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The Informational Economy Functional: A Variational Principle for Decoherence and Classical Emergence

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Abstract

The emergence of classicality through quantum decoherence is commonly described from complementary perspectives emphasizing stability (environment-induced superselection), objectivity (Quantum Darwinism), or physical feasibility (information thermodynamics). In realistic open quantum systems, however, these aspects coexist and compete under finite physical resources. In this work we argue that classical structure selection is most naturally understood as a resource-constrained, multi-objective process. We introduce the Informational Economy Functional (IEF), an effective accounting framework that places loss of distinguishability, energetic dissipation, and the generation of redundantly accessible records on equal footing. The associated Principle of Informational Economy characterizes emergent classical structures as those achieving an optimal compromise among stability, objectivity, and energetic feasibility. Classicality is thus neither maximally stable, nor maximally redundant, nor maximally energy-efficient, but instead reflects a Pareto-optimal balance shaped by environmental constraints. The IEF yields falsifiable predictions concerning pointer-structure variability, redundancy deformation, and resource-sensitive trade-offs, and suggests concrete experimental tests in continuously monitored quantum platforms. Classical reality is thereby reinterpreted as the most economical configuration in which information can stably form, propagate, and persist.

Keywords: quantum decoherence; Quantum Darwinism; open quantum systems; information thermodynamics; pointer states; redundant information; resource constraints

1. Introduction

1.1. Is Quantum Decoherence Selective?

Quantum decoherence is widely regarded as the central mechanism mediating the transition from quantum behavior to classical reality. It is often described, at an intuitive level, as an unavoidable form of environmental noise [1,2]: uncontrolled interactions with surrounding degrees of freedom rapidly suppress quantum coherence and erase nonclassical features. This picture raises a deeper question: Is decoherence truly blind and indiscriminate, acting as a structureless noise that washes out all quantum properties equally, or does it exhibit an intrinsic form of selectivity that determines which aspects of a quantum system are able to persist and ultimately define the classical world we observe?

A growing body of theoretical and experimental work supports the latter view [3,4]. Decoherence is not merely a random disturbance imposed by the environment; rather, it displays systematic tendencies that privilege certain structures over others. Several influential frameworks have been developed to articulate different facets of this selectivity.

A first and foundational perspective is provided by environment-induced superselection, or einselection [1,5]. In this framework, the interaction between a system and its

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environment dynamically stabilizes a preferred set of quantum states—so-called pointer states—that are least sensitive to environmental perturbations. These states minimize the loss of predictability under environmental monitoring and therefore survive decoherence more robustly than arbitrary superpositions. Einselection thus provides a clear and compelling criterion for dynamical stability, explaining why certain states can persist long enough to exhibit classical behavior. However, by construction, it does not address how information about these stable states becomes accessible, shared, or confirmed by observers through the environment.

Complementing this emphasis on stability, Quantum Darwinism highlights a distinct and equally essential feature of classical reality: objectivity [3,6]. According to this framework, classicality arises when information about certain system properties is redundantly proliferated into many independent fragments of the environment. Multiple observers can then independently infer the same system state by accessing different environmental fragments, without disturbing the system or communicating with one another. Quantum Darwinism therefore explains how classical information, once established, becomes publicly accessible and intersubjectively agreed upon. At the same time, it typically treats the existence of pointer states as a given. It describes how information about selected states becomes objective, but does not fundamentally explain why those particular states serve as the sources of information in the first place.

A third line of insight is provided by predictive and information thermodynamics, which draws attention to the energetic costs associated with information processing [7–9]. From this perspective, decoherence is not only an informational process but also a thermodynamic one: the loss and redistribution of information are constrained by heat dissipation and entropy production. Thermodynamic principles thus impose fundamental feasibility conditions on decoherence pathways, ruling out processes that would require prohibitive energetic resources. However, these principles function primarily as constraints rather than selection rules. They can exclude energetically inefficient or impossible processes, but they do not, by themselves, determine which among the many thermodynamically allowed outcomes will be realized. Moreover, thermodynamic cost is largely insensitive to the semantic content of information, treating the loss of classically relevant information and the loss of inaccessible microscopic details on equal footing.

Each of these three approaches is internally consistent and highly successful within its respective domain. Together, they reveal that decoherence exhibits clear tendencies toward stability, objectivity, and physical feasibility. Yet these frameworks have largely been developed in parallel, and are often applied in isolation. In realistic physical settings, however, these tendencies are not separable. Stability, redundancy, and energetic cost arise simultaneously during system–environment interactions and may compete with one another.

This coexistence naturally gives rise to tension. A structure that is maximally stable under environmental coupling may encode information that is difficult or costly to broadcast. Conversely, a structure whose information is easily and rapidly proliferated into the environment may lack long-term dynamical robustness. Likewise, a structure that appears optimal from an informational standpoint may be energetically prohibitive, rendering its realization physically implausible. When such conflicts arise, no single criterion alone can determine which classical structure will prevail.

This observation leads to a more fundamental challenge. If decoherence is neither blind nor governed by a single dominant tendency, how are these competing requirements reconciled in practice? What principle governs the trade-offs between stability, objectivity, and energetic cost, and determines which classical structures ultimately leave persistent, objective traces in the world?

1.2. Decoherence as a Resource-Constrained Multi-Objective Selection Process

In this work, we address the question raised above by proposing a unified framework for understanding how stability, objectivity, and physical feasibility are jointly balanced during decoherence. We argue that decoherence can be fruitfully formulated as a resource-constrained multi-objective selection process, in which classical structures emerge not by optimizing a single criterion, but by achieving an effective compromise among competing informational and physical requirements.

From this perspective, classicality arises when certain structures simultaneously exhibit robustness against environmental disturbance, enable efficient and redundant dissemination of information into the environment, and remain compatible with thermodynamic constraints on energy dissipation [1,3,9]. To make this idea precise, we introduce the *Principle of Informational Economy* and an associated functional, which we call the *Informational Economy Functional* (IEF). The IEF provides a unified accounting framework in which stability, objectivity, and energetic cost appear as competing contributions to a single effective quantity. For a given candidate classical structure Π, the functional takes the schematic form

$$\mathcal{J}[\Pi; t] \sim \dot{C}_{\text{stab}} + \lambda_1 \dot{Q}_{\text{out}} - \lambda_2 \dot{I}_{\text{brd}}, \tag{1}$$

where $\dot{C}_{\text{stab}}$ characterizes the rate at which the distinguishability of the alternatives defined by Π is degraded under the system–environment interaction, $\dot{Q}_{\text{out}}$ quantifies the associated energetic cost, and $\dot{I}_{\text{brd}}$ measures the rate at which information about the structure Π is disseminated into the environment and becomes accessible. The coefficients λ_1 and λ_2 encode properties of the environment and the monitoring architecture, and play the role of effective conversion factors between informational and physical resources.

Within this framework, the classical structures selected by decoherence are identified as those that extremize the IEF over an appropriate time scale. Importantly, this variational formulation is not introduced as a teleological assumption about nature "optimizing" a goal. Rather, it emerges as an effective and equivalent description of how stable, objective records can form under simultaneous informational and thermodynamic constraints. Structures that dissipate information or energy without producing redundantly accessible records are naturally suppressed, while those that convert available resources into persistent, shareable information traces are favored.

A key consequence of this viewpoint is that the main existing approaches to decoherence and classical emergence appear as limiting cases within a single framework. Einselection emphasizes the minimization of information loss [1,5], Quantum Darwinism highlights the maximization of redundant information proliferation [3,6], and predictive thermodynamics constrains the energetic feasibility of information processing [8,9]. The IEF therefore situates them within a common structure that makes their mutual tensions and trade-offs explicit.

The remainder of this paper is organized as follows. In Section 2, we introduce the Informational Economy Functional in detail and clarify the physical meaning of each of its components, as well as the notion of a candidate classical structure. In Section 3, we derive the IEF as an effective variational principle starting from global unitarity, standard thermodynamic inequalities for open quantum systems, and an operational notion of redundant information. Section 4 explores the broader interpretation of decoherence as an informational economy, highlighting the role of trade-offs and Pareto-optimal structures. In Section 5, we discuss experimentally accessible consequences of the framework and propose concrete scenarios in which environment-induced transitions between different pointer structures can be observed. We conclude in Section 6 with a summary and outlook. A fully worked simulation model illustrating the emergence of the IEF and its predictive power is presented in Appendix A.

2. The Informational Economy Functional: Definition and Physical Meaning

2.1. Candidate Structures: What Is Being Selected?

A central step in formulating a unified description of decoherence-driven classical emergence is to clarify what, precisely, is being selected by the system–environment interaction. In much of the traditional literature, this question is framed in terms of pointer states: a distinguished set of quantum states that remain stable under environmental monitoring and thus survive decoherence [1,5]. While this notion has proven highly effective in identifying dynamically robust states, it is not sufficiently general for capturing the full range of classical structures that arise in realistic open quantum systems.

In many physically relevant situations, what remains stable under decoherence is not an individual quantum state, but an entire structure of distinguishability. Classical descriptions are typically associated with coarse-grained variables rather than sharp quantum states: spatial regions rather than exact position eigenstates, phase-space cells rather than precise points, or families of approximately commuting observables rather than a single preferred basis. In such cases, decoherence suppresses interference between different coarse-grained alternatives while leaving internal structure within each alternative largely intact. This suggests that the outcome of decoherence should be understood not as the selection of a single state, but as the stabilization of a classically interpretable structure [2,10].

Motivated by this observation, we adopt a more general notion of a candidate classical structure, denoted by Π. Conceptually, Π specifies an operationally defined set of distinguishable alternatives, corresponding to an effective coarse-grained description of the system that can support classical records and is realizable as a physically implementable monitoring structure. In this way, Π encodes which distinctions are effectively monitored by the environment and thus which degrees of freedom can serve as carriers of objective classical records. This notion admits complementary interpretations at several levels.

At the most abstract, information-theoretic level, Π represents a distinguishability structure: a coarse-graining of the system's state space into equivalence classes that define which microscopic differences are regarded as irrelevant and which distinctions are retained. From this perspective, Π specifies the informational resolution at which the system is described.

At the operational level, Π can be represented by a measurement structure, such as a projective decomposition, a positive-operator-valued measure (POVM), or a coarse-grained observable. In this sense, Π defines a set of questions that can be meaningfully asked about the system, together with the corresponding classical outcomes. Importantly, this does not imply that an actual measurement is performed; rather, it characterizes the effective observables that are implicitly monitored through the system–environment interaction [3,6].

At the dynamical level, Π is tied to the geometry of system–environment coupling. Different interaction Hamiltonians selectively couple different system degrees of freedom to the environment, thereby determining which observables are effectively read out, amplified, and recorded. For instance, scattering environments tend to monitor position-like observables, thermal reservoirs favor energy eigenstates, and spin environments may select specific spin components—each of these cases corresponds to a different choice of Π. From this viewpoint, Π captures the readout geometry imposed by the environment: it specifies which aspects of the system's state are most readily imprinted into environmental degrees of freedom and thus become candidates for classical records [4]. In this sense, different choices of Π can be associated with different effective monitoring channels, and hence with different induced open-system dynamics.

Crucially, Π does not represent a specific quantum state. Instead, it labels a candidate classical structure that determines which degrees of freedom are effectively monitored and which distinctions can be redundantly recorded by the environment. Different choices of Π

correspond to different potential classical descriptions of the same underlying quantum system, and decoherence can be understood as a process that selectively stabilizes some of these structures while suppressing others.

This perspective also clarifies the physical meaning of environmental "preference" in decoherence. The environment does not interpret or evaluate quantum states; it simply interacts with the system through specific couplings. These couplings naturally privilege certain observables and coarse-grainings by efficiently dispersing their associated information into many environmental degrees of freedom. In this sense, Π encodes what the environment effectively measures, amplifies, and renders objective through interaction alone [3].

In the following, we will treat Π as the fundamental variable over which classical emergence is assessed. The role of decoherence is then to mediate a competition among different candidate structures—understood here as physically implementable monitoring configurations—favoring those that best satisfy the combined requirements of stability, redundant information transfer, and physical feasibility. This viewpoint sets the stage for introducing a quantitative criterion that compares different structures on equal footing.

2.2. The Informational Economy Functional

Having clarified what is meant by a candidate classical structure, we now introduce the central quantitative object of this work: the Informational Economy Functional (IEF). The purpose of the IEF is to provide a compact and physically transparent way of comparing different candidate structures on equal footing, taking into account the multiple constraints that govern decoherence-driven classical emergence.

For a given candidate structure Π, we define the Informational Economy Functional as

$$\mathcal{J}[\Pi; t] \;:=\; \dot{C}_{\mathrm{stab}}(\Pi; t) \;+\; \lambda_1 \dot{Q}_{\mathrm{out}}(\Pi; t) \;-\; \lambda_2 \dot{I}^{(\delta)}_{\mathrm{brd}}(\Pi; t), \tag{2}$$

where all quantities are evaluated at time t. Several features of this definition are worth emphasizing from the outset. First, $\mathcal{J}$ is an instantaneous rate functional: it characterizes the local-in-time balance between loss of distinguishability, energetic cost, and information gain associated with a given structure. Second, $\mathcal{J}$ depends explicitly on the choice of Π, reflecting the fact that different candidate structures interact with the environment in different ways. Accordingly, all three contributions entering $\mathcal{J}$—including $\dot{C}_{\mathrm{stab}}$, $\dot{Q}_{\mathrm{out}}$, and $\dot{I}^{(\delta)}_{\mathrm{brd}}$—are, in general, Π-dependent. Third, the coefficients λ_1 and λ_2 encode properties of the environment and the monitoring architecture, and set the relative weight with which physical and informational resources are accounted for. Here $\dot{C}_{\mathrm{stab}}$ and $\dot{I}^{(\delta)}_{\mathrm{brd}}$ are measured in the same informational units (nats per unit time), while $\dot{Q}_{\mathrm{out}}$ has units of energy per unit time; accordingly, λ_1 carries units of inverse energy, whereas λ_2 is dimensionless.

Each term in Equation (2) has a clear physical interpretation.

2.2.1. Information Loss and Structural Stability

The first term, $\dot{C}_{\mathrm{stab}}(\Pi; t)$, represents a structure-specific stability cost, quantifying the loss of distinguishability between the alternatives defined by Π and thus characterizing the persistence of Π-conditioned distinguishability under open-system dynamics.

For a projective or coarse-grained realization $\Pi = \{\Pi_i\}$, one may associate an ensemble $\mathcal{E}_\Pi(t) = \{p_i(t), \rho_i(t)\}$ with $p_i(t) = \mathrm{Tr}[\Pi_i \rho(t)]$ and $\rho_i(t) = \Pi_i \rho(t) \Pi_i / p_i(t)$. The distinguishability of these alternatives can then be characterized by the Holevo quantity

$$\chi_\Pi(t) = S\left(\sum_i p_i(t)\rho_i(t)\right) - \sum_i p_i(t) S(\rho_i(t)), \tag{3}$$

and a natural stability cost is given by

$$\dot{C}_{\text{stab}}(\Pi;t) := \left[-\dot{\chi}_{\Pi}(t)\right]_{+}, \tag{4}$$

where $[x]_{+} := \max\{x, 0\}$.

This notion is closely related to predictability-sieve ideas [1,11], but is here formulated in a structure-dependent manner appropriate for comparing different candidate classical descriptions.

2.2.2. Energetic Cost and Physical Feasibility

The second term, $\lambda_1 \dot{Q}_{\text{out}}(\Pi;t)$, accounts for the energetic cost associated with information processing during decoherence. Information transfer and entropy production are inevitably accompanied by heat flow into the environment, and not all information-processing pathways are physically affordable [7,9]. This term enforces thermodynamic feasibility by penalizing structures whose stabilization and monitoring require excessive energy dissipation. The coefficient λ_1 converts heat flow into an informational currency, setting the effective exchange rate between energetic and informational resources. Crucially, a structure that is informationally efficient is not necessarily energetically cheap, and this term makes such trade-offs explicit.

2.2.3. Information Broadcast and Objectivity

The final term, $-\lambda_2 \dot{I}^{(\delta)}_{\text{brd}}(\Pi;t)$, represents the gain associated with the broadcast of information about the structure Π into the environment. Here, $\dot{I}^{(\delta)}_{\text{brd}}$ quantifies the rate at which information about Π becomes accessible through environmental degrees of freedom that can, in principle, be independently interrogated.

To formalize this notion, we first introduce a thresholded broadcast measure capturing the total amount of accessible information distributed across independent fragments. For a given tolerance parameter $\delta \in (0, 1)$, we define

$$I^{(\delta)}_{\text{brd}}(\Pi;t) := \sum_i \min\left\{ I(Z_\Pi : F_i;t),\ (1-\delta)H(Z_\Pi;t) \right\}, \tag{5}$$

where Z_Π is the classical readout variable associated with the structure Π, and the sum runs over independently accessible environmental fragments $\{F_i\}$. Each fragment contributes at most $(1-\delta)H(Z_\Pi;t)$, ensuring that only fragments carrying near-complete information about Z_Π contribute fully to the broadcast measure.

The broadcast rate entering the Informational Economy Functional is then defined as the time derivative

$$\dot{I}^{(\delta)}_{\text{brd}}(\Pi;t) := \frac{d}{dt} I^{(\delta)}_{\text{brd}}(\Pi;t). \tag{6}$$

Such broadcast is a necessary precursor to classical objectivity: only information that becomes widely accessible across multiple independent fragments can support an intersubjectively agreed classical description. The negative sign reflects the fact that information broadcast is beneficial within the informational economy, offsetting the costs associated with stability loss and energy dissipation.

In regimes where the environment exhibits sufficient fragmentation and the broadcast persists over time, one expects the emergence of multiple disjoint fragments, each carrying near-complete records, giving rise to conventional redundancy plateaus. In this sense, information broadcast, as quantified by $\dot{I}^{(\delta)}_{\text{brd}}$, provides a dynamical and operational measure of the conditions under which redundancy emerges in Quantum Darwinism.

2.2.4. Balance of Physical and Informational Resources

The coefficients λ_1 and λ_2 determine how the three contributions entering the Informational Economy Functional are balanced against one another. They reflect physical properties of the system–environment setting.

For a system coupled to a single thermal reservoir, λ_1 is naturally related to the inverse temperature β, as suggested by thermodynamic constraints such as Spohn's inequality. In more general settings involving structured or non-equilibrium environments, λ_1 encodes an effective thermodynamic cost determined by the underlying interaction and coarse-graining.

The coefficient λ_2 reflects properties of the environment's monitoring architecture, such as its degree of fragmentation and the efficiency with which information about Π becomes accessible. In particular, it characterizes how effectively information about Π can be broadcast across independently accessible environmental degrees of freedom.

In this sense, both coefficients are determined by physical characteristics of the environment and interaction geometry.

Taken together, the three terms in Equation (2) provide a unified accounting framework for decoherence. The IEF places environment-induced superselection, Quantum Darwinism, and predictive thermodynamics within a single quantitative structure, in which their mutual tensions and trade-offs can be explicitly analyzed. This interplay of information loss, energetic cost, and redundant record formation is schematically illustrated in Figure 1.

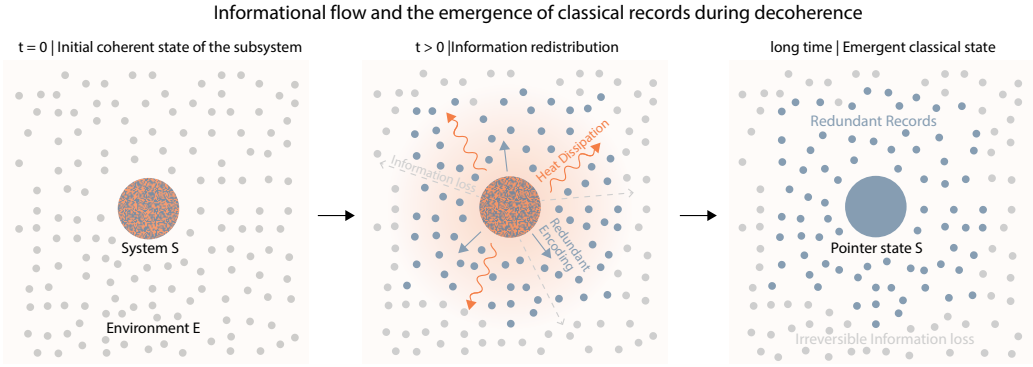


Figure 1. Schematic illustration of informational flow and the emergence of classical records during decoherence. An initially coherent system S interacts with an environment E, leading to concurrent processes: (i) loss of locally accessible distinguishability associated with candidate structures Π, captured by the stability cost $\dot{C}_{\mathrm{stab}}$; (ii) dissipation of energy into environmental degrees of freedom, quantified by $\dot{Q}_{\mathrm{out}}$; and (iii) the broadcast of information about Π into multiple independently accessible environmental fragments, quantified by $\dot{I}^{(\delta)}_{\mathrm{brd}}$. These competing processes jointly determine the emergence of stable, redundantly accessible classical records. The Informational Economy Functional provides a unified accounting framework that captures this trade-off.

In the following, the IEF will serve as the basis for formulating an effective selection principle for classical structures.

2.3. The Principle of Informational Economy

The Informational Economy Functional introduced above provides a quantitative way of comparing different candidate classical structures at the level of instantaneous information and energy flows. To elevate this comparison into a general statement about classical emergence, it is necessary to clarify how such local-in-time balances translate into stable, observable structures over physically relevant time scales. This motivates the formulation of the Principle of Informational Economy (PIE).

Decoherence is not necessarily a uniform or steady process. Depending on the system, environment, and coupling, it may occur as a rapid transient event or as a sustained dynamical process extending over long times. Classical records, however, are defined neither by instantaneous fluctuations nor by momentary optimality, but by their persistence and accessibility over a characteristic recording time. For this reason, the relevant quantity for assessing classical emergence is not the instantaneous value of the IEF, but its average over an appropriate time window.

Accordingly, we define the classical structure selected by decoherence as

$$\Pi^* = \arg\min_{\Pi} \lim_{T\to\infty} \frac{1}{T} \int_0^T \mathcal{J}[\Pi; t]\, \mathrm{d}t, \tag{7}$$

where the long-time average represents the theoretical default for stable record formation. In experimental or operational settings, this definition can be straightforwardly generalized to a finite time window comparable to the decoherence or recording time, without altering the conceptual content of the principle.

The physical meaning of the Principle of Informational Economy is deliberately modest. It provides an effective variational description of which structures can leave persistent, redundantly accessible traces under simultaneous informational and thermodynamic constraints. Candidate structures that dissipate information or energy without producing robust, shareable records are naturally suppressed, while those that convert available resources into stable and widely accessible information traces are favored.

In this sense, the PIE plays a role analogous to other emergent variational principles in physics. Much like free-energy minimization in nonequilibrium thermodynamics, it does not replace the underlying microscopic dynamics, but offers a compact and predictive way of characterizing the long-term outcomes of complex open-system interactions [9,12].

At the same time, we emphasize that decoherence, in its most general sense, does not necessarily always involve energetic dissipation [13]. The present work, however, focuses on decoherence processes that give rise to classicalization: the emergence of stable, widely accessible and observer-independent records [3,6]. The establishment of such objective records requires more than the suppression of interference. It demands (i) the broadcast of information about selected degrees of freedom into multiple, independently accessible environmental channels, and (ii) the stabilization of these records against subsequent environmental noise. Physically, the broadcast of information entails correlating the system with many environmental degrees of freedom, which typically requires work to excite them out of their initial states. The subsequent stabilization of these records relies on the relaxation of these excited environmental modes, leading to irreversible energy dissipation into the environment's internal degrees of freedom [1,2]. Consequently, within this operational regime of record formation, the irreversible loss of distinguishability of the alternatives defined by Π and the associated energetic dissipation are not incidental but represent the unavoidable physical resources expended to "pay for" objectivity.

A further important aspect of this classicalizing regime concerns the role of nonlinearity. While the underlying open-system dynamics considered here are linear at the level of density operators (e.g., GKLS evolution), the informational quantities relevant for classicality—such as entropy, mutual information, and distinguishability measures—are inherently nonlinear functionals of the quantum state. In addition, the operational description of classical alternatives involves conditional states and coarse-grained structures, which introduce further nonlinearities through normalization and selection [13]. In the present framework, these nonlinear features enter not at the level of the dynamical equations, but through the informational quantities that define the stability and broadcast terms of the Informational Economy Functional. The IEF could therefore be understood as an

effective scalarization of nonlinear informational constraints, consistent with resource-based perspectives on quantum coherence and information [14]. As a result, the optimal structure Π^* may exhibit sharp changes as environmental parameters are varied, reflecting an effective nonlinearity at the level of emergent classical structures rather than in the underlying microscopic dynamics.

The Informational Economy Functional and the associated Principle of Informational Economy are therefore proposed as an effective description of this resource-consuming classicalizing regime. They quantify the trade-off between the cost of information loss and energy dissipation, and the benefit of redundant information gain.

3. From Microscopic Balance to an Effective Variational Principle

3.1. Microscopic Ingredients and Physical Assumptions

In this section, we show how the Informational Economy Functional (IEF) and the associated Principle of Informational Economy (PIE) emerge naturally from well-established microscopic considerations. Under physically standard assumptions, the problem of classical structure selection admits a compact and interpretable effective formulation.

The logical starting point consists of three ingredients that are independently well understood. First, the global dynamics of a closed system–environment composite is unitary, which implies exact conservation laws for entropy and correlations. Second, when the environment acts as a thermodynamic reservoir, open-system dynamics are subject to nontrivial constraints arising from entropy production and heat flow. Third, within the framework of Quantum Darwinism, only a specific subset of system–environment correlations—those that become widely accessible and redundantly recorded across multiple observers—can give rise to objective classical records. The present section shows how these ingredients can be combined into a single accounting structure that exposes the resource trade-offs underlying classicalization.

Throughout this section, we work under the following assumptions, which delimit the physical regime addressed by the IEF and PIE.

(A1) Thermodynamic structure.

The system interacts with one or more large environments that may be approximated as near-equilibrium reservoirs. The reduced dynamics of the system can therefore be described, at an effective level, by GKLS/Davies-type generators or, more generally, by dynamics satisfying a Spohn entropy production inequality [13,15]. In this setting, one can define an entropy production rate,

$$\sigma(t) = \dot{S}_S(t) - \sum_\alpha \beta_\alpha \dot{Q}_{\text{in}}^{(\alpha)}(t) \geq 0, \tag{8}$$

where $\dot{Q}_{\text{in}}^{(\alpha)}$ denotes the heat current flowing *into* the system from reservoir α, and β_α is the corresponding inverse temperature. For a single thermal reservoir this reduces to the familiar inequality

$$\dot{S}_S(t) - \beta \dot{Q}_{\text{in}}(t) \geq 0. \tag{9}$$

In the present framework, this thermodynamic structure constrains the energetic cost term in the Informational Economy Functional.

(A2) Environment fragmentation.

The environment can be operationally coarse-grained into a collection of fragments $\{F_i\}$ whose mutual correlations are sufficiently weak to allow independent access by different observers. This is the standard assumption underlying Quantum Darwinism:

objectivity is defined with respect to observers who each probe only a small fraction of the environment and do not communicate with one another [3,6].

(A3) Implementable readout structures.

Candidate classical structures Π are not treated as arbitrary mathematical decompositions of the system Hilbert space. Instead, each Π represents a physically implementable readout geometry, corresponding to a monitoring channel that specifies which degrees of freedom of the system are effectively interrogated and recorded by the environment [1,2]. Throughout the present analysis, we consider a fixed underlying physical setup, specified by the available system–environment couplings and reservoir properties. Within this setup, different candidate structures Π correspond to different implementable monitoring channels, and therefore induce different effective open-system dynamics. Accordingly, the quantities entering the Informational Economy Functional—including $\dot{C}_{\mathrm{stab}}(\Pi;t)$, $\dot{Q}_{\mathrm{out}}(\Pi;t)$, and $\dot{I}^{(\delta)}_{\mathrm{brd}}(\Pi;t)$—are, in general, Π-dependent.

With these assumptions in place, we now derive an exact correlation ledger based on global unitarity, incorporate thermodynamic constraints as a physical currency, and identify the portion of system–environment correlations that can contribute to objective classical records. This preparation sets the stage for interpreting classicality as a resource-constrained information task in the subsequent sections.

3.2. Correlation Accounting, Thermodynamic Constraints, and Information Broadcast

We begin from the microscopic description of a closed system–environment composite. The total state $\rho_{SE}(t)$ evolves unitarily according to

$$\dot{\rho}_{SE}(t) = -\frac{i}{\hbar}[H_T, \rho_{SE}(t)], \qquad H_T = H_S \otimes I_E + I_S \otimes H_E + H_{\mathrm{int}}. \tag{10}$$

As a direct consequence of unitarity, the von Neumann entropy of the composite system is conserved:

$$S(\rho_{SE}(t)) = \text{const.} \tag{11}$$

To make the redistribution of information explicit, we consider the quantum mutual information between system and environment:

$$I(S:E) = S(\rho_S) + S(\rho_E) - S(\rho_{SE}). \tag{12}$$

Taking a time derivative and using the constancy of $S(\rho_{SE})$, one obtains the exact identity

$$\dot{I}(S:E) = \dot{S}_S + \dot{S}_E. \tag{13}$$

Equation (13) provides a strict correlation ledger: any change in the system entropy must be accompanied by a corresponding change in the entropy of the environment, such that their sum accounts for the rate at which correlations are created or redistributed. In this sense, decoherence does not entail the disappearance of information, but rather its transfer into system–environment correlations and environmental degrees of freedom [13,16].

While Equation (13) is exact, it is purely informational. By itself, it does not distinguish physically feasible correlation pathways from those that are thermodynamically forbidden or prohibitively costly. To discuss informational economy, a physical currency must therefore be introduced.

Under assumption (A1), the reduced dynamics of the system obey a Spohn entropy production inequality [12,15]. In particular, for each implementable monitoring structure Π, the rate of change of the system's von Neumann entropy and the heat flow to the

environment are constrained by a non-negative entropy production rate. For a single thermal reservoir, this implies the standard system-side second-law inequality,

$$\dot{S}_S(\Pi;t) + \beta \dot{Q}_{\text{out}}(\Pi;t) \geq 0, \tag{14}$$

For each candidate structure Π, this inequality is understood to apply to the corresponding effective open-system dynamics induced by the associated monitoring structure, where $\dot{Q}_{\text{out}}(\Pi;t)$ denotes the heat flow into the environment. In the present context, this inequality provides a non-arbitrary physical origin for the energetic cost term appearing in the Informational Economy Functional.

At the same time, the total mutual information $I(S:E)$ generated under these constraints contains a vast amount of microscopic detail and, by itself, does not characterize classical objectivity. A central insight of Quantum Darwinism is that only the portion of system–environment correlations that can be accessed independently by many observers constitutes objective classical information [3,6].

Operationally, the environment is partitioned into fragments:

$$E = \bigsqcup_i F_i. \tag{15}$$

Given a candidate structure Π, we associate with it a readout variable Z_Π on the system, represented by an observable, POVM outcome, or coarse-grained classical label. Conceptually, Π specifies which question about the system is being answered by the environment, potentially in a redundant manner across multiple fragments.

To characterize the information about Z_Π that becomes accessible through the environment, we employ the thresholded broadcast quantity $I^{(\delta)}_{\text{brd}}(\Pi;t)$ introduced in Section 2.2. This quantity captures both the total accessible information and its distribution across independently accessible environmental fragments.

To relate this broadcast quantity to the underlying information-theoretic structure of system–environment correlations, we now consider the total mutual information between the readout variable Z_Π and the environment.

By the data processing inequality, since Z_Π is obtained from the system by a quantum channel [16], one has

$$I(Z_\Pi : E) \leq I(S:E), \tag{16}$$

which provides an upper bound on the total information about Z_Π that can be accessed from the environment. Combining this bound with the broadcast quantity introduced in Section 2.2, it is natural to define the complement of accessible information as

$$I_{\text{irr}}(\Pi;t) := I(Z_\Pi : E) - I^{(\delta)}_{\text{brd}}(\Pi;t) \geq 0. \tag{17}$$

Here I_{irr} represents correlations that, relative to the chosen fragmentation and access model, are not redundantly readable and therefore cannot support stable, multi-observer records. These correlations have not vanished at the microscopic level; rather, they are dispersed into fine-grained environmental degrees of freedom that do not contribute to classical objectivity.

The decomposition

$$I(Z_\Pi : E) = I^{(\delta)}_{\text{brd}}(\Pi;t) + I_{\text{irr}}(\Pi;t) \tag{18}$$

thus makes explicit the tension between useful, objective records and informational waste. This tension will play a central role in the task-based interpretation of classicality developed in the next subsection.

In this sense, information broadcast provides the dynamical mechanism by which redundancy, and hence classical objectivity, can emerge under appropriate environmental conditions.

3.3. From Inequalities to an Effective Variational Principle

The relations established so far—including the exact correlation ledger implied by global unitarity, the thermodynamic lower bounds on irreversible open-system dynamics, and the decomposition of system–environment correlations into redundantly accessible and inaccessible components—collectively set the stage for an operational characterization of classicality.

If classical emergence is understood as the outcome of competing informational and physical requirements, it is then natural to formulate it in terms of an explicit information-theoretic task that the system–environment dynamics may or may not successfully accomplish.

3.3.1. Objective-Record Task

Fix a time window τ and a candidate structure Π, associated with a classical readout variable Z_Π. We say that Π forms an objective classical record over the time window τ if the following conditions are satisfied:

- Multi-observer access: there exist $R \gg 1$ disjoint environmental fragments $\{F_i\}$, each accessed by a different observer;
- No communication: observers do not exchange information or coordinate their inferences;
- Reliable readout: each observer can infer the same value of Z_Π with error probability at most ε;
- Temporal stability: the inferred value of Z_Π remains consistent throughout the time window τ.

In physical terms, this definition captures the minimal operational meaning of classicality: a variable is classical if multiple observers, acting independently and without coordination, can reliably read out the same information over a relevant timescale. This formulation may be viewed as a minimal axiomatization of Quantum Darwinism, with the additional requirement of temporal persistence [3,6].

By standard information-theoretic bounds (e.g., Fano-type inequalities [17]), reliable inference with bounded error probability requires each observer to access a fragment carrying a minimum amount of mutual information about Z_Π. If R observers independently succeed at the task, the environment must therefore host many disjoint fragments that each encode sufficient information about the same variable. Successful completion of the objective-record task thus requires the proliferation of redundantly accessible information.

In the present framework, the proliferation of redundantly accessible records is enabled by sustained information broadcast into the environment. Accordingly, we quantify task performance operationally by the net amount of thresholded broadcast accumulated over the time window τ, while the success of the objective-record task remains defined by the formation of sufficiently many disjoint fragments carrying near-complete information about Z_Π.

$$\Delta I^{(\delta)}_{\mathrm{brd}}(\Pi;\tau) := \int_{t_0}^{t_0+\tau} \dot{I}^{(\delta)}_{\mathrm{brd}}(\Pi;t)\, dt, \tag{19}$$

Large values of $\Delta I^{(\delta)}_{\mathrm{brd}}(\Pi;\tau)$ indicate sustained formation of redundantly accessible records over the time window, with contributions arising from multiple independently

accessible fragments, each carrying near-complete information about Z_Π up to the threshold δ. In this sense, $\Delta I^{(\delta)}_{\mathrm{brd}}$ provides an operational measure of redundancy formation and directly underpins the successful completion of the objective-record task by quantifying the sustained buildup of independently accessible records.

Not every candidate structure can complete the objective-record task. Two classes of constraints are fundamental.

1. **Stability constraint.** If the variable Z_Π drifts rapidly or is erased on timescales shorter than τ, different observers will obtain inconsistent readouts and the task fails. Within the present framework, stability is characterized by the persistence of distinguishability associated with the candidate structure Π. This is quantified by the structure-dependent stability cost $\dot{C}_{\mathrm{stab}}(\Pi; t)$, which measures the rate of loss of distinguishability between the alternatives defined by Π. Bounding $\dot{C}_{\mathrm{stab}}(\Pi; t)$ therefore constrains the rate at which information about Z_Π is degraded and ensures temporal consistency of the recorded variable.
2. **Energetic constraint.** The formation of redundant records is not free. Broadcasting information into multiple environmental degrees of freedom necessarily involves entropy increase and heat flow. Within the Markovian/Davies/Spohn framework, irreversible dynamics obey the system-side second-law inequality

$$\dot{S}_S(\Pi; t) + \beta \dot{Q}_{\mathrm{out}}(\Pi; t) \geq 0, \tag{20}$$

which provides a lower bound on the energetic cost associated with record formation. Energetic feasibility is therefore a necessary condition for completing the objective-record task.

3.3.2. Constrained Optimization Formulation

With these ingredients, the emergence of classicality can be formulated as a constrained optimization problem. For a fixed underlying physical setup and environmental conditions, different candidate structures Π induce different effective monitoring dynamics, and the task is to identify those structures that most efficiently convert available physical resources into redundant records.

At the level of time-integrated quantities, we define

$$\Delta C_{\mathrm{stab}}(\Pi; \tau) := \int_{t_0}^{t_0+\tau} \dot{C}_{\mathrm{stab}}(\Pi; t)\, dt, \qquad Q_{\mathrm{out}}(\Pi; \tau) := \int_{t_0}^{t_0+\tau} \dot{Q}_{\mathrm{out}}(\Pi; t)\, dt, \tag{21}$$

$$\Delta I^{(\delta)}_{\mathrm{brd}}(\Pi; \tau) := \int_{t_0}^{t_0+\tau} \dot{I}^{(\delta)}_{\mathrm{brd}}(\Pi; t)\, dt. \tag{22}$$

The constrained optimization problem can then be expressed as

$$\max_{\Pi}\ \Delta I^{(\delta)}_{\mathrm{brd}}(\Pi; \tau) \quad \text{subject to} \quad \Delta C_{\mathrm{stab}}(\Pi; \tau) \leq C_0, \qquad Q_{\mathrm{out}}(\Pi; \tau) \leq Q_0, \tag{23}$$

where C_0 and Q_0 represent, respectively, the admissible loss of distinguishability and energetic budgets set by the dynamics and environment.

Introducing Lagrange multipliers yields the corresponding Lagrangian,

$$\mathcal{L}[\Pi] = \Delta C_{\mathrm{stab}}(\Pi; \tau) + \lambda_1 Q_{\mathrm{out}}(\Pi; \tau) - \lambda_2 \Delta I^{(\delta)}_{\mathrm{brd}}(\Pi; \tau), \tag{24}$$

which, in continuous time, can be written as

$$\mathcal{L}[\Pi] = \int_{t_0}^{t_0+\tau} \left(\dot{C}_{\mathrm{stab}}(\Pi; t) + \lambda_1 \dot{Q}_{\mathrm{out}}(\Pi; t) - \lambda_2 \dot{I}^{(\delta)}_{\mathrm{brd}}(\Pi; t) \right) dt. \tag{25}$$

This naturally identifies the instantaneous Informational Economy Functional,

$$\mathcal{J}[\Pi;t] = \dot{C}_{\text{stab}}(\Pi;t) + \lambda_1 \dot{Q}_{\text{out}}(\Pi;t) - \lambda_2 \dot{I}^{(\delta)}_{\text{brd}}(\Pi;t), \tag{26}$$

and leads to the effective selection principle

$$\Pi^* = \arg\min_{\Pi} \frac{1}{\tau} \int_{t_0}^{t_0+\tau} \mathcal{J}[\Pi;t]\, dt. \tag{27}$$

Here λ_1 and λ_2 play the role of shadow prices, encoding how stability and energetic resources are traded against informational gain. Importantly, this formulation does not posit that nature "optimizes" a goal; rather, it provides an equivalent bookkeeping description of optimal strategies for completing the objective-record task under resource constraints.

An equivalent perspective follows from the decomposition established in Section 3.2,

$$I(Z_\Pi : E) = I^{(\delta)}_{\text{brd}}(\Pi;t) + I_{\text{irr}}(\Pi;t), \tag{28}$$

together with the observation that, for given Π-induced effective dynamics, the total accessible information $I(Z_\Pi : E)$ is bounded by the underlying system–environment correlations. Maximizing the broadcast component therefore necessarily minimizes the inaccessible remainder. From this viewpoint, classical structures are those that channel as much correlation as possible into widely accessible environmental records, thereby minimizing the fraction of information dispersed into unreadable microscopic detail. Under appropriate fragmentation conditions, such broadcast leads to the emergence of redundantly accessible records in the sense of Quantum Darwinism [3].

The resulting Principle of Informational Economy is thus an effective variational principle. Within the Quantum Darwinism regime and the dynamical assumptions adopted here, objective record formation is equivalent to the successful completion of a well-defined information-theoretic task under physical resource constraints. Outside this regime—for example, in strongly non-Markovian dynamics or non-thermal environments—the same logic applies, but the relevant constraints and accounting terms must be appropriately modified.

Taken together, global unitarity supplies an exact correlation ledger; open-system thermodynamics introduces unavoidable resource costs; and environmental fragmentation distinguishes useful records from inaccessible detail. By formulating classicality as an operational task, the Informational Economy Functional and its associated variational principle arise as an effective and physically motivated description of how stable, widely accessible records can form under resource constraints. In this sense, classical structures can be understood as those that achieve an efficient conversion of physical resources into objective information, within the modeling framework adopted here.

4. Towards an Informational Economics of Decoherence

Once stability, redundant accessibility, and physical feasibility are treated on the same footing, decoherence can be viewed as a process in which objective information is produced under finite physical constraints. In this sense, information in an open quantum system is not an abstract quantity that can be stored or copied without limit; rather, the production of robust and widely accessible records is constrained by energetic budgets, environmental noise, and the effective capacity of the environment to host distinguishable imprints. This perspective is consistent with the view that classical information arises through environment-mediated record formation rather than abstract copying [3].

This perspective naturally motivates a Pareto-type description. For a given environment and monitoring architecture, each implementable candidate structure Π induces a

triple of competing performance measures over the relevant time window: (i) a stability cost associated with the loss of distinguishability of the alternatives defined by Π (captured by $C_{\mathrm{stab}}(\Pi; t)$ or its rate), (ii) an energetic cost associated with dissipation into the environment (captured by $Q_{\mathrm{out}}(\Pi; t)$ or its rate), and (iii) an informational gain associated with the broadcast of information into independently accessible environmental degrees of freedom (captured by $I^{(\delta)}_{\mathrm{brd}}(\Pi; t)$ or its rate). A structure may then be called Pareto optimal if none of these objectives can be improved without worsening at least one of the others. This notion follows the standard definition of Pareto optimality in multi-objective optimization [18]. From this viewpoint, classical structures are not characterized by an extremum of a single criterion, but are naturally associated with Pareto-optimal trade-offs among stability, energetic feasibility, and redundant accessibility.

Within this economic interpretation, the coefficients λ_1 and λ_2 in the Informational Economy Functional admit a clear meaning as effective shadow prices: they encode how the environment and platform weight energetic expenditure and stability costs against informational gain. Such Lagrange-multiplier interpretations are standard in variational formulations of statistical and information-theoretic physics [19]. Different physical conditions (temperature, coupling strength, noise spectra, fragmentation structure) correspond to different effective price systems, and therefore to different preferred trade-offs.

In this sense, distinct environments may stabilize distinct "classical realities"—not because the underlying principles differ, but because the available resources and their effective exchange rates differ. This situation is directly analogous to familiar phase transitions in condensed-matter physics: the same microscopic constituents can support qualitatively different macroscopic phases when the control parameters are varied. Likewise, the quantum–classical boundary is not absolute, but tunable. Concrete examples are already well known. In superconducting qubit platforms, for instance, the preferred pointer structures can shift between energy eigenstates, phase states, or rotated bases depending on the dominant noise channel, measurement backaction, and dissipation rates [20,21].

From the present viewpoint, these changes reflect a reweighting of the effective shadow prices: what counts as an economical classical record depends on which informational pathways are cheapest to stabilize and amplify under the given experimental conditions.

Seen through the lens of the IEF, several influential approaches can be recovered as boundary regimes within a single coordinate system. First, in a stability-dominated regime—for instance when redundant records have little value or are difficult to form (effectively $\lambda_2 \to 0$)—the selection principle reduces to the predictability-sieve intuition associated with environment-induced superselection: the preferred structures are those that minimize the loss of predictability, i.e., those with small $\dot{C}_{\mathrm{stab}}(\Pi; t)$. Second, in a broadcast-dominated regime—when many weakly correlated fragments are available and noise is sufficiently low, so that redundant accessibility is strongly rewarded (effectively large λ_2)—the production of widely broadcast information that can give rise to redundant records becomes the primary organizing principle, in the spirit of Quantum Darwinism. Third, in an energetically constrained regime—when dissipation budgets are stringent (large λ_1)—high-cost recording pathways are suppressed, and feasibility constraints carve out the admissible region of structures, consistent with the emphasis of information thermodynamics.

Overall, the Informational Economy Functional can be viewed as embedding these otherwise separate criteria into a unified accounting framework. In that framework, classical structures emerge along a Pareto frontier determined by the environmentally induced trade-offs between stability, feasibility, and information broadcast and redundancy formation. This organization is schematically illustrated in Figure 2, which highlights how classical

records occupy the Pareto-efficient boundary separating stable, information-producing structures from information-wasting alternatives.

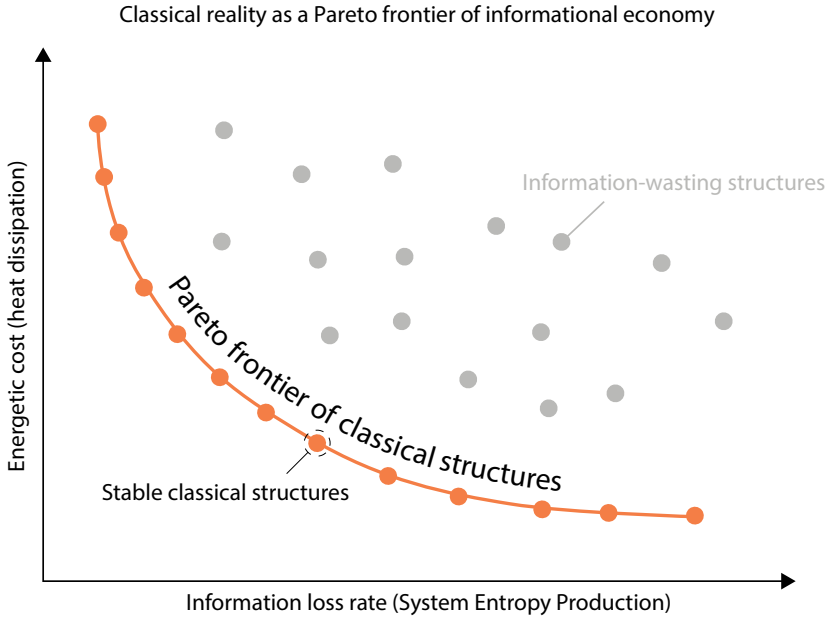


Figure 2. Schematic illustration of classical reality as a Pareto frontier in the informational economy of decoherence. Each candidate structure is represented as a point characterized by competing objectives: stability (loss of distinguishability of the alternatives), energetic cost (heat dissipation into the environment), and informational gain (production of widely accessible records in the environment). Structures lying on the Pareto frontier represent optimal trade-offs, for which no objective can be improved without worsening at least one other. Information-wasting structures lie off the frontier, dissipating physical resources without producing stable, objective records.

5. Falsifiability and Experimental Scenarios

The IEF framework naturally leads to structural predictions about how classicality reorganizes itself under changing resource constraints. These predictions go beyond both environment-induced superselection and Quantum Darwinism, and admit clear experimental refutation.

5.1. General Falsifiable Predictions of the IEF

5.1.1. Prediction I: Resource-Driven Variability of the Selected Pointer Structure

In the IEF framework, a pointer structure is not treated as an intrinsic attribute of the system alone. Rather, the selected structure Π^* is the one that best satisfies the joint requirements of stability, energetic feasibility, and redundant accessibility under the prevailing environmental conditions. A key consequence is that, even within a fixed open-system dynamical setting, the optimal structure Π^* can shift when the effective resource conditions change [21–23]—for example when the accessibility and fragmentation of environmental records are altered, or when the effective energetic and informational "prices" encoded by λ_1 and λ_2 are varied.

This yields a direct falsifiable prediction: as experimentally controllable resource conditions are tuned continuously, the optimal pointer structure selected by the Informational Economy Functional should exhibit either smooth deformations or, in certain regimes, sharp reorientations. Operationally, this corresponds to a continuous rotation/distortion or an abrupt switching of the preferred readout variable Z_Π (and its associated effective measurement basis). Importantly, the IEF does not merely assert that pointer bases can depend on details of the coupling; rather, it predicts a systematic reorganization of the selected structure

under changes in resource conditions—a behavior that can be tested by scanning the accessible readout geometry while holding the underlying dynamical model fixed.

5.1.2. Prediction II: Systematic Deformation of Redundancy Plateaus with Resource Prices

Quantum Darwinism emphasizes the existence of redundancy plateaus [3,24]. The IEF strengthens this picture by predicting how the geometry of redundancy plateaus responds to changes in resource constraints. Specifically, varying the effective energetic price λ_1 (e.g., by tightening dissipation budgets or increasing the cost of maintaining excited environmental modes) should generically suppress or delay the formation of robust redundant records, reducing the plateau height and/or slowing its buildup. Conversely, increasing the effective informational value λ_2 (e.g., by improving fragment accessibility, detection efficiency, or fragment multiplicity) should favor higher and more rapidly formed plateaus, but only at the expense of increased stability cost and/or energetic outflow quantified by $\dot{C}_{\mathrm{stab}}(\Pi; t)$ and $\dot{Q}_{\mathrm{out}}(\Pi; t)$. Thus, the IEF predicts not only that redundancy may appear, but that plateau height, formation rate, and persistence deform in a controlled and correlated manner with the measured stability and dissipation costs. This cost–shape correlation provides a quantitative experimental signature: if redundancy plateaus deform under parameter scans yet show no systematic co-variation with the inferred $\dot{C}_{\mathrm{stab}}(\Pi; t)$ and $\dot{Q}_{\mathrm{out}}(\Pi; t)$, the IEF accounting is disfavored.

5.1.3. Prediction III: Price-Sensitive Trade-Offs and the Absence of Globally Dominant Structures

A central implication of the IEF is that stability, energetic feasibility, and redundant broadcasting define competing objectives whose trade-offs are controlled by the effective prices (λ_1, λ_2). Accordingly, the framework predicts that the identity of the optimal structure is generally price-sensitive: scanning (λ_1, λ_2) should change which Π minimizes $\langle \mathcal{J} \rangle_\tau$, and improvements in redundancy should be accompanied by measurable increases in at least one of the associated costs [9,12]. Equivalently, there should be no single structure that remains simultaneously near-optimal in stability and dissipation while also maximizing the production of widely broadcast information leading to redundant records across a broad range of resource conditions.

This prediction admits a clear falsification protocol. Fix a family of candidate structures $\Pi(\theta)$ (e.g., a continuous rotation of the readout axis) and experimentally tune the effective prices by changing dissipation budgets, fragment accessibility, or temperature. If, over wide scans of these resource conditions, the same θ^* continues to (i) maximize redundant record production while also (ii) minimizing both $\dot{C}_{\mathrm{stab}}(\Pi; t)$ and $\dot{Q}_{\mathrm{out}}(\Pi; t)$ (or, more conservatively, yields simultaneous improvements of all three metrics without any detectable trade-off), then the informational-economy hypothesis is undermined. In contrast, the IEF predicts a robust *Pareto-type* organization: moving toward higher redundancy necessarily incurs increased stability loss and/or energetic cost, and the selected Π^* should track these changing trade-offs as the effective prices are varied.

5.2. Candidate Experimental Platforms

Among currently available platforms, superconducting qubits subject to continuous weak measurement provide a particularly direct and versatile testbed for the Informational Economy Functional. Continuous measurement techniques in these systems are highly mature, allow real-time monitoring of individual quantum trajectories, and—crucially—permit controlled rotation of the effective measurement axis [20,21,25]. This tunability makes the candidate structure Π experimentally addressable as a continuously adjustable parameter, rather than a fixed property imposed by the system–environment coupling.

An important advantage of superconducting platforms is that measurement backaction, engineered dissipation, and thermal effects can be modeled and, to a significant extent, tuned independently. This separation enables controlled exploration of informational and energetic contributions to record formation, which is essential for testing the resource-based predictions of the IEF [13,20,26].

More concretely, this separation allows experimentally controllable parameters to be mapped in a transparent way onto the quantities entering the Informational Economy Functional:

- Measurement axis angle $\theta \rightarrow$ candidate structure $\Pi(\theta)$;
- Measurement strength $k \rightarrow$ rate of information extraction and redundant record formation;
- Engineered dissipation channels or additional damping $\rightarrow$ heat flow $\dot{Q}_{\text{out}}$;
- Environmental temperature $T \rightarrow$ controls the effective energetic price λ_1 (typically scaling with $\beta = 1/k_B T$);
- Number of probes, detection efficiency, bandwidth, or fragment resolution $\rightarrow$ controls the effective informational value λ_2 via accessibility and redundancy capacity.

All three rates entering the IEF are, in principle, experimentally accessible, albeit at different levels. The stability cost $\dot{C}_{\text{stab}}(\Pi; t)$ can be inferred from the decay of distinguishability associated with the candidate structure, for example via coherence decay, contrast loss, or observable-specific variance dynamics reconstructed from quantum trajectories. The heat flow $\dot{Q}_{\text{out}}$ may be accessed either indirectly—via calibrated dissipators, jump statistics, or well-characterized master-equation models—or, in suitably engineered devices, through direct calorimetric readout [12,27,28]. Accordingly, experimental tests of the IEF can be carried out at different levels of resolution, ranging from model-based inference to direct energetic measurements. Finally, the broadcast information rate $\dot{I}^{(\delta)}_{\text{brd}}(\Pi; t)$ can be reconstructed, at an operational level, from multi-probe mutual information measurements across independently accessible environmental channels, with suitable thresholding to approximate the broadcast quantity introduced in the main text. Together, these controls enable systematic scans of the IEF landscape as experimental resource conditions are varied.

While superconducting qubits provide a natural and highly flexible starting point, related tests could also be implemented in trapped ions, cavity QED systems, or solid-state spin platforms such as NV centers, where measurement channels, environmental fragmentation, and dissipation can be engineered with high precision. These complementary platforms reinforce that the IEF does not rely on a platform-specific mechanism, but probes a general resource-based structure underlying decoherence and classical record formation.

More broadly, these considerations also clarify how the experimental agenda implied by the IEF differs from existing tests inspired by Quantum Darwinism. Most Quantum Darwinism experiments focus on establishing the existence of redundancy plateaus—demonstrating that information about a preferred observable is copied into many environmental fragments [3,6,29]. In such experiments, the pointer observable is typically assumed to be fixed, and energetic costs are not explicitly monitored.

The IEF framework leads to a qualitatively different experimental agenda. Rather than asking whether redundancy exists, it asks under what energetic and stability costs redundancy is produced, and how its structure reorganizes when those costs are varied. Instead of holding the pointer structure fixed, the IEF approach actively scans the space of candidate structures Π. Finally, it treats stability cost, energetic dissipation, and information broadcast (leading to redundancy) on equal footing by directly probing their trade-offs. In this sense, while Quantum Darwinism identifies signatures of objectivity, the IEF predicts how objectivity reshapes itself under changing resource constraints.

Taken together, the Informational Economy Functional leads to predictions that admit clear and direct experimental refutation. In particular, the framework would be challenged if one were to observe (i) a persistent absence of pointer reorientation under broad variations of environmental and resource conditions; (ii) the simultaneous optimization of stability, energetic efficiency, and information broadcast and the resulting redundant record formation over wide parameter ranges; or (iii) redundancy plateaus whose structure and dynamics remain insensitive to changes in energetic or stability costs.

The observation of any such behavior would undermine the central claim of the IEF: that classical structures do not arise as fixed features of the system alone, but instead emerge as economical compromises shaped by finite informational and physical resources.

6. Conclusions

On the question of whether quantum decoherence exhibits an intrinsic tendency, the major existing frameworks—environment-induced superselection, Quantum Darwinism, and predictive or information thermodynamics—each capture a genuine and important aspect of the quantum-to-classical transition. Respectively, they emphasize stability, objectivity, and physical feasibility. In realistic physical settings, however, these aspects do not operate in isolation. They coexist, compete, and constrain one another within the same open-system dynamics.

In this work, we have argued that the selection of classical structures by decoherence is most naturally understood as a resource-constrained, multi-objective selection process. By introducing the Informational Economy Functional, we provided a quantitative framework that places information loss, energetic expenditure, and redundant information broadcasting on equal footing. The associated Principle of Informational Economy then characterizes the emergent classical structures as those that realize an optimal compromise among stability, objectivity, and feasibility, rather than extremizing any single criterion in isolation.

This perspective unifies previously distinct approaches within a single coherent framework. Environment-induced superselection, Quantum Darwinism, and predictive thermodynamics emerge as complementary limits of the same underlying informational economy, corresponding to different regions of the trade-off space. Classicality, in this view, is neither maximally stable, nor maximally redundant, nor maximally energy-efficient. A perfectly stable structure would be dynamically frozen and informationally inert; a maximally redundant one would demand prohibitive energetic resources; a maximally energy-saving one would fail to produce any distinguishable records. The classical world we observe is instead "good enough" across all three dimensions.

Such classical compromise is not a deficiency, but a structural consequence of competing physical constraints. It is precisely the hallmark of a Pareto-optimal solution: a configuration that cannot be improved along one axis without degrading performance along another. From this standpoint, our familiar classical reality may be understood as residing within the compromise region permitted by the laws of physics and the environmental conditions in which records are formed and maintained.

What we call the classical world may therefore be nothing more—and nothing less—than the most economical balance at which information can stably exist, be shared, and persist.

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Appendix A. Minimal Simulation Model for IEF-Induced Pointer Reorientation

This appendix provides a minimal, fully reproducible dynamical setting in which the Informational Economy Functional (IEF) can be evaluated across a one-parameter family of candidate readout geometries $\Pi(\theta)$. Within this controlled model, a reorientation of the optimal pointer structure $\theta^*(\lambda_1)$ is observed as the energetic cost parameter λ_1 is varied, illustrating the core mechanism discussed in the main text.

We consider a single qubit system S with Hamiltonian (setting $\hbar = 1$)

$$H = \frac{\omega}{2}\,\sigma_z. \tag{A1}$$

The environment is modeled effectively by two Markovian channels: (i) a continuously monitored (pure-dephasing) channel along a tunable measurement axis, and (ii) a finite-temperature amplitude-damping bath that provides an unambiguous energetic flux.

The tunable monitored observable is defined as

$$\sigma_\theta := \cos\theta\,\sigma_z + \sin\theta\,\sigma_x, \qquad \theta \in [0, \pi/2], \tag{A2}$$

with associated Lindblad operator

$$L_\theta = \sigma_\theta. \tag{A3}$$

Varying θ is interpreted as varying an implementable monitoring channel or readout geometry, corresponding to a family of candidate structures $\Pi(\theta)$ in the sense of the main text.

Appendix A.1. GKLS Dynamics and Initial State

The density matrix $\rho(t)$ evolves according to a standard GKLS master equation,

$$\dot{\rho} = -i[H, \rho] + k\,\mathcal{D}[L_\theta]\rho + \gamma(n+1)\,\mathcal{D}[\sigma_-]\rho + \gamma n\,\mathcal{D}[\sigma_+]\rho, \tag{A4}$$

where $\mathcal{D}[A]\rho := A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$, $k \geq 0$ is the monitoring strength, $\gamma \geq 0$ is the damping rate, and $n \geq 0$ is the thermal excitation parameter.

Initial state

The system is initialized at time t_0 in a pure superposition state

$$\rho(t_0) = |+\rangle\langle+|, \qquad |+\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle + |1\rangle). \tag{A5}$$

which is unbiased with respect to both σ_z and σ_x. This choice is made for illustrative purposes, as it maximizes the sensitivity of both informational and energetic contributions to the monitoring angle θ. The qualitative behavior reported below—in particular the reorientation of the optimal structure $\theta^*(\lambda_1)$ under increasing energetic cost—is robust under variations of the initial state, although the precise crossover value of λ_1 depends on initialization, averaging window, and proxy definitions.

Appendix A.2. Measured Quantities and Proxies

We emphasize that this simulation model is not intended to provide a microscopic realization of the full thresholded broadcast quantity $I_{\mathrm{brd}}^{(\delta)}$ introduced in the main text. Rather, it supplies a minimal and computable setting in which operational surrogates for the three terms of the IEF can be evaluated consistently across a family of candidate structures $\Pi(\theta)$. The purpose is therefore illustrative rather than theorem-level: to show how competing stability, energetic, and broadcast tendencies can reorganize the preferred readout structure within a controlled open-system model.

(i) Stability cost proxy.

In the main text, the stability term is defined in terms of the loss of distinguishability associated with the candidate structure Π. In the present simulation model, we adopt a computable proxy for this quantity based on the monitored observable σ_θ. Since the distinguishability of the alternatives selected by $\Pi(\theta)$ is encoded in the spread of outcomes of σ_θ, we use the decay of the corresponding variance as a proxy for the loss of distinguishability.

$$\dot{C}_{\mathrm{stab}}^{(\mathrm{proxy})}(\theta;t) := -\frac{d}{dt}\,\mathrm{Var}_{\rho(t)}(\sigma_\theta), \qquad \mathrm{Var}_{\rho(t)}(\sigma_\theta) = \langle \sigma_\theta^2 \rangle_{\rho(t)} - \langle \sigma_\theta \rangle_{\rho(t)}^2. \tag{A6}$$

We then define its time-window average as

$$\overline{\dot{C}}_{\mathrm{stab}}(\theta;\tau) := \frac{1}{\tau}\int_{t_0}^{t_0+\tau} \dot{C}_{\mathrm{stab}}^{(\mathrm{proxy})}(\theta;t)\,dt. \tag{A7}$$

This proxy is not claimed to be unique or exact; it is intended only to capture, in a transparent and computable manner, the rate at which the alternatives associated with $\Pi(\theta)$ lose operational distinguishability.

(ii) Energetic cost from the thermal bath.

Defining the bath-induced part of the dynamics as

$$\dot{\rho}_{\mathrm{bath}} := \gamma(n+1)\,\mathcal{D}[\sigma_-]\rho + \gamma n\,\mathcal{D}[\sigma_+]\rho, \tag{A8}$$

we define the heat flow into the environment as

$$\dot{Q}_{\mathrm{out}}(t) := -\mathrm{Tr}(H\,\dot{\rho}_{\mathrm{bath}}(t)). \tag{A9}$$

As the energetic contribution enters the IEF as a cost, we use the time-window average of the magnitude of the heat flow:

$$\overline{\left|\dot{Q}_{\mathrm{out}}\right|}(\theta;\tau) := \frac{1}{\tau}\int_{t_0}^{t_0+\tau} \left|\dot{Q}_{\mathrm{out}}(t)\right| \mathrm{d}t. \tag{A10}$$

This should be understood as a simplified energetic proxy appropriate to the present minimal model; a more complete treatment would separately track the sign of the heat current and any energetic overhead associated with the monitoring channel itself.

(iii) Broadcast-information proxy.

To represent the environment-side gain term, we adopt a simple proxy for the broadcast rate associated with the monitoring channel. In this minimal model, the monitored observable σ_θ continuously imprints information into the environment at a rate controlled by the monitoring strength k. We therefore use the variance of σ_θ as an operational proxy for the rate at which information about the candidate structure $\Pi(\theta)$ becomes accessible,

$$\dot{I}_{\mathrm{brd}}^{(\delta),(\mathrm{proxy})}(\theta;t) = \frac{Mk}{\ln 2}\,\mathrm{Var}_{\rho(t)}(\sigma_\theta) = \frac{Mk}{\ln 2}\Big(1 - \langle \sigma_\theta \rangle_{\rho(t)}^2\Big), \tag{A11}$$

and define its time-window average as

$$\bar{\dot{I}}^{(\delta)}_{\mathrm{brd}}(\theta;\tau) := \frac{1}{\tau}\int_{t_0}^{t_0+\tau} \dot{I}^{(\delta),(\mathrm{proxy})}_{\mathrm{brd}}(\theta;t)\,\mathrm{d}t. \tag{A12}$$

This expression is not intended as a literal evaluation of the full thresholded broadcast quantity used in the main text. Rather, it serves as an operationally transparent surrogate that is sufficient to capture qualitative trends and structural transitions in the present simulation setting.

Appendix A.3. IEF Landscape and Pointer-Structure Reorientation

For each $\theta \in [0, \pi/2]$, we construct a time-window averaged Informational Economy Functional,

$$\bar{J}(\theta;\tau) = \bar{\dot{C}}_{\mathrm{stab}}(\theta;\tau) + \lambda_1 \overline{|\dot{Q}_{\mathrm{out}}|}(\theta;\tau) - \lambda_2 \bar{\dot{I}}^{(\delta)}_{\mathrm{brd}}(\theta;\tau), \tag{A13}$$

and define the optimal pointer structure as

$$\theta^*(\lambda_1) = \arg\min_{\theta\in[0,\pi/2]} \bar{J}(\theta;\tau). \tag{A14}$$

Reproducible parameter set (example).

In this appendix, the symbols $\bar{\dot{C}}_{\mathrm{stab}}$ and $\bar{\dot{I}}^{(\delta)}_{\mathrm{brd}}$ should therefore be understood as model-dependent surrogates for the corresponding terms in the main-text IEF, rather than exact microscopic evaluations of those quantities.

We use

$$\omega = 3, \quad k = 0.25, \quad \gamma = 1, \quad n = 0.1, \quad M = 10, \quad \tau = 2, \quad dt = 0.02, \qquad \lambda_2 = 4.0. \tag{A15}$$

Observed numerical behavior (illustrative).

For the parameter set in Equation (A15) and the initial state (A5), one observes that the optimal structure satisfies $\theta^* \approx \pi/2$ for $\lambda_1 = 0, 1, 5, 20, 60, 70$, while for sufficiently large energetic cost (e.g., $\lambda_1 \gtrsim 80$) the minimum shifts to $\theta^* \approx 0$, as illustrated in Figure A1.

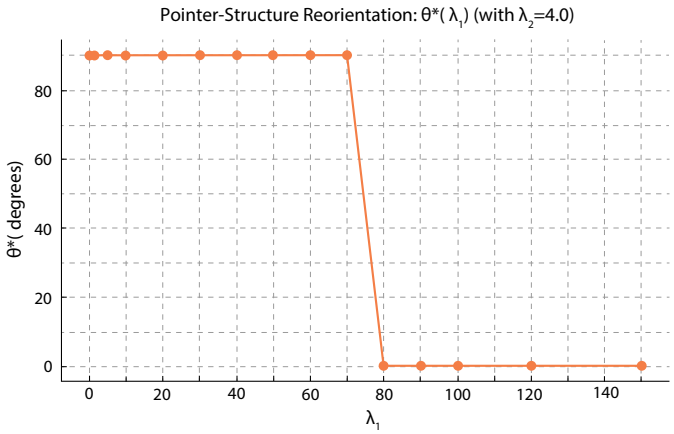


Figure A1. Illustrative pointer-structure reorientation in the simulation model. Shown is the optimal readout angle $\theta^*(\lambda_1)$ minimizing the time-averaged Informational Economy Functional $\bar{J}(\theta;\tau)$ for the parameter set specified in Appendix A. As the energetic price λ_1 is increased, the selected structure undergoes a sharp reorientation from $\theta^* \approx \pi/2$ to $\theta^* \approx 0$. This behavior demonstrates the existence of regime-dependent pointer selection within the IEF framework; the sharpness of the transition is model- and proxy-dependent and is not claimed to be universal.

This produces the characteristic "plateau + jump" profile discussed in the main text. While the precise location of the crossover depends on initialization, averaging window, and proxy choice, the existence of a sharp reorientation of the preferred structure under increasing energetic cost is generic within this model.

We emphasize that the purpose of this simulation model is not to predict quantitative thresholds, nor to provide a full microscopic realization of the main-text broadcast and stability measures, but to demonstrate the qualitative existence of an IEF-driven reorganization of preferred classical structures under competing stability, energetic, and broadcast tendencies.

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