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A Scalar Particle Under Effects of a Magnetic Field Induced by the Lorentz Symmetry Violation

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Abstract

We investigate the effects of Lorentz symmetry violation (LSV) on a scalar particle via a non-minimal coupling in the Klein–Gordon equation within the charge–parity–time CPT-odd gauge sector. Through an analytical approach, we derive bound-state solutions for two distinct anisotropic backgrounds: time-like and space-like. In the time-like case, the LSV induces an effective centrifugal potential, modifying the angular momentum spectrum. When a hard-wall confining potential is included, discrete energy levels emerge, explicitly dependent on the LSV parameters. In the space-like scenario, the particle becomes confined by a Coulomb-type potential induced by the LSV, leading to a quantized energy spectrum that reduces to the free-particle limit when the LSV parameters vanish. Our results illustrate how spacetime anisotropies, encoded in a background vector field, can significantly alter the quantum dynamics of scalar particles in the presence of a magnetic field.

Keywords: Lorentz symmetry violation; scalar particle; magnetic field induced; solutions of bound states

1. Introduction

The Standard Model of particle physics (SM), established in the latter half of the XX century, provides a robust theoretical framework for describing fundamental particles and their interactions. It successfully unifies three of the four known fundamental forces—electromagnetic, weak, and strong nuclear interactions—within a quantum field theory consistent with special relativity [1,2]. However, it does not incorporate gravity, and despite its remarkable empirical success, several experimental anomalies and theoretical limitations have prompted searches for physics beyond the SM. Among the most studied extensions is the possibility of Lorentz symmetry violation (LSV), which may arise in theories attempting to unify gravity with quantum mechanics, such as string theory or non-commutative geometry. In this context, the exploration of LSV effects in quantum systems has become a fertile ground for probing fundamental physics at energy scales currently inaccessible to direct experimental investigation [3].

Parameters used to adjust the SM and some theoretical predictions about fundamental interactions have become targets of questioning. One example is the potential variation of the fine-structure constant [4,5]; recall that this constant is expressed in terms of three other fundamental constants: the elementary charge of the electron, Planck’s constant, and the speed of light in vacuum. Which of these might actually be changing? In addition,



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an anomaly observed in the ^8Be nuclear transition has been reported that could point to a fifth fundamental interaction [6]. There is also the controversy that the proton radius may differ between measurements with muonic hydrogen [7] and measurements with electron scattering [8]. Here are some questions that lead to the search for more precise experiments and/or theoretical models capable of explaining these discrepancies or anomalies.

Accordingly, various theoretical extensions have been proposed to address discrepancies between SM predictions and experimental observations. Representative examples include noncommutative field theories [9], supersymmetry [10], and frameworks incorporating LSV [11]. Over the past decades, LSV has provided a principal conceptual setting for the development of the so-called Extended Standard Model. As indicated by its designation, this framework consists of an extension of the SM in which non-minimal coupling terms are introduced into the SM Lagrangian density.

The concept of LSV first emerges prominently in the context of string theory, where efforts to formulate a quantum field-theoretical description that consistently includes gravity lead to spontaneous symmetry breaking [12]. This breaking is mediated by background fields of tensorial character, i.e., tensor fields that select preferred spacetime directions and thereby induce anisotropies. Subsequently, the study of LSV becomes even more relevant through its incorporation into generalized Chern–Simons-type theories, which are then confronted with observational data, thereby enabling the derivation of constraints on the parameters associated with Lorentz symmetry breaking [13].

Since then, the LSV has been the subject of study across all sectors of SM [14] to address existing gaps in its understanding. Despite being a potent theoretical tool, there have been no signs of its existence. This is because its existence lies at the Planck energy scale, a scale still too remote for current fundamental physics laboratories. Even so, LSV has been a prominent theoretical backdrop due to its ramifications in fundamental sectors (photonic, fermionic, bosonic, flavor, color, and others) [15,16].

In the same way that LSV has been a theoretical extension to try to estimate or adjust the anomalies of the electric [17] and magnetic [18] dipoles of the electron, LSV can be adopted to investigate the magnetic anomaly of the muon, since there is a discrepancy between the theoretical value given by the SM and the experimental result [19].

Anisotropic effects have been investigated in relativistic quantum systems characterized by a background field that violates Lorentz symmetry. These violation terms are inserted into wave equations via non-minimal coupling [11,20]. There are studies involving the Dirac equation, such as geometric phases [21–23], Landau-type quantization [24,25] and on Dirac oscillator [26]. In the case of scalar particles, there are studies on central potentials [27], Landau quantization [28,29], solutions of bound states [30] and quantum scattering [31]. It is widely accepted that the surfaces of compact astrophysical objects, such as neutron stars and white dwarfs, can sustain extremely intense magnetic fields, on the order of 10^{12} G, which significantly alter atomic structure and, consequently, atomic properties [32,33]. Furthermore, these stellar magnetic fields are sufficiently strong to deflect the trajectories of high-energy charged particles (cosmic rays). As a consequence, observational data indicate that the spatial distribution of these particles exhibits anisotropies, i.e., they tend to arrive preferentially from specific directions [34], rather than being uniformly distributed as is expected in an isotropic scenario. Motivated by these possible magnetic effects on relativistic particles, we undertake in this paper the study of a scalar particle subjected to a spatially varying magnetic field generated by a background that explicitly violates Lorentz symmetry, characterized by the presence of a privileged direction, either time-like or space-like.

The structure of this paper is as follows. In Section 2, we present the Klein–Gordon equation modified through non-minimal coupling, which incorporates the presence of

a background field that characterizes the LSV; in the Section 3, we introduce a possible electromagnetic field configuration capable of inducing a variable magnetic field that decreases linearly with the axial coordinate, which can define a time-like (Section 4) or space-like anisotropy (Section 5). In Section 6, the conclusions are given.

2. Modified Klein–Gordon Equation

The theoretical foundation of the SM rests crucially on three discrete symmetries: parity (P), time reversal (T), and charge conjugation (C). These symmetries combine to form CPT symmetry, a fundamental theorem in quantum field theory that ensures invariance under the combined transformation of charge conjugation, parity inversion, and time reversal [1]. Within the framework of the Standard Model Extension (SME), these symmetries may be preserved or violated depending on the chosen background fields. It is important to note that LSV does not necessarily imply CPT violation; however, the converse holds: any violation of CPT symmetry necessarily implies Lorentz violation [11].

In the photonic sector, there are two LSV terms, one of which violates CPT symmetry and the other does not. This characteristic defines the first one as an CPT-odd term [15,16], while the second one is known as a CPT-even term [13,35]. In particular, CPT-odd terms have been investigated on electromagnetic propagation in continuous matter [36], in cold plasma modes [37] and on chiral parameters with pulsar timing [38]. While CPT-odd terms have been extensively studied in the photonic sector, their effects on scalar fields remain less explored, particularly in curved or anisotropic backgrounds. To bridge this gap, we consider a scalar particle subject to a CPT-odd Lorentz-violating coupling in a static cylindrical spacetime. This geometry is chosen for two key reasons: first, it naturally accommodates the axial symmetry of the magnetic field configuration we will introduce; second, it allows the background vector field which encodes the Lorentz violation—to select privileged directions (time-like or space-like) in a mathematically tractable setting. Within this framework, the modified Klein–Gordon equation incorporating the non-minimal coupling can be solved analytically, revealing how Lorentz violation influences bound states and energy spectra.

Therefore, let us consider the line element with the following symmetry (with the speed of light and the reduced Planck constant set to unity: $c = \hbar = 1$)

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 d\varphi^2 + dz^2, \quad (1)$$

with $0 \leq \rho = \sqrt{x^2 + y^2} < \infty$ being the axial coordinate, $0 \leq \varphi \leq 2\pi$ the azimuthal coordinate, $-\infty < z < \infty$ the z -axis perpendicular to the xy plane and t is the time variable. Since we are dealing with a relativistic scalar particle, we must adopt the Klein–Gordon equation

$$(\square - m^2)\phi = 0, \quad (2)$$

where $\square = \partial_\mu \partial^\mu = -\partial_t^2 + \nabla^2$, with the Greek letter indices taken the values 0 (for time) and 1, 2, and 3 (for spatial components), $\partial_\mu \equiv \partial/\partial x_\mu$ and ϕ is the scalar field associated to the relativistic particle with mass m .

In Ref. [13], Sean Carroll, George Field, and Roman Jackiw proposed a non-minimal Chern–Simons type coupling, which carries in its structure a background vector field that violates CPT symmetry and, consequently, violates Lorentz symmetry. Inspired by this coupling, authors modified this term adopting the dual electromagnetic tensor in order to investigate the consequences of the inversion of electric and magnetic fields in the mathematical structure, something pertinent from a theoretical point of view. For example, there are studies of this type of non-minimal coupling in analysis of the hydrogen atom [21], and on the induction of geometric phases [11]. Based on Ref. [20], one of the authors of

this paper and Humberto Belich [29,39,40] proposed the non-minimal coupling into Klein–Gordon equation with a background vector field that governs LSV, $v_\mu: \partial_\mu \rightarrow \partial_\mu - ig\tilde{F}_{\mu\nu}v^\nu$, where g is a coupling constant, $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$, with $\epsilon_{\mu\nu\alpha\beta}$ the Levi-Civita symbol, is the dual electromagnetic tensor with $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$. Hence, the Klein–Gordon equation, with this coupling, becomes

$$(\partial_\mu - ig\tilde{F}_{\mu\alpha}v^\alpha)(\partial^\mu - ig\tilde{F}^{\mu\beta}v_\beta)\phi - m^2\phi = 0 \quad (3)$$

which can be rewritten in a more explicit form

$$\square\phi - 2ig(\tilde{F}_\alpha^\mu v^\alpha)\partial_\mu\phi - ig(\partial_\mu\tilde{F}^{\mu\alpha})v_\alpha\phi - g^2(\tilde{F}_{\mu\alpha}v^\alpha)(\tilde{F}_\beta^\mu v^\beta)\phi - m^2\phi = 0. \quad (4)$$

The third term of Equation (4) represents Maxwell’s homogeneous equations. Therefore, Equation (4) becomes

$$\square\phi - 2ig(\tilde{F}_\alpha^\mu v^\alpha)\partial_\mu\phi - g^2(\tilde{F}_{\mu\alpha}v^\alpha)(\tilde{F}_\beta^\mu v^\beta)\phi - m^2\phi = 0. \quad (5)$$

We can take Equation (5) further in order to explicitly express it in terms of magnetic ($\vec{B}$) and electric ($\vec{E}$) fields, that is,

$$\square\phi + 2ig(\vec{v} \cdot \vec{B})\partial_0\phi + 2igv_0(\vec{B} \cdot \vec{\nabla})\phi - 2ig(\vec{v} \times \vec{E}) \cdot \vec{\nabla}\phi + 2gv_0\vec{v} \cdot (\vec{E} \times \vec{B}) + g^2v_0^2B^2\phi - g^2(\vec{v} \cdot \vec{B})^2\phi - g^2[(v_2^2 + v_3^2)E_1^2 + (v_1^2 + v_3^2)E_2^2 + (v_1^2 + v_2^2)E_3^2]\phi - m^2\phi = 0. \quad (6)$$

Equation (6) is the Klein–Gordon equation modified by the non-minimal coupling of odd-CPT nature LSV. We can now investigate some possible theoretical LSV scenarios for numerous background and electromagnetic field configurations. It should be noted that the vector background field is considered a fixed and non-dynamic external field. Therefore, it has no degrees of freedom.

Observational astrophysical data, such as vacuum birefringence in light from distant galaxies, dictate that the components of v_μ are negligible. There is a current upper limit to the magnitude of this background vector field, which is on the order of $|v^\mu| < 10^{-42}$ GeV [41,42] or even more restrictive, reaching 10^{-45} GeV [14,43] in some analyses of cosmic microwave background radiation.

3. Variable Magnetic Field Induced by LSV

Equation (6) provides us with numerous and variable possible scenarios for breaking the Lorentz symmetry on a scalar particle. These numerous anisotropic backgrounds are established by combinations of the background vector and electromagnetic fields. In this sense, let us establish the following electromagnetic field configuration:

$$\vec{E} = 0; \quad \vec{B}(\rho) = \frac{\mu I}{2\pi\rho}\hat{\phi} = \frac{\lambda}{\rho}\hat{\phi}, \quad (7)$$

where $\lambda = \mu I/(2\pi)$, with μ being the magnetic permeability of the medium, $\hat{\phi}$ is the azimuthal unit vector and I is the electric current. We chose this configuration because it is simpler from an analytical point of view, but also because it is known that in our Universe there are several scenarios of variable and intense magnetic fields. For example, as mentioned above on surface of compact cosmic objects, there may be strong magnetic fields on the order of 10^{12} G, which would radically alter the atomic structure and, consequently, its atomic properties [32,33]. Furthermore, magnetars, one of the most magnetic objects in the Universe due to their gigantic magnetic energy, are capable of warping the surrounding space-time, creating super-anisotropic and energetic environments [44]. Although the

magnetic field given in Equation (7) represents a field generated by an infinite wire, it can be seen as a generic and variable magnetic field, which decreases with the axial distance. In this case, Equation (6) becomes

$$\square\phi + 2ig(\vec{v} \cdot \vec{B})\partial_0\phi + 2igv_0(\vec{B} \cdot \vec{\nabla})\phi + g^2v_0^2B^2\phi - g^2(\vec{v} \cdot \vec{B})^2\phi - m^2\phi = 0. \quad (8)$$

Equation (8) describes the quantum dynamics of a scalar particle under the effects of a magnetic field induced by the LSV, taking into account the general background field, that is, time-like plus space-like. We will analyze these characteristics separately for the electromagnetic configuration given in Equation (7).

4. Time-like Anisotropy

In this Section, we consider a scalar particle under the effects of a magnetic field induced by the LSV in a background characterized by time-like anisotropy (TA). This TA is characterized by the following background vector field configuration:

$$v_\mu = (-v_0, \vec{0}); \quad v_0 = a, \quad (9)$$

where $a = \text{const.}$ This nomenclature is common for aether-like LSV fields [45–50]. From a physical point of view, Lorentz covariance is violated; however, spatial isotropy is maintained, meaning that space still appears the same in all directions. In particular, in Ref. [21], through corrections to the spectrum of the hydrogen atom, an estimate is given for the product ga , that is, $ga \leq 10^{-20}(\text{eV})^{-1}$. Hence, for the line element given in Equation (1) and by substituting Equations (7) and (9) into Equation (8), we obtain

$$-\frac{\partial^2\phi}{\partial t^2} + \frac{\partial^2\phi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial\phi}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2} + \frac{2ig\lambda a}{\rho^2}\frac{\partial\phi}{\partial\varphi} + \frac{g^2a^2\lambda^2}{\rho^2}\phi - m^2\phi = 0. \quad (10)$$

4.1. Induced Centrifugal Potential

To solve Equation (10), let us consider the solution

$$\phi(\rho, \varphi, z, t) = R(\rho)e^{-i(\mathcal{E}t - l\varphi - kz)}, \quad (11)$$

which $\mathcal{E}, l = 0, \pm 1, \pm 2, \dots$ and $-\infty < k < \infty$ represent the eigenvalues of energy, angular momentum, and linear momentum in the z -direction, respectively. Therefore, by substituting Equation (11) into Equation (10), we obtain the axial wave equation

$$\frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} - \frac{l_g^2}{\rho^2}R + \alpha^2R = 0, \quad (12)$$

where we define the parameters

$$\alpha^2 = E^2 - m^2 - k^2 \quad \text{and} \quad l_g^2 = l^2 + 2gla\lambda - g^2a^2\lambda^2. \quad (13)$$

Equation (12) is Bessel differential equation [51] and its solution is given in terms of Bessel and Neumann function, $J_{|l_g|}(\alpha\rho)$ and $N_{|l_g|}(\alpha\rho)$, respectively:

$$R(\rho) = C_1J_{|l_g|}(\alpha\rho) + C_2N_{|l_g|}(\alpha\rho), \quad (14)$$

where C_1 and C_2 are constants. It is worth remembering that the Neumann function diverges at the origin, which goes against the necessary conditions for the existence and physical validity of a wave function. In this sense, we must impose that $C_2 = 0$. Therefore, the general and physically possible solution for Equation (12) is given in the form

$$R(\rho) = C_1 J_{|l_g|}(\alpha\rho). \quad (15)$$

Equation (15) represents the axial wave function of a free scalar particle under effects of a magnetic field induced by LSV in an environment with TA. One can note that there are influences of the background on the relativistic particle. This feature can be seen through the induction of a centrifugal potential in the structure of the axial wave equation, that is,

$$V(\rho)_{\text{eff}} = V(\rho)_c + V(\rho)_{\text{ind}}; \quad V(\rho)_c = \frac{l^2}{\rho^2}, \quad V(\rho)_{\text{ind}} = \frac{2g\lambda la - g^2 a^2 \lambda^2}{\rho^2}, \quad (16)$$

which $V(\rho)_c$ is the centrifugal potential and $V(\rho)_{\text{ind}}$ is the induced potential by LSV. Consequently, the angular momentum values undergo somewhat correction due to TA, as defined in Equation (13). By setting $a \rightarrow 0$ or $\lambda \rightarrow 0$, one recovers the relativistic quantum dynamics of a free scalar particle in Minkowski spacetime, that is, a homogeneous medium. One can see that $|l_g| = \sqrt{l^2 + 2g\lambda la - g^2 a^2 \lambda^2} > 0$. To ensure the positive sign of l_g , one has to study the sign of the inequality, which gives $l > 0.414ag\lambda$ and $l < -2.414ag\lambda$.

4.2. Hard-Wall Potential

Discontinuous potentials are used in academic studies for the mathematical understanding of quantum wave equations, especially in the non-relativistic case, in the study of the Schrödinger equation [52,53]. In this case, one of the most important discontinuous potentials is the one-dimensional infinite potential well due to the obtaining of energy levels [52]. In three dimensions, this potential is also investigated in spherical symmetry, known as a spherical box [53], where energy levels are also obtained. This type of potential has been investigated in symmetries and non-trivial backgrounds in more complex quantum systems; for example, in cylindrical symmetry, the cylindrical box, also known as the hard-wall potential (HP), has been investigated in relativistic quantum particles in non-inertial frames of reference [54–56].

The HP is mathematically defined by a condition

$$R(\alpha\rho_0) = 0, \quad (17)$$

where $\rho_0 = \text{const}$. Furthermore, let us consider the particular case $\alpha\rho_0 \gg 1$. In this case, the Bessel function can be rewritten in the following form [57]:

$$J(\alpha\rho) \sim \sqrt{\frac{2}{\pi\alpha\rho_0}} \cos\left(\alpha\rho_0 - \frac{|l_g|\pi}{2} - \frac{\pi}{4}\right). \quad (18)$$

Then, by substituting the Bessel function (18) into Equation (15), one obtains from the boundary condition (17) that

$$\mathcal{E}_{l,n} = \pm \sqrt{m^2 + k^2 + \frac{\pi^2}{\rho_0^2} \left(n + \frac{|l_g|}{2} + \frac{3}{4}\right)^2}. \quad (19)$$

Equation (19) gives us the energy levels of a relativistic scalar particle under the effects of a high velocity (HP) plus a magnetic field induced by LSV in a TA background. Let us note that the energy profile of the relativistic system is influenced by the LSV through the redefinition of angular momentum in terms of the parameters a and λ . By taking $a \rightarrow 0$ or $g \rightarrow 0$ into Equation (19) we recover the relativistic energy levels of a scalar particle are recovered under effects of HP in Minkowski spacetime.

5. Space-like Anisotropy

In this Section, we consider the following background vector field configuration:

$$v_\mu = (0, \vec{v}), \quad \vec{v} = b\hat{\phi}, \quad (20)$$

where $b = \text{const.}$ In this configuration, it is noted that there is no temporal part, but only the spatial part: the azimuthal angular part. That is, it is a configuration that characterizes anisotropy of a spatial nature, so space-like anisotropy (SA). There are two reasons to choosing this configuration. The first is to facilitate the analytical execution of the problem. The second is the induction of a Coulomb-type potential. In addition to these two reasons, this feature breaks spatial isotropy [13]. Based on analyses of cosmic microwave background radiation, the estimated limit for $|\vec{v}|$ is $|\vec{v}| < 10^{-45} \text{ GeV}$ [43,44].

Induced Coulomb-Type Potential

Therefore, by substituting Equations (7) and (20) into Equation (8), and by considering Equation (11), one has

$$\frac{d^2 R}{d\rho^2} + \frac{1}{\rho} \frac{dR}{d\rho} - \frac{l^2}{\rho^2} R + \frac{2gb\lambda\mathcal{E}}{\rho} R - \beta^2 R = 0, \quad (21)$$

where we introduce the new parameters

$$l^2 = l^2 + g^2 b^2 \lambda^2 \quad \text{and} \quad \beta^2 = m^2 + k^2 - \mathcal{E}^2. \quad (22)$$

In Equation (21), an axial wave equation, one can note the presence of a Coulomb-type potential, which is induced by the background vector and electromagnetic field configurations, both originating from the LSV. Let us replace the variable to $\xi = 2b\rho$ in Equation (21):

$$\frac{d^2 R}{d\xi^2} + \frac{1}{\xi} \frac{dR}{d\xi} - \frac{l^2}{\xi^2} R + \frac{\gamma}{\xi} R - \frac{1}{4} R = 0, \quad (23)$$

with

$$\gamma = \frac{gb\lambda\mathcal{E}}{\beta}. \quad (24)$$

By imposing the conditions of continuity, finiteness, and univocity of the wave function through the analysis of the limits $\xi \rightarrow 0$ ($\rho \rightarrow 0$) and $\xi \rightarrow \infty$ ($\rho \rightarrow \infty$), one obtains the general solution

$$R(\xi) = \xi^{|\iota|} e^{-\frac{\xi}{2}} f(\xi), \quad (25)$$

where $f(\xi)$ is so far an unknown function. By substituting Equation (25) into Equation (23), one has

$$\xi \frac{d^2 f}{d\xi^2} + (2|\iota| + 1 - r) \frac{df}{d\xi} + \left(\gamma - |\iota| - \frac{1}{2} \right) f = 0. \quad (26)$$

Equation (26) is confluent hypergeometric equation [57] and its solutions is a power series in a form of the confluent hypergeometric function:

$$f(r) = {}_1F_1 \left(\left| \iota \right| + \frac{1}{2} - \gamma, 2|\iota| + 1; \xi \right). \quad (27)$$

To ensure the conditions for the existence of eigenfunctions, the confluent hypergeometric series has to be transformed into a polynomial. To this end, the series (27) has to be truncated using the condition

$$|\iota| + \frac{1}{2} - \gamma = -n, \quad (28)$$

with $n = 0, 1, 2, \dots$. Hence, from the condition (28), we obtain the expression

$$\mathcal{E}_{l,n} = \pm \sqrt{\frac{m^2 + k^2}{1 + \frac{g^2 b^2 \lambda^2}{(n + |\iota| + \frac{1}{2})^2}}}, \quad (29)$$

which represents the relativistic energy profile of a scalar particle under effects of a magnetic field in a background with SA. It can be observed that SA drastically influenced the system to the point of confining the particle through a Coulomb-type potential, unlike TA. Furthermore, the relativistic energy levels of the particle are influenced by the background due to the presence of the parameters associated with LSV, g and b . By taking $b \rightarrow$ and/or $g \rightarrow 0$, the total energy of a free particle in Minkowski spacetime, $\mathcal{E}_{l,n} \rightarrow \mathcal{E} = \pm \sqrt{m^2 + k^2}$, is recovered.

6. Conclusions

We have investigated a scalar particle under LSV effects through a non-minimal coupling of the CPT-odd gauge sector, which is characterized by the presence of a constant background vector field. After proposing a possible electromagnetic field configuration, through a purely analytical analysis, we determined bound-state solutions for two possible background field scenarios: TA and SA.

In the case of TA, we saw that a centrifugal potential is induced by the LSV, influencing the solution of the axial wave equation, providing eigenfunctions defined in terms of the Bessel function of the first kind rewritten in terms of the parameters associated with Lorentz breaking. Next, we imposed the presence of an HP in order to confine the scalar particle and determine the energy levels of the system, which depend on the parameters governing the LSV.

In the spacetime with SA, the scalar particle becomes confined by a Coulomb-like potential induced by the LSV, which determines the system's energy profile. We note that these energy levels depend on the parameters associated with the LSV.

In all cases analyzed, it is possible to recover the total energy of a free relativistic particle in Minkowski spacetime at the limit of the parameters associated with LSV set to zero, that is, the absence of anisotropies, whether TA or SA.

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