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The Very Long Lifetime of ^{14}C in the Shell Model

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Abstract: This is a fitting memory for our late friend and colleague Aldo Covello. For many years, he was our host in the series of Spring Seminars which he organized. In these conferences, the shell model was a central subject which was taken very seriously. This paper is written after 70 years of successful shell model calculations of nuclear energies and also various transitions. The beta decay of ^{14}C has been an enigma. The history and present situation are described. The importance check of any theory to yield the strength of the mirror transition of ^{14}O is pointed out.

Keywords: the shell model; ^{14}C beta decay; mirror decay of ^{14}O

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In this paper, I look back at 70 years of the nuclear shell model. Actually, there is a prehistoric part, which is close to 100 years old. Atomic nuclei were discovered by Rutherford in 1911. For several years, their composition was a mystery until the neutron was discovered by Chadwick in 1932. In the same year, Heisenberg published a paper in which he showed that nuclei are composed of protons and neutrons.

In those early days before mass-spectroscopy was used to measure nuclear binding energies, physicists used various transitions and reactions to determine that certain nuclei are more stable than others. In atoms, extra stability is associated with closed shells. In the same year, 1932, Bartlett suggested a similar structure for some nuclei [1]. He suggested that, in ^{4}He, there are closed 1s shells of protons and neutrons and in ^{16}O there was also a closed 1p shell. Not many papers followed Bartlett's idea. Most thorough and systematic ones were written by Elsasser [2–6]. By studying experimental data, he discovered several nuclei with extra stability. It was difficult for most physicists to understand how a system with a rather large number of particles interacting by strong short-range forces may be described by an independent particle model. In addition, the nature of the magic numbers was baffling. The lowest of them, 2, 8 and 20, could somehow make sense. Higher magic numbers, discovered by Elsasser, 50, 82 and 126, could be obtained only from very strange central potentials.

In a comprehensive review article, Bethe and Bacher [7] gave a description of nuclear physics in 1936. They present the shell model, arguments against it but also a case where only it seems to explain the data. A very devastating paper against any shell model was published in the same year by no lesser person than Niels Bohr [8]. He wrote: "In the atom and in the nucleus we have indeed to do with two extreme cases of mechanical many-body problems for which a procedure of approximation resting on a combination of one-body problems, so effective for the former case, loses any validity in the latter".

During World War II, nuclear physicists in major countries were occupied with work on nuclear weapons. In 1948, Maria Mayer published a detailed study [9] in which she showed that nuclei whose proton and/or neutron numbers were found by Elsasser to be magic have indeed extra stability. Mayer's paper revived the interest in the shell model. Feenberg and Hammack [10] and Nordheim [11] published detailed papers in which they tried to reproduce the new data in models similar to that of Elsasser. The shells which proposed contained certain orbits characterized by the orbital angular momenta l of the nucleons.

Maria Mayer [12] and, independently, Jensen et al. [13] introduced a novel idea. The shell structure, taken to be that of a harmonic oscillator central potential, is modified by a strong spin–orbit interaction. The l orbit is split into a lower $j = l + 1/2$ orbit and a higher $j = l - 1/2$ one. Both orbits remain in the same major shell with an important exception.

The spin–orbit interaction,

$$2a(\mathbf{s}.\mathbf{l}) = a[(\mathbf{s}+\mathbf{l}).(\mathbf{s}+\mathbf{l}) - \mathbf{l}.\mathbf{l} - \mathbf{s}.\mathbf{s}] = a[j(j+1) - l(l+1) - 3/4]\,, \tag{1}$$

is equal to al for $j = l + 1/2$ and to $-a(l+1)$ for $j = l - 1/2$. The $j = l + 1/2$ orbit with the highest value of l in a shell is mostly affected. If its energy is sufficiently pushed down (by a negative value of a), it may join the shell below it. This effect leads to the observed magic numbers. For example, the closed shells $1s, 1p, 1d2s, 1f2p$ contain 40 protons or neutrons but only when joined by the 10 protons (or neutrons) in the $1g_{9/2}(l = 4, j = 9/2)$ orbit is the magic number 50 reached. The simple Mayer–Jensen shell model has been widely accepted and used by experimentalists and theorists. Wigner, who made seminal contributions to nuclear physics, remained skeptical. He could not understand the origin of the interaction (1).

Apart from the radial dependence, wave functions in the shell model are well defined for ground states of closed shells nuclei. They remain well defined also if a single nucleon is added or removed from such nuclei. If there are several nucleons outside closed shells (valence nucleons), they may couple in several states (with the exception of two identical $j = 1/2$ nucleons or holes). To calculate energies and wave functions, it is necessary to calculate eigenvalues and eigenstates of the sub-matrix of the Hamiltonian, which is defined by states of the shell. Maria Mayer was aware of this situation and stated coupling rules for the spins of ground states. They were $J = 0$ for even-even nuclei of which there are no exceptions and J equal to j of one of the orbits in the shell for odd-even (or even-odd) nuclei. There are some exceptions to the second rule.

Mayer tried to find some theoretical basis for her rules. She looked at some j^n configurations of neutrons and of protons and "for simplicity", calculated their energy levels with a delta interaction [14]. The calculated ground state spins agreed with her rules! In addition, the pairing energy emerged from the calculation. She thought that the exceptions to her rule are due to the finite range of the interaction. Her student Dieter Kurath wrote a short paper [15] on this subject and so did I, a student of Pauli in Zurich [16].

In spite of the schematic nature of the zero range delta interaction, Mayer's novel approach made an important impact. In earlier calculations, it was assumed that the interaction between nucleons may be approximated by a constant over the nucleus. Short range interactions were not easy to handle in standard spectroscopy. Matrix elements of two-body interactions were expanded in terms of Slater integrals, each obtained from a term in the expansion of the integrand in terms of the particle coordinates r_1 and r_2 . If the interaction is constant where the wave functions do not vanish, only the $k = 0$ Slater integral does not vanish. If, however, the interaction is a delta function, all Slater integrals need not vanish; each is proportional to $2k + 1$.

Kurath in Ref. [15] showed that an argument of Racah against a ground state spin calculated in the shell model [17] is based on the $k = 0$ Slater integral. Racah created modern spectrometry for the earlier version of the nuclear shell model, but, when it became unpopular, he moved to atomic spectroscopy.

I had been looking for an expansion in which matrix elements of the delta interaction will have only one term. Instead of the Slater expansion of the interaction, the product of wave functions could be expanded in terms of the center-of-mass $\mathbf{R} = (\mathbf{r_1} + \mathbf{r_2})/2$ and relative coordinate $\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$. A simple and finite expansion occurs only for the kinetic energy and for the harmonic oscillator potential. The results should be independent of $\mathbf{R}$ since the interaction is translationally invariant [18]. This transformation is used now everywhere.

As mentioned above, for an efficient use of the shell model, it is necessary to know the two-body interaction. In the early days, people tried to use the interaction between free nucleons with or without some theoretical modifications. Even if agreement with some experimental data was obtained, no agreement with other ones could be reached. The standard excuse was "configuration mixing"—disagreements were blamed on the effect of shell model configurations not included. Such a procedure may lead to good agreement, but it has no meaning. An amusing example of failure of such a calculation is offered by dealing with calcium isotopes. In 1955, Ford and Levinson published a series of three papers on levels of ^{43}Ca. They started from levels of ^{42}Ca of which the first excited level, with $J = 2$, was known. Above it, two higher levels were measured, but their spins were unknown. Ford and Levinson took those spins to be $J = 4$ and $J = 6$, as in some other nuclei and, using their energies with those of the measured states, calculated positions of ^{43}Ca levels—so far, so good. However, they did not get correctly the position of the first excited $J = 5/2$ state. This situation was known to several authors who left this problem. Ford and Levinson did not give up but called for help with the mixing of other configurations with the $(f_{7/2})^n$ one. As if by miracle, the calculated position of the $J = 5/2$ state came out very close to the measured value and so was the case with some other observables. In the fall of 1956, there was an international conference on theoretical physics in Seattle, WA, USA. Jensen gave a talk on the nuclear shell model and the Ford–Levinson work seemed to be one of its successes. Not much later, the spins of the 2 levels, taken by Ford and Levinson to have $J = 4$ and $J = 6$, were measured to have spins $J = 0$ and $J = 2\ldots$ They do not belong to the $(f_{7/2})^2$ configuration. They are "intruder states" from some other configuration.

Slowly, the realization that the operators which should be used in a model may be very different from the real ones dawned on nuclear physicists. This was pointed out very clearly by Keith Brueckner and his collaborators. The problem was how to determine the effective interaction for the shell model in nuclei where the shell model seems to give a reasonable description. Clearly, no theoretical derivation seems easy for such a complex system. I have been looking for a case where the shell model description will indicate a simple configuration. I chose a case which seemed simple and in summer 1954 asked a student to study it and determine the effective two-body interaction. The system we considered was the four low lying states of ^{40}K which have spins obtained by coupling a $1d_{3/2}$ proton hole with a $1f_{7/2}$ neutron ($J = 2, 3, 4, 5$).

To check the consistency of our calculation, we calculated energies of states of another simple related shell model configuration. We chose the energy levels of the $1d_{3/2}$ proton—$1f_{7/2}$ neutron configuration ($J = 2, 3, 4, 5$) expected in ^{38}Cl. We looked at the levels published by 1954, and the only agreement was in the spin $J = 2$ of the ground state. We were disappointed but not surprised. We used pure jj-coupling wave functions which may have been rather extreme. In addition, it was not clear that it is a good approximation to use matrix elements from one nucleus in another one. Only in 1956 were accurate measurements of ^{38}Cl levels published, and they were in very good agreement with our calculated ones [19]. A few days after Ref. [19] was published, the case considered there appeared in a paper by Pandya [20]. He derived the Pandya relation expressing analytically particle–hole interaction as a linear combination of particle–particle interactions. He looked for an example and found the ^{38}Cl—^{40}K case.

The work and results of Ref. [19] started a new period in shell model calculations. I gave a talk on this work in a meeting of the Israel Physical Society and Racah said that this is the beginning of nuclear spectroscopy. Matrix elements of the effective interaction, diagonal and non-diagonal, were extracted from complex nuclei [21]. It took some time, but the successful calculations were convincing. When I was advocating this method, Arthur Kerman told me that, while he was a student at Caltech (California Institute of Technology, CA, USA), Richard Feynman was trying to use this method on light nuclei (probably in the $1p$ shell). In 1962, I gave a colloquium talk in Caltech. After the talk, Feynman told me that he tried this approach until, for a certain nucleus, his prediction was that four low lying levels are almost degenerate. He thought that this is very unlikely and dropped this approach. Perhaps Feynman talked about ^{16}N in which measured energies of the four lowest states are below 0.4 MeV.

Eigenvalues of the Hamiltonian obey the variational principle and hence may be calculated even with approximate wave functions. This method has been widely applied in calculations of energies and other observables [21]. Only one of the earlier papers and books, where this approach is discussed, is mentioned here. It referred to the specific case which is discussed in the following [22]. It is the ^{14}C beta decay which should be an allowed Gamow–Teller beta decay with a half life of a few days. Its observed lifetime is more than 5000 years. It has been a great gift to archaeology and a great enigma to nuclear structure physics.

The $J = 0$ ground state of ^{14}C, according to the shell model, is due to the two proton holes in the $1p$ orbit. This configuration has two possible $J = 0$ states, one with $S = 0$, $L = 0(^1S_0)$ and the other with $S = 1, L = 1(^3P_0)$. The single nucleon spin–orbit interaction Equation (1), which gives rise to the shell model, leads to a linear combination of these two states:

$$x|^1S_0> +y|^3P_0> . \tag{2}$$

This is an isospin $T = 1$ level. It decays by emitting an electron and neutrino to the $T = 0$ ground state of ^{14}N which has $J = 1$. In the p^{-2} configuration, there are three independent $T = 0$ states in which any state with $J = 1$ may be expressed as a linear combination of them. Thus, the ^{14}N ground state may be expressed as

$$\alpha|^3S_1> +\beta|^1P_1> +\gamma|^3D_1> . \tag{3}$$

The operator of the allowed beta decay is $\sigma = 2\,\mathbf{s}$, which has non-vanishing matrix elements only between states with the same part which depends on $\mathbf{r}$. Thus, in the case considered here, it is equal to a linear combination of αx and βy. Its precise value is

$$\sqrt{6}(\alpha x - \beta y/\sqrt{3})\,. \tag{4}$$

Inglis noted that, if the effective forces are central and even in the presence of spin-orbit interaction, the matrix element (4) cannot vanish [23]. He argued, as shown below, that the coefficients x and y in Equation (2) have the same signs. In the case of central-and spin-orbit interactions, the $L = 2$ D state has a non-vanishing non-diagonal matrix element only with the $^1P_1$ state. This is due to the single particle spin–orbit interaction. The α and β coefficients in Equation (3) have opposite signs. As a result, the matrix element (4) cannot vanish. Inglis suggested that the near vanishing of the matrix element is due to mixing of higher configurations into states of the $1p$ shell.

This is the situation, in the extreme case of jj-coupling. The coefficients of the $(1p_{1/2})^{-2}$ state with $J = 0, T = 1$ are $x = \sqrt{1/3}$ and $y = \sqrt{2/3}$. The coefficients of the $J = 1, T = 0$ state are $\alpha = -\sqrt{1/27}, \beta = \sqrt{6/27}$ and $\gamma = \sqrt{20/27}$ as in Ref. [23].

In 1954, Jancovici, a graduate student in Princeton and I, a visitor there, noticed that the situation is changed if tensor forces are included in the shell model interactions. The $^3D_1$ state may then strongly interact with the $^3S_1$ state in addition to its spin–orbit interaction with the $^1P_1$ state. If this interaction is sufficiently strong, the signs of the α and β become equal and cancellation or near cancellation may be possible [24]. That work was carried out in the period when various interactions were used taken from attempts to extract them from analysis of nucleon–nucleon reactions. Ref. [24] was no exception, and this was the case with the several publications which followed it. One conclusion was clear: tensor forces may play an important role in ^{14}C beta decay.

Cohen and Kurath [22] carried out shell model calculations in the $1p$ shell. Following our paper [24] and its followers, they determined the effective interaction by attempting to achieve the best fit to measured energy levels. No attempt was made to obtain the longevity of ^{14}C, but they refer to it. They write in their comments on their results on ^{14}N: "The major point of interest here concerns the beta decay of ^{14}C which has been difficult for the shell model … actually, changing the ^{14}N ground state to one which has an over-lap of 0.998 with the state from the latter case would produce the nearly exact cancellation indicated by experiment". They mention a possible effect of mixing with configurations from a higher shell. The experimental situation is still complex and the results of Ref. [22] support the view that the longevity of ^{14}C may have a simple explanation in the shell model.

The wave-function [2] is an eigenstate of the sub-matrix of the Hamiltonian

$$\begin{array}{c|cc|} {}^3P_0 & E(^3P_0) & -2a\sqrt{2} \\ {}^1S_0 & -2a\sqrt{2} & E(^1S) \end{array} = \begin{vmatrix} E(^3P_0)-E' & -2a\sqrt{2} \\ -2a\sqrt{2} & E(^1S)-E' \end{vmatrix} + \begin{vmatrix} E' & 0 \\ 0 & E' \end{vmatrix} \tag{5}$$

In the matrices (5), $E(^3P_0)$ and $E(^1S_0)$ are the energies of the states before diagonalization. E'' is the lower $J = 0$ eigenvalue of Hamiltonian (5), either of the left or right side of it. The higher eigenvalue, E' , lies 13.75 MeV above E''. To diagonalize the matrix on the left, it is sufficient to diagonalize the left matrix on the right. Diagonalization of the matrix on the left yields the two eigenvalues E'' and E'. Hence, diagonalization of the matrix on the right yields the corresponding eigenvalues 0 and E''–E' . This difference is taken here to be –13.75 MeV.

The difference between the two eigenvalues of the spin–orbit interaction (1) of a single nucleon is

$$E_{l+1/2} - E_{l-1/2} = a(2l+1)\,. \tag{6}$$

The value of a in the case considered here can be obtained from the difference of the single hole states in ^{15}N. The ground state of ^{15}N is a $J = 1/2^-$ state, and a $J = 3/2^-$ state is 6.324 MeV above it. Thus, the value of a is 2.108 MeV, the coefficient in Equation (1) is 4.216 and the non-diagonal matrix element in Hamiltonian (5) is $-4.216\sqrt{2} = -5.962$ MeV.

The matrix to be diagonalized:

$$\begin{array}{c|cc} {}^3P_0 & U & -4.216\sqrt{2} \\ {}^1S_0 & -4.216\sqrt{2} & W \end{array} \tag{7}$$

Since one of its eigenvalues is 0, its determinant vanishes so that $UW = 8 \times 2.108^2 = 8 \times 4.442 = 35.54$. The trace of matrix (7) is equal to the trace of the diagonalized matrix. Thus, $U + W = (E'' - E') + (E' - E') = -13.75$ MeV. From these values follows $U = -10.3$ and $W = -3.45$ MeV. The derivations above may be summarized by a simple expression of U and W. The matrix (7) has an eingenvalue 0 and hence its determinant vanishes:

$$UW - 8a^2 = 0\,. \tag{8}$$

The other eigenvalue, N, is equal to the trace of the matrix, $E = U + W$. Thus, Equation (8) may be rewritten as

$$U(N-U) - 8a^2 = 0 = U^2 - UN + 8a^2 = (U - N/2)^2 - N^2/4 + 8a^2\,. \tag{9}$$

From Equation (9), the explicit expression for the lower diagonal matrix element follows:

$$U = \frac{1}{2}(E - \sqrt{(E^2 - 4(8a^2))}\,. \tag{10}$$

The higher element is given by

$$W = \frac{1}{2}(E + \sqrt{(E^2 - 4(8a^2))}\,. \tag{11}$$

The sum of Equations (10) and (11) is $U + W = N$ and their product is

$$UW = (E - \sqrt{(E^2 - 4(8a^2))})(E + \sqrt{(E^2 - 4(8a^2))})/4 = (E^2 - (E^2 - 4(8a^2)))/4 = 8a^2\,. \tag{12}$$

The coefficients of the eigenstate (3) can be obtained from the matrix (5) by

$$\begin{vmatrix} \overline{-10.3 - 5.962} & \overline{y} \\ -5.962 - 3.45 & x \end{vmatrix} = -13.75 \begin{vmatrix} \overline{y} \\ x \end{vmatrix} \tag{13}$$

The relation (8) is satisfied by

$$x = (13.75 - 10.3)y/5.962 = 0.579y \text{ or } x = 5.962y/(13.75 - 3.45) = 0.579y. \tag{14}$$

The normalized coefficients are equal, to a good approximation, to $x = 0.5$ and $y = \sqrt{0.75}$. The wave function (2) with these coefficients has an overlap larger than 0.995 with the $J = 0$ state of two $p_{1/2}$ holes.

The amplitudes in the ^{14}N ground state, the coefficients of Equation (4), may be obtained by diagonalization of the shell model sub-matrix

$$\begin{array}{c|lll} ^3S_1 & V(S) & a\sqrt{2/3} & V_T \\ ^1P_1 & a\sqrt{2/3} & V(P) & -a\sqrt{5/6} \\ ^3D_1 & V_T & -a\sqrt{5/6} & V(D) - a(3/2) \end{array} \tag{15}$$

As shown in Ref. [24], due to the tensor forces, the coefficients α and β may have the same sign. If it is absent, they have different signs due to the positive non-diagonal element of the spin–orbit interaction between the $^3S_1$ and the $^1P_1$ states.

The matrix element of the Gamow–Teller beta decay is given by Equation (4) as

$$\sqrt{6}(x\alpha - y\beta/\sqrt{3}). \tag{16}$$

Due to the values of x and y obtained above, $\sqrt{0.75}/\sqrt{3} = \sqrt{0.25} = 0.5$ and this matrix element vanishes for any equal coefficients, $\alpha = \beta$. The experimental situation in ^{14}N is more complicated than in ^{14}C. In the following, we assume that the matrix element (11) is strongly reduced. Using plausible coefficients, it is possible to check this mechanism by looking at the mirror beta decay. The faster decay of ^{14}O to the ^{14}N ground state should be due to the difference in Coulomb energies. The values of α and β in obtaining 0 or close to them may occur in the actual nuclei. The measured matrix element in the ^{14}C is actually very small but not exactly zero.

As stated above, the near cancellation of the beta decay matrix element in ^{14}C, should not occur for the mirror transition of ^{14}O. In the case considered above, states are of two proton holes interacting also by the Coulomb interaction. In ^{14}O, there are neutron holes and the difference [10] is slightly smaller, 6.176 MeV. Hence, a = 2.059, the coefficient in Equation (1) is $2a$ = 4.118 and the non-diagonal matrix element in Hamiltonian (5), in the present case, is $-4.118\sqrt{2} = -5.824$ MeV.

In matrix (5), E($^3P_0$) includes -2α, and its energy should be increased by 0.1 MeV. Using Equation (10) in this case yields $U = \frac{1}{2}(-13.65 - \sqrt{13.65^2 - 4 \times 5.824^2}) = -10.38$ MeV and $W = -3.27$ MeV. In the case considered now, $x = 3.27y/5.824 = 0.561$ and $x = 10.38y/13.65 = 0.561$. The normalized coefficients are x = 0.489 and y = 0.872. To calculate the beta decay matrix element (9), it is not sufficient to take $\alpha = \beta$. A plausible choice, consistent with a large value of γ, is $\alpha = \beta = 0.3$. With this value, the square of the element (9) becomes equal to $0.09 \times 6(0.489 - 0.872/\sqrt{3})^2 = 0.000104$. The ft value is obtained by dividing 5300 by this number [25]. Thus, $\log ft = \log(5300/0.000104) = 7.7$, which is in the region of experimental results.

The importance of ^{14}C dating in archaeology is demonstrated by the University of Cambridge which publishes a journal called *Radicarbon*. Most of the articles in it deal with applications of ^{14}C, but, several years ago, an issue was devoted to the nuclear physics behind the phenomenon. The paper "The half-life of ^{14}C—Why is it so long?" by Kutschera was published online by Cambridge University Press [26]. It contains a detailed bibliography on this subject—Ref. [24]—as well as recent large scale computations. No results on beta decay of ^{14}O are presented. In the Introduction, Kutschera states that, in the approach of Ref. [24], using the simple shell model, some reduction of the rate of ^{14}C beta decay is obtained but not sufficiently. No reference is quoted. This approach may turn out to be not the correct one but not for the reason given in *Radiocarbon.*

Only three of the many papers listed in Ref. [27] are mentioned here. Fayache, Zamick, and Muther considered central, spin–orbit and tensor forces [28]. They considered, however, also mixing of nearby configurations and various values of the interactions. They found a set of values which fits the data. They quote a theoretical derivation of these values (if "the enhancement of the small component of the Dirac spinors of the nucleons is taken into account").

Negret et al. (38 authors, all listed in the References) present experimental results relevant to beta decay from which information on A = 14 nuclei may be deduced. The theoretical analysis is based on calculations in which shell model wave functions were used, but no central potential (core) is assumed, NCSM. Not all low levels appear and the big reduction of ^{14}C beta decay is not reproduced. The important result is that the main component of the $J = 1$, $T = 0$ ground state of ^{14}N is indeed $^3D_1$. The authors, like some others, believe that clustering is an important ingredient that should be included. The authors of the next paper do not share this opinion.

Maris, Vary, Navratil, Ormand, Nam, and Dean use "ab initio no-core shell model" in their calculation [29]. The Hamiltonian is taken from "the chiral effective field theory including three-nucleon force terms". They find that the latter have a large effect leading to the large reduction of the beta decay rate of ^{14}C. If this sounds simple, the order of the matrix with which they deal is 872,999,912. The number of non-vanishing, diagonal and non-diagonal 3NF (3-nucleon-forces) matrix elements is about 2×10^{13}. The shell model is supposed to give some simplicity from the complexity of the nuclear many body system. The results of this paper are far from simple.

To find if the shell model is sufficiently detailed to yield the results of beta decay of ^{14}C and ^{14}O, it may be necessary to understand the level structure of these nuclei and of ^{14}N. It may be necessary to consider possible mixing between levels of the $1f2$p configuration and some $1g2$d levels. Even small admixtures may affect the values of the small decay rates. At this time, it is too early to give up the hope.

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References

1. Bartlett, J.N. Nuclear structure. *Nature* **1932**, *130*, 165. [CrossRef]
2. Elsasser, W.M. Sur le principe de Pauli dans les noiyaux. *J. Phys. Radium* **1933**, *4*, 549–556. [CrossRef]
3. Elsasser, W.M. Remarques sur la constitution des noyaux atomiques. I. *J. Phys. Radium* **1934**, *5*, 253–256. [CrossRef]
4. Elsasser, W.M. Sur le principe de Pauli dans les noiyaux. II. *J. Phys. Radium* **1934**, *5*, 389–397. [CrossRef]
5. Elsasser, W.M. Remarques sur la constitution des noyaux atomiques. II. *J. Phys. Radium* **1934**, *5*, 475–485. [CrossRef]
6. Elsasser, W.M. Sur le principe de Pauli dans les noiyaux. III. *J. Phys. Radium* **1934**, *5*, 635–639. [CrossRef]
7. Bethe, H.A.; Bacher, R.F. Nuclear Physics. A. Stationary states of nuclei. *Rev. Mod. Phys.* **1936**, *8*, 82–229. [CrossRef]
8. Bohr, N. Neutron capture and nuclear constitution. *Nature* **1936**, *137*, 344–348. [CrossRef]
9. Mayer, M.G. On closed shells in nuclei. *Phys. Rev.* **1948**, *74*, 235–239. [CrossRef]
10. Feenberg, E.; Hammack, K.C. Nuclear shell structure. *Phys. Rev.* **1949**, *75*, 1877–1893. [CrossRef]
11. Nordheim, L. On spins, moments, and shells in nuclei. *Phys. Rev.* **1949**, *75*, 1894–1901. [CrossRef]
12. Mayer, M.G. On closed shells in nuclei. II. *Phys. Rev.* **1949**, *75*, 1969–1970. [CrossRef]
13. Haxel, O.; Jensen, J.H.D.; Suess, H.E. Zur interpretation der ausgezeichneten Nucleonzahlen im Bau der Atomkerne. *Naturwiss* **1948**, *5*, 376. (Received on 12 February 1949). [CrossRef]
14. Mayer, M.G. Nuclear configurations in the spin-orbit coupling model II. Theoretical considerations. *Phys. Rev.* **1950**, *78*, 22–23. [CrossRef]
15. Kurath, D. Effect of finite range interactions in the *jj* coupling shell model. *Phys. Rev.* **1950**, *80*, 98–99. [CrossRef]
16. Talmi, I. On the spin of the nuclear ground state in the *jj*-coupling scheme. *Phys. Rev.* **1951**, *82*, 101–102. [CrossRef]

17. Racah, G. Nuclear coupling and shell model. *Phys. Rev.* **1950**, *78*, 622–623. [CrossRef]
18. Talmi, I. Nuclear spectroscopy with harmonic oscillator wave-functions. *Helv. Phys. Acta* **1952**, *25*, 185–234. Available online: https://www.e-periodica.ch/digbib/view?pid=hpa-001 (accessed on 3 August 2022)
19. Goldstein, S.; Talmi, I. Related *jj*-coupling configurations in K^{40} and Cl^{38}. *Phys. Rev.* **1956**, *102*, 589–590. [CrossRef]
20. Pandya, S.P. Nucleon–hole interaction in *jj* coupling. *Phys. Rev.* **1956**, *103*, 956–957. [CrossRef]
21. Talmi, I. Effective interactions and coupling schemes in nuclei. *Rev. Mod. Phys.* **1962**, *34*, 704–722. [CrossRef]
22. Cohen, S.; Kurath, D. Effective interactions for the 1p shell. *Nucl. Phys.* **1965**, *73*, 1–24. [CrossRef]
23. Inglis, D.R. The energy levels and the structure of light nuclei. *Rev. Mod. Phys*. **1953**, *25*, 390–450. [CrossRef]
24. Jancovici, B.; Talmi, I. Tensor forces and the β decay of ^{14}C and ^{14}O. *Phys. Rev.* **1954**, *95*, 289–291. [CrossRef]
25. Winther, A.; Kofoed-Hansen, O. On the coupling constants of β-decay. Evidence from allowed transitions. *Mat. Fys. Medd. Dan. Vid. Selsk.* **1953**, *27/14*, 1–22. Available online: http://gymarkiv.sdu.dk/MFM/kdvs/mfm (accessed on 3 August 2022).
26. Kutschera, W. The half-life of ^{14}C——Why is it so long? *Radiocarbon* **2019**, *61*, 1135–1142. [CrossRef]
27. Fayache, M.S.; Zamick, L.; Müther, H. Vanishing Gamow-Teller rate for $A = 14$ and the nucleon–nucleon interaction in the medium. *Phys. Rev. C* **1999**, *60*, 067305. [CrossRef]
28. Negret, A.; Adachi, T.; Barrett, B.R.; Baumer, C.; van den Berg, A.M.; Berg, G.P.A.; Von Brentano, P.; Frekers, D.; De Frenne, D.; Fujita, H.; et al. Gamow-Teller strengths in the $A = 14$ multiplet: A challenge to the shell model. *Phys. Rev. Lett.* **2006**, *97*, 062502. [CrossRef]
29. Maris, P.; Vary, J.P.; Navratil, P.; Ormand, W.E.; Nam, H.; Dean, D.J. Origin of the anomalous long lifetime of ^{14}C. *Phys. Rev. Lett.* **2011**, *106*, 202502. [CrossRef]