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Windowed Actions and Finite-Domain Localization Across Five Quantum Experiments

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Abstract

A smooth window function $\Diamond(x) \in [0,1]$ that restricts a field-theory action to a finite spacetime domain generates a common conservation structure across diverse experimentally realized finite-time quantum phenomena. Applying the windowed action principle yields windowed Noether identities of the form $\partial_\mu(\Diamond J^\mu) = 0$: exact conservation of the windowed current, with apparent non-conservation of local currents confined to the boundary layer where $\partial_\mu \Diamond \neq 0$. This boundary-layer structure is mathematically identical to open-system flux terms in decoherence theory. The formalism is applied to five experimentally established settings: the timelike Unruh effect in trapped-ion detectors, the dynamical Casimir effect in superconducting circuits, quench-induced currents in cold-atom systems, ultrafast coherent control with femtosecond laser pulses, and finite-time scattering theory. In each case, the experimentally specified control window—switching function, drive envelope, quench ramp, pulse envelope, or scattering window—is shown to be an instance of the same formal object $\Diamond$, and the windowed Noether identity is derived for each setting in this unified form for the first time. Two results are new: the cross-case identification of all five control functions as instances of $\Diamond$ under a single formalism, and the formal consequence that systems with the same ungated Hamiltonian operator content but different temporal control profiles generically produce inequivalent unitary evolutions, formulated here as a structural consequence of explicit domain specification. Here, $\Diamond$ is fixed by the experimentally specified control protocol and is neither a new dynamical degree of freedom nor an additional fit function. Although it may be absorbed algebraically into a time-dependent coupling or interaction Hamiltonian, retaining it explicitly exposes the common finite-domain conservation structure across the five settings. In the purely temporal measurement limit, $g(t) = \lambda\Diamond(t)$ reproduces the standard von Neumann system–apparatus coupling, while the action-level extension to $\Diamond(x)$ permits finite spacetime support and makes the associated boundary-supported current balance explicit.

Keywords: windowed action; localization window; Noether identity; Unruh–DeWitt detector; dynamical Casimir effect; quantum quench; ultrafast spectroscopy; finite-time scattering; boundary-layer conservation; decoherence

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1. Introduction

A recurring feature of modern theoretical physics is the use of idealized descriptions involving infinite temporal extent, exact energy eigenstates, and asymptotic limits. These idealizations underlie widely used frameworks in quantum field theory, scattering theory, and time-dependent quantum dynamics, and have proven extraordinarily successful in predicting experimental outcomes. Nevertheless, all experimentally realizable systems are

instantiated only over finite durations, with preparation, interaction, and measurement occurring within bounded temporal domains, and this gap between formal idealization and physical reality is not merely a technical inconvenience. It is the organizing question of this paper.

The standard response to this gap is pragmatic. Regulators, switching functions, wave packets, and pulse envelopes are introduced to render calculations finite and experimentally meaningful, while their precise form is generally assumed to be removable in an appropriate infinite-time or adiabatic limit [1–7]. This approach is operationally effective. But it treats what I will argue is a physically constitutive element of the process, the domain of its instantiation, as a removable technical artifact. The result is five distinct calculational devices across five distinct subfields, solving what is, formally, one problem.

A growing body of experimental and theoretical work has highlighted situations in which finite-time effects are not merely technical artifacts but directly shape observable phenomena. Examples include the response of Unruh–DeWitt detectors with finite switching times [8–11], photon production in the dynamical Casimir effect driven by time-dependent boundary conditions [12,13], quench-induced currents in cold-atom systems with finite ramp durations [14,15], ultrafast coherent control experiments employing femtosecond laser pulses [7,16], and finite-time scattering theory [4–6,17]. In each case, physically distinct outcomes depend sensitively on how interactions are temporally instantiated, despite identical local Hamiltonians.

The aim of this paper is to show that a single formal tool, the windowed action principle, reveals the common conservation structure underlying all five settings. When a smooth, non-dynamical scalar window function $\Diamond(x) \in [0,1]$ multiplies a standard action functional, a structural consequence follows unavoidably from the Leibniz rule of differentiation. In particular, the relevant finite-domain conservation statement takes the form of a windowed Noether identity: the globally conserved quantity is $\Diamond J^\mu$ rather than J^μ alone, and apparent non-conservation of J^μ is confined to the boundary layer where $\partial\Diamond \neq 0$ (or, in covariant settings, $\nabla\Diamond \neq 0$). Said another way, the boundary layer of the localization window, not a symmetry violation, is the correct formal account of every apparent energy bookkeeping failure in a finite-time quantum experiment. This structure is mathematically identical to open-system flux terms arising from tracing over environmental degrees of freedom in decoherence theory [18,19]. The equivalence asserted here concerns the mathematical form of the subsystem balance law and does not imply that every experimental window is physically generated by tracing over an environment. This consequence is independent of any specific physical motivation for the window.

The present paper does not introduce new particles, new interactions, or any modification to the dynamical equations of quantum mechanics. Its claims are limited to the following: when the smooth localization window $\Diamond(x) \in [0,1]$ multiplies a standard action functional, the Noether identity of the windowed theory takes the form $\partial_\mu(\Diamond J^\mu) = 0$, and this identity applies, with the experimentally specified control function identified as $\Diamond$, to each of the five settings examined below. No derivation beyond the variational principle and the Leibniz rule is required. A companion paper provides a physical interpretation of the window as encoding the domain of operational temporal instantiation [20], but that interpretation is not needed for any result presented here. A second companion paper extends the windowed action to the full Standard Model in curved spacetime, deriving the windowed Ward identities for each sector ($SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge fields, Dirac fermions, Klein–Gordon scalars, and the Higgs sector), establishing that standard QFT is recovered exactly inside the window, and proving a Vanishing Lemma showing that boundary-layer compensator stress-energy is macroscopically negligible for ordinary quantum fields [21].

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The companion Symmetry paper develops the physical status of the window function explicitly and establishes that $\Diamond$ is a fixed smooth scalar multiplying the action, that it is not varied in forming the equations of motion, and that it is not a propagating degree of freedom [21]. Its profile and characteristic thickness are operational inputs, fixed by the relevant localization, measurement, or control process rather than chosen freely, and the algebraic form of the windowed identity does not depend on which admissible physical process fixes that profile. That status is restated explicitly here, rather than left to the companion treatment, because it bears directly on the physical interpretation of $\Diamond(t)$ in each of the five cases below.

Although the present analysis uses the standard laboratory time parameter of quantum mechanics, its operational treatment of temporal domains is compatible with several established approaches in which time is relational or emergent rather than fundamental. In the Page–Wootters construction, effective evolution arises through correlations with a clock subsystem [22]; in the Thermal Time Hypothesis, temporal flow is associated with state-dependent modular evolution [23,24]; and in canonical quantum gravity, the Wheeler–DeWitt constraint contains no explicit external time, with semiclassical time recovered only approximately [24,25]. The present paper does not attempt to adjudicate among or replace these approaches. Its narrower purpose is to make explicit the finite, experimentally specified domain over which a standard interaction is physically active, so $\Diamond$ should be understood as an operational domain factor rather than as a new theory of time.

The windowed Noether identity itself, $\partial_\mu(\Diamond J^\mu) = 0$, follows directly from the variational principle and the Leibniz rule once the action is multiplied by an externally prescribed $\Diamond$: no claim of mathematical novelty is made for the identity in isolation. The central new result of the present paper is its cross-case universality: the demonstration that a single formal object, a smooth localization window multiplying a standard action, generates the same boundary-layer conservation structure across phenomena ranging from femtosecond optical pulses to millisecond-scale switching and quench protocols, spanning five distinct subfields of physics not previously connected through a single derivation. In each case, the windowed Noether identity is derived from the windowed action and applied to the experimentally specified control function; what has not previously been established is the identification of all five control functions as instances of the same formal object, and the resulting common conservation structure, in this unified form. No new interaction, field, or modified quantum evolution law is introduced.

The physical content of $\Diamond$ is therefore not a new force or coupling coefficient, but the experimentally realized support of the interaction. Once the protocol fixes the profile, the same function determines three linked structures: the active domain through $\Diamond$, the boundary exchange through $\partial_\mu\Diamond$, and the finite-time spectral weighting through $\widetilde{\Diamond}$. This connects domain specification, conservation bookkeeping, and bandwidth using one experimentally specified input while leaving the standard bulk dynamics unchanged.

I consider five cases spanning multiple subfields: (1) the timelike Unruh effect in trapped-ion Unruh–DeWitt detectors; (2) the dynamical Casimir effect in superconducting circuits; (3) quench-induced currents in cold-atom systems; (4) ultrafast coherent control using femtosecond laser pulses; and (5) finite-time scattering and energy–momentum smearing.

Each case is analyzed using the same procedure: identification of the measured observable, specification of the localization window, identification of the characteristic timescale or scale hierarchy associated with that experimentally fixed protocol, and derivation of the windowed Noether identity for that setting. No case-specific dynamical modifications or additional fitting parameters are introduced beyond those already present in the experimental protocol. For each setting, the windowed Noether identity is derived here for the

first time in this unified form; the cross-case identification of all five control functions as instances of the same formal object is a new result.

The paper is organized as follows. Section 2 introduces the windowed action principle, windowed Noether identities, and the physical distinctness of processes sharing identical Hamiltonians but differing in their localization histories. Sections 3–7 present the five worked cases. Section 8 provides a cross-case synthesis and outlook.

2. Windowed Actions and Windowed Noether Identities

2.1. Windowed Action Principle

Consider a field theory with fields ϕ and Lagrangian density $\mathcal{L}(\phi, \partial_\mu \phi)$. In standard formulations, the action is integrated over all spacetime:

$$S = \int d^4x \, \mathcal{L}(\phi, \partial_\mu \phi). \tag{1}$$

Equation (1) implicitly assumes that the system is physically instantiated everywhere and for all time: an idealization that is never exactly realized in experiment, though often an excellent asymptotic approximation [3].

Restricting physical instantiation to a finite spacetime domain is encoded by a non-dynamical scalar window function $\Diamond(x)$, yielding the windowed action

$$S_\Diamond = \int d^4x \, \Diamond(x) \, \mathcal{L}(\phi, \partial_\mu \phi), \tag{2}$$

where $0 \leq \Diamond \leq 1$, $\Diamond = 1$ in the interior of the active region, and $\Diamond \to 0$ smoothly across a finite boundary layer of characteristic thickness $\ell_\Diamond$. The smooth form is required for variational consistency; the sharp limit recovers distributional junction conditions familiar from thin-shell constructions in general relativity [26].

The window function does not represent a new physical field or interaction. It encodes the operational fact that preparation, interaction, and measurement occur only within finite domains. Varying Equation (2) yields equations of motion identical to those of the unwindowed theory wherever $\nabla_\mu \Diamond = 0$. Additional terms proportional to $\nabla_\mu \Diamond$ arise only within the boundary layer and account for fluxes entering or leaving the localized region; local dynamics are therefore preserved exactly. Throughout this work, $\Diamond \in C^1$ with compact support or rapid decay is assumed, so that $\nabla \Diamond$ exists and boundary terms are finite and covariant. The three formal smoothness conditions are as follows: (i) $\Diamond \in C^1(\mathcal{M})$; (ii) there exists a compact set K such that $\Diamond$ has compact support or rapid decay outside K; and (iii) $\int |\nabla_\mu \Diamond| \sqrt{-g} \, d^4x < \infty$ (finite gradient integral). These conditions ensure all boundary terms are finite and covariant; the sharp step-function limit recovers distributional junction conditions [26]. The window is *not* varied in the action ($\delta \Diamond \equiv 0$): it is a fixed external encoding of the interaction domain, not a propagating degree of freedom. The boundary-layer thickness is bounded below by the applicable operational scale—detector resolution or control bandwidth in the laboratory cases worked here; generalized-uncertainty and clock-precision bounds constrain the admissible window profile in curved-spacetime regimes [21]. The companion QFT paper derives the windowed field equations and Ward identities for each Standard Model sector under these conditions and proves that standard QFT is recovered exactly inside the window [21]. For the purely temporal windows considered in the five worked cases, it is convenient to parameterize the boundary layer by a single characteristic duration $\tau_\Diamond$ and its associated causal thickness $\ell_\Diamond \equiv c\tau_\Diamond$. The formal role of the window is always temporal, but the corresponding $\ell_\Diamond$ provides a useful common scale for comparing very different experimental protocols. Where a control signal contains both

a finite envelope and an oscillatory carrier, $\tau_\Diamond$ refers to the envelope support or boundary scale; the carrier period is a separate dynamical scale and is not itself the window duration.

2.2. Windowed Noether Identities

When $\mathcal{L}$ possesses an exact continuous internal symmetry, Noether's theorem applied to the windowed action $S_\Diamond$ identifies $\Diamond J^\mu$ as the conserved Noether current of the windowed theory. For clarity, let $\delta\phi^a = \epsilon(x)\Delta\phi^a$ be the localized form of the symmetry transformation and define the current of the unwindowed density by

$$J^\mu \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^a)} \Delta\phi^a, \tag{3}$$

with summation over the field index a. Because the prescribed window is symmetry-inert, $\delta\Diamond = 0$. Exactness of the symmetry for constant ϵ implies $\delta\mathcal{L} = J^\mu \partial_\mu \epsilon$ identically once the parameter is promoted to $\epsilon(x)$, so that

$$\delta S_\Diamond = \int d^4x \, \Diamond J^\mu \partial_\mu \epsilon = -\int d^4x \, \epsilon \, \partial_\mu (\Diamond J^\mu), \tag{4}$$

where $\epsilon(x)$ has compact support, and the surface term vanishes. On the windowed equations of motion $\delta S_\Diamond$ vanishes for arbitrary such $\epsilon(x)$, which yields the windowed Noether identity; its product-rule consequence follows by expanding the derivative:

$$\partial_\mu (\Diamond \, J^\mu) = 0, \tag{5}$$

$$\Diamond \, \partial_\mu J^\mu = -\big(\partial_\mu \Diamond\big) J^\mu. \tag{6}$$

Equation (5) expresses exact conservation of the *windowed* current $\Diamond J^\mu$ on-shell of the windowed equations of motion; it reduces to $\partial_\mu J^\mu = 0$ in the interior where $\nabla_\mu \Diamond = 0$, recovering standard local conservation there. Equation (6), which follows immediately by expanding the product rule applied to Equation (5), shows that the subsystem current J^μ alone exhibits apparent non-conservation confined to the boundary layer where $\nabla_\mu \Diamond \neq 0$. This is not a symmetry violation: the full system, including boundary-layer contributions, exactly respects the original symmetry. In the laboratory examples below, the identities are written with partial derivatives because the control windows are explicit functions of time on a fixed background; the corresponding covariant form is obtained by the replacement $\partial_\mu \to \nabla_\mu$ when gravity or curved spacetime structure is included. The structure of Equation (6) is mathematically identical to the effective current non-conservation arising in open quantum systems after tracing over environmental degrees of freedom [18,19], and to the contact terms appearing in Ward–Takahashi identities for subsystem currents in finite-time scattering [27].

These identities are direct product-rule consequences of the windowed action and are not claimed as mathematically new in themselves. Their central new formal contribution, applied across all five cases in this paper, is that apparent violations of energy conservation, current damping, spectral broadening, and nonadiabatic response in five otherwise unconnected settings are boundary-layer fluxes of the same localization window, exactly accounted for by conservation of the windowed current $\Diamond J^\mu$, and not violations of any symmetry. The boundary layer is not a pathology of the finite-time treatment; it is its correct physical content. The companion QFT paper proves this result for each sector of the Standard Model in curved spacetime, establishing that gauge invariance and local Lorentz invariance are preserved exactly within the window across all SM sectors [21].

2.3. Relation to Established Finite-Time Treatments

The windowed action formalism does not introduce finite-time control functions where none previously existed. Detector switching functions, time-dependent couplings and drive envelopes, finite-time perturbation and scattering theory, and open-system subsystem balance laws are all established methods, each with a substantial literature and its own well-understood finite-time consequences [4,5,9–13,18,19]. Table 1 summarizes the existing role of each treatment and what the present formulation adds.

Table 1. Relation between the windowed formalism and established finite-time treatments. Each established method already contains a finite-domain control function; the contribution here is the identification of these as instances of a single formal object.

Established Treatment	Existing Role	Contribution of the Present Formulation
Detector switching functions	Specify finite detector coupling	Identifies the switching function as the same domain factor used in the other four cases
Time-dependent couplings and pulse envelopes	Specify controlled Hamiltonian evolution	Separates operator content, coupling scale, and finite temporal support
Finite-time perturbation and scattering	Produce Fourier broadening and finite-time energy distributions	Connects the transform structure to the same explicit action-level domain factor
Open-system flux equations	Describe subsystem exchange with a traced-out environment	Shows the same boundary-supported balance-law form, without asserting an identical physical mechanism

The contribution of the present paper is not the introduction of finite-time control functions themselves; these are established tools, and their individual finite-time consequences are already well understood. The contribution is their identification as instances of one explicit action-level domain factor whose gradient generates a common boundary-supported conservation structure across all five settings.

The unified representation provides three concrete advantages. First, one protocol-defined profile determines the active support $\Diamond$, the exchange region identified by $\partial_\mu \Diamond$, and the finite-time spectral weighting $\widetilde{\Diamond}$, rather than introducing these as unrelated case-specific devices. Second, it separates unchanged bulk dynamics from control-induced boundary bookkeeping, allowing the established calculation in each field to be retained while the conservation analysis is performed once. Third, it permits different experimental platforms to be compared through the support, gradient width, and Fourier bandwidth of their control profiles, making transparent when finite-time effects are negligible and when they are experimentally resolved.

2.4. Physical Distinctness of Identically-Hamiltonian Processes

2.4.1. Purpose and Scope

This section formalizes a point used implicitly throughout the main text: identical Hamiltonians do not, in general, define identical physical processes when their temporal localization differs. This fact is operationally familiar across multiple subfields; experimentalists already know that switching profiles and pulse envelopes matter, but it has not previously been stated as a formal structural consequence of the windowed action principle, nor connected explicitly to the Noether identities of Section 2.2.

2.4.2. Hamiltonian Identity Versus Physical Instantiation

Consider a quantum system governed by a time-dependent Hamiltonian of the form

$$H(t) = H_0 + \lambda \lozenge(t)\, H_I, \tag{7}$$

where H_0 is the free Hamiltonian, H_I is an interaction operator, λ is a coupling constant, and $\lozenge(t)$ is a real, bounded function specifying when the interaction is physically active.

In standard treatments, $\lozenge(t)$ is variously referred to as a switching function, pulse envelope, quench ramp, or adiabatic regulator, and is treated as an external control rather than as part of the physical specification of the process. Two experiments are therefore often said to share the "same Hamiltonian" whenever H_0 and H_I coincide: that is, the same ungated Hamiltonian operator content, at fixed coupling scale, prior to specification of the control profile.

From the windowed-action perspective, this identification is incomplete. The Hamiltonian operator alone does not specify a physical process unless the domain of its instantiation is also given. The pair $(H_I, \lozenge)$ together defines the physically realized interaction: the two protocols share the same ungated Hamiltonian operator content but have different complete time-dependent generators, because their control profiles $\lozenge(t)$ differ. This distinction underlies the behavior observed in Unruh–DeWitt detectors [8,10,11], ultrafast coherent control experiments [7,16], cold-atom quenches [14,15], and finite-time scattering theory [4–6,17].

2.4.3. Physical Status and Experimental Determination of the Window

Formally, the windowed identity of Equations (5) and (6) holds for any fixed, bounded, sufficiently smooth admissible profile $\lozenge(t)$: no further physical assumption about the origin of the profile is required for the algebraic identity itself.

In a physical application, however, the profile is not arbitrary. It is fixed, before the observable of interest is calculated, by the reported experimental protocol: the detector switching function in the Unruh–DeWitt case of Section 3, the boundary-drive envelope in the dynamical Casimir case of Section 4, the ramp profile in the cold-atom quench of Section 5, the pulse envelope in the ultrafast-control case of Section 6, and the preparation, interaction, or detection interval in the finite-time scattering treatment of Section 7. In every case $\lozenge$ is read off from the apparatus or control sequence used to produce the reported data, and it introduces no fit parameter beyond those already contained in the experimental protocol.

A window profile chosen post hoc to improve agreement with measured results would constitute an additional fit function and is not part of the framework used here.

2.4.4. Equivalent Time-Dependent Parameterizations

Equation (7) may equally be written with the window absorbed into an overall time-dependent coupling, or into a time-dependent interaction operator:

$$H(t) = H_0 + \lambda \lozenge(t)\, H_I = H_0 + \lambda(t)\, H_I, \qquad \lambda(t) \equiv \lambda \lozenge(t), \tag{8}$$

or, equivalently, by defining

$$H_I(t) \equiv \lozenge(t)\, H_I. \tag{9}$$

These are algebraically equivalent parameterizations of the same physical content: whichever of $\lozenge(t)$, $\lambda(t)$, or $H_I(t)$ is regarded as fundamental, the predictions are identical whenever the three definitions specify the same complete time-dependent generator $H(t)$.

Reabsorbing $\Diamond$ into the coupling or into the interaction operator introduces no new physics, and is not disputed here.

The window is nonetheless kept separate throughout this paper, for three reasons, none of which asserts that the factorization changes the underlying physics. First, it distinguishes the interaction operator H_I, fixed by the microscopic coupling, from the coupling magnitude λ and the finite temporal support $\Diamond(t)$, which are set by the experimental protocol. Second, it exposes directly the region where $\partial_t \Diamond \neq 0$, which is exactly the boundary layer responsible for the apparent non-conservation of Equation (6); this region is obscured once $\Diamond$ is folded into $\lambda(t)$ or $H_I(t)$. Third, it permits the same action-level and boundary-layer analysis to be applied across five subfields that conventionally use different notation for what is, formally, the same object. Keeping $\Diamond$ explicit does not change the microscopic dynamics, but its role is not physically empty. The apparatus fixes the support of the interaction; the complete evolution and spectral response depend on that support, and $\partial_t \Diamond$ identifies the interval in which the localized subsystem exchanges the relevant conserved quantity with the control apparatus. The factorization therefore makes physical domain specification explicit while also providing a common analytical organization; it introduces no new interaction or degree of freedom.

2.4.5. The von Neumann Measurement Limit

In the standard von Neumann measurement model, a system observable $\hat{A}$ couples to the pointer momentum $\hat{P}$ through the interaction

$$H_{\text{int}}(t) = g(t)\, \hat{A} \otimes \hat{P}, \tag{10}$$

where $g(t)$ has support during the measurement interval [28]. Equation (7) contains this construction as the purely temporal special case

$$H_I = \hat{A} \otimes \hat{P}, \qquad g(t) = \lambda \Diamond(t). \tag{11}$$

The present framework therefore contains, rather than replaces, the standard von Neumann coupling and does not modify its measurement dynamics or the projection postulate. In this sense the present construction generalizes the von Neumann picture by localizing the interaction on a finite region of spacetime rather than only on a finite time interval.

The extension made here concerns domain specification and the level of formulation. Writing

$$S_{\text{int},\Diamond} = \int d^4x\, \Diamond(x) \mathcal{L}_{\text{int}}(x) \tag{12}$$

permits the prescribed interaction support to be temporal or spacetime-localized and makes its boundary-supported current balance explicit. The same action-level construction then applies not only to system–apparatus measurements, but also to driven boundaries, quantum quenches, pulse-controlled transitions, and finite-time scattering.

2.4.6. Time Evolution and Window Dependence

The unitary time-evolution operator generated by Equation (7) is

$$U(t_f, t_i) = \mathcal{T} \exp\left[-\frac{i}{\hbar} \int_{t_i}^{t_f} dt\, \left(H_0 + \lambda\, \Diamond(t)\, H_I \right) \right], \tag{13}$$

where $\mathcal{T}$ denotes time ordering. Even when H_0 and H_I are fixed, different choices of $\Diamond(t)$ yield inequivalent unitary operators and therefore inequivalent physical evolutions:

$$U_{\Diamond_1}(t_f, t_i) \neq U_{\Diamond_2}(t_f, t_i), \qquad \text{for generic } \Diamond_1 \neq \Diamond_2. \tag{14}$$

This formal fact explains why different switching profiles in Unruh–DeWitt detector models produce distinct excitation spectra even for identical detector Hamiltonians and trajectories [8,10,11], and why different pulse envelopes in ultrafast spectroscopy yield different transition probabilities for the same system Hamiltonian [7,16]. When the Hamiltonian terms commute at different times, the time-ordered exponential reduces to an ordinary exponential, but different profiles still yield different evolutions unless their integrated couplings are equal.

More generally, whenever a process is instantiated through a finite window $\Diamond(t)$, the leading spectral structure is controlled by the Fourier transform of the window or its gradient, schematically $\widetilde{\Diamond}(\omega)$ or $\widetilde{\partial_t \Diamond}(\omega)$. The dominant bandwidth is therefore set by the characteristic temporal width of the window profile, $\Delta\omega \sim O(1/\tau_\Diamond)$, with profile-dependent order-unity coefficients. The five worked cases below differ in their detailed dynamics, but all inherit this same window-defined bandwidth structure.

2.4.7. Energy–Time Structure and Boundary Contributions

For an initial state $|i\rangle$ and final state $|f\rangle$, first-order perturbation theory yields

$$A_{fi} = -\frac{i\lambda}{\hbar} \int dt \, \Diamond(t) \, e^{i\omega_{fi} t} \, \langle f|H_I|i\rangle, \tag{15}$$

where $\omega_{fi} = (E_f - E_i)/\hbar$. Equation (15) shows that the spectral content of the transition is determined by the Fourier transform of the localization window. This structure underlies the following: switching-dependent excitation probabilities in Unruh–DeWitt detectors [8,10,11]; pulse-duration-dependent spectral broadening in ultrafast coherent control [7,16]; and energy–momentum smearing in finite-time scattering theory [4–6,17].

For systems whose unwindowed stress-energy satisfies the usual local conservation law in the absence of external switching, the corresponding stress-energy bookkeeping takes the analogous finite-domain form:

$$\partial_\mu(\Diamond \, T^{\mu\nu}) = 0, \tag{16}$$

$$\Diamond \, \partial_\mu T^{\mu\nu} = -\big(\partial_\mu \Diamond\big) T^{\mu\nu}. \tag{17}$$

For internal symmetries, Equation (5) is the direct Noether identity of the windowed action. For spacetime translations and energy–momentum bookkeeping, the same structure should be read as the finite-domain conservation statement for the localized system together with the externally specified control window. The subsystem stress-energy alone need not be conserved across the boundary layer; its apparent non-conservation is precisely the flux term proportional to $\partial_\mu \Diamond$. In the experimental examples below, this flux is supplied by the preparation, drive, ramp, or detection protocol that defines the window.

2.4.8. Physical Distinctness of Identical Hamiltonians

Two processes governed by the same Hamiltonian operator but instantiated with different $\Diamond(t)$ are physically distinct in the same sense that two processes with different initial states are physically distinct. This is experimentally manifest in the following: Unruh–DeWitt detectors, where switching-dependent excitation spectra are observed [8,10,11]; ultrafast coherent control, where envelope-dependent yields and interference patterns arise [7,16]; cold-atom quenches, where ramp-rate-dependent currents are measured [14,15]; and finite-time scattering, where preparation-dependent energy distributions are recorded [4,5].

Standard treatments accommodate these differences pragmatically. The windowed formalism identifies the localization domain as a shared formal element across all five cases.

2.4.9. Formal Significance of Domain Specification

The Hamiltonian specifies *what* dynamics occur; the localization window specifies *when and where* those dynamics are physically active. This distinction is not optional: the windowed action formalism makes domain specification structurally consequential, so that the Noether identities, the conservation structure, and the spectral content of transitions all depend on the window, not on the Hamiltonian alone. Treating $\Diamond$ as a removable artifact discards this structure and forces case-specific workarounds, including regulators, wave packets, and adiabatic switching, in place of a single derivation that covers all five cases simultaneously. In simple terms: localization constraints $\rightarrow$ windowed Noether identity $\rightarrow$ boundary-layer flux $\rightarrow$ physical inequivalence of identical Hamiltonians.

3. Case 1: Timelike Unruh Effect in Trapped-Ion Detectors

3.1. Experimental Observable and Standard Formulation

The Unruh effect predicts that a uniformly accelerated detector interacting with a quantum field in its vacuum state responds thermally with an effective temperature proportional to its proper acceleration [1,2]. While direct observation at macroscopic accelerations remains impractical, a related thermal response (the timelike Unruh effect) can occur for inertial detectors confined to future or past light cones, due to timelike entanglement in the quantum vacuum [29,30]. Analog realizations using trapped ions provide controlled platforms in which Unruh–DeWitt (UDW) detector dynamics can be probed experimentally [8]. In the Luo et al. experiment, the ion's internal spin serves as the two-level detector and the vibrational mode encodes the effective quantum field; the experiment constitutes a proof-of-principle analog simulation in which the single-mode bosonic two-point function plays the role of the Wightman function, rather than a direct measurement in a relativistic quantum field vacuum. Here, the phonon is the quantized collective vibrational mode of the trapped-ion simulator, which plays the role of the simulated bosonic field [31].

In these systems, a two-level detector is coupled to a quantized field along a prescribed trajectory. The interaction Hamiltonian is conventionally written as

$$H_I(\tau) = \lambda\, \chi(\tau)\, m(\tau)\, \phi[x(\tau)], \tag{18}$$

where τ is the detector's proper time, $m(\tau)$ is the detector monopole operator, ϕ is the field operator evaluated along the trajectory $x(\tau)$, λ is a small coupling constant, and $\chi(\tau)$ is a switching function that turns the interaction on and off [8]. (Equation (18) represents the model interaction Hamiltonian used for clarity of exposition. The full Hamiltonian of the trapped-ion system may be found in [8]).

The experimentally measured quantities are the excitation and de-excitation probabilities $P_{g\to e}$ and $P_{e\to g}$ as functions of effective acceleration, coupling duration, and detector gap frequency. Standard treatments emphasize that without $\chi(\tau)$ the detector response function diverges; $\chi(\tau)$ is therefore introduced to regularize the theory, yet is typically treated as a parametrized calculational necessity rather than as a physically constitutive element of the process. Different choices of $\chi(\tau)$ lead to quantitatively different responses, particularly at short times, as analyzed in detail in [9–11].

3.2. Windowed Interpretation: Switching Function as Localization Domain

In the windowed action formalism, the switching function $\chi(\tau)$ is not an auxiliary regulator; it is the temporal localization window through which the detector–field interaction is physically instantiated. The distinction matters: a regulator is removed at the end of a calculation; a localization window is fixed by the physical setup. For this case, I therefore identify

$$\Diamond(\tau) \equiv \chi(\tau). \tag{19}$$

The leading-order excitation probability is then

$$P_{g\to e} = \lambda^2 \int d\tau \int d\tau' \, \Diamond(\tau)\, \Diamond(\tau')\, e^{-i\omega(\tau-\tau')}\, W(\tau,\tau'), \tag{20}$$

where ω is the detector energy gap and $W(\tau,\tau')$ is the Wightman function of the field evaluated along the detector trajectory.

In the windowed interpretation, the detector–field interaction exists only within the interval where $\Diamond(\tau) \neq 0$. The boundary-layer thickness $\ell_\Diamond$ defines the temporal resolution with which energy exchange is physically active. No modification is made to the field theory or detector dynamics; the Unruh response is recovered in the same way as in standard calculations, once the physical domain of interaction is correctly specified. Divergences associated with infinite interaction time are removed by the same compact temporal support that standard treatments introduce as a regulator; the new formal result of the windowed approach is to establish that compact support as the physically constitutive interaction domain, from which the windowed Noether identity follows as a consequence rather than as an auxiliary prescription.

3.3. Quantitative Correspondence and Dominant Scale

In trapped-ion experiments, excitation and de-excitation probabilities exhibit oscillatory structure and saturation effects that depend sensitively on the switching profile and duration [8]. In the proof-of-principle trapped-ion realization of Luo et al., the switching profile is taken to be exponential with characteristic time $T_d = 0.2$ ms, while the phonon-mode frequencies are $\omega_p/(2\pi) = 25$ and 50 kHz, the spin frequency is $\omega_q/(2\pi) = 200$ kHz, and the coupling scale is $g_0/(2\pi) = 5$ kHz [8]. The exponential profile and its characteristic time T_d are fixed in advance by the programmed control sequence of the ion-trap apparatus, not adjusted after the excitation probability is measured. Identifying the window scale with the experimental switching time gives

$$\tau_\Diamond^{(\mathrm{UDW})} = T_d = 2.0 \times 10^{-4}\ \mathrm{s}, \qquad \ell_\Diamond^{(\mathrm{UDW})} = cT_d \approx 6.0 \times 10^{4}\ \mathrm{m}. \tag{21}$$

The corresponding boundary frequency scale is $\tau_\Diamond^{-1} \approx 5$ kHz, matching the measured detector–field coupling scale in order of magnitude. The large value of $\ell_\Diamond$ should therefore be understood as an equivalent causal distance associated with the switching time, not as a literal detector size. Once this dominant timescale is fixed by the experimental pulse shape, the windowed Noether identity of Equations (5) and (6) enforces consistent energy bookkeeping across the boundary layer and constrains the de-excitation behavior without additional adjustment. This explains why different switching profiles produce distinct excitation spectra for identical Hamiltonians and trajectories [9–11]: the switching function is part of the physical specification of the interaction domain, not an arbitrary regulator.

3.4. Comparison with the Standard Description

Table 2 summarizes the correspondence between the standard UDW treatment and the windowed description for this case.

Table 2. Case 1: Timelike Unruh Effect—Windowed versus Standard Description.

Feature	Standard UDW	Windowed Description
Switching function	Mathematical regulator	Physical localization window
Divergences	Removed by regularization	Controlled by compact temporal support
Profile dependence	Regulator/profile choice	Physically constitutive control profile
Free parameters	Profile plus duration choice	Dominant timescale from switching profile
Conservation	Subsystem bookkeeping implicit	Exact windowed current

4. Case 2: Dynamical Casimir Effect in Superconducting Circuits

4.1. Experimental Observable and Standard Formulation

The dynamical Casimir effect (DCE) arises when time-dependent boundary conditions convert vacuum fluctuations into real photons. In superconducting circuit implementations, a transmission line terminated by a SQUID allows rapid modulation of the effective boundary condition, producing measurable photon flux and two-mode squeezing correlations [12,13].

The two implementations considered here use distinct SQUID architectures. Wilson et al. employ an open coplanar transmission line terminated by a flux-tunable SQUID; modulation of the SQUID Josephson inductance changes the effective electrical length and boundary condition of the line [12]. Lähteenmäki et al. employ a 4 mm coplanar-waveguide Josephson metamaterial containing 250 SQUIDs and operated as a 5.4 GHz microwave cavity; flux modulation changes the propagation velocity and effective electrical length of the cavity [13]. In both configurations, the physical circuit and static boundary remain present, while the applied flux pump supplies the externally driven time dependence.

Standard treatments model the boundary as an effectively moving mirror with prescribed trajectory or impedance and, in some formulations, employ explicit UV cutoffs or frequency-dependent regularization to control divergences induced by mode-mixing [12,13].

4.2. Windowed Formulation: Localized Boundary Instantiation

In the windowed formalism, the time-dependent modulation of the boundary is active only during the finite temporal interval in which the SQUID flux pump is applied. This is represented by a temporal window $\Diamond(t)$ multiplying the boundary interaction. More explicitly, the time-dependent flux pump may be written schematically as

$$\delta\Phi_{\mathrm{ext}}(t) = \delta\Phi_0 \, \Diamond_{\mathrm{env}}(t) \cos(\omega_d t + \varphi), \tag{22}$$

where $\Diamond(t) \equiv \Diamond_{\mathrm{env}}(t)$ is the finite pump envelope, and ω_d is the microwave carrier frequency. Let T_{env} denote the protocol-dependent duration for which the envelope is appreciably nonzero; the rise and fall times determine its boundary-layer widths. The envelope determines the finite temporal support, while the carrier drives the standard mode mixing within that support.

The full Bogoliubov transformation relating incoming and outgoing field modes involves mixing coefficients $\alpha_{kk'}$ and $\beta_{kk'}$ determined by the pump profile; photon creation arises from the $\beta_{kk'}$ coefficients, which mix positive- and negative-frequency modes. In schematic form, the windowed Bogoliubov transformation may be written as

$$a_k^{\mathrm{out}} = \sum_{k'} \left[\alpha_{kk'}[\Diamond_{\mathrm{env}}, \omega_d] \, a_{k'}^{\mathrm{in}} + \beta_{kk'}[\Diamond_{\mathrm{env}}, \omega_d] \, a_{k'}^{\mathrm{in}\,\dagger} \right], \tag{23}$$

where the coefficients $\alpha_{kk'}$ and $\beta_{kk'}$ are determined by the pump envelope, carrier frequency, circuit parameters, and the finite temporal support of the boundary modulation. The resulting β coefficients quantify photon creation in the usual way. This schematic form represents the distinct roles of the temporal envelope and carrier in the Bogoliubov transformation; the full calculation requires the standard mode-function matching at the driven boundary [12]. The pump envelope determines the Fourier width associated with finite activation and deactivation, while the carrier frequency supplies the pair-production condition and the circuit response selects the populated modes. Keeping these roles separate makes the finite interaction domain explicit without replacing the standard mode-mixing calculation.

4.3. Comparison with Experimental Observations

Observed photon spectra and squeezing correlations depend strongly on the temporal profile of the boundary modulation [12,13]. In the Josephson-metamaterial realization of Lähteenmäki et al., the cavity resonance is $f_c = 5.4$ GHz and the boundary is driven at $f_d = 10.8$ GHz, while Wilson et al. report modulation near 11 GHz in the original SQUID-terminated transmission-line experiment [12,13]. The drive frequency and modulation waveform are fixed by the flux-pump signal applied to the SQUID before the photon spectrum is recorded, not tuned afterward to match the observed spectrum. For a harmonically driven boundary, the carrier timescale is the inverse angular drive frequency,

$$\tau_d^{(\mathrm{DCE})} = \omega_d^{-1} = (2\pi f_d)^{-1} \approx 1.47 \times 10^{-11}\ \mathrm{s}, \qquad \ell_d^{(\mathrm{DCE})} = c\tau_d \approx 4.4\ \mathrm{mm}. \tag{24}$$

Equation (24) characterizes the microwave carrier, not the duration or boundary-layer thickness of $\Diamond_{\mathrm{env}}$; both figures follow from the Lähteenmäki et al. drive frequency and are reported here as carrier-equivalent scales only.

The finite temporal support is fixed separately, by the flux-pump and acquisition protocol rather than by the carrier. Wilson et al. operated the drive in continuous-wave mode for the frequency scans, but for the broadband photon-spectrum measurements the drive was chopped between on and off states of 50 ms each [12]. The chopped protocol therefore supplies a directly reported finite window,

$$T_{\mathrm{env}}^{(\mathrm{DCE})} = T_{\mathrm{on}} = 5.0 \times 10^{-2}\ \mathrm{s}, \qquad L_{\mathrm{on}}^{(\mathrm{DCE})} = cT_{\mathrm{on}} \approx 1.5 \times 10^{7}\ \mathrm{m}. \tag{25}$$

Here, $L_{\mathrm{on}}^{(\mathrm{DCE})}$ is the causal-equivalent extent of the active interval, not the physical size of the circuit. It is also distinct from the boundary-layer thickness $\ell_\Diamond$ used elsewhere in this paper: that quantity is set here by the rise and fall time of the pump, which Wilson et al. do not report and which is therefore left unassigned. Three scales are thus distinguished in this case: the carrier τ_d of Equation (24), the active-window duration T_{env} of Equation (25), and the unreported edge scale τ_{edge}.

The separation is wide. The envelope frequency scale $T_{\mathrm{on}}^{-1} = 20$ Hz stands against a carrier near 11 GHz in the same experiment [12], a ratio of order 10^{-9}. The windowed description therefore predicts that the finite-time correction to the spectral selection rule is negligible for this protocol and that $\widetilde{\Diamond}_{\mathrm{env}}(\Delta E)$ is indistinguishable from the idealized $\delta(\Delta E)$ at the resolution of the measurement. This is a consistency check rather than a new prediction: the formalism locates the window correctly and then correctly returns the standard result in the regime where the window is long compared with the dynamics it contains. The continuous-wave scans represent the complementary limit, in which no independent finite pump-envelope duration is specified by the protocol at all. That the same device supports both protocols illustrates directly that the window is a property of the experimental procedure rather than of the Hamiltonian or the hardware.

4.4. Comparison with the Standard Description

In the windowed formalism, the DCE remains the standard boundary-mode-mixing process, with the finite flux-pump envelope made explicit as the localization window. The envelope specifies when the effective boundary modulation is active, the carrier drives mode mixing within that support, and the circuit determines which modes are populated. These roles are collected in Table 3.

Table 3. Case 2: Dynamical Casimir Effect—Windowed versus Standard Description.

Feature	Standard DCE	Windowed Description
Boundary modulation	Effective moving-boundary model	Flux-pump envelope with carrier-driven modulation
Bandwidth control	Drive-profile and boundary-model dependent	Envelope support separated from carrier scale
Parameter inputs	Drive waveform and circuit parameters	Pump-envelope support ($T_{\mathrm{on}} = 50$ ms chopped) and carrier frequency ($f_d \sim 11$ GHz)
Interpretation	Driven boundary mode mixing	Finite-domain activation of standard mode mixing

5. Case 3: Quench-Induced Currents in Cold-Atom Systems

5.1. Experimental Observables and Standard Description

Cold-atom platforms provide an exceptionally clean setting for studying nonequilibrium quantum dynamics. In many experiments, system parameters such as lattice depth, interaction strength, or spin–orbit coupling are changed over a finite time interval, producing quench-induced currents. Measured observables include time-dependent particle currents, spin currents, and associated relaxation profiles following the quench. Representative settings include quench-induced spontaneous currents in rings of ultracold fermionic atoms, spin-current generation and relaxation in a quenched spin–orbit-coupled Bose–Einstein condensate, and related optical-lattice quench protocols for probing equilibrium current patterns [14,15,32].

In standard treatments, these phenomena are analyzed using time-dependent Hamiltonians of the form

$$H(t) = H_0 + \lambda(t)\, H_1, \tag{26}$$

where $\lambda(t)$ is a prescribed ramp function. Nonadiabatic excitations are attributed to Landau–Zener-type transitions, while damping is modeled using phenomenological terms or coupling to external baths. While successful phenomenologically, this approach typically treats the ramp profile as an external control parameter without deeper physical interpretation.

5.2. Windowed Interpretation: Quench Ramps as Localization Windows

In the windowed formalism, the quench ramp defines a temporal localization boundary: the system transitions over a finite interval from one physical configuration to another, and the rate of that transition is set by the ramp profile. The boundary layer of the localization window is identified with this ramp interval. For a smooth switch-on from pre-quench to post-quench configuration, the window gradient $\partial_t \Diamond$ is concentrated in the ramp region of thickness τ_q:

$$\Diamond(t) = \frac{1}{2}\left[1 + \tanh\left(\frac{t - t_0}{\tau_q}\right)\right], \tag{27}$$

where τ_q is the characteristic ramp duration. Note that Equation (27) is a one-sided switch-on: $\Diamond \to 0$ before the quench and $\Diamond \to 1$ after it, so $\partial_t \Diamond$ is nonzero only during the ramp

interval $|t - t_0| \lesssim \tau_q$ and vanishes outside it. This profile captures the localization boundary of the quench transition; the post-quench system is fully instantiated within the window. Unlike the compact-support windows of Cases 1, 2, and 5, this is a permanent switch-on: the boundary layer is concentrated near $t = t_0$ and the windowed formalism applies to the transient transition interval rather than a bounded active domain.

The windowed Noether identity then provides a formal analogy for the transient boundary-layer flux generated during the quench:

$$\Diamond\, \partial_\mu J^\mu = -(\partial_\mu \Diamond) J^\mu, \tag{28}$$

with the right-hand side concentrated in the ramp interval where $\partial_t \Diamond \neq 0$. It is important to note that standard quench theory fully accounts for transient currents via Landau–Zener-type nonadiabatic dynamics: the quench generates superpositions of post-quench energy eigenstates that produce oscillating transient currents. The windowed Noether identity does not replace this dynamical account; rather, it establishes a new formal result for the quench setting: the dominant transient timescale hierarchy is governed by the Fourier width of the window gradient $\partial_t \Diamond$, which is $\sim 1/\tau_q$, and the boundary-layer flux of Equation (6) holds exactly over the ramp interval. Transient currents are concentrated in the ramp interval, consistent with Equation (5), and their qualitative scaling with τ_q is captured by the window structure without requiring a separate case-specific conservation analysis. The detailed amplitude and late-time relaxation remain those of the standard quench dynamics.

5.3. Quantitative Correspondence

Experiments show that both the amplitude and decay time of quench-induced currents scale with the quench duration τ_q [14,15]. A representative experimental scale is provided by the spin-orbit-coupled Bose–Einstein condensate quench studied by Li et al., where the Raman coupling is ramped to its final value in $t_E = 1$ ms and the reported thermalization and spin-current decay constants span the millisecond range, from $0.4(1)$ to $5.1(8)$ ms [32]. The ramp duration t_E is fixed by the programmed Raman-coupling ramp protocol before the spin current is measured, not selected afterward to match the observed decay. The programmed value $t_E = 1$ ms is the sole experimental input used to set $\tau_\Diamond^{(\mathrm{q})}$ below. The values from $0.4(1)$ to $5.1(8)$ ms are independently measured decay and thermalization constants quoted only to show that the response occupies the same millisecond hierarchy; they are not averaged with t_E, fitted, or used to derive Equation (27) or the numerical window scale. Identifying the window width with the ramp time gives

$$\tau_\Diamond^{(\mathrm{q})} = \tau_q = t_E = 1.0 \times 10^{-3}\ \mathrm{s}, \qquad \ell_\Diamond^{(\mathrm{q})} = c\tau_q \approx 3.0 \times 10^5\ \mathrm{m}. \tag{29}$$

The corresponding frequency scale is $\tau_q^{-1} \approx 1$ kHz, which lies in the same ms-scale hierarchy as the measured transient response times. As in Case 1, the large value of $\ell_\Diamond$ is an equivalent causal length associated with the temporal ramp, not a literal cloud size. In the windowed formalism, the observed scaling follows directly from the Fourier width of $\partial_t \Diamond(t)$, which is $\sim 1/\tau_q$: a slower quench (τ_q large) produces a narrower frequency content and suppresses high-frequency excitations, while a sharper quench (τ_q small) broadens the excitation spectrum. Once τ_q is fixed by the experimental ramp profile, the window gradient sets the characteristic timescale hierarchy of the transient response without introducing additional fitting parameters. This scaling holds across different quench shapes, reflecting the generality of the Fourier relationship between ramp duration and excited-frequency bandwidth.

5.4. Comparison with the Standard Description

Table 4 sets the ramp-based windowed picture against the standard nonadiabatic treatment.

Table 4. Case 3: Quench-Induced Currents—Windowed versus Standard Description.

Feature	Standard Quench Theory	Windowed Description
Ramp function	External control	Physical instantiation window
Nonadiabaticity	Landau–Zener-type dynamics	Boundary-layer flux form
Damping/relaxation	Standard dynamics and baths	Dominant transient scale from window
Conservation laws	Usually implicit	Windowed conservation form
Parameters	Ramp-profile inputs	Dominant scale τ_q

6. Case 4: Ultrafast Coherent Control and Finite Laser Pulses

6.1. Experimental Observables and Standard Description

Ultrafast spectroscopy and coherent control experiments employ femtosecond or attosecond laser pulses to drive transitions in atoms, molecules, and solids. Observables include photoelectron yields, excitation probabilities, and interference fringes arising from multi-pulse sequences [7,16].

In standard theory, the transition amplitude between states $|i\rangle$ and $|f\rangle$ driven by a laser pulse with envelope $E(t)$ is written as

$$A_{fi} = \int_{-\infty}^{\infty} dt\; E(t)\, e^{i\omega_{fi}t}\, V_{fi}, \tag{30}$$

where $\omega_{fi} = (E_f - E_i)/\hbar$ and V_{fi} is the dipole matrix element (with $E(t)$ absorbed into the normalization of V_{fi}). Finite pulse duration is essential to obtain finite results and is typically interpreted as a practical experimental constraint. The standard account invokes energy–time uncertainty relations heuristically to explain spectral broadening and temporal interference effects [7,16].

6.2. Windowed Interpretation: Pulse Envelopes as Time Instantiation

In the windowed formalism, the pulse envelope is identified with the localization window via (This identification uses the pulse envelope $E(t) \geq 0$, not the rapidly oscillating physical electric field $E_{\text{phys}}(t) = E(t)\cos(\omega_0 t + \phi)$, which takes both positive and negative values. The window $\Diamond(t) = E(t)/E_{\max}$ satisfies $0 \leq \Diamond \leq 1$ because the envelope is non-negative by construction. In practice, the envelope is extracted from the experimental pulse characterization before applying this identification.)

$$\Diamond(t) \equiv E(t)/E_{\max}. \tag{31}$$

The transition amplitude becomes

$$A_{fi} = \int dt\; \Diamond(t)\, e^{i\omega_{fi}t}\, \tilde{V}_{fi}, \tag{32}$$

where $\tilde{V}_{fi} = E_{\max} V_{fi}$ absorbs the normalization, and all physically meaningful time evolution is confined to the support of $\Diamond(t)$. Energy spread arises not from abstract uncertainty arguments but from the Fourier structure of the localization window itself; observed interference effects in pump–probe and coherent control experiments reflect the overlap between multiple localization windows.

6.3. Agreement with Observed Behavior

Ultrafast experiments show that shorter pulses produce broader spectra, pulse shaping controls interference visibility, and phase-locked pulse sequences generate controllable fringes [7,16]. A representative numerical example is provided by Matselyukh et al., who use a 5.2 fs FWHM optical pulse spanning 500–1000 nm to initiate strong-field ionization dynamics in silane, with attosecond soft-X-ray absorption used to probe the resulting ultrafast molecular dynamics [33]. The localization window is identified with the optical pump envelope, which spans 500–1000 nm and constitutes the physical interaction domain for the matter-light coupling. The pulse envelope is fixed by independent characterization of the optical pump pulse before the ionization dynamics are recorded, not inferred from the measured dynamics themselves. For a Gaussian pump envelope, the corresponding standard deviation is

$$\tau_{\Diamond}^{(\mathrm{ultra})} = \frac{\Delta t_{\mathrm{FWHM}}}{2\sqrt{2\ln 2}} = \frac{5.2\ \mathrm{fs}}{2\sqrt{2\ln 2}} \approx 2.21\ \mathrm{fs}, \tag{33}$$

so that

$$\ell_{\Diamond}^{(\mathrm{ultra})} = c\tau_{\Diamond} \approx 6.6 \times 10^{-7}\ \mathrm{m} = 0.66\ \mu\mathrm{m}. \tag{34}$$

This inferred window thickness lies directly inside the reported optical wavelength span, showing that the boundary layer extracted from the pulse duration is of the same scale as the physically active optical field. The corresponding transform-limited bandwidth, $\Delta\nu_{\mathrm{FWHM}} \approx 0.44/\Delta t_{\mathrm{FWHM}} \approx 8.5 \times 10^{13}$ Hz, is likewise the observed few-cycle spectral scale. The windowed Noether identity established here for the ultrafast setting gives a new formal result: the spectral broadening and interference structure follow directly from the Fourier content of $\Diamond(t)$, replacing the heuristic energy–time uncertainty argument with a precise windowed conservation statement. Once the pulse envelope is specified, no additional free parameters are required to organize the dominant spectral width and interference trends.

6.4. Comparison with the Standard Description

The pulse-envelope window and the standard treatment of this case are contrasted in Table 5.

Table 5. Case 4: Ultrafast Coherent Control—Windowed versus Standard Description.

Feature	Standard Coherent Control	Windowed Description
Pulse envelope	Experimental control input	Domain-restriction window
Spectral width	Pulse bandwidth/uncertainty	Window Fourier width
Interference	Wave packet/pulse overlap	Window overlap
Parameters	Pulse-shape dependent	Dominant scale from envelope

7. Case 5: Finite-Time Scattering and Windowed Energy–Momentum Conservation

7.1. Standard Formulation and Its Idealizations

Relativistic and nonrelativistic scattering theory occupies a central place in quantum mechanics and quantum field theory. In its canonical formulation, transition amplitudes between asymptotic in- and out-states are computed by integrating interaction terms over infinite time [4–6,17]. This procedure yields exact energy–momentum conservation expressed through Dirac delta distributions (Standard notation: $\mathcal{A}_{fi}$ is the transition amplitude from initial state i to final state f; E_i, E_f are total initial and final energies; $\mathbf{p}_i$, $\mathbf{p}_f$ are total initial and final momenta; δ is the Dirac delta function; and $\delta^{(3)}$ its three-dimensional counterpart. Throughout Case 5, $\hbar$ appears explicitly in dynamical amplitudes and Fourier

phase factors; the windowed conservation identities are classical bookkeeping relations and contain no standalone $\hbar$ prefactors.)

$$\mathcal{A}_{fi} \propto \delta(E_f - E_i)\, \delta^{(3)}(\mathbf{p}_f - \mathbf{p}_i), \tag{35}$$

which arise mathematically from the identity

$$\int_{-\infty}^{\infty} dt\; e^{i(E_f - E_i)t/\hbar} = 2\pi\hbar\, \delta(E_f - E_i). \tag{36}$$

This idealization underpins the S-matrix formalism and is extraordinarily successful for calculating cross sections. However, its assumptions are never physically realized: real scattering experiments involve finite-duration beam preparation, finite interaction regions, and detectors with limited temporal resolution. Measured energy and momentum distributions always exhibit finite widths rather than exact delta-function support.

Standard treatments accommodate this by introducing regulators, wave packets, or switching functions under the rubric of "adiabatic switching" or "finite-time effects," typically justified pragmatically without deeper physical significance [4–6,17].

7.2. Windowed Interpretation: Scattering as a Localized Temporal Process

In the windowed formalism, finite-time scattering is not merely a technical correction to the infinite-time idealization. It reflects the physical fact that scattering interactions are instantiated within finite preparation, interaction, and detection windows.

The transition amplitude is written as

$$\mathcal{A}_{fi} = -\frac{i}{\hbar} \int dt\; \Diamond(t)\, e^{i(E_f - E_i)t/\hbar}\, \mathcal{M}_{fi}, \tag{37}$$

where $\mathcal{M}_{fi}$ is the invariant matrix element and $\Diamond(t)$ is the temporal localization window encoding the duration over which the interaction is physically active. For a compact window approximated by a top-hat profile of duration T, integrating Equation (37) gives

$$\mathcal{A}_{fi} = -\frac{i}{\hbar}\, \mathcal{M}_{fi} \int_{-T/2}^{T/2} dt\; e^{i\Delta E\, t/\hbar} = -\frac{iT}{\hbar}\, \mathcal{M}_{fi}\, \mathrm{sinc}\!\left(\frac{\Delta E\, T}{2\hbar}\right), \tag{38}$$

where $\Delta E = E_f - E_i$ and $\mathrm{sinc}(x) \equiv \sin(x)/x$, replacing the exact energy delta function of Equation (35) with a sinc distribution whose width is set by the localization scale T. In the infinite-time limit $T \to \infty$, $T\,\mathrm{sinc}(\Delta E\, T/2\hbar) \to 2\pi\hbar\, \delta(\Delta E)$, recovering the standard result. (Here the temporal window is written explicitly, so Equation (38) displays the finite-time replacement of the exact energy delta function. Momentum-space smearing is obtained in the same way when the spatial support of the beam, wave packet, or detector acceptance is included).

In the windowed formalism, this smearing is not interpreted as a failure of energy conservation. Rather, it reflects the stress-energy version of the windowed Noether identity:

$$\Diamond\, \partial_\mu T^{\mu\nu} = -\big(\partial_\mu \Diamond\big)\, T^{\mu\nu}, \tag{39}$$

which guarantees exact conservation of the *windowed* stress-energy $\Diamond\, T^{\mu\nu}$ while allowing apparent subsystem non-conservation localized to the boundary layer. The physically relevant spectral "selection rule" is not the exact $\delta(\Delta E)$ but the window transform $\widetilde{\Diamond}(\Delta E)$, with the delta function recovered only in the idealized limit of infinite localization duration.

7.3. Physical Meaning and Universality

This interpretation clarifies several long-standing features of scattering theory: (1) energy spread is unavoidable because exact energy conservation requires infinite temporal support, which is physically unrealizable; (2) wave packets are not optional refinements but operational necessities reflecting localization; and (3) the $\Diamond$ used here is a genuine finite-time window, distinct from the adiabatic regulator $e^{-\epsilon|t|}$ employed in the standard Gell-Mann–Low formalism [34], where $\epsilon \to 0$ is taken at the end to project out the vacuum-to-vacuum amplitude. The adiabatic regulator is removed in the limit and carries no physical content about interaction duration; the localization window $\Diamond$ is not removed but fixed by the physical preparation of the scattering experiment.

Importantly, the windowed formalism makes no new predictions for total cross sections in regimes where the infinite-time approximation is valid. A representative example in which the interaction-localization window is directly tied to the beam structure is provided by proton bunches in the HL-LHC. The bunch length σ_z is fixed by the accelerator's beam optics and injection parameters before any cross section is measured, independent of the scattering process under study. With rms bunch length $\sigma_z = 7.55$ cm, the corresponding temporal width is

$$\tau_{\Diamond}^{(\mathrm{scat})} = \sigma_z/c \approx 2.5 \times 10^{-10}\ \mathrm{s}, \qquad \ell_{\Diamond}^{(\mathrm{scat})} = \sigma_z = 7.55\ \mathrm{cm}, \tag{40}$$

so that the associated Fourier width is $\Delta E \sim \hbar/\tau_{\Diamond} \approx 2.6 \times 10^{-6}$ eV, microscopically tiny compared with TeV collision energies [35]. This illustrates the asymptotic regime in which finite-time effects are present formally but negligible experimentally.

A complementary example in which finite pulse and time-of-flight structure is experimentally relevant is the CERN n_TOF facility. The facility uses a white neutron source produced by 20 GeV/c protons and an experimental area located near the end of a roughly 200 m beam line; the associated resolution function is explicitly non-Gaussian and energy dependent rather than an exact delta function [36,37]. The facility review reports relative energy resolutions ranging from about 3×10^{-4} below 10 eV to about 5×10^{-3} near 1 MeV, demonstrating directly that finite pulse duration, flight path, and moderation-time structure remain operationally relevant even in a precision time-of-flight setting [36]. In this case the same localization logic appears through the experimentally imposed pulse and flight-time window, but the observed width is most naturally expressed through the facility's resolution function rather than the simple estimate $\Delta E \sim \hbar/\tau_{\Diamond}$. Together, the HL-LHC and n_TOF examples show both regimes: one in which the Fourier width is microscopically negligible and one in which finite temporal structure is experimentally resolved.

7.4. Comparison with the Standard Description

This case is particularly significant because it demonstrates that the same localization principle underlying Unruh detectors, the dynamical Casimir effect, quenches, and ultrafast control also governs the most foundational formalism of quantum theory. The windowed Noether identity $\partial_\mu(\Diamond\, T^{\mu\nu}) = 0$ established here for the scattering window is the formal counterpart of what standard treatments achieve piecemeal through adiabatic switching, wave packets, and regulators; the new result is that all of these are instances of a single derivation. Table 6 tabulates the comparison in detail.

Table 6. Case 5: Finite-Time Scattering—Windowed versus Standard Description.

Feature	Standard Scattering Theory	Windowed Interpretation
Time domain	Infinite, idealized	Finite, localized
Energy support	Exact delta function in asymptotic limit	Windowed conservation at finite time
Finite-time treatment	Adiabatic switching/wave packets	Finite-time physical support
Wave packets	Standard localization tool	Operational domain specification
Interpretation	Asymptotic abstraction	Finite-window domain

8. Cross-Case Synthesis, Summary, and Outlook

8.1. Cross-Case Synthesis

The five cases examined here span quantum field detectors, vacuum radiation phenomena, nonequilibrium many-body dynamics, strong-field and ultrafast optical control, and foundational scattering theory. In none of them are additional dynamical modifications required. What changes between cases is only the profile of the window, $\Diamond$, the particular shape of the control function that experimentalists already specify as part of their protocol, and the characteristic scale or scale hierarchy that protocol defines. The windowed Noether identity, $\partial_\mu(\Diamond J^\mu) = 0$, is the same formal statement in every case. Said another way: five communities have been solving the same formal problem with five different tools. At the finite-domain level, all five tools are $\Diamond$. The common output of the formulation may therefore be summarized by the triplet $\{\Diamond, \partial_\mu\Diamond, \widetilde{\Diamond}\}$, which respectively records the interaction support, boundary exchange, and finite-time spectral weighting.

8.2. Global Summary

The repeated emergence of the same windowed structure across these cases suggests that finite-domain localization is not merely an auxiliary feature of specific experimental protocols but a formal unifying structure across physically distinct regimes. Using representative experimental parameters, causal lengths span roughly thirteen orders of magnitude: from the boundary-layer scales $\ell_\Diamond \sim 0.66$ μm (ultrafast pulses) and 7.55 cm (HL-LHC bunch localization) through 6.0×10^4 m (trapped-ion timelike Unruh switching) and 3.0×10^5 m (cold-atom quenches), to the active-window extent $L_{\mathrm{on}}^{(\mathrm{DCE})} \approx 1.5 \times 10^7$ m (chopped dynamical Casimir drive). These values are not apparatus sizes; they are the causal lengths associated with the experimentally imposed temporal windows, read either as a boundary-layer thickness or, in the DCE case, as the extent of the active interval itself. The separate figure $c\omega_d^{-1} \approx 4.4$ mm is a carrier-equivalent distance rather than a window scale, and is not interpreted as a boundary-layer length. Table 7 collects these results case by case.

8.3. Interpretation

A natural objection is that localization windows merely encode good calculational practice rather than physical structure. I argue that this framing has the dependency backwards. In each of the five cases, the window is not introduced after the fact to regularize an otherwise divergent calculation; it is specified at the outset by the experimental protocol. The window is constitutive of the process, not a cleanup device applied to an idealized description of it.

Table 7. Summary of all five cases in the windowed formalism. Each case is characterized by an experimentally specified temporal window profile and the characteristic scale or scale hierarchy relevant to that protocol.

Case	Platform	Observable	Window	Characteristic Scale(s)	Standard Interpretation
I	Unruh detectors	Detector excitation	Temporal	T_d	Switching regularization
II	Dynamical Casimir	Photon spectrum	Temporal	$T_{\text{env}} = 50$ ms (chopped); carrier ω_d^{-1}	Moving boundary
III	Quench currents	Transient currents	Temporal	τ_q	Nonadiabatic dynamics
IV	Ultrafast spectroscopy	Spectral width	Temporal	Δt_{FWHM}	Finite pulse envelope
V	Scattering theory	Energy spread	Temporal	σ_z/c or Δt	Adiabatic switching/wave packets

In each case, the windowed description introduces no additional dynamical parameters beyond those already present in the experimental protocol. Standard treatments invoke case-specific regulator, profile, or control-language choices to achieve the same calculations; the windowed formalism replaces these with a single formal derivation. The new results are as follows: (i) the windowed Noether identity $\partial_\mu(\Diamond J^\mu) = 0$, derived from the windowed action for each of the five settings; (ii) the cross-case identification that the switching function, drive envelope, quench ramp, pulse envelope, and scattering window are all instances of the same formal object $\Diamond$; and (iii) the formal result developed in Section 2.4 that processes sharing identical Hamiltonians but having different localization histories are physically inequivalent in a precise, quantifiable sense.

This does not mean that a single scalar timescale exhausts the detailed phenomenology of every case. Profile shape, standard dynamical response, and system-specific transfer functions still matter. The narrower claim is that once the experimentally specified window profile is fixed, its characteristic scale hierarchy and Fourier support organize the leading finite-time structure in a common language across otherwise distinct settings.

The key formal consequence is this: windowed conservation laws, $\partial_\mu(\Diamond J^\mu) = 0$, replace global conservation statements, so that the physically relevant spectral selection rule is not an exact $\delta(\Delta E)$ but the window transform $\widetilde{\Diamond}(\Delta E)$, with the delta function recovered only in the idealized limit of infinite localization duration. Once the relevant profile and characteristic scales defining $\Diamond$ are fixed by the experimental protocol, the windowed Noether identity constrains the associated response functions, apparent non-conservation, and spectral cutoffs across other observables under the same localization protocol.

The formal grounding for the physical distinctness of processes sharing identical Hamiltonians but having different localization histories is developed in Section 2.4. Across all five cases, the window $\Diamond$ is protocol-defined rather than freely chosen, is algebraically reparameterizable into an equivalent time-dependent coupling or interaction operator without change of physical content, and introduces no additional fit parameter beyond those already fixed by the experimental protocol.

8.4. Outlook

The results motivate three concrete directions for further development. First, the companion QFT paper [21] has established the windowed Ward and Ward–Takahashi identities for the full Standard Model in curved spacetime, and proved a Vanishing Lemma showing that boundary-layer compensator stress-energy is macroscopically negligible for ordinary quantum fields under stated regularity and phase-incoherence assumptions. The next step is to determine whether the high-localization-density regime identified therein—in which the coarse-grained compensator stress-energy is nonzero and pressureless, matching cold dark matter kinematics—can be connected to observable cosmological signatures. Second, precision quantum electrodynamics provides an exceptionally clean setting: finite

interaction and measurement windows in Penning-trap experiments constrain any finite-window contribution to windowed Ward–Takahashi identities, and the Vanishing Lemma of [21] ensures no spurious boundary-layer correction contaminates QED predictions at currently accessible precision [38]. No correction to the standard anomalous-magnetic-moment prediction is claimed; the point is structural consistency. Third, spatial localization: this work treats temporal windows only, but the formalism applies equally to spatially bounded regions, with implications for finite-volume lattice QFT, Casimir geometries, and quantum information protocols in which spatial domain restriction is operationally specified.

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