal transformation and matched exposures described above. For independent parallel paths, a separate recombination analysis is required.

Therefore, Equation (12) is a cancellation result for specified error channels together with the implementation of the non-orientability.

Finally, let us comment about a possible connection of the topological phase reversal property here with the spin-echo phenomenon (or Hahn echo) [33], in which the pulse acts as a phase-reversing mirror inverting the phase accumulated by each individual spin, transforming an expanding disorder (dephasing) state into a synchronized convergence state (rephasing).

For quasi-static dephasing, spin echo reverses the error generator through a π rotation, yielding $X\sigma_z X = -\sigma_z$. In our explicit model, a similar refocusing algebra does appear, but with the internal transformation implemented at the Möbius seam and cancellation occurring over two matched traversals. It can therefore be interpreted as a boundary-implemented echo mechanism. The Klein-bottle picture offers a geometric analogy; however, identifying an actual Klein-bottle topology would require additional boundary identifications, which are not assumed here.

4. Boundary Cancellation and Quantum Gates

We now discuss, in this context, the boundary cancellation and the quantum gates. Let U_g be a quantum gate acting locally or sequentially on the chain, and let U_M denote the global Möbius propagation operator. A circuit layer may be represented schematically as

$$U_{\mathrm{layer}} = U_M U_g. \tag{33}$$

The Möbius operation is not a replacement for the logical gate. Rather, it acts as a topological boundary filter that modifies the way coherent errors accumulate.

If a gate suffers from a coherent over-rotation,

$$U_g \longrightarrow e^{-i(\theta+\delta\theta)H}, \tag{34}$$

then, an ordinary repeated circuit accumulates the systematic shift $\delta\theta$.

On the contrary, in a Möbius chain, the topological return can pair the over-rotation with a reversed or twisted (with opposite sign) contribution. The leading residual error is then determined by the mismatch between the two branches,

$$\delta\theta_{\text{eff}} \sim \frac{1}{2}(\delta\theta - \delta\theta_M). \tag{35}$$

For $\delta\theta_M = \delta\theta$, the leading coherent contribution does cancel.

In realistic implementations, the cancellation will be approximate, and its quality will depend on the degree to which the Möbius branch reproduces the same systematic error with the exact required topological sign or orientation relation, Figure 4. However, and in any case, the topological Möbius mechanism produces an efficient coherent error mitigation effect.

Coherent gate over-rotations require an explicit unitary calculation, and the relationship between two error contributions must follow from a specified physical operation rather than from an assumed subtraction. We therefore now formulate the cancellation condition at the operator level and provide an explicit finite-chain model with an explicit Möbius boundary operation.

4.1. Transformation of a Coherent Gate Error

Let us consider a rotation generated by a dimensionless Hermitian operator A,

$$R_A(\theta) = e^{-i\theta A/2}, \tag{36}$$

with an additive coherent angular error δ:

$$\widetilde{R}_A(\theta, \delta) = e^{-i(\theta+\delta)A/2}. \tag{37}$$

If the implemented twist satisfies

$$T^\dagger A T = -A, \tag{38}$$

then

$$T^\dagger \widetilde{R}_A(\theta, \delta) T = e^{+i(\theta+\delta)A/2}. \tag{39}$$

Thus, conjugation reverses both the intended rotation and its coherent error. This observation is essential: cancellation of an error does not automatically imply preservation of a nontrivial target gate.

For example, a paired implementation that preserves a net intended rotation can be written as

$$U_{\text{pair}} = \left[T^\dagger \widetilde{R}_A(-\theta, \delta_M) T\right] \widetilde{R}_A(\theta, \delta_o). \tag{40}$$

Since all factors involve the same generator,

$$U_{\text{pair}} = R_A(2\theta + \delta_o - \delta_M). \tag{41}$$

Relative to the intended gate $R_A(2\theta)$, the residual error is therefore

$$U_{\text{res}} = R_A(\delta_o - \delta_M). \tag{42}$$

The difference between the errors is now derived from the conjugation rule. Equal additive errors cancel exactly in this single-generator model.

However, this condition must be checked against the physical error model. For a common multiplicative calibration error,

$$\widetilde{R}_A(\theta) = R_A[(1 + \alpha)\theta], \tag{43}$$

reversing the commanded angle also reverses the additive angular error. The same construction then gives

$$U_{\text{pair}} = R_A[2(1 + \alpha)\theta], \tag{44}$$

so the calibration error survives. Consequently, the proposed cancellation cannot be claimed for arbitrary gate over-rotations.

4.2. Explicit Finite-Chain Model

Let us consider a finite chain of N sites with a two-dimensional internal state,

$$\mathcal{H} = \mathrm{span}\{|j\rangle : j = 0, \ldots, N-1\} \otimes \mathbb{C}^2. \tag{45}$$

We define the unitary translation with an internal boundary operation T:

$$S_T = \sum_{j=0}^{N-2} |j+1\rangle\langle j| \otimes I + |0\rangle\langle N-1| \otimes T. \tag{46}$$

Periodic closure corresponds to $T = I$. A concrete two-channel Möbius identification is implemented by

$$T = \sigma_x, \tag{47}$$

which exchanges the internal channels at the seam.

During each step, let the internal state experience a coherent perturbation generated by

$$H_{\mathrm{err}} = \frac{\hbar\omega}{2}\sigma_z. \tag{48}$$

For an exposure time Δt, its unitary is

$$E(\delta) = e^{-iH_{\mathrm{err}}\Delta t/\hbar} = e^{-i\delta\sigma_z/2}, \qquad \delta = \omega\Delta t. \tag{49}$$

One step of the discrete-time evolution is

$$W_T(\delta) = S_T[I_{\mathrm{pos}} \otimes E(\delta)]. \tag{50}$$

This specifies a finite, unitary transport model with an explicit coherent perturbation and boundary operation. It does not assume that an arbitrary static tight-binding Hamiltonian produces ideal directed translation.

Starting immediately after the seam, one complete traversal produces the internal operation

$$U_T(\delta) = TE(\delta)^N. \tag{51}$$

For periodic closure,

$$U_P(\delta)^2 = E(\delta)^{2N} = e^{-iN\delta\sigma_z}. \tag{52}$$

For Möbius closure,

$$U_M(\delta)^2 = \sigma_x E(\delta)^N \sigma_x E(\delta)^N = I, \tag{53}$$

because

$$\sigma_x\sigma_z\sigma_x = -\sigma_z. \tag{54}$$

The cancellation therefore follows from the explicitly implemented channel exchange and the matched perturbation on successive traversals. It is exact for this commuting error model. The ideal operation after two traversals is the identity, so the stored quantum information during transport is preserved.

Figure 5 makes the two-channel boundary identification explicit. The blue and orange nodes label the internal channels a and b; the solid arrows indicate the directed translation of the model. At the seam, $T = \sigma_x$ exchanges the channels: $|N-1, a\rangle \mapsto |0, b\rangle$ and $|N-1, b\rangle \mapsto |0, a\rangle$.

The dashed vertical line locates this seam and does not denote an additional propagation path. The rightmost nodes repeat the initial sites to display the closure; they are not additional sites. A scalar boundary phase is set to zero in this explicit model. The crossing is a boundary wiring identification, not a physical scattering junction. For a full strip realization, the transverse connectivity must also be specified.

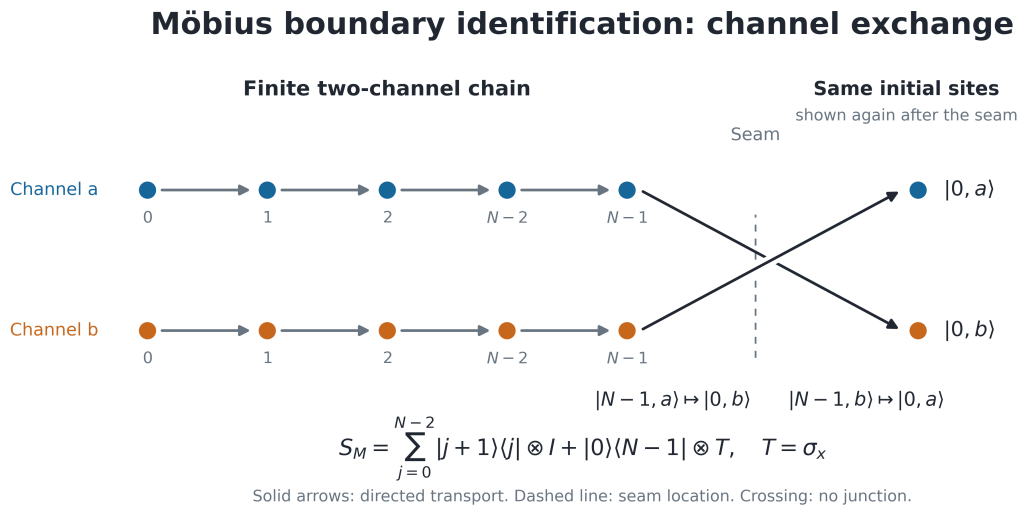


Figure 5. Two-channel realization of a Möbius boundary identification. Blue and orange nodes denote internal channels a and b, exchanged at the seam by $T = \sigma_x$. Solid arrows show directed transport; the dashed line marks the seam. The repeated initial sites at the right display the twisted closure. One circuit returns the state to its initial spatial position while exchanging its internal channels; two circuits restore the ideal internal configuration. The crossing has no junction.

4.3. Symmetry Breaking and Quantitative Fidelity

To quantify departures from matched exposures, let the angular perturbation per step be δ on the first traversal and $(1+\eta)\delta$ on the second. Here, η parametrizes a traversal-to-traversal mismatch. The two-traversal operations are

$$U_P^{(2)} = E[(1+\eta)\delta]^N E(\delta)^N \tag{55}$$

$$= \exp\left[-\frac{i}{2}N(2+\eta)\delta\sigma_z\right], \tag{56}$$

and

$$U_M^{(2)} = \sigma_x E[(1+\eta)\delta]^N \sigma_x E(\delta)^N \tag{57}$$

$$= \exp\left[+\frac{i}{2}N\eta\delta\sigma_z\right]. \tag{58}$$

For a systematic mismatch repeated over m two-traversal cycles, define the entanglement fidelity relative to the ideal identity operation:

$$F_e(U) = \frac{|\operatorname{Tr} U|^2}{4}. \tag{59}$$

Then

$$F_{e,P} = \cos^2\left[\frac{mN(2+\eta)\delta}{2}\right], \tag{60}$$

whereas

$$F_{e,M} = \cos^2\left[\frac{mN\eta\delta}{2}\right]. \tag{61}$$

For small accumulated residual angles,

$$1 - F_{e,P} \simeq \frac{m^2N^2(2+\eta)^2\delta^2}{4}, \tag{62}$$

$$1 - F_{e,M} \simeq \frac{m^2N^2\eta^2\delta^2}{4}. \tag{63}$$

These expressions provide quantitative predictions as functions of perturbation strength, number of traversals, and symmetry breaking. Direct numerical propagation of W_T can be compared with these analytical results. Even perturbations under the twist, such as a term proportional to σ_x, provide a useful control case because their cancellation is not predicted.

The physical conclusion is therefore conditional and testable: the Möbius boundary implementation supplies a specified internal transformation, while suppression requires an error generator that is odd under that transformation and sufficiently matched exposures. Non-orientability alone does not guarantee cancellation of arbitrary coherent gate errors.

In the explicit model, one traversal denotes one circuit of N sites, returning to the initial spatial position while applying the internal twist $T = \sigma_x$. The spatial return does not generally restore the internal state. Two traversals restore the ideal internal configuration because $T^2 = I$, up to an irrelevant global phase. The cancellation described here occurs between these two successive traversals; it does not require interference between independent paths.

Let the integrated error angles on the successive traversals be a and b. For an error generator that is odd under T, the residual angle is $a - b$, whereas periodic closure gives $a + b$. Thus, in the small-error regime, the ratio of entanglement infidelities is

$$\frac{1 - F_{e,M}}{1 - F_{e,P}} \simeq \left(\frac{a - b}{a + b}\right)^2. \tag{64}$$

Writing $b = (1 + \eta)a$, this becomes

$$\frac{1 - F_{e,M}}{1 - F_{e,P}} \simeq \frac{\eta^2}{(2 + \eta)^2} \simeq \frac{\eta^2}{4}, \qquad |\eta| \ll 1. \tag{65}$$

Suppression therefore depends on matched integrated exposures. Slowly varying systematic errors can satisfy this condition over successive traversals. Spatial correlation is only helpful when paired exposures remain correlated after the twist mapping. For zero-mean random angles with equal variance and correlation coefficient ρ, the corresponding ratio of mean infidelities is approximately $(1 - \rho)/(1 + \rho)$. Uncorrelated errors ($\rho = 0$) receive no suppression in this model.

Figure 6 directly connects the boundary operation here to coherent-error suppression. Each traversal is one circuit of the same chain, and the diagram is read chronologically from left to right; it does not represent two independent interfering paths or a reversal of propagation direction. For $E(c) = \exp(-ic\sigma_z/2)$, the periodic closure gives $U_P = E(b)E(a) = E(a + b)$. The Möbius closure instead gives $U_M = TE(b)TE(a) = E(a - b)$ because $T = \sigma_x$ and $T\sigma_z T = -\sigma_z$. Matched integrated error angles, $a = b$, therefore yield identity after two traversals in this commuting single-generator model. The sign reversal follows from the internal channel exchange, not from reversing an integration direction.

Thus, the periodic and Möbius seams implement I and $T = \sigma_x$, respectively, giving accumulated error angles $a + b$ and $a - b$, and this makes explicit that the sign reversal requires an error generator that is odd under the implemented twist.

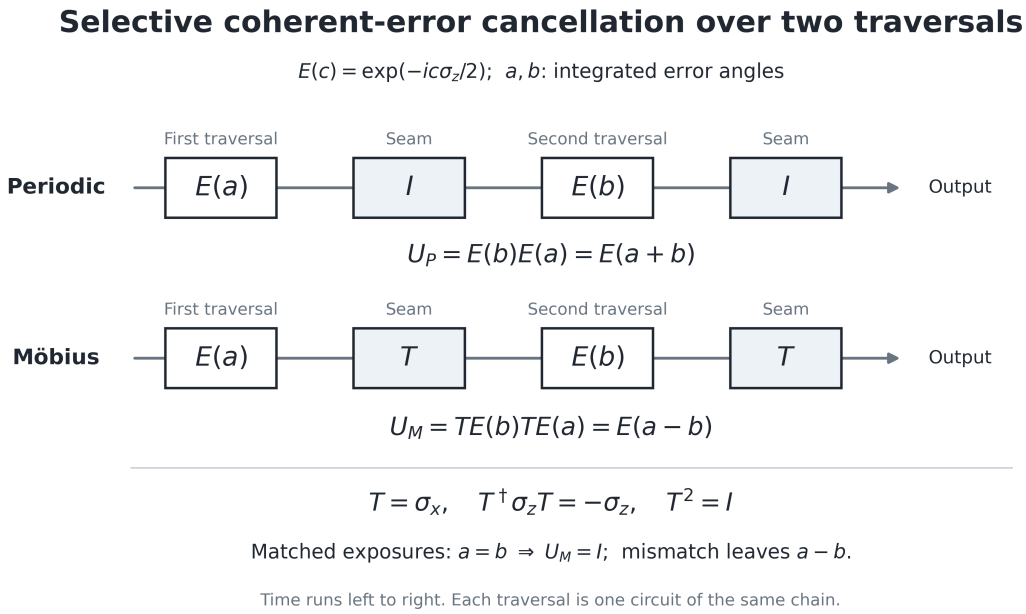


Figure 6. Selective coherent-error cancellation over two successive traversals. The periodic seam acts as I and the Möbius seam as $T = \sigma_x$. The dimensionless integrated error angles a and b generate $E(a)$ and $E(b)$. Since $T^\dagger \sigma_z T = -\sigma_z$ and $T^2 = I$, the Möbius sequence gives $E(a-b)$, whereas periodic closure gives $E(a+b)$. Cancellation occurs for matched exposures $a = b$; mismatch leaves a residual angle $a - b$. Even errors under T are not canceled.

5. Comparison with Other Error-Suppression Mechanisms

The Möbius mechanism is distinct from standard stabilizer quantum error correction. It does not require syndrome extraction, measurement feedback, or redundant logical encoding.

It is also distinct from dynamical decoupling, where time-dependent pulse sequences average unwanted interactions. On the contrary, in our topological framework, error suppression arises from static global boundary constraints and topology enters through boundary identification and global phase cancellation. The Möbius chain may therefore be viewed as a boundary-engineered passive error-suppression layer.

Our paper is related in spirit to decoherence-free subspaces, because both approaches use structure to reduce sensitivity to selected perturbations. However, decoherence-free subspaces rely on symmetry in system–environment coupling and do encode information in protected subspaces. The Möbius mechanism here instead uses a non-orientable boundary identification to compare two topologically related error contributions.

The framework of this paper is also related to geometric and topological approaches, but it differs from the known topological computing requiring anyonics or braiding. We also notice that our quantum Möbius approach can be used to complement and generalize such standard known approaches as anyonics, braiding and others.

Finally, as we mentioned in the Section 1 [14], Google (QAI and DeepMind) used a reinforcement-learning strategy for quantum error correction control very recently: error detection signals are used to continuously compensate for processor drift. Our quantum approach is theoretical and conceptual rather than engineering and experimental; its practical and quantitative implementation, and combination with the new Google AI error control [14] deserves investigation.

These results emphasize the growing importance of adaptive autonomous error-control mechanisms and provide a timely experimental context for exploring different and comple-

mentary approaches—such as the Möbius boundary mechanism proposed here—in which error suppression arises intrinsically from geometry, topology and quantum interferences.

6. Physical Interpretation and Possible Implementations

Main items of the cancellation mechanism accounted in this paper can be summarized as follows:

(i) In an ordinary (topologically trivial) closed chain, a coherent error accumulated over a full loop returns with the same orientation and can reinforce itself.
(ii) In a Möbius chain, the returning amplitude is identified with a twisted copy of the initial amplitude. This twist reverses or compares the contribution of symmetric coherent perturbations. The error is therefore converted from an accumulated global phase into a difference between two topologically related branches of different signs.
(iii) The mechanism exploits a global non-orientable boundary identification together with a compatible transformation of the error dynamics. The Möbius identification relates propagation along the chain to a twisted counterpart. The resulting cancellation reflects how the boundary transformation acts on the relevant error generators, rather than on the identification alone.
(iv) This mechanism is expected to be most effective for slowly varying, systematic, and spatially correlated errors. It is not expected to cancel arbitrary local stochastic noise.
(v) Therefore, Möbius error cancellation should be regarded as complementary to, not a replacement for, conventional quantum error correction.
(vi) Possible physical settings include photonic waveguide arrays with engineered path topology, superconducting circuits with synthetic boundary conditions, cold atoms in programmable optical potentials, trapped-ion chains with controlled phase-space loops, and topological materials supporting non-trivial boundary modes.
(vii) In each case, the essential requirement is not the literal construction of a physical Möbius strip, but the implementation of an effective non-orientable boundary identification in the quantum dynamics.
(viii) The explicit model provided in Section 4 is particularly useful because it distinguishes three elements: the boundary operation defined by (T), the error that changes sign, and the evolution that pairs the two exposures. Furthermore, it provides exact formulas, without confusing a cancellation demonstrated for transport with an universal protection for quantum gates.

7. Conclusions

We have formulated a Möbius boundary mechanism for passive suppression of selected coherent quantum errors under specified symmetry and evolution conditions. This paper extends the Möbius Quantum Computation program introduced in Ref. [13] by investigating a concrete application: coherent error cancellation in a quantum chain with a non-orientable boundary identification.

The central distinction is between the topology specifying the global boundary identification and the dynamics determining its action on quantum states and error generators. A Möbius identification alone does not guarantee error cancellation. Cancellation requires a compatible boundary transformation that reverses the relevant error contribution, together with matched propagation along the paired evolution segments. Under these conditions, the central result is the cancellation relation

$$\Delta_M = 0 \tag{66}$$

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for symmetric coherent perturbations. This result shows that global topology can act as a passive error-suppression resource. For matched commuting phase-error generators, opposite accumulated error phases yield exact cancellation. More generally, exact cancellation requires the error propagators of the paired segments to be mutual inverses. A reversal of the leading-order error contribution alone does not ensure this inverse relation when time ordering or noncommuting generators are relevant.

Departures from the ideal cancellation conditions produce residual errors. In the commuting phase-error model, the residual is determined by the mismatch between the paired accumulated phases. In more general dynamics, higher-order commutator contributions may also survive. Error components that do not reverse sign under the boundary transformation are not canceled.

The proposed mechanism shares with spin echo the principle of reversing a coherent error contribution. Its distinctive feature is that the reversal is associated with the prescribed boundary transformation rather than an externally timed refocusing pulse sequence.

The term passive refers to operation under a fixed engineered boundary coupling in the idealized model; realizing and maintaining that coupling remains a physical implementation requirement.

The analysis therefore identifies non-orientable boundary conditions as a potential resource for coherent-error suppression. The topological character of the identification does not, by itself, establish robustness against arbitrary perturbations, deformation-invariant cancellation, or universal topological protection. The resulting suppression depends on the symmetry and matching conditions of the chosen dynamical realization.

The present work is theoretical and intentionally conceptual to illustrate the essential principle. It provides a proof of principle of a new topological mechanism for passive quantum error suppression within the stated model and assumptions. It does not yet establish hardware-level performance or include comprehensive device-dependent noise models, finite-temperature effects, or realistic imperfections in the boundary couplings. Dedicated numerical studies and experimental tests are required to quantify these effects and determine the regime in which useful suppression persists.

Possible implementation routes may be investigated in photonic circuits, superconducting qubits, trapped ions, cold atoms, and synthetic quantum matter platforms, provided that the required connectivity and internal-state transformation can be engineered. Their feasibility and performance must be assessed separately for each platform.

Beyond its application to coherent error suppression, the Möbius framework suggests that non-orientable topological boundary conditions may constitute a broader design principle for quantum information processing. This perspective opens new directions connecting topology, quantum dynamics, and quantum computation.

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