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Geometry of State-Update Processes and Wave Function Collapse

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Abstract

We develop an information-geometric framework for describing quantum state-update processes associated with measurement and statistical distinguishability. The approach is based on the quantum relative entropy and the quantum Fisher information metric, which together induce a natural Riemannian geometry on the manifold of quantum states. Using the second-order expansion of relative entropy, we show how the Fisher metric governs the local structure of distinguishability between nearby states and defines a corresponding thermodynamic length. This geometric structure provides an effective description of finite quantum state transitions in terms of fluctuation geometry and information-space distance. The formalism is applied to thermal two-level systems and harmonic oscillator states, illustrating how the Fisher metric encodes susceptibilities, fluctuations, and geometric transition costs. We also discuss the relation between thermodynamic length, dissipation bounds, and optimal paths in state space. Within this framework, wave function collapse is interpreted not as a microscopic dynamical mechanism, but as an effective state-update process that admits a geometric characterization in the manifold of density operators. The resulting perspective unifies concepts from quantum information theory, thermodynamics, and differential geometry within a common operational framework based on statistical distinguishability. Possible connections with quantum speed limits, entanglement geometry, and holographic relations between relative entropy and gravitational dynamics are briefly discussed.

Keywords: geometry; wave funcion collapse; entropy

1. Introduction

The nature of wave function collapse remains one of the central open questions in the foundations of quantum mechanics. In the standard formulation, collapse is introduced as a non-unitary projection associated with measurement, without a clear dynamical or energetic description [1].

Information-theoretic approaches provide a complementary perspective. The relative entropy between quantum states quantifies statistical distinguishability and plays a central role in quantum information theory [2,3]. Its local expansion defines the Fisher information metric, which endows the space of states with a Riemannian structure [4].

Moreover, in holographic frameworks, the second-order variation of relative entropy has been identified with canonical energy in the bulk gravitational theory [5,6]. This relation establishes a deep link between information geometry and dynamical response.

In this work, we suggest that wave function collapse might be viewed as a state-update process. For such processes we discuss the underlying geometry, based on Fisher's associated metrics. We speak of a finite transition in the information-geometric manifold, characterized by a non-zero relative entropy and governed by the Fisher metric.

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2. Information Geometry and Relative Entropy

A central concept in the present work is the geometric structure induced by statistical distinguishability in the space of quantum states. This structure emerges naturally from the quantum relative entropy and its local expansion, which defines the quantum Fisher information metric (See Appendix A).

2.1. Quantum Relative Entropy

Let ρ and σ be two density operators acting on the same Hilbert space. The quantum relative entropy is defined as

$$S(\rho\|\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)].$$

The relative entropy is non-negative,

$$S(\rho\|\sigma) \geq 0,$$

and vanishes if and only if $\rho = \sigma$. It therefore provides a natural measure of distinguishability between quantum states [2,3].

Unlike a true metric, relative entropy is not symmetric under exchange of its arguments and does not satisfy the triangle inequality. Nevertheless, its local second-order expansion induces a genuine Riemannian metric structure on the manifold of quantum states.

2.2. Local Expansion and Fisher Metric

Consider a smooth family of density operators parametrized by coordinates θ^i:

$$\rho(\theta).$$

For two nearby states,

$$\rho(\theta) \quad \text{and} \quad \rho(\theta + d\theta),$$

the relative entropy admits the expansion

$$S(\rho(\theta + d\theta)\|\rho(\theta)) = \frac{1}{2}F_{ij}(\theta)\, d\theta^i d\theta^j + O(d\theta^3),$$

where

$$F_{ij}$$

is the quantum Fisher information matrix.

The Fisher matrix therefore defines the local metric tensor on the information manifold:

$$ds^2 = F_{ij}(\theta)\, d\theta^i d\theta^j.$$

This metric quantifies the infinitesimal distinguishability between nearby quantum states and provides the fundamental geometric structure underlying the present analysis.

For thermal equilibrium states parametrized by inverse temperature β, the Fisher information reduces to the energy variance,

$$F_{\beta\beta} = \langle(\Delta H)^2\rangle,$$

with

$$\Delta H = H - \langle H\rangle.$$

Equivalently,

$$F_{\beta\beta} = \frac{\partial^2 \ln Z}{\partial \beta^2},$$

where

$$Z = \mathrm{Tr}(e^{-\beta H})$$

is the partition function.

Thus, the geometry of the statistical manifold is directly governed by thermodynamic fluctuations.

2.3. Thermodynamic Length

The Fisher metric induces a natural notion of distance between quantum states known as the thermodynamic length,

$$L = \int_\gamma \sqrt{F_{ij}(\theta)\, d\theta^i d\theta^j},$$

where γ denotes a path in parameter space.

For a one-parameter family of states parametrized by θ, this reduces to

$$L = \int_{\theta_i}^{\theta_f} \sqrt{F(\theta)}\, d\theta.$$

The thermodynamic length measures the geometric extent of a transition in the information manifold and plays an important role in finite-time thermodynamics and dissipation theory [7,8].

In particular, the Fisher metric determines the local cost associated with driving a system through state space. This geometric viewpoint provides a natural bridge between statistical distinguishability and nonequilibrium thermodynamic behavior.

2.4. Geometric Cost Functional

Given a trajectory $\theta^i(t)$ in parameter space, one may define the quadratic functional:

$$\mathcal{E} = \int_0^\tau F_{ij}(\theta)\dot{\theta}^i\dot{\theta}^j\, dt.$$

This expression appears naturally in finite-time thermodynamics, where it governs dissipation and excess work for slowly driven processes.

Motivated by this structure, we interpret $\mathcal{E}$ as an effective information-geometric cost associated with transitions between quantum states. Importantly, this quantity should not be viewed as a fundamental microscopic energy, but rather as a geometric functional determined by the distinguishability structure of the state manifold.

The positivity of the Fisher metric guarantees

$$\mathcal{E} \geq 0,$$

reflecting the irreversible character of finite state-space transformations.

2.5. Geometric Interpretation of State Updates

Within the present framework, transitions between quantum states may be represented geometrically as finite displacements in the information manifold. The relative entropy quantifies the global distinguishability between initial and final states, while the Fisher metric governs the local geometric structure along interpolating paths.

This viewpoint does not modify the standard quantum measurement postulates, nor does it introduce a microscopic dynamical model of wave function collapse. Rather, it

provides an effective geometric description of quantum state updates in terms of distinguishability, fluctuations, and thermodynamic length.

The resulting framework unifies several closely related concepts:

$$\text{Relative entropy} \longleftrightarrow \text{Statistical distinguishability};$$

$$\text{Fisher information} \longleftrightarrow \text{local fluctuation geometry};$$

$$\text{thermodynamic length} \longleftrightarrow \text{geometric cost of transitions}.$$

In this sense, the geometry of the quantum state manifold provides a natural language for characterizing the structure of finite quantum state transformations.

3. Interpretational Scope and Relation to Quantum Measurement Theory

The purpose of the present work is not to propose a modification of quantum mechanics or a new dynamical theory of wave function collapse. Rather, our goal is to develop an information-geometric framework for describing transitions between quantum states associated with measurement processes.

In standard quantum mechanics, the measurement postulate states that, following a projective measurement, the state of the system is updated according to the Born rule. This update is commonly represented as an instantaneous projection onto an eigenstate of the measured observable [1]. The present approach does not attempt to derive this postulate from microscopic dynamics, nor does it replace the standard formalism.

Instead, we focus on the geometric structure associated with the distinguishability between pre- and post-measurement states. The central objects in our analysis—relative entropy, Fisher information, and thermodynamic length—quantify statistical separation in the manifold of density operators. Within this framework, the transition between initial and final states may be represented as a finite displacement in information space.

It is important to emphasize that the trajectories introduced in the preceding sections should not be interpreted as literal microscopic paths followed by the quantum system during collapse. Rather, they constitute effective geometric interpolations in the manifold of quantum states, providing a convenient description of distinguishability, fluctuation structure, and associated energetic bounds.

From this perspective, the thermodynamic length does not describe a physical spacetime distance, but instead measures the geometric extent of a transition in the statistical manifold. Likewise, the quadratic form

$$F_{ij}\dot{\theta}^i\dot{\theta}^j$$

should be understood as an effective geometric cost functional rather than a fundamental dynamical energy.

The interpretation adopted here is therefore operational and geometric rather than ontological. The framework characterizes how distinguishable two quantum states are and quantifies the geometric structure associated with transitions between them, without asserting that collapse itself is a continuous physical process.

3.1. Relation to Decoherence and Other Interpretations

The present approach is complementary to existing interpretations and models of quantum measurement.

First, unlike decoherence theory, the framework developed here does not attempt to describe the dynamical suppression of interference through environmental interactions. Decoherence explains the emergence of classical behavior by tracing over environmental degrees of freedom, leading to effectively diagonal reduced density matrices .

In contrast, our analysis focuses on the information geometry associated with transitions between quantum states once such transitions are specified.

Second, the present framework differs fundamentally from objective collapse models such as the Ghirardi–Rimini–Weber (GRW) theory, since no stochastic modification of Schrödinger dynamics is introduced. No new physical collapse mechanism is postulated here.

Likewise, the formalism does not rely on branching structures as in the many-worlds interpretation. The information-geometric quantities employed in this work are independent of any particular ontological interpretation of quantum mechanics.

The framework is therefore best viewed as an effective geometric description of quantum state updates, formulated in terms of distinguishability measures and fluctuation geometry.

3.2. Energetic Interpretation and Limitations

A central idea explored in this work is that the Fisher information metric induces a natural quadratic form associated with transitions in state space. Motivated by known holographic relations between relative entropy and canonical energy [6], we have interpreted this quadratic structure as an effective energetic cost of state-space evolution.

However, this interpretation should not be regarded as universally equivalent to canonical energy in the strict gravitational sense. Outside holographic settings, the quantity

$$E_c \sim \int F_{ij}(\theta)\dot{\theta}^i\dot{\theta}^j \, dt$$

should instead be understood as an information-geometric functional analogous to the dissipation functionals appearing in nonequilibrium thermodynamics [7,8].

Accordingly, the energetic interpretation proposed here is heuristic and effective rather than fundamental. Its significance lies primarily in revealing a common mathematical structure linking distinguishability, geometry, fluctuations, and dissipation.

3.3. Experimental Accessibility

Although the present work is primarily theoretical, several quantities appearing in the formalism are experimentally accessible in principle. For thermal quantum systems, the Fisher information reduces to the variance of the Hamiltonian,

$$F_{\beta\beta} = \langle (\Delta H)^2 \rangle,$$

which is directly related to measurable susceptibilities and heat capacities.

Similarly, relative entropy and quantum Fisher information are actively studied in quantum metrology and quantum information experiments [9]. Consequently, the geometric quantities discussed here are not merely formal constructs, but are connected to observable fluctuation properties of quantum systems.

The present framework therefore suggests the possibility of exploring measurement-induced state transitions from an operational geometric viewpoint, particularly in controllable quantum platforms such as two-level systems, cold atoms, and Gaussian optical states.

4. Explicit Example: Two-Level System

To illustrate the general framework, we consider a two-level quantum system, which provides the simplest nontrivial setting where all relevant quantities can be computed explicitly.

4.1. Hamiltonian and Thermal State

We take the Hamiltonian to be

$$H = \frac{\omega}{2}\sigma_z, \tag{1}$$

with eigenstates $|0\rangle$ and $|1\rangle$ satisfying

$$H|0\rangle = -\frac{\omega}{2}|0\rangle, \qquad H|1\rangle = +\frac{\omega}{2}|1\rangle. \tag{2}$$

The thermal (Gibbs) state at inverse temperature β is

$$\rho_i = \frac{e^{-\beta H}}{Z}, \tag{3}$$

where the partition function is

$$Z = \mathrm{Tr}\, e^{-\beta H} = e^{\beta\omega/2} + e^{-\beta\omega/2} = 2\cosh\left(\frac{\beta\omega}{2}\right). \tag{4}$$

In the energy eigenbasis, the density matrix is diagonal,

$$\rho_i = \begin{pmatrix} p_0 & 0 \\ 0 & p_1 \end{pmatrix}, \tag{5}$$

with occupation probabilities

$$p_0 = \frac{e^{\beta\omega/2}}{Z}, \qquad p_1 = \frac{e^{-\beta\omega/2}}{Z}. \tag{6}$$

4.2. Collapse Process

We consider a projective measurement in the energy basis that collapses the system into the excited state,

$$\rho_f = |1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{7}$$

This transition represents a maximal change in the state, from a mixed thermal distribution to a pure eigenstate.

4.3. Relative Entropy

The relative entropy between the final and initial states is

$$S(\rho_f \| \rho_i) = \mathrm{Tr}(\rho_f \log \rho_f - \rho_f \log \rho_i). \tag{8}$$

Since ρ_f is a projector onto $|1\rangle$, we have

$$\rho_f \log \rho_f = 0, \tag{9}$$

and therefore

$$S(\rho_f \| \rho_i) = -\langle 1| \log \rho_i |1\rangle. \tag{10}$$

Using the diagonal form of ρ_i,

$$\langle 1|\rho_i|1\rangle = p_1 = \frac{e^{-\beta\omega/2}}{Z}, \tag{11}$$

we obtain

$$S(\rho_f \| \rho_i) = -\log p_1 = \log Z + \frac{\beta\omega}{2}. \tag{12}$$

Substituting the partition function,

$$S(\rho_f \| \rho_i) = \log\left[2\cosh\left(\frac{\beta\omega}{2}\right)\right] + \frac{\beta\omega}{2} \tag{13}$$

This quantity is strictly positive and increases with β, reflecting the growing distinguishability between the thermal state and the excited state at low temperatures.

4.4. Fisher Information and Local Geometry

For the Gibbs family $\rho(\beta)$, the Fisher information is given by

$$F_{\beta\beta} = \langle (\Delta H)^2 \rangle. \tag{14}$$

The expectation value of the energy is

$$\langle H \rangle = -\frac{\partial}{\partial\beta} \ln Z = -\frac{\omega}{2} \tanh\left(\frac{\beta\omega}{2}\right), \tag{15}$$

and, as before,

$$\langle H^2 \rangle = \frac{\omega^2}{4}. \tag{16}$$

Thus,

$$F_{\beta\beta} = \frac{\omega^2}{4} \mathrm{sech}^2\left(\frac{\beta\omega}{2}\right). \tag{17}$$

This shows that the local geometry is controlled by energy fluctuations, which decrease as the system approaches a pure state.

4.5. Geometric and Energetic Interpretation

The collapse process $\rho_i \to \rho_f$ corresponds to a finite displacement in the information manifold. The relative entropy computed above measures the total distinguishability between the initial and final states, while the Fisher information governs the local structure along any interpolating path.

In particular, the quadratic form

$$E_c \sim \delta\beta^2 F_{\beta\beta} \tag{18}$$

defines the local energetic cost of variations in β, linking the collapse process to the canonical energy discussed in previous sections.

This example thus provides a concrete realization of the general framework: the collapse is characterized by a finite relative entropy, while its local geometric and energetic properties are determined by the Fisher metric.

4.6. Limiting Cases

It is instructive to consider two limiting regimes:

High-temperature limit ($\beta \to 0$): The state approaches the maximally mixed state, and

$$F_{\beta\beta} \to \frac{\omega^2}{4}, \qquad S(\rho_f \| \rho_i) \to \log 2. \tag{19}$$

Low-temperature limit ($\beta \to \infty$): The system approaches the ground state, and

$$F_{\beta\beta} \to 0, \qquad S(\rho_f \| \rho_i) \to \beta\omega. \tag{20}$$

These limits highlight the interplay between fluctuations and distinguishability: as fluctuations vanish, the Fisher metric becomes degenerate, while the global distinguishability between states increases.

4.7. Discussion

This simple model captures the essential features of the proposed interpretation. The collapse from a thermal state to a pure eigenstate is associated with a finite relative entropy, while the Fisher metric encodes the local response properties and determines the energetic cost of variations.

The example demonstrates explicitly how distinguishability, fluctuations, and geometry are unified within a single framework, supporting the general interpretation of collapse as a finite process in the information manifold.

5. Example: Harmonic Oscillator and Gaussian States

We consider a quantum harmonic oscillator with a Hamiltonian:

$$H = \omega\left(a^\dagger a + \frac{1}{2}\right). \tag{21}$$

Let the system be initially in a thermal (Gaussian) state

$$\rho_i = \frac{e^{-\beta H}}{Z}, \tag{22}$$

with partition function $Z = (1 - e^{-\beta\omega})^{-1}$.

Suppose that a measurement induces a small change in inverse temperature, $\beta \to \beta + \delta\beta$, leading to a nearby state ρ_f.

5.1. Relative Entropy

For two nearby thermal states, the relative entropy admits the expansion

$$S(\rho_f \| \rho_i) \approx \frac{1}{2} F_{\beta\beta} (\delta\beta)^2, \tag{23}$$

where the Fisher information is given by

$$F_{\beta\beta} = \langle (\Delta H)^2 \rangle. \tag{24}$$

For the harmonic oscillator, one finds

$$\langle (\Delta H)^2 \rangle = \omega^2 \frac{e^{\beta\omega}}{(e^{\beta\omega} - 1)^2}. \tag{25}$$

5.2. Interpretation

Thus, the relative entropy between nearby Gaussian states is determined by energy fluctuations, as in the discrete case. The corresponding quadratic form defines the Fisher metric and governs the local geometry of the state space.

The associated canonical energy is therefore

$$E_c = F_{\beta\beta} (\delta\beta)^2, \tag{26}$$

which again reflects the fluctuation structure of the system.

5.3. Discussion

This example shows that the proposed framework applies equally to continuous-variable systems, where Gaussian states play a central role. The unification of distinguishability, fluctuations, and energy thus holds beyond finite-dimensional Hilbert spaces.

6. Thermodynamic Length and Collapse

A natural geometric quantity associated with the Fisher metric is the thermodynamic length, defined as

$$\mathcal{L} = \int_{\theta_i}^{\theta_f} \sqrt{F_{ij}(\theta)\, d\theta^i d\theta^j}. \tag{27}$$

For a single-parameter family, such as thermal states parametrized by β, this reduces to

$$\mathcal{L} = \int_{\beta_i}^{\beta_f} \sqrt{F_{\beta\beta}(\beta)}\, d\beta. \tag{28}$$

6.1. Explicit Computation: Two-Level System

We consider again a two-level system described by the Hamiltonian

$$H = \frac{\omega}{2}\sigma_z, \tag{29}$$

with eigenvalues $E_\pm = \pm\frac{\omega}{2}$. For section-completeness, we repeat here some formulas given already above.

The thermal (Gibbs) state at inverse temperature β is

$$\rho(\beta) = \frac{e^{-\beta H}}{Z}, \tag{30}$$

with partition function

$$Z = \mathrm{Tr}\, e^{-\beta H} = e^{-\beta\omega/2} + e^{\beta\omega/2} = 2\cosh\left(\frac{\beta\omega}{2}\right). \tag{31}$$

6.1.1. Fisher Information

For a Gibbs family parametrized by β, the Fisher information reduces to the energy variance:

$$F_{\beta\beta} = \langle (\Delta H)^2 \rangle. \tag{32}$$

The expectation value of the energy is

$$\langle H \rangle = -\frac{\partial}{\partial\beta} \ln Z = -\frac{\omega}{2} \tanh\left(\frac{\beta\omega}{2}\right). \tag{33}$$

The second moment is

$$\langle H^2 \rangle = \frac{\omega^2}{4}, \tag{34}$$

since $H^2 = \frac{\omega^2}{4}\,\mathbb{I}$.

Thus, the variance is

$$\langle (\Delta H)^2 \rangle = \langle H^2 \rangle - \langle H \rangle^2 = \frac{\omega^2}{4}\left[1 - \tanh^2\left(\frac{\beta\omega}{2}\right)\right]. \tag{35}$$

Using the identity $1 - \tanh^2 x = \operatorname{sech}^2 x$, we obtain

$$F_{\beta\beta} = \frac{\omega^2}{4} \operatorname{sech}^2\left(\frac{\beta\omega}{2}\right). \tag{36}$$

6.1.2. Thermodynamic Length

The thermodynamic length between two states β_i and β_f is

$$\mathcal{L} = \int_{\beta_i}^{\beta_f} \sqrt{F_{\beta\beta}(\beta)}\, d\beta. \tag{37}$$

Substituting the explicit form of the Fisher information,

$$\mathcal{L} = \frac{\omega}{2} \int_{\beta_i}^{\beta_f} \operatorname{sech}\left(\frac{\beta\omega}{2}\right) d\beta. \tag{38}$$

Introducing the dimensionless variable

$$x = \frac{\beta\omega}{2}, \qquad d\beta = \frac{2}{\omega} dx, \tag{39}$$

we obtain

$$\mathcal{L} = \int_{x_i}^{x_f} \operatorname{sech}(x)\, dx. \tag{40}$$

The integral is elementary and yields

$$\int \operatorname{sech}(x)\, dx = 2 \arctan\left(\tanh \frac{x}{2}\right). \tag{41}$$

Thus, the thermodynamic length is

$$\mathcal{L} = 2\left[\arctan\left(\tanh \frac{x}{2}\right)\right]_{x_i}^{x_f}. \tag{42}$$

Returning to the original variable,

$$\mathcal{L} = 2\left[\arctan\left(\tanh \frac{\beta\omega}{4}\right)\right]_{\beta_i}^{\beta_f} \tag{43}$$

6.1.3. Physical Interpretation

This explicit result provides a closed-form expression for the geometric distance between thermal states of the two-level system. The dependence on β reflects the underlying fluctuation structure encoded in the Fisher metric.

In particular, the thermodynamic length remains finite for all finite temperature intervals, reflecting the bounded nature of the state space. Moreover, the integrand $\sqrt{F_{\beta\beta}}$ is proportional to the energy fluctuations, showing that the geometry is directly controlled by the susceptibility of the system.

Within the present framework, this length quantifies the geometric extent of the collapse trajectory, providing a concrete measure of the distance traversed in the information manifold during the transition.

7. Geodesics and Dissipation Bounds

7.1. Geodesic Equation

The Fisher information matrix defines a Riemannian metric on the parameter manifold:

$$ds^2 = F_{ij}(\theta)\, d\theta^i d\theta^j. \tag{44}$$

The geodesics of this manifold are obtained by extremizing the thermodynamic length,

$$\mathcal{L} = \int \sqrt{F_{ij}\dot{\theta}^i\dot{\theta}^j}\, dt, \tag{45}$$

which leads to the Euler–Lagrange equations

$$\ddot{\theta}^k + \Gamma^k_{ij}\dot{\theta}^i\dot{\theta}^j = 0, \tag{46}$$

where the Christoffel symbols are

$$\Gamma^k_{ij} = \frac{1}{2}F^{kl}\Big(\partial_i F_{jl} + \partial_j F_{il} - \partial_l F_{ij}\Big). \tag{47}$$

These equations define the optimal paths in parameter space that minimize the thermodynamic length.

7.2. Single-Parameter Case

For a one-dimensional manifold parametrized by θ, the metric reduces to $ds^2 = F(\theta)\, d\theta^2$. In this case, the geodesic equation simplifies to

$$\ddot{\theta} + \frac{1}{2}\frac{d\ln F(\theta)}{d\theta}\,\dot{\theta}^2 = 0. \tag{48}$$

This equation admits a first integral:

$$\sqrt{F(\theta)}\,\dot{\theta} = \text{const.} \tag{49}$$

Thus, geodesics correspond to trajectories of constant thermodynamic speed.

7.3. Application: Two-Level System

For the two-level system, the Fisher information is

$$F(\beta) = \frac{\omega^2}{4}\,\mathrm{sech}^2\left(\frac{\beta\omega}{2}\right). \tag{50}$$

The geodesic condition becomes

$$\frac{\omega}{2}\,\mathrm{sech}\left(\frac{\beta\omega}{2}\right)\dot{\beta} = \text{const.} \tag{51}$$

This equation determines the optimal evolution of $\beta(t)$ that minimizes the thermodynamic length between two states.

7.4. Thermodynamic Length and Dissipation

A key result in nonequilibrium thermodynamics is that the thermodynamic length controls the minimal dissipation incurred during a finite-time process [7,8]. In particular, for slowly driven processes, the excess work satisfies the bound:

$$W_{\text{diss}} \geq \frac{\mathcal{L}^2}{\tau}, \tag{52}$$

where τ is the duration of the protocol.

More precisely, the excess work can be expressed as a quadratic functional of the control parameters,

$$W_{\text{diss}} \sim \int_0^\tau F_{ij}(\theta)\, \dot{\theta}^i \dot{\theta}^j \, dt, \tag{53}$$

which is minimized by geodesic paths.

7.5. Implications for Collapse

Within the present framework, wave function collapse can be interpreted as a finite-time process in the information manifold. The associated thermodynamic length then provides a lower bound on the energetic cost of the transition:

$$E_c \gtrsim \frac{\mathcal{L}^2}{\tau}. \tag{54}$$

This relation suggests that collapse is constrained by geometric bounds analogous to those governing nonequilibrium thermodynamic transformations.

In this sense, the Fisher metric not only determines distinguishability and canonical energy, but also sets fundamental limits on the dissipation associated with quantum state transitions.

7.6. Geodesic Optimality

While generic quantum state transitions need not follow geodesic paths, the Fisher information metric singles out a distinguished class of trajectories that minimize thermodynamic length and the associated dissipation. These geodesics therefore provide a natural benchmark for collapse processes, representing the most efficient transitions between given initial and final states within the information-geometric framework.

While idealized collapse in standard quantum mechanics is instantaneous, any physical implementation of state reduction necessarily involves a finite-time process. In such cases, the information-geometric framework implies that geodesic trajectories minimize dissipation and therefore represent optimally efficient realizations of the transition.

Geodesic efficiency acquires operational significance whenever quantum state transitions are implemented as finite-time physical processes, providing a criterion for optimality in terms of minimal dissipation.

In the two-level system, the energetic cost of state transitions reflects a competition between global distinguishability, quantified by relative entropy, and local geometric cost, determined by the Fisher metric. This interplay suggests the existence of an intermediate temperature regime in which transitions can be implemented most efficiently.

8. Information-Geometric Expansion of Relative Entropy

In this section we summarize the fundamental relation between relative entropy and the quantum Fisher information metric, which constitutes the geometric basis of the present framework. Although the underlying expansion is well known in information geometry and quantum estimation theory [4,9], it plays a central conceptual role in connecting distinguishability, fluctuations, and geometric structure.

8.1. Local Expansion Around a Reference State

Let

$$\rho(\theta)$$

be a smooth family of full-rank density operators depending on a set of continuous parameters

$$\theta^i.$$

Consider two nearby states,

$$\rho(\theta) \quad \text{and} \quad \rho(\theta + \delta\theta).$$

The quantum relative entropy between these states is

$$S(\rho(\theta + \delta\theta)\|\rho(\theta)) = \mathrm{Tr}[\rho(\theta + \delta\theta)(\log \rho(\theta + \delta\theta) - \log \rho(\theta))].$$

Expanding around the reference state

$$\rho(\theta),$$

the linear contribution vanishes due to the normalization condition

$$\mathrm{Tr}(\delta\rho) = 0.$$

The leading nontrivial contribution is quadratic and yields

$$S(\rho(\theta + \delta\theta)\|\rho(\theta)) = \frac{1}{2}\,\delta\theta^i \delta\theta^j F_{ij}(\theta) + O(\delta\theta^3),$$

where

$$F_{ij}(\theta)$$

is the quantum Fisher information matrix.

Thus, relative entropy locally induces a positive-definite quadratic form on the manifold of quantum states.

8.2. Quantum Fisher Information Metric

The quantum Fisher information matrix defines the Riemannian metric tensor of the information manifold,

$$ds^2 = F_{ij}(\theta)\, d\theta^i d\theta^j.$$

This metric quantifies the infinitesimal distinguishability between neighboring quantum states and provides the natural geometric structure associated with statistical fluctuations.

In the symmetric logarithmic derivative (SLD) formalism, the Fisher matrix is given by

$$F_{ij} = \mathrm{Tr}\left[\rho L_i L_j\right],$$

where the SLD operators satisfy

$$\partial_i \rho = \frac{1}{2}(L_i \rho + \rho L_i).$$

The positivity of relative entropy implies

$$F_{ij} \geq 0,$$

ensuring that the induced geometry is Riemannian.

8.3. Thermal States and Fluctuation Geometry

For equilibrium Gibbs states,

$$\rho(\beta) = \frac{e^{-\beta H}}{Z},$$

the Fisher information with respect to inverse temperature reduces to the variance of the Hamiltonian:

$$F_{\beta\beta} = \langle (\Delta H)^2 \rangle.$$

Equivalently,

$$F_{\beta\beta} = \frac{\partial^2 \ln Z}{\partial \beta^2}.$$

Hence the local geometry of the thermal manifold is governed directly by energy fluctuations.

This relation establishes a close connection between information geometry and thermodynamic response theory: large fluctuations imply large statistical distinguishability between nearby thermal states.

8.4. Geometric Cost Functional

Given a trajectory

$$\theta^i(t)$$

in parameter space, one may define the quadratic functional

$$\mathcal{C} = \int_0^\tau F_{ij}(\theta)\dot{\theta}^i\dot{\theta}^j \, dt.$$

This quantity naturally appears in finite-time thermodynamics and optimal control theory, where analogous quadratic forms govern dissipation and excess work [7,8].

Motivated by these parallels, we interpret

$$\mathcal{C}$$

as an effective geometric cost associated with transitions in the space of quantum states.

It is important to emphasize that this functional should not be regarded as a fundamental microscopic energy in general quantum systems. Rather, it provides an information-geometric measure of the cost associated with finite state-space transformations.

The positivity of the Fisher metric guarantees

$$\mathcal{C} \geq 0,$$

reflecting the irreversible character of distinguishable state updates.

8.5. Geometric Interpretation

The expansion of relative entropy reveals that the same geometric object—the Fisher information matrix—simultaneously determines:

local distinguishability;

fluctuation structure;

thermodynamic length;

and

effective geometric cost.

Within the present framework, transitions between quantum states may therefore be represented geometrically as finite displacements in the information manifold.

Importantly, this interpretation is operational rather than ontological. No modification of standard quantum mechanics is introduced, and the formalism does not attempt to provide a microscopic dynamical theory of wave function collapse.

Instead, the information-geometric framework characterizes the structure of quantum state updates in terms of distinguishability and fluctuation geometry, thereby unifying several concepts from quantum information theory, statistical mechanics, and thermodynamics within a common mathematical language.

9. Conclusions

We have developed an information-geometric framework for describing quantum state transitions associated with measurement processes. The relative entropy quantifies distinguishability between initial and final states, while the Fisher information metric governs the local geometric structure of the corresponding state manifold.

We see that state-update processes admit a natural geometric characterization in terms of distinguishability and fluctuation structure.

Outlook

The information-geometric interpretation developed here suggests several intriguing extensions. First, since the Fisher metric bounds the speed of state evolution, the thermodynamic length associated with collapse may be related to quantum speed limits [10,11], indicating that collapse processes are subject to fundamental bounds on their duration. Second, the geometric structure underlying distinguishability is closely tied to entanglement properties of quantum states, suggesting that collapse may be viewed as a trajectory in an emergent entanglement geometry [12], where changes in correlations are encoded in the Fisher metric. Finally, given the established relation between relative entropy and gravitational dynamics in holographic settings [5,6], the energetic cost associated with collapse may reflect underlying gravitational constraints, hinting at a deeper connection between quantum measurement, spacetime geometry, and the thermodynamics of information. These perspectives open a route toward embedding collapse phenomena within a broader geometric and dynamical framework.

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Appendix A. Supplementary Remarks on Relative Entropy and Fisher Geometry

The purpose of this appendix is merely to summarize several standard results from information geometry and quantum estimation theory used throughout the main text. No claim of originality is made regarding these mathematical relations; they are included only for completeness and to establish notation.

Appendix A.1. Second-Order Expansion of Relative Entropy

Let

$$\rho(\theta)$$

be a smooth family of density operators depending on parameters

$$\theta^i.$$

For two neighboring states,

$$\rho(\theta) \quad \text{and} \quad \rho(\theta + \delta\theta),$$

the quantum relative entropy is

$$S(\rho(\theta + \delta\theta) \| \rho(\theta)) = \mathrm{Tr}[\rho(\theta + \delta\theta)(\log \rho(\theta + \delta\theta) - \log \rho(\theta))].$$

Expanding around the reference state

$$\rho(\theta),$$

the linear contribution vanishes because density matrices remain normalized,

$$\mathrm{Tr}(\delta\rho) = 0.$$

The leading nontrivial contribution is quadratic:

$$S(\rho(\theta + \delta\theta) \| \rho(\theta)) = \frac{1}{2} F_{ij}(\theta)\, \delta\theta^i \delta\theta^j + O(\delta\theta^3),$$

where

$$F_{ij}$$

is the quantum Fisher information matrix.

Thus the Fisher metric naturally emerges as the local Hessian structure associated with statistical distinguishability.

Appendix A.2. Symmetric Logarithmic Derivative Formalism

The quantum Fisher information may be expressed in terms of the symmetric logarithmic derivative (SLD). Defining operators

$$L_i$$

through

$$\partial_i \rho = \frac{1}{2}(L_i \rho + \rho L_i),$$

the Fisher matrix is

$$F_{ij} = \mathrm{Tr}\left[\rho L_i L_j\right].$$

This construction guarantees that

$$F_{ij}$$

is positive semidefinite and therefore defines a Riemannian metric on the manifold of quantum states.

For pure states,

$$\rho = |\psi\rangle\langle\psi|,$$

the metric reduces to the familiar Fubini–Study form.

Appendix A.3. Thermal Equilibrium States

For Gibbs states,

$$\rho(\beta) = \frac{e^{-\beta H}}{Z},$$

the Fisher information with respect to inverse temperature is directly related to equilibrium fluctuations:

$$F_{\beta\beta} = \langle (\Delta H)^2 \rangle.$$

Equivalently,

$$F_{\beta\beta} = \frac{\partial^2 \ln Z}{\partial \beta^2}.$$

This relation establishes the connection between information geometry and thermodynamic response theory.

In particular, near critical points where fluctuations become large, the Fisher metric develops singular behavior, leading to divergent thermodynamic curvature and large thermodynamic lengths.

Appendix A.4. Thermodynamic Length

The Fisher metric induces the thermodynamic length

$$L = \int_\gamma \sqrt{F_{ij}(\theta)\, d\theta^i d\theta^j},$$

which measures the geometric extent of a transformation in state space.

For a one-parameter trajectory,

$$\theta = \theta(t),$$

this becomes

$$L = \int \sqrt{F(\theta)}\, d\theta.$$

Thermodynamic length plays an important role in finite-time thermodynamics, optimal control, and dissipation theory, where it provides lower bounds on irreversible entropy production.

Appendix A.5. Interpretational Remarks

The information-geometric quantities discussed in this work quantify distinguishability and fluctuation structure in the manifold of quantum states. The geometric trajectories considered throughout the paper should therefore be interpreted as effective paths in information space rather than literal microscopic dynamical evolutions.

Accordingly, the framework developed here does not replace the standard measurement postulates of quantum mechanics and does not constitute a dynamical theory of wave function collapse. Instead, it provides an information-geometric characterization of quantum state transitions in terms of relative entropy, Fisher information, and thermodynamic length.

For further details on quantum information geometry and statistical manifolds, see Refs. [3,4,9].

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