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Correction

Correction: Gorban et al. The Asymmetric Dynamical Casimir Effect. *Physics* 2023, *5*, 398–422

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There was an error in the original paper [1], which occurred in the calculation of the DCE spectrum from the time-dependant perturbations on $\lambda(t)$. Specifically, the matching conditions computed in Ref. [1] were solved incorrectly, when actually these conditions do not appear to be generally solvable by any method known to the authors. A correction is made to Section 2.4 of Ref. [1]. The content following Equation (62) to be replaced with the following two paragraphs:

- From these continuity equations, it becomes understandable that unlike the matching conditions in Equations (18) and (19), general matching conditions for $\lambda(t)$ cannot be found using this approach. This is due to the presence of the convolution integral between $\mathcal{L}(\omega - \omega')$ and $\partial_x \Phi(\omega')$ in Equation (61). This convolution ultimately leads to nonlinear mixing of different frequency terms.
  To illustrate this difficulty straightforwardly, the form of $f(t)$ used in prior Sections (see Equation (34)) was employed in the continuity equations to investigate the resulting scattering coefficients, assuming the preservation of linearity a priori. The result is that the scattering coefficients in the frequency domain become dependent on $\omega \pm \omega_0$ modes ($s_\pm(\omega \pm \omega_0)$, $r_\pm(\omega \pm \omega_0)$). A detailed derivation of these scattering terms can be seen in Appendix A. To that end, work is currently underway to apply the Bogoliubov approach to this problem; however, those results are reserved for a future paper.

Additionally, the following paragraph to be added to the end of Section 2.3 of Ref. [1]:

- There is an important caveat we must address with regard to general scattering. These similarities only hold when the mechanism driving scattering affects the position or some material property related to the plasma frequency. This is because such mechanisms act by causing the strength of the δ function in the potential to become time-dependent. Such considerations do not extend straightforwardly to allowing the strength of the δ' term, which is addressed in Section 2.4 just below.

We have also modified the discussion of λ_0 fluctuations in Section 6 (Conclusions). The second sentence now reads:

- Fluctuations on λ_0 were explored and we discussed obstructions to analyzing linear scattering in this case.

Appendix A is added, as given below, to highlight the difficulties in calculating the time-dependant perturbations on $\lambda(t)$.

Lastly, we have corrected erroneous inclusions of w instead of the appropriate ω in Equations (6)–(9), (18), (19), (22) and (23), as well as the definition of ξ in Section 3.2.2., which now reads $\xi = \gamma_0 \omega_0$.

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Appendix A. $\lambda(t)$ Linear Scattering

Here, we provide a derivation of the scattering terms for $f(t)$ chosen such that the resulting expressions for matching conditions are as simple as possible. This allows us straightforward illustration of the way in which we are obstructed from deriving scattering matrix elements as we did in the rest of this paper.

Starting from Equations (61) and (62),

$$
\begin{aligned}
&-\partial_x\Phi(\omega 0^+) + \partial_x\Phi(\omega, 0^-) + \mu[\Phi(\omega, 0^+) + \Phi(\omega, 0^-)] \\
&- \int \frac{d\omega'}{2\pi}\mathcal{L}(\omega-\omega')(\partial_x\Phi(\omega', 0^+) + \partial_x\Phi(\omega', 0^-)) = 0
\end{aligned}
$$

and

$$
-\Phi(\omega, 0^+) + \Phi(\omega, 0^-) + \int \frac{d\omega'}{2\pi}\mathcal{L}(\omega-\omega')(\Phi(\omega', 0^+) + \Phi(\omega', 0^-)) = 0,
$$

it becomes seen that a general form of the matching conditions cannot be derived due to convolution Fourier transforms. To demonstrate the difficulty these integrals provide for the matching conditions, we take specific form $\lambda(t) = \lambda_0[1 + \epsilon f(\omega_0 t, |t|/\tau)]$, where f is assumed for now to have the same type of functional dependence found in Equation (34). We note, though, that we do not specify an explicit functional definition for f. Instead, making the general assumption that in the limit where $\tau \to \infty$, one has a "monochromatic-like" limit where its Fourier transform satisfies

$$
\lim_{\tau\to\infty}\mathcal{F}(\omega) = b[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)], \tag{A1}
$$

where b is some normalization constant for the Dirac delta distributions. Using this, one then has the Fourier transform of $\lambda(t)$ as

$$
\mathcal{L}(\omega) = \lambda_0[\delta(\omega) + \epsilon\mathcal{F}(\omega)], \tag{A2}
$$

where in what follows we assume we already computed the limit on τ whenever evaluating integrals.

Substituting $\mathcal{L}(\omega-\omega')$ into Equations (61) and (62), one has:

$$
\begin{aligned}
&-\partial_x\Phi(\omega 0^+) + \partial_x\Phi(\omega, 0^-) + \mu[\Phi(\omega, 0^+) + \Phi(\omega, 0^-)] \\
-\frac{\lambda_0}{2\pi}[\partial_x\Phi(\omega, 0^+) + \partial_x\Phi(\omega, 0^-)] &- \lambda_0\epsilon\int \frac{d\omega'}{2\pi}\mathcal{F}(\omega-\omega')(\partial_x\Phi(\omega', 0^+) + \partial_x\Phi(\omega', 0^-)) = 0
\end{aligned} \tag{A3}
$$

and

$$
\begin{aligned}
&-\Phi(\omega, 0^+) + \Phi(\omega, 0^-) + \frac{\lambda_0}{2\pi}(\Phi(\omega, 0^+) + \Phi(\omega, 0^-)) \\
&+\lambda_0\epsilon\int \frac{d\omega'}{2\pi}\mathcal{F}(\omega-\omega')(\Phi(\omega', 0^+) + \Phi(\omega', 0^-)) = 0.
\end{aligned} \tag{A4}
$$

Now, explicitly evaluating these integrals under the above limits and assumptions, one obtains:

$$
\begin{aligned}
&-\partial_x\Phi(\omega 0^+) + \partial_x\Phi(\omega, 0^-) + \mu[\Phi(\omega, 0^+) + \Phi(\omega, 0^-)] - \frac{\lambda_0}{2\pi}[\partial_x\Phi(\omega, 0^+) + \partial_x\Phi(\omega, 0^-)] \\
&-\frac{\lambda_0\epsilon b}{2\pi}[\partial_x\Phi(\omega-\omega_0, 0^+) + \partial_x\Phi(\omega-\omega_0, 0^-) + \partial_x\Phi(\omega+\omega_0, 0^+) + \partial_x\Phi(\omega+\omega_0, 0^-)] = 0
\end{aligned} \tag{A5}
$$

and

$$
\begin{aligned}
-\Phi(\omega,0^+) + \Phi(\omega,0^-) + \frac{\lambda_0}{2\pi}(\Phi(\omega,0^+) + \Phi(\omega,0^-))\\
+\frac{\lambda_0 \epsilon b}{2\pi}[\Phi(\omega-\omega_0,0^+) + \Phi(\omega-\omega_0,0^-) + \Phi(\omega+\omega_0,0^+) + \Phi(\omega+\omega_0,0^-)] = 0.
\end{aligned}
\tag{A6}
$$

Next, we further assume that the ingoing and outgoing fields are linearly related as before, giving

$$\Phi_+(\omega,x) = s_-(\omega)e^{-i\omega x}\Theta(-x) + (e^{-i\omega x} + r_-(\omega)e^{i\omega x})\Theta(x)$$

and

$$\Phi_-(\omega,x) = (e^{i\omega x} + r_+(\omega)e^{-i\omega x})\Theta(-x) + s_+(\omega)e^{i\omega x}\Theta(x).$$

Now, Equations (A5) and (A6) can be re-expressed explicitly in terms of transmission and reflection coefficients, offering

Φ_+:

$$
\begin{aligned}
-i\omega(1+\frac{\lambda_0}{2\pi})(r_-(\omega)-1) - i\omega(1-\frac{\lambda_0}{2\pi})s_-(\omega)\\
+\mu[1+r_-(\omega)+s_-(\omega)] = \frac{\lambda_0\epsilon b}{2\pi}[i(\omega-\omega_0)(r_-(\omega-\omega_0)-1)\\
+i(\omega-\omega_0)s_-(\omega-\omega_0) + i(\omega+\omega_0)(r_-(\omega+\omega_0)-1) + i(\omega+\omega_0)s_-(\omega+\omega_0)],
\end{aligned}
\tag{A7}
$$

and

$$
\begin{aligned}
(\frac{\lambda_0}{2\pi}-1)(1+r_-(\omega)) + (\frac{\lambda_0}{2\pi}+1)s_-(\omega)\\
= -\frac{\lambda_0\epsilon b}{2\pi}[2+r_-(\omega-\omega_0)+s_-(\omega-\omega_0)+r_-(\omega+\omega_0)+s_-(\omega+\omega_0)].
\end{aligned}
\tag{A8}
$$

Φ_-:

$$
\begin{aligned}
-i\omega(1+\frac{\lambda_0}{2\pi})s_+(\omega) + i\omega(1-\frac{\lambda_0}{2\pi})(1-r_+(\omega)) + \mu[1+s_+(\omega)+r_+(\omega)]\\
= \frac{\lambda_0\epsilon b}{2\pi}[i(\omega-\omega_0)s_+(\omega-\omega_0) + i(\omega-\omega_0)(1-r_+(\omega-\omega_0))\\
+i(\omega+\omega_0)s_+(\omega+\omega_0) + i(\omega+\omega_0)(1-r_+(\omega+\omega_0))],
\end{aligned}
\tag{A9}
$$

and

$$
\begin{aligned}
(\frac{\lambda_0}{2\pi}-1)s_+(\omega) + (\frac{\lambda_0}{2\pi}+1)(1+r_+(\omega))\\
= -\frac{\lambda_0\epsilon b}{2\pi}[2+s_+(\omega-\omega_0)+r_+(\omega-\omega_0)+s_+(\omega+\omega_0)+r_+(\omega+\omega_0)].
\end{aligned}
\tag{A10}
$$

Equations (A7)–(A10) provide four coupled equations, with 12 unknown terms: four scattering terms for each frequency argument appearing (ω, $\omega \pm \omega_0$). Therefore, there are not enough constraints on the fields to produce a definitive solution to the $\lambda(t)$ perturbation for the (1 + 1)D mirror in this scattering approach. The authors are not aware of any technique within this linear scattering framework that would allow for one to solve problems of this type. Additionally, this result seems to suggest that there may be some general obstruction that prevents this type of linear scattering framework from solution when the potential contains a δ' potential with time-dependent strength. This is because potentials in this form typically couple different frequencies together in a way that prevents the matching conditions from being solvable. The authors are still optimistic than an approach based upon Bogoliubov transformations may be more successful, but such an approach requires substantial development which is reserved for future work.

Addition of Authors

Patrick M. Brown and Jacob A. Matulevich were not included as authors in the original paper [1]. The corrected Author Contributions statement to read as appears here.

Author Contributions: Conceptualization, M.J.G.; methodology, M.J.G.; software, M.J.G. and W.D.J.; validation, M.J.G., W.D.J., P.M.B. and J.A.M.; formal analysis, M.J.G., W.D.J., P.M.B. and J.A.M.; investigation, M.J.G. and W.D.J.; resources, G.B.C.; data curation, M.J.G. and W.D.J.; writing—original draft preparation, M.J.G. and W.D.J.; writing—review and editing, P.M.B., J.A.M. and G.B.C.; supervision, G.B.C.; project administration, G.B.C. All authors have read and agreed to the published version of the manuscript.

Reference

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