

Article

Cosmology of a Polynomial Model for de Sitter Gauge Theory Sourced by a Fluid

Jia-An Lu ^{1,2}

¹ School of Data and Computer Science, Guangdong Peizheng College, Guangzhou 510830, China; ljagdgz@163.com

² School of Physics, Sun Yat-sen University, Guangzhou 510275, China

Abstract: In the de Sitter gauge theory (DGT), the fundamental variables are the de Sitter (dS) connection and the gravitational Higgs/Goldstone field ζ^A , where A is a 5 dimensional index. Previously, a model for DGT was analyzed, which generalizes the MacDowell–Mansouri gravity to have a variable cosmological constant, $\Lambda = 3/l^2$, where l is related to ζ^A by $\zeta^A \zeta_A = l^2$. It was shown that the model sourced by a perfect fluid does not support a radiation epoch and the accelerated expansion of the parity invariant universe. In this paper, I consider a similar model, namely, the Stelle–West gravity, and couple it to a modified perfect fluid, such that the total Lagrangian 4-form is polynomial in the gravitational variables. The Lagrangian of the modified fluid has a nontrivial variational derivative with respect to l , and as a result, the problems encountered in the previous study no longer appear. Moreover, to explore the elegance of the general theory, as well as to write down the basic framework, I perform the Lagrange–Noether analysis for DGT sourced by a matter field, yielding the field equations and the identities with respect to the symmetries of the system. The resulted formula are dS covariant and do not rely on the existence of the metric field.

Keywords: Stelle–West gravity; gauge theory of gravity; cosmic acceleration



Citation: Lu, J.-A. Cosmology of a Polynomial Model for de Sitter Gauge Theory Sourced by a Fluid. *Physics* **2022**, *4*, 1168–1179. <https://doi.org/10.3390/physics4040076>

Received: 9 July 2022

Accepted: 15 September 2022

Published: 2 October 2022

Publisher’s Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The gauge theories of gravity (GTG) aims at treating gravity as a gauge field, in particular, constructing a Yang–Mills-type Lagrangian, which reduces to general relativity (GR) in some limiting cases, while providing some novel falsifiable predictions. A well-founded subclass of GTG is the Poincaré gauge theory (PGT) [1–5], in which the gravitational field consists of the Lorentz connection and the co-tetrad field. Moreover, the PGT can be reformulated as de Sitter gauge theory (DGT), in which the Lorentz connection and the co-tetrad field are united into a de Sitter (dS) connection [6,7]. In fact, before the idea of DGT is realized, a related Yang–Mills-type Lagrangian for gravity was proposed by MacDowell and Mansouri (MM) [8], and reformulated into a dS-invariant form by West [9], which reads:

$$\begin{aligned} \mathcal{L}^{\text{MM}} &= \epsilon_{ABCDE} \zeta^E \mathcal{F}^{AB} \wedge \mathcal{F}^{CD} \\ &= \epsilon_{\alpha\beta\gamma\delta} (l R^{\alpha\beta} \wedge R^{\gamma\delta} - 2l^{-1} R^{\alpha\beta} \wedge e^\gamma \wedge e^\delta + l^{-3} e^\alpha \wedge e^\beta \wedge e^\gamma \wedge e^\delta), \end{aligned} \quad (1)$$

where ϵ_{ABCDE} and $\epsilon_{\alpha\beta\gamma\delta}$ are the 5-dimensional (5d) and 4d Levi–Civita symbols, ζ^A is a dS vector constrained by $\zeta^A \zeta_A = l^2$, l is a positive constant, \mathcal{F}^{AB} is the dS curvature, $R^{\alpha\beta}$ is the Lorentz curvature, and e^α is the orthonormal co-tetrad field. The 5d indexes are denoted by capital Latin letters and take on the values 0, 1, 2, 3, 4, and the 4d indexes are denoted by Greek letters and take on the values 0 (time), 1, 2, 3 (space). This theory is equivalent to the Einstein–Cartan (EC) theory with a cosmological constant $\Lambda = 3/l^2$ and a Gauss–Bonnet (GB) topological term, as seen in Equation (1).

Note that some special gauges with the residual Lorentz symmetry can be defined by $\zeta^A = \delta^A_4 l$, where δ^A_B is the Kronecker delta. Henceforth, ζ^A is akin to an unphysical

Goldstone field. To make ζ^A physical, and become the gravitational Higgs field, one may replace the constant l by a dynamical l , resulting in the Stelle–West (SW) theory [7]. The theory is further explored in Refs. [10,11] (see also the review [12]), in which the constraint $\zeta^A \zeta_A = l^2$ is completely removed, in other words, $\zeta^A \zeta_A$ needs not to be positive. Suppose that $\zeta^A \zeta_A = \sigma l^2$, where $\sigma = \pm 1$. When $l \neq 0$, the metric field can be defined by $g_{\mu\nu} = (\tilde{D}_\mu \zeta^A)(\tilde{D}_\nu \zeta_A)$, where $\tilde{D}_\mu \zeta^A = \tilde{\delta}^A_B D_\mu \zeta^B$, $\tilde{\delta}^A_B = \delta^A_B - \zeta^A \zeta_B / \sigma l^2$, $D_\mu \zeta^A = d_\mu \zeta^A + \Omega^A_{B\mu} \zeta^B$, and $\Omega^A_{B\mu}$ is the dS connection. It was shown [11] that $\sigma = \pm 1$ corresponds to the Lorentz/Euclidean signature of the metric field, and the signature changes when $\zeta^A \zeta_A$ changes its sign.

On the other hand, it remains to check whether the SW gravity is viable. Although the SW Lagrangian reduces to the MM Lagrangian when l is a constant, the field equations do not. In the SW theory, there is an additional field equation coming from the variation with respect to l , which is nontrivial even when l is a constant. Actually, a recent study [13] presents some negative results for a related model, whose Lagrangian is equal to the SW one times $(-l/2)$. For a homogeneous and isotropic universe with parity-invariant torsion, it is found that l being a constant implies the energy density of the material fluid being a constant, and so l should not be a constant in the general case. Moreover, in the radiation epoch, the l equation forces the energy density to be equal to zero; while in the matter epoch, a dynamical l only works to renormalize the gravitational constant by some constant factor, and hence, the cosmic expansion decelerates as in GR.

In this paper, it is shown that the SW gravity suffers from similar problems encountered in the model considered in Ref. [13]. Furthermore, I solve these problems by using a new fluid with the Lagrangian being a polynomial in the gravitational variables. The merits of a Lagrangian polynomial in some variables are that it is simple and nonsingular with respect to those variables. In Refs. [14,15], the polynomial Lagrangian for gravitation and other fundamental fields were proposed, while in this paper, the polynomial Lagrangian for a perfect fluid is proposed, which reduces to the Lagrangian of a usual perfect fluid when l is a constant. It turns out that, in contrast to the case with an ordinary fluid, the SW gravity coupled with the new fluid supports the radiation epoch and naturally drives the cosmic acceleration. In addition, when writing down the basic framework of DGT, a Lagrangian–Noether analysis is performed, which generalizes the results of Ref. [16] to the cases with arbitrary matter field and arbitrary ζ^A .

The article is organized as follows. In Section 2.1, a Lagrangian–Noether analysis is conducted for the general DGT sourced by a matter field. In Section 2.2, I reduce the analysis of Section 2.1 in the Lorentz gauges, and show how the two Noether identities in PGT can be elegantly unified into one identity in DGT. In Section 3.1, the SW model of DGT is introduced, with the field equations derived both in the general gauge and the Lorentz gauges. Further, the matter source is discussed in Section 3.2, where a modified perfect fluid with the Lagrangian polynomial in the gravitational variables is constructed, and a general class of perfect fluids is defined, which contains both the usual and modified perfect fluids. Then, I couple the SW gravity with the class of fluids and study the coupling system in the homogeneous, isotropic, and parity-invariant universe. The field equations are deduced in Section 4.1, solved in Section 4.2 for the vacuum case, and, in Section 4.3, for the material case. In Section 4.4, the above results are compared with observations, which determines the value of the coupling constant. In the last section, I give some conclusions, and discuss the remaining problems, possible solutions, and extensions.

2. De Sitter Gauge Theory

2.1. Lagrangian–Noether Machinery

The DGT sourced by a matter field is described by the Lagrangian 4-form:

$$\mathcal{L} = \mathcal{L}(\psi, D\psi, \zeta^A, D\zeta^A, \mathcal{F}^{AB}), \tag{2}$$

where ψ is a p -form valued at some representation space of the dS group $SO(1,4)$, $D\psi = d\psi + \Omega^{AB} T_{AB} \wedge \psi$ is the covariant exterior derivative, T_A^B are representations of the dS

generators, ζ^A is a dS vector, $D\zeta^A = d\zeta^A + \Omega^A_B \zeta^B$, Ω^A_B is the dS connection 1-form, and $\mathcal{F}^A_B = d\Omega^A_B + \Omega^A_C \wedge \Omega^C_B$ is the dS curvature 2-form. The variation of \mathcal{L} resulted from the variations of the explicit variables reads:

$$\delta\mathcal{L} = \delta\psi \wedge \partial\mathcal{L}/\partial\psi + \delta D\psi \wedge \partial\mathcal{L}/\partial D\psi + \delta\zeta^A \times \partial\mathcal{L}/\partial\zeta^A + \delta D\zeta^A \wedge \partial\mathcal{L}/\partial D\zeta^A + \delta\mathcal{F}^{AB} \wedge \partial\mathcal{L}/\partial\mathcal{F}^{AB}, \tag{3}$$

where $(\partial\mathcal{L}/\partial\psi)_{\mu_{p+1}\dots\mu_4} \equiv \partial\mathcal{L}_{\mu_1\dots\mu_p\mu_{p+1}\dots\mu_4}/\partial\psi_{\mu_1\dots\mu_p}$, and the other partial derivatives are similarly defined. The variations of $D\psi$, $D\zeta^A$, and \mathcal{F}^{AB} can be transformed into the variations of the fundamental variables ψ , ζ^A , and Ω^{AB} , leading to:

$$\delta\mathcal{L} = \delta\psi \wedge V_\psi + \delta\zeta^A \times V_A + \delta\Omega^{AB} \wedge V_{AB} + d(\delta\psi \wedge \partial\mathcal{L}/\partial D\psi + \delta\zeta^A \times \partial\mathcal{L}/\partial D\zeta^A + \delta\Omega^{AB} \wedge \partial\mathcal{L}/\partial\mathcal{F}^{AB}), \tag{4}$$

where,

$$V_\psi \equiv \delta\mathcal{L}/\delta\psi = \partial\mathcal{L}/\partial\psi - (-1)^p D\partial\mathcal{L}/\partial D\psi, \tag{5}$$

$$V_A \equiv \delta\mathcal{L}/\delta\zeta^A = \partial\mathcal{L}/\partial\zeta^A - D\partial\mathcal{L}/\partial D\zeta^A, \tag{6}$$

$$V_{AB} \equiv \delta\mathcal{L}/\delta\Omega^{AB} = T_{AB}\psi \wedge \partial\mathcal{L}/\partial D\psi + \partial\mathcal{L}/\partial D\zeta^A \times \zeta_B + D\partial\mathcal{L}/\partial\mathcal{F}^{AB}. \tag{7}$$

The symmetry transformations in DGT consist of the diffeomorphism transformations and the dS transformations. For the diffeomorphism transformations, they can be promoted to a gauge-invariant version [16,17], namely, the parallel transports in the fiber bundle with the gauge group as the structure group. The action of an infinitesimal parallel transport on a variable is a gauge-covariant Lie derivative (the gauge-covariant Lie derivative has been used in the metric-affine gauge theory of gravity [18]) $L_v \equiv v \rfloor D + Dv \rfloor$, where v is the vector field, which generates the infinitesimal parallel transport, and \rfloor denotes a contraction, for example, $(v \rfloor \psi)_{\mu_2\dots\mu_p} = v^{\mu_1} \psi_{\mu_1\mu_2\dots\mu_p}$. Put $\delta = L_v$ in Equation (3), utilize the arbitrariness of v , then one obtains the chain rule:

$$v \rfloor \mathcal{L} = (v \rfloor \psi) \wedge \partial\mathcal{L}/\partial\psi + (v \rfloor D\psi) \wedge \partial\mathcal{L}/\partial D\psi + (v \rfloor D\zeta^A) \times \partial\mathcal{L}/\partial D\zeta^A + (v \rfloor \mathcal{F}^{AB}) \wedge \partial\mathcal{L}/\partial\mathcal{F}^{AB}, \tag{8}$$

and the first Noether identity:

$$(v \rfloor D\psi) \wedge V_\psi + (-1)^p (v \rfloor \psi) \wedge DV_\psi + (v \rfloor D\zeta^A) \times V_A + (v \rfloor \mathcal{F}^{AB}) \wedge V_{AB} = 0. \tag{9}$$

On the other hand, the dS transformations are defined as vertical isomorphisms on the fiber bundle. The actions of an infinitesimal dS transformation on the fundamental variables are as follows:

$$\delta\psi = B^{AB} T_{AB}\psi, \quad \delta\zeta^A = B^{AB} \zeta_B, \quad \delta\Omega^{AB} = -DB^{AB}, \tag{10}$$

where B^A_B is a dS algebra-valued function, which generates the infinitesimal dS transformation. Substitute Equation (10) and $\delta\mathcal{L} = 0$ into Equation (4), and make use of Equation (7) and the arbitrariness of B^{AB} , one arrives at the second Noether identity:

$$DV_{AB} = -T_{AB}\psi \wedge V_\psi - V_{[A} \times \zeta_{B]}. \tag{11}$$

The above analyses are so general that they do not require the existence of a metric field. In the special case with a metric field being defined, $\zeta^A \zeta_A$ equating to a positive constant, and $p = 0$, the above analyses coincide with those in Ref. [16].

2.2. Reduction in the Lorentz Gauges

Consider the case with $\zeta^A \zeta_A = l^2$, where l is a positive function. Then, one may define the projector $\tilde{\delta}^A_B = \delta^A_B - \zeta^A \zeta_B / l^2$, the generalized tetrad $\tilde{D}\zeta^A = \tilde{\delta}^A_B D\zeta^B$, and

a symmetric rank-2 tensor (this formula has been given in Refs. [11,19], and is different from that originally proposed by Stelle and West [7] by a factor $(l_0/l)^2$, where l_0 is the vacuum expectation value of l),

$$g_{\mu\nu} = \eta_{AB}(\tilde{D}_\mu \zeta^A)(\tilde{D}_\nu \zeta^B), \tag{12}$$

which is a localization of the dS metric, $\hat{g}_{\mu\nu} = \eta_{AB}(d_\mu \hat{\zeta}^A)(d_\nu \hat{\zeta}^B)$, where η_{AB} is the 5d Minkowski metric, and $\hat{\zeta}^A$ are the 5d Minkowski coordinates on the 4d dS space.

Though Equation (12) seems less natural than the choice $g_{\mu\nu}^* = \eta_{AB}(D_\nu \zeta^A)(D_\nu \zeta^B)$, it coincides with another natural identification (15) (the relation between Equations (12) and (15) is discussed below in this Section). If $g_{\mu\nu}$ is non-degenerate, it is a metric field with Lorentz signature, and one may define $\tilde{D}^\mu \zeta_A \equiv g^{\mu\nu} \tilde{D}_\nu \zeta_A$. Put $v^\mu = \tilde{D}^\mu \zeta_A$ in Equation (9) and utilize $(\tilde{D}_\mu \zeta^A)(\tilde{D}^\mu \zeta_B) = \tilde{\delta}^A_B$, one obtains:

$$\begin{aligned} \tilde{V}_A = & -(\tilde{D}\zeta_A]D\psi) \wedge V_\psi - (-1)^p(\tilde{D}\zeta_A]\psi) \wedge DV_\psi - (\tilde{D}\zeta_A]d \ln l) \times V_C \zeta^C \\ & -(\tilde{D}\zeta_A]\mathcal{F}^{CD}) \wedge V_{CD}, \end{aligned} \tag{13}$$

where $\tilde{V}_A = \tilde{\delta}^B_A V_B$. When l is a constant, Equation (13) implies that the ζ^A equation ($\tilde{V}_A = 0$ for this case) can be deduced from the other field equations ($V_\psi = 0$ and $V_{CD} = 0$), as pointed out in Ref. [19]. Substitute Equation (13) into Equation (11), and make use of $\tilde{V}_{[A} \times \zeta_{B]} = V_{[A} \times \zeta_{B]}$ and $\tilde{D}\zeta_{[A} \times \zeta_{B]} = D\zeta_{[A} \times \zeta_{B]}$, one attains

$$\begin{aligned} DV_{AB} = & -T_{AB}\psi \wedge V_\psi + (D\zeta_{[A} \times \zeta_{B]}]D\psi) \wedge V_\psi + (-1)^p(D\zeta_{[A} \times \zeta_{B]}]\psi) \wedge DV_\psi \\ & + (D\zeta_{[A} \times \zeta_{B]}]d \ln l) \times V_C \zeta^C + (D\zeta_{[A} \times \zeta_{B]}]\mathcal{F}^{CD}) \wedge V_{CD}. \end{aligned} \tag{14}$$

When l is a constant, Equation (14) coincides with the corresponding result in Ref. [16]. As shown later in this Section, Equation (14) unifies the two Noether identities in PGT.

To see this, let us define the Lorentz gauges by the condition $\zeta^A = \delta^A_4 l$ [7]. If $h^A_B \in SO(1,4)$ preserves these gauges, then $h^A_B = \text{diag}(h^\alpha_\beta, 1)$, where h^α_β belongs to the Lorentz group $SO(1,3)$. In the Lorentz gauges, Ω^α_β transforms as a Lorentz connection, and Ω^{α_4} transforms as a co-tetrad field. Therefore, one may identify Ω^α_β as the space-time connection Γ^α_β , and Ω^{α_4} as the co-tetrad field e^α divided by some quantity with the dimension of length, a natural choice for which is l . Resultantly, Ω^{AB} is identified with a combination of geometric quantities as follows:

$$\Omega^{AB} = \begin{pmatrix} \Gamma^{\alpha\beta} & l^{-1}e^\alpha \\ -l^{-1}e^\beta & 0 \end{pmatrix}. \tag{15}$$

In the case with constant l , this formula is given in Refs. [7,20], and, in the case with varying l , it is given in Refs. [10,19]. In the Lorentz gauges, $\tilde{D}\zeta^4 = 0$, $\tilde{D}\zeta^\alpha = \Omega^\alpha_4 l = e^\alpha$ (where Equation (15) is used), and so $g_{\mu\nu}$ defined by Equation (12) satisfies $g_{\mu\nu} = \eta_{\alpha\beta} e^\alpha_\mu e^\beta_\nu$, implying that Equation (12) coincides with Equation (15). Moreover, according to Equation (15), one finds the expression for \mathcal{F}^{AB} in the Lorentz gauges as follows [19]:

$$\mathcal{F}^{AB} = \begin{pmatrix} R^{\alpha\beta} - l^{-2}e^\alpha \wedge e^\beta & l^{-1}[S^\alpha - d \ln l \wedge e^\alpha] \\ -l^{-1}[S^\beta - d \ln l \wedge e^\beta] & 0 \end{pmatrix}, \tag{16}$$

where $R^\alpha_\beta = d\Gamma^\alpha_\beta + \Gamma^\alpha_\gamma \wedge \Gamma^\gamma_\beta$ is the spacetime curvature, and $S^\alpha = de^\alpha + \Gamma^\alpha_\beta \wedge e^\beta$ is the spacetime torsion.

Now one can interpret the results in Section 2.1 in the Lorentz gauges. In those gauges, $D\psi = D^\Gamma \psi + 2l^{-1}e^\alpha T_{\alpha 4} \wedge \psi$, $D\zeta^\alpha = e^\alpha$, $D\zeta^4 = dl$, and so Equation (2) becomes:

$$\mathcal{L} = \mathcal{L}^L(\psi, D^\Gamma \psi, l, dl, e^\alpha, R^{\alpha\beta}, S^\alpha), \tag{17}$$

where $D^\Gamma \psi = d\psi + \Gamma^{\alpha\beta} T_{\alpha\beta} \wedge \psi$. It is the same as a Lagrangian 4-form in PGT [21], with the fundamental variables being $\psi, l, \Gamma^{\alpha\beta}$, and e^α . The relations between the variational

derivatives with respect to the PGT variables and those with respect to the DGT variables can be deduced from the following equality:

$$\delta\zeta^A \times V_A + 2\delta\Omega^{\alpha 4} \wedge V_{\alpha 4} = \delta l \times \Sigma_l + \delta e^\alpha \wedge \Sigma_\alpha, \tag{18}$$

where $\Sigma_l \equiv \delta\mathcal{L}^L/\delta l$ and $\Sigma_\alpha \equiv \delta\mathcal{L}^L/\delta e^\alpha$. Explicitly, the relations are:

$$\Sigma_\psi \equiv \delta\mathcal{L}^L/\delta\psi = V_\psi, \tag{19}$$

$$\Sigma_l = V_4 - 2l^{-2}e^\alpha \wedge V_{\alpha 4}, \tag{20}$$

$$\Sigma_{\alpha\beta} \equiv \delta\mathcal{L}^L/\delta\Gamma^{\alpha\beta} = V_{\alpha\beta}, \tag{21}$$

$$\Sigma_\alpha = 2l^{-1}V_{\alpha 4}. \tag{22}$$

It is remarkable that the DGT variational derivative V_{AB} unifies the two PGT variational derivatives $\Sigma_{\alpha\beta}$ and Σ_α . With the help of Equations (19)–(22), the $\alpha\beta$ components and $\alpha 4$ components of Equation (14) are found to be:

$$D^\Gamma\Sigma_{\alpha\beta} = -T_{\alpha\beta}\psi \wedge \Sigma_\psi + e_{[\alpha} \wedge \Sigma_{\beta]}, \tag{23}$$

$$\begin{aligned} D^\Gamma\Sigma_\alpha &= D^\Gamma_\alpha\psi \wedge \Sigma_\psi + (-1)^p(e_\alpha \rfloor \psi) \wedge D^\Gamma\Sigma_\psi + \partial_\alpha l \times \Sigma_l \\ &\quad + (e_\alpha \rfloor R^{\beta\gamma}) \wedge \Sigma_{\beta\gamma} + (e_\alpha \rfloor S^\beta) \wedge \Sigma_\beta, \end{aligned} \tag{24}$$

which are just the two Noether identities in PGT [21], with both ψ and l as the matter fields; $\partial_\alpha l = e_\alpha \rfloor dl$. This completes our proof for the earlier statement that the DGT identity (14) unifies the two Noether identities in PGT.

3. Polynomial Models for DGT

3.1. Stelle–West Gravity

It is natural to require that the Lagrangian for DGT is regular with respect to the fundamental variables. The simplest regular Lagrangians are polynomial in the variables, and, in order to recover the EC theory, the polynomial Lagrangian should be at least linear in the gauge curvature. Moreover, to ensure $\mathcal{F}^{AB} = 0$ is naturally a vacuum solution, the polynomial Lagrangian should be at least quadratic in \mathcal{F}^{AB} (when the Lagrangian is linear in \mathcal{F}^{AB} , one may add some ‘constant term’ (independent of \mathcal{F}^{AB}) to ensure $\mathcal{F}^{AB} = 0$ is a vacuum solution, but this way is not so natural). The general Lagrangian quadratic in \mathcal{F}^{AB} reads:

$$\begin{aligned} \mathcal{L}^G &= (\kappa_1 \epsilon_{ABCDE} \zeta^E + \kappa_2 \eta_{AC} \zeta_B \zeta_D + \kappa_3 \eta_{AC} \eta_{BD}) \mathcal{F}^{AB} \wedge \mathcal{F}^{CD} \\ &= \kappa_1 \mathcal{L}^{SW} + \kappa_2 (S^\alpha \wedge S_\alpha - 2S^\alpha \wedge d \ln l \wedge e_\alpha) \\ &\quad + \kappa_3 [R^{\alpha\beta} \wedge R_{\alpha\beta} + d(2l^{-2}S^\alpha \wedge e_\alpha)], \end{aligned} \tag{25}$$

where the κ_1 term is the SW Lagrangian, the κ_2 and κ_3 terms are parity odd, and the κ_3 term is a sum of the Pontryagin and modified Nieh–Yan topological terms. This quadratic Lagrangian is a special case of the, at most, quadratic Lagrangian proposed in Refs. [10,22], and one should note that the quadratic Lagrangian satisfies the requirement mentioned above about the vacuum solution, while the, at most, quadratic Lagrangian does not always satisfy that requirement.

Among the three terms in Equation (25), the SW term is the only one that can be reduced to the EC Lagrangian in the case with positive and constant $\zeta^A \zeta_A$. Thus, the SW Lagrangian is the simplest choice for the gravitational Lagrangian, which (i) is regular with respect to the fundamental variables; (ii) can be reduced to the EC Lagrangian; (iii) ensures $\mathcal{F}^{AB} = 0$ is naturally a vacuum solution.

For the above reason, the gravitational Lagrangian is taken to be \mathcal{L}^{SW} , i.e., put $\kappa_1 = 1$ and $\kappa_2 = \kappa_3 = 0$ in Equation (25). The SW Lagrangian 4-form \mathcal{L}^{SW} takes the same form as \mathcal{L}^{MM} in the first line of Equation (1), while ζ^A is not constrained by any condition. Substitute Equation (1) into Equations (6)–(7), make use of $\partial\mathcal{L}^{SW}/\partial\mathcal{F}^{AB} = \epsilon_{ABCDE}\zeta^E\mathcal{F}^{CD}$ and the Bianchi identity $D\mathcal{F}^{AB} = 0$, one obtains the gravitational field equations:

$$-\kappa\epsilon_{ABCDE}\mathcal{F}^{AB}\wedge\mathcal{F}^{CD} = \delta\mathcal{L}^m/\delta\zeta^E, \tag{26}$$

$$-\kappa\epsilon_{ABCDE}D\zeta^E\wedge\mathcal{F}^{CD} = \delta\mathcal{L}^m/\delta\Omega^{AB}, \tag{27}$$

where \mathcal{L}^m is the Lagrangian of the matter field coupled to the SW gravity, with κ as the coupling constant. In the vacuum case, Equation (27) is given in Ref. [22] by direct computation, while here, Equation (27) is obtained from the general Equation (7).

In the Lorentz gauges, \mathcal{L}^{SW} takes the same form as \mathcal{L}^{MM} in the second line of Equation (1), while l becomes a dynamical field. The gravitational field equations read:

$$-(\kappa/4)\epsilon_{\alpha\beta\gamma\delta}\epsilon^{\mu\nu\sigma\rho}e^{-1}R^{\alpha\beta}{}_{\mu\nu}R^{\gamma\delta}{}_{\sigma\rho} - 4\kappa l^{-2}R + 72\kappa l^{-4} = \delta S_m/\delta l, \tag{28}$$

$$-\kappa\epsilon_{\alpha\beta\gamma\delta}\epsilon^{\mu\nu\sigma\rho}e^{-1}\partial_\nu l \times R^{\gamma\delta}{}_{\sigma\rho} + 8\kappa e_{[\alpha}{}^\mu e_{\beta]}{}^\nu\partial_\nu l^{-1} + 4\kappa l^{-1}T^\mu{}_{\alpha\beta} = \delta S_m/\delta\Gamma^{\alpha\beta}{}_\mu, \tag{29}$$

$$-8\kappa l^{-1}(G^\mu{}_\alpha + \Lambda e_\alpha{}^\mu) = \delta S_m/\delta e^\alpha{}_\mu, \tag{30}$$

where $e = \det(e^\alpha{}_\mu)$, R is the scalar curvature, $G^\mu{}_\alpha$ is the Einstein tensor, $T^\mu{}_{\alpha\beta} = S^\mu{}_{\alpha\beta} + 2e_{[\alpha}{}^\mu S^\nu{}_{\beta]\nu}$, and S_m is the action of the matter field. Although when l is a constant \mathcal{L}^{SW} reduces to the EC Lagrangian with a cosmological constant and a GB topological term, the field equations do not reduce to those of EC with a cosmological constant. The reason lies in the existence of Equation (28), which is nontrivial, even when l is a constant. As a result, the coupling constant κ cannot be fixed by simply comparing Equations (29) and (30) with the EC equations. As shown below in Section 4.4, κ could be determined by a comparison between the theory and cosmological observations.

3.2. Polynomial dS Fluid

For the same reason of choosing a polynomial Lagrangian for DGT, I intend to use those matter sources with polynomial Lagrangian. It has been shown that the Lagrangian of fundamental fields can be reformulated into polynomial forms [14,15]. However, when describing the universe, it is more adequate to use a fluid as the matter source. The Lagrangian of an ordinary perfect fluid (PF) [23] can be written in a Lorentz-invariant form:

$$\mathcal{L}_{\mu\nu\rho\sigma}^{PF} = -\epsilon_{\alpha\beta\gamma\delta}e^\alpha{}_\mu e^\beta{}_\nu e^\gamma{}_\rho e^\delta{}_\sigma \rho + \epsilon_{\alpha\beta\gamma\delta}J^\alpha e^\beta{}_\nu e^\gamma{}_\rho e^\delta{}_\sigma \wedge \partial_\mu\phi, \tag{31}$$

where ϕ is a scalar field, J^α is the particle number current which is Lorentz covariant and satisfies $J^\alpha J_\alpha < 0$, $\rho = \rho(n)$ is the energy density, and $n \equiv \sqrt{-J^\alpha J_\alpha}$ is the particle number density. The Lagrangian (31) is polynomial in the PGT variable $e^\alpha{}_\mu$, but it is not polynomial in the DGT variables when it is reformulated into a dS-invariant form, in which case the Lagrangian reads:

$$\begin{aligned} \mathcal{L}_{\mu\nu\rho\sigma}^{PF} = & -\epsilon_{ABCDE}(D_\mu\zeta^A)(D_\nu\zeta^B)(D_\rho\zeta^C)(D_\sigma\zeta^D)(\zeta^E/l)\rho \\ & +\epsilon_{ABCDE}J^A(D_\nu\zeta^B)(D_\rho\zeta^C)(D_\sigma\zeta^D)\wedge(\zeta^E/l)\partial_\mu\phi, \end{aligned} \tag{32}$$

where J^A is a dS-covariant particle number current, which satisfies $J^A J_A < 0$ and $J^A\zeta_A = 0$, $\rho = \rho(n)$ and $n \equiv \sqrt{-J^A J_A}$. Because l^{-1} appears in Equation (32), the Lagrangian is not polynomial in ζ^A .

A straightforward way to modify Equation (32) into a polynomial Lagrangian is to multiply it by l . In the Lorentz gauges, $J^4 = 0$, and one may define the invariant $J^\mu \equiv J^\alpha e_\alpha{}^\mu$. Then, the modified Lagrangian $\mathcal{L}_{\mu\nu\rho\sigma}^{PF} = -\epsilon\epsilon_{\mu\nu\rho\sigma}\rho l + e\epsilon_{\mu'\nu\rho\sigma}J^{\mu'} \wedge l \times \partial_\mu\phi$. It can be verified that this Lagrangian violates the particle number conservation law

$\nabla_\mu J^\mu = 0$, where ∇_μ is the linearly covariant, metric-compatible and torsion-free derivative. To preserve the particle number conservation, we may replace $l \times \partial_\mu \phi$ by $\partial_\mu(l\phi)$, and the corresponding dS-invariant Lagrangian is:

$$\begin{aligned} \mathcal{L}_{\mu\nu\rho\sigma}^{\text{DF}} = & -\epsilon_{ABCDE}(D_\mu \zeta^A)(D_\nu \zeta^B)(D_\rho \zeta^C)(D_\sigma \zeta^D) \zeta^E \rho(n) \\ & + \epsilon_{ABCDE} J^A (D_\nu \zeta^B)(D_\rho \zeta^C)(D_\sigma \zeta^D) \wedge \left(\frac{1}{4} D_\mu \zeta^E \times \phi + \zeta^E \partial_\mu \phi \right). \end{aligned} \tag{33}$$

The perfect fluid depicted by the above Lagrangian is called the polynomial dS fluid, or dS fluid (DF) for short. In the Lorentz gauges,

$$\begin{aligned} \mathcal{L}_{\mu\nu\rho\sigma}^{\text{DF}} = & -e\epsilon_{\mu\nu\rho\sigma} \rho l + \epsilon_{\alpha\beta\gamma\delta} J^\alpha e^\beta_\nu e^\gamma_\rho e^\delta_\sigma \wedge (\partial_\mu l \times \phi + l \partial_\mu \phi) \\ = & -e\epsilon_{\mu\nu\rho\sigma} \rho l + e\epsilon_{\mu'\nu\rho\sigma} J^{\mu'} \wedge \partial_\mu(l\phi), \end{aligned} \tag{34}$$

which is equivalent to Equation (31) when l is a constant.

Define the Lagrangian function \mathcal{L}_{DF} by $\mathcal{L}_{\mu\nu\rho\sigma}^{\text{DF}} = \mathcal{L}_{\text{DF}} e\epsilon_{\mu\nu\rho\sigma}$, then $\mathcal{L}_{\text{DF}} = -\rho l + J^\mu \partial_\mu(l\phi)$. To compare the polynomial dS fluid with the ordinary perfect fluid, let us consider a general model with the Lagrangian function:

$$\mathcal{L}_m = -\rho l^k + J^\mu \partial_\mu(l^k \phi), \tag{35}$$

where $k \in \mathbb{R}$. When $k = 0$, it describes the ordinary perfect fluid; when $k = 1$, it describes the polynomial dS fluid. The variation of $S_m \equiv \int dx^4 e \mathcal{L}_m$ with respect to ϕ gives the particle number conservation law $\nabla_\mu J^\mu = 0$. The variation with respect to J^α yields $\partial_\mu(l^k \phi) = -\mu U_\mu l^k$, where $\mu \equiv d\rho/dn = (\rho + p)/n$ is the chemical potential, $p = p(n)$ is the pressure, and $U^\mu \equiv J^\mu/n$ is the 4-velocity of the fluid particle. Making use of these results, one may check that the on-shell Lagrangian function is equal to $p l^k$, and the variational derivatives:

$$\delta S_m / \delta l = -k \rho l^{k-1}, \tag{36}$$

$$\delta S_m / \delta \Gamma^{\alpha\beta}_\mu = 0, \tag{37}$$

$$\delta S_m / \delta e^\alpha_\mu = (\rho + p) l^k U^\mu U_\alpha + p l^k e^\alpha_\mu. \tag{38}$$

One can see that $\delta S_m / \delta l = 0$ for the ordinary perfect fluid, while $\delta S_m / \delta l = -\rho$ for the polynomial dS fluid.

Finally, it should be noted that the polynomial dS fluid does not support a signature change corresponding to $\zeta^A \zeta_A$ varying from negative to positive. The reason is that when $\zeta^A \zeta_A < 0$, there exists no J^A , which satisfies $J^A J_A < 0$ and $J^A \zeta_A = 0$.

4. Cosmological Solutions

4.1. Field Equations for the Universe

In this Section, the coupling system of the SW gravity and the fluid model (35) is analyzed in the homogenous, isotropic, parity-invariant and spatially flat universe characterized by the following ansatz [13]:

$$e^0_\mu = d_\mu t, \quad e^i_\mu = a d_\mu x^i, \tag{39}$$

$$S^0_{\mu\nu} = 0, \quad S^i_{\mu\nu} = b e^0_\mu \wedge e^i_\nu, \tag{40}$$

where a and b are functions of the cosmic time t , and $i = 1, 2, 3$. On account of Equations (39) and (40), the Lorentz connection $\Gamma^{\alpha\beta}_\mu$ and curvature $R^{\alpha\beta}_{\mu\nu}$ can be calculated [13]. Further, assume that $U^\mu = e_0^\mu$, then $U_\mu = -e^0_\mu$, and so $U_\alpha = -\delta^0_\alpha$. Now, the reduced form of each term of Equations (28)–(30) can be attained. In particular,

$$\epsilon_{\alpha\beta\gamma\delta} \epsilon^{\mu\nu\rho\sigma} e^{-1} R^{\alpha\beta}_{\mu\nu} R^{\gamma\delta}_{\sigma\rho} = 96(ha) \cdot a^{-1} h^2, \tag{41}$$

$$R = 6[(ha) \cdot a^{-1} + h^2], \tag{42}$$

$$\epsilon_{0i\gamma\delta} \epsilon^{\mu\nu\sigma\rho} e^{-1} \partial_\nu l \times R^{\gamma\delta}{}_{\sigma\rho} = -4h^2 \dot{l} e_i^\mu, \tag{43}$$

$$\epsilon_{ij\gamma\delta} \epsilon^{\mu\nu\sigma\rho} e^{-1} \partial_\nu l \times R^{\gamma\delta}{}_{\sigma\rho} = 0, \tag{44}$$

$$T^\mu{}_{0i} = -2b e_i^\mu, \quad T^\mu{}_{ij} = 0, \tag{45}$$

$$G^\mu{}_0 = -3h^2 e_0^\mu, \tag{46}$$

$$G^\mu{}_i = -[2(ha) \cdot a^{-1} + h^2] e_i^\mu, \tag{47}$$

$$\delta S_m / \delta e^0{}_\mu = -\rho l^k e_0^\mu, \tag{48}$$

$$\delta S_m / \delta e^i{}_\mu = p l^k e_i^\mu, \tag{49}$$

where dot on top of a quantity or being a superscript denotes the differentiation with respect to t , and $h = \dot{a}/a - b$. Substitution of the above equations into Equations (28)–(30) leads to:

$$(ha) \cdot a^{-1} (h^2 + l^{-2}) + l^{-2} (h^2 - \Lambda) = k\rho l^{k-1} / 24\kappa, \tag{50}$$

$$(h^2 + l^{-2}) \dot{l} - 2b l^{-1} = 0, \tag{51}$$

$$8\kappa l^{-1} (-3h^2 + \Lambda) = \rho l^k, \tag{52}$$

$$8\kappa l^{-1} [-2(ha) \cdot a^{-1} - h^2 + \Lambda] = -p l^k, \tag{53}$$

which constitute the field equations for the universe.

Generally, if the requirement of parity invariance is removed, then the ansatz (40) should be replaced by [24]:

$$S^0{}_{\mu\nu} = 0, \quad S^i{}_{\mu\nu} = b(t) e^0{}_\mu \wedge e^i{}_\nu + c(t) e^i{}_{jk} e^j{}_\mu \wedge e^k{}_\nu. \tag{54}$$

Correspondingly, Equations (50)–(53) are generalized to be:

$$(ha) \cdot a^{-1} (h^2 - c^2 + l^{-2}) - 2hc(ca) \cdot a^{-1} + (h^2 - c^2 - 3l^{-2}) l^{-2} = \frac{k}{24\kappa} \rho l^{k-1}, \tag{55}$$

$$(h^2 - c^2 + l^{-2}) \dot{l} - 2b l^{-1} = 0, \tag{56}$$

$$c(h\dot{l} + l^{-1}) = 0, \tag{57}$$

$$-3(h^2 - c^2) + \Lambda = \frac{l^{k+1}}{8\kappa} \rho, \tag{58}$$

$$-\left[2(ha) \cdot a^{-1} + h^2 - c^2\right] + \Lambda = -\frac{l^{k+1}}{8\kappa} p. \tag{59}$$

When $c = 0$, the above equations reduce to Equations (50)–(53). In virtue of Equation (57), there are two branches of solutions—one is parity even ($c = 0$) and the other is parity odd ($c \neq 0$), which satisfies $h\dot{l} + l^{-1} = 0$. In this paper, only the parity-even case is considered.

4.2. The Vacuum Solution

In the vacuum, $\rho = p = 0$, then Equations (50)–(53) read:

$$(ha) \cdot a^{-1} (h^2 + l^{-2}) + l^{-2} (h^2 - \Lambda) = 0, \tag{60}$$

$$(h^2 + l^{-2}) \dot{l} - 2b l^{-1} = 0, \tag{61}$$

$$-3h^2 + \Lambda = 0, \tag{62}$$

$$-2(ha) \cdot a^{-1} - h^2 + \Lambda = 0. \tag{63}$$

It can be shown that Equations (60) and (63) can be deduced from Equations (61) and (62), and the solution for the latter reads:

$$a/a_0 = (l/l_0)e^{\int_{t_0}^t (\pm l^{-1})dt}, \tag{64}$$

$$b = l^{-1}\dot{l}, \tag{65}$$

where l is an arbitrary positive function, and a_0 and l_0 are the values of a and l at some moment t_0 . In particular, if l is a constant, then:

$$a = a_0e^{H(t-t_0)}, b = 0, \tag{66}$$

where $H = \dot{a}/a = \pm l^{-1}$ is a constant. This solution is just the dS space, which describes an inflationary universe.

4.3. The Material Solution

In the general case with matter, let us first derive the continuity equation from the field Equations (50)–(53). Rewrite Equation (52) as:

$$h^2 = l^{-2} - \rho l^{k+1}/24\kappa. \tag{67}$$

Substituting Equation (67) into Equation (53) yields:

$$(ha)\cdot a^{-1} = l^{-2} + (\rho + 3p)l^{k+1}/48\kappa. \tag{68}$$

Multiply Equation (68) by $2h$, making use of Equation (67) and $h = \dot{a}/a - b$, one gets:

$$2h\dot{h} = (\rho + p)l^{k+1}\dot{a}a^{-1}/8\kappa - 2b(ha)\cdot a^{-1}, \tag{69}$$

in which, according to Equations (50), (51), and (67),

$$2b(ha)\cdot a^{-1} = \dot{l}[(k + 1)\rho l^k/24\kappa + 2l^{-3}]. \tag{70}$$

Differentiate Equation (67) with respect to t , and compare it with Equations (69) and (70), one arrives at the continuity equation:

$$\dot{\rho} + 3(\rho + p)\dot{a}a^{-1} = 0, \tag{71}$$

which is, unexpectedly, the same as the usual one. Suppose that $p = w\rho$, where w is a constant. Then, Equation (71) has the solution:

$$\rho = \rho_0(a/a_0)^{-3(1+w)}, \tag{72}$$

where a_0 and ρ_0 are the values of a and ρ at some moment t_0 .

Now, one can solve Equations (50)–(52), while Equation (53) is replaced by Equation (71) with the solution (72). First, substitute Equations (67) and (68) into Equation (50), one finds:

$$\rho l^{k+3} = 48\kappa(3w - k - 1)/(3w + 1). \tag{73}$$

Assume that $\kappa < 0$, then according to the above relation, $\rho l^{k+3} > 0$ implies $(3w - k - 1)/(3w + 1) < 0$. The only concern are the cases with $k = 0, 1$, and assume that $k + 1 > -1$, then $\rho l^{k+3} > 0$ constrains w by:

$$-\frac{1}{3} < w < \frac{k + 1}{3}. \tag{74}$$

For the ordinary fluid ($k = 0$), the pure radiation ($w = 1/3$) cannot exist. In fact, on account of Equation (73), $\rho l^3 = 0$ in this case, which is unreasonable. This problem

is similar to that appeared in Ref. [13]. On the other hand, for the dS fluid ($k = 1$), Equation (74) becomes $-1/3 < w < 2/3$, which contains both the cases with pure matter ($w = 0$) and pure radiation ($w = 1/3$). Generally, the combination of Equations (72) and (73) yields:

$$l = l_0(a/a_0)^{\frac{3(w+1)}{k+3}}, \tag{75}$$

where l_0 is the value of l when $t = t_0$, and is related to ρ_0 by Equation (73).

Second, substituting Equation (67) into Equation (51), and utilizing Equations (73) and (75), one obtains:

$$b = \frac{3(w+1)(k+2)}{(3w+1)(k+3)} \dot{a} a^{-1}, \tag{76}$$

and hence,

$$h = \frac{3w - 2k - 3}{(3w+1)(k+3)} \dot{a} a^{-1}. \tag{77}$$

Third, substitution of Equations (73) and (77) into Equation (52) leads to:

$$\dot{a} a^{-1} = H_0(l_0/l), \tag{78}$$

where $H_0 \equiv (\dot{a} a^{-1})_{t_0}$ is the Hubble constant, being related to l_0 by:

$$H_0 = \sqrt{\frac{3w+1}{-3w+2k+3}} \times (k+3)l_0^{-1}. \tag{79}$$

Here, note that Equation (74) implies that $3w+1 > 0$, $-3w+k+1 > 0$, $k+1 > -1$, and so $-3w+2k+3 > 0$. In virtue of Equations (75), (76), and (78), one has:

$$b = b_0(a_0/a)^{\frac{3(w+1)}{k+3}}, \tag{80}$$

where b_0 is related to H_0 by Equation (76). Moreover, substitute Equation (75) into Equation (78) and solve the resulting equation, one attains:

$$(a/a_0)^{\frac{3(w+1)}{k+3}} - 1 = \frac{3(w+1)}{k+3} \times H_0(t - t_0). \tag{81}$$

In conclusion, the solutions for the field Equations (50)–(53) are given by Equations (72), (75), (80), and (81), with the independent constants a_0 , H_0 , and t_0 .

4.4. Comparison with Observations

If k is specified, one can determine the value of the coupling constant κ from the observed values of $H_0 = 67.4 \text{ km} \times \text{s}^{-1} \times \text{Mpc}^{-1}$ and $\Omega_0 \equiv 8\pi\rho_0/3H_0^2 = 0.315$ [25]. For example, put $k = 1$, then according to Equation (79) (with $w = 0$), one has:

$$l_0 = 4/\sqrt{5}H_0 = 8.19 \times 10^{17} \text{ s}. \tag{82}$$

Substitution of Equation (82) and $\rho_0 = 3H_0^2\Omega_0/8\pi = 1.79 \times 10^{-37} \text{ s}^{-2}$ into Equation (73) yields:

$$\kappa = -\rho_0 l_0^4/96 = -8.41 \times 10^{32} \text{ s}^2. \tag{83}$$

This value is an important reference for the future work, which will explore the viability of the model in the solar system scale.

Furthermore, the deceleration parameter $q \equiv -a\ddot{a}/\dot{a}^2$ derived from the above models can be compared with the observed one. With the help of Equations (78) and (75), one finds $\dot{a} \sim a^{(k-3w)/(k+3)}$, then $\ddot{a} = \frac{k-3w}{k+3} \times \dot{a}^2 a^{-1}$, and so:

$$q = \frac{3w - k}{k + 3}. \tag{84}$$

Putting $w = 0$, one can find that the universe accelerates ($q < 0$) if $k > 0$, linearly expands ($q = 0$) if $k = 0$, and decelerates ($q > 0$) if $k < 0$. In particular, for the model with an ordinary fluid ($k = 0$), the universe expands linearly (this result is different from that of Ref. [13], where the cosmological solution describes a decelerating universe; as stated before, the gravitational Lagrangian in Ref. [13] is equal to $(-1/2)\mathcal{L}^{\text{SW}}$, which is not equivalent to \mathcal{L}^{SW}); while for the model with a dS fluid ($k = 1$), the universe accelerates with $q = -1/4$, which is consistent with the observational result $-1 \leq q_0 < 0$ [26–28], where q_0 is the present-day value of q . It should be noted that Equation (84) implies that q is a constant when w is a constant, and so the models cannot describe the transition from deceleration to acceleration when w is a constant.

5. Remarks

It is shown that the requirement of regular Lagrangian may be crucial for DGT, as it is shown that the SW gravity coupled with an ordinary perfect fluid (whose Lagrangian is not regular with respect to ζ^A when $\zeta^A \zeta_A = 0$) does not permit a radiation epoch and the acceleration of the universe, while the SW gravity coupled with a polynomial dS fluid (whose Lagrangian is regular with respect to ζ^A) is out of these problems. Yet, in the latter model, there exists the problem that it cannot describe the transition from deceleration to acceleration in the matter epoch. Actually, only the parity-even branch of the model is analyzed here. One may further analyze the parity-odd branch and check whether the transition problem exists in that case.

Moreover, there are two possible ways to refine the present model. The first is to modify the gravitational part to be the general quadratic model (25), which is a special case of the, at most, quadratic model proposed in Refs. [10,22], but the coupling of which with the polynomial dS fluid is unexplored. It is unknown whether the effect of the κ_2 term could solve the problem encountered in the SW gravity.

The second way is to modify the matter part. Although the Lagrangian of the polynomial dS fluid is regular with respect to ζ^A , it is not regular with respect to J^A when $\zeta^A \zeta_A = 0$, in which case there should be $J^A J_A \geq 0$, and so the number density $n \equiv \sqrt{-J^A J_A}$ is not regular. One could find a new fluid model whose Lagrangian is regular with respect to all variables, based on the polynomial models for fundamental fields proposed in Refs. [14,15].

Moreover, the present study may be extended to the inflationary epoch. As was shown in Section 4.2, in the vacuum, the theory contains the dS solution, which describes an inflationary universe. Then there should be a transition from the inflationary epoch to the radiation epoch. As usual, this might be achieved by introducing a particle production rate Γ given by some quantum theory. In GR, with the help of energy conservation, the contribution of Γ to the effective pressure p_{eff} can be derived [29,30]. As indicated in Section 4.3, energy conservation in the present theory takes the same form as that in GR, and so it could be believed that the derivation of p_{eff} also applies to the present theory. Replacing p by p_{eff} in the dS fluid, one could further explore the corresponding dynamics.

Funding: This research is supported by the National Natural Science Foundation for Young Scientists of China under Grant No. 12005307.

Data Availability Statement: Not applicable.

Acknowledgments: I would like to thank S.-D. Liang and Z.-B. Li for their abiding help. Furthermore, thanks are due to the anonymous referees who asked enlightening questions. Lastly, I owe my thanks to my parents and wife.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Kibble, T.W.B. Lorentz invariance and the gravitational field. *J. Math. Phys.* **1961**, *2*, 212–221. [[CrossRef](#)]
2. Sciama, D.W. On the analogy between charge and spin in general relativity. In *Infeld Festschrift: Recent Developments in General Relativity. A Collection of Papers Dedicated to Leopold Infeld*; Pergamon Press: Oxford, UK, 1962; pp. 415–439

3. Blagojević, M.; Hehl, F.W. (Eds.) *Gauge Theories of Gravitation. A Reader with Commentaries*; Imperial College Press: London, UK, 2013.
4. Ponomarev, V.N.; Barvinsky, A.O.; Obukhov, Y.N. *Gauge Approach and Quantization Methods in Gravity Theory*; Nauka: Moscow, Russia, 2017. Available online: <http://en.ibrae.ac.ru/pubtext/259/> (accessed on 10 September 2022).
5. Mielke, E.W. *Geometrodynamics of Gauge Fields*; Springer International Publishing Switzerland: Cham, Switzerland, 2017. [[CrossRef](#)]
6. Stelle, K.S.; West, P.C. De Sitter gauge invariance and the geometry of the Einstein–Cartan theory. *J. Phys. A Math. Theor.* **1979**, *12*, L205–L210. [[CrossRef](#)]
7. Stelle, K.S.; West, P.C. Spontaneously broken de Sitter symmetry and the gravitational holonomy group. *Phys. Rev. D* **1980**, *21*, 1466–1488. [[CrossRef](#)]
8. MacDowell, S.W.; Mansouri, F. Unified geometric theory of gravity and supergravity. *Phys. Rev. Lett.* **1977**, *38*, 739–742. [[CrossRef](#)]
9. West, P.C. A geometric gravity Lagrangian. *Phys. Lett. B* **1978**, *76*, 569–570. [[CrossRef](#)]
10. Westman, H.; Złośnik, T. Exploring Cartan gravity with dynamical symmetry breaking. *Class. Quant. Grav.* **2014**, *31*, 95004. [[CrossRef](#)]
11. Magueijo, J.; Rodríguez-Vázquez, M.; Westman, H.; Złośnik, T. Cosmological signature change in Cartan Gravity with dynamical symmetry breaking. *Phys. Rev. D* **2014**, *89*, 63542. [[CrossRef](#)]
12. Westman, H.; Złośnik, T. An introduction to the physics of Cartan gravity. *Ann. Phys.* **2015**, *361*, 330–376. [[CrossRef](#)]
13. Alexander, S.; Cortès, M.; Liddle, A.; Magueijo, J.; Sims, R.; Smolin, L. The cosmology of minimal varying Lambda theories. *Phys. Rev. D* **2019**, *100*, 83507. [[CrossRef](#)]
14. Pagels, H.R. Gravitational gauge fields and the cosmological constant. *Phys. Rev. D* **1984**, *29*, 1690–1698. [[CrossRef](#)]
15. Westman, H.; Złośnik, T. Cartan gravity, matter fields, and the gauge principle. *Ann. Phys.* **2013**, *334*, 157–197. [[CrossRef](#)]
16. Lu, J.-A. Energy, momentum and angular momentum conservation in de Sitter gravity. *Class. Quantum Grav.* **2016**, *33*, 155009. [[CrossRef](#)]
17. Hehl, F.W.; von der Heyde, P.; Kerlick, G.D.; Nester, J.M. General relativity with spin and torsion: Foundations and prospects. *Rev. Mod. Phys.* **1976**, *48*, 393–416. [[CrossRef](#)]
18. Hehl, F.W.; McCrea, J.D.; Mielke, E.W.; Ne’eman, Y. Metric-affine gauge theory of gravity: Field equations, Noether identities, world spinors, and breaking of dilation invariance. *Phys. Rep.* **1995**, *258*, 1–171. [[CrossRef](#)]
19. Lu, J.-A.; Huang, C.-G. Kaluza–Klein-type models of de Sitter and Poincaré gauge theories of gravity. *Class. Quantum Grav.* **2013**, *30*, 145004. [[CrossRef](#)]
20. Guo, H.-Y. The local de Sitter invariance. *Kexue Tongbao (Chin. Sci. Bull.)* **1976**, *21*, 31–34.
21. Obukhov, Y.N. Poincaré gauge gravity: Selected topics. *Int. J. Geom. Meth. Mod. Phys.* **2006**, *3*, 95–138. [[CrossRef](#)]
22. Westman, H.; Złośnik, T. Gravity, Cartan geometry, and idealized waywisers. *arXiv* **2012**, arXiv:1203.5709. [[CrossRef](#)]
23. Brown, J.D. Action functionals for relativistic perfect fluids. *Class. Quant. Grav.* **1993**, *10*, 1579–1606. [[CrossRef](#)]
24. Magueijo, J.; Złośnik, T. Parity violating Friedmann Universes. *Phys. Rev. D.* **2019**, *100*, 84036. [[CrossRef](#)]
25. Planck Collaboration. Planck 2018 results. VI. Cosmological parameters. *Astron. Astrophys.* **2020**, *641*, A6. [[CrossRef](#)]
26. Riess, A.G.; Filippenko, A.V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P.M.; Gilliard, R.L.; Hogan, C.J.; Jsa, S.; Kirshner, R.P. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron. J.* **1998**, *116*, 1009–1038. [[CrossRef](#)]
27. Schmidt, B.; Suntzeff, N.B.; Phillips, M.M.; Schommer, R.A.; Clocchiatti, A.; Kirshner, R.P.; Garnavich, P.; Challis, P.; Leibundgut, B.; Spyromilio, J.; et al. The high-Z supernova search: Measuring cosmic deceleration and global curvature of the universe using type IA supernovae. *Astrophys. J.* **1998**, *507*, 46–63. [[CrossRef](#)]
28. Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R.A.; Nugent, P.; Castro, P.G.; Deustua, S.; Fabbro, S.; Goobar, A.; Hook, I.M.; et al. Measurements of Omega and Lambda from 42 high redshift supernovae. *Astrophys. J.* **1999**, *517*, 565–586. [[CrossRef](#)]
29. Prigogine, I.; Geheñiau, J.; Gunzig, E.; Nardone, P. Thermodynamics and cosmology. *Gen. Relativ. Gravit.* **1989**, *21*, 767–776. [[CrossRef](#)]
30. Calvão, M.O.; Lima, J.A.S.; Wagal, I. On the thermodynamics of matter creation in cosmology. *Phys. Lett. A* **1992**, *162*, 223–226. [[CrossRef](#)]