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Review

Modeling Turbulence in Permeable Media: The Double-Decomposition Concept Revisited

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Abstract: In this article, a concept named double decomposition, which is used to model turbulent flows in porous media, is examined. This concept is based on the idea that in a turbulent flow through a porous matrix, local instantaneous variables can be averaged in time and space, simultaneously. Depending on how these operators are applied, averaged equations take different forms. In this article, instantaneous local equations are averaged using both operators and a different set of equations resulting from such operations are commented upon. Additional terms proposed for the averaged equations are discussed.

Keywords: turbulent flow; modeling; double decomposition; permeable structures; porous media

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1. Introduction

Modeling of macroscopic transport for laminar flows in permeable media can be based on the volume-average methodology [1]. When properties of the fluid vary with time in addition to spatial deviations, two possible methodologies can be followed in order to obtain macroscopic equations. One can first apply time averaging [2], or else, one can start with volume averaging [3]. These two approaches lead to distinct sets of equations. However, the order of the averaging is immaterial if the variables are split using the double-decomposition theory, which is fully described in [4] and briefly revisited here. This new concept sheds some light on the existing controversy about which order mathematical operators should be applied to governing equations when double averaging the equation set.

The double-decomposition idea was initially developed for the flow variables in porous media and extended to several flows and cases. A review of all these cases is beyond the scope of the present paper and, to the interested reader, the fundamentals of the double-decomposition concept can be found in [5] in addition to its extension to moving media, which is detailed in [6]. Examples including double diffusion [7] and turbulent combustion in porous media [8] are mentioned here as applications of the ideas compiled in [4]. Other studies in the literature dealing with modeling turbulence in porous media include a review on methodologies [9], Direct Numerical Simulations (DNS) [10], two-scale models [11] and flow in channels with permeable walls [12].

Further, the study of turbulence though porous media can find application in several areas, including engineering and environmental research. Examples of the former are studies in porous combustors [13], moving bed reactors [14], thermal energy systems [15], volumetric solar collectors [16], impinging jets for cooling or heating [17], among others. Applications of the study of turbulence in porous media can also be useful to environmental research such as simulation of forest fires [18] and flows over vegetations [19].

Here, a brief review of this new double-decomposition concept is revisited, but with no much of details, which can be found in [4–8].

2. Flow Equations

The local instantaneous continuity and momentum equations are given as [4–6]:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\rho \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho\, \mathbf{g}, \tag{2}$$

where $\mathbf{u}$, ρ, p, μ and $\mathbf{g}$ are the velocity vector, density, pressure, fluid viscosity and gravity acceleration vector, respectively.

2.1. Averaging Operators

Any general quantity φ can be averaged in time as:

$$\overline{\varphi} = \frac{1}{\Delta t} \int_t^{t+\Delta t} \varphi\, dt, \tag{3}$$

giving then

$$\varphi = \overline{\varphi} + \varphi', \tag{4}$$

where φ' is the time fluctuation of φ around its average value, $\overline{\varphi}$, with $\overline{\varphi'} = 0$.

Likewise, the volume average of φ over a Representative Elementary Volume (REV) in a porous medium is given by [20]

$$\langle \varphi \rangle^v = \frac{1}{\Delta V} \int_{\Delta V} \varphi\, dV, \tag{5}$$

being,

$$\left\langle \varphi_f \right\rangle^v = \phi \left\langle \varphi_f \right\rangle^i, \tag{6}$$

where $\phi = \Delta V_f / \Delta V$ is the porosity of the medium, ΔV_f is the volume occupied by the fluid and ΔV is the total volume (fluid plus solid). Further,

$$\varphi = \langle \varphi \rangle^i + {}^i\varphi, \ \left\langle {}^i\varphi \right\rangle^i = 0, \tag{7}$$

where ${}^i\varphi$ is the spatial deviation of ϕ with respect to the volume average, $\langle \varphi \rangle^i$.

Gradient, divergent and time rate are also volume averaged as [1,20,21]:

$$\langle \nabla \varphi \rangle^v = \nabla(\phi \langle \varphi \rangle^i) + \frac{1}{\Delta V} \int_{A_i} \mathbf{n} \varphi dS, \tag{8}$$

$$\langle \nabla \cdot \boldsymbol{\varphi} \rangle^v = \nabla \cdot (\phi \langle \boldsymbol{\varphi} \rangle^i) + \frac{1}{\Delta V} \int_{A_i} \mathbf{n} \cdot \boldsymbol{\varphi}\, dS, \tag{9}$$

$$\langle \frac{\partial \varphi}{\partial t} \rangle^v = \frac{\partial}{\partial t}(\phi \langle \varphi \rangle^i) - \frac{1}{\Delta V} \int_{A_i} \mathbf{n} \cdot (\mathbf{u}_i \varphi) dS, \tag{10}$$

where A_i is the interfacial area, $\mathbf{u}_i$ is the velocity of phase i, $\mathbf{n}$ is the unit vector normal to A_i, φ is a scalar and $\boldsymbol{\varphi}$ a vector.

2.2. Averaged Balance Equations

By applying time averaging to Equations (1) and (2), one gets:

$$\nabla \cdot \bar{\mathbf{u}} = 0, \tag{11}$$

$$\rho\nabla\cdot(\bar{\mathbf{u}}\bar{\mathbf{u}}) = -(\nabla\,\overline{p})^{*} + \mu\nabla^{2}\bar{\mathbf{u}} + \nabla\cdot(-\rho\,\overline{\mathbf{u}'\mathbf{u}'}), \tag{12}$$

with

$$-\rho\overline{\mathbf{u}'\mathbf{u}'} = \mu_t 2\overline{\mathbf{D}} - \frac{2}{3}\rho k\,\mathbf{I}, \tag{13}$$

where $\overline{\mathbf{D}}$, k, μ_t and $\mathbf{I}$ are the mean deformation tensor, the turbulent kinetic energy per unit mass, the turbulent viscosity and the unity tensor, respectively, and $P_k = -\rho\overline{\mathbf{u}'\mathbf{u}'} : \nabla\bar{\mathbf{u}}$ is the production of k due to the gradients of $\mathbf{u}$.

3. Double Decomposition

Using the operators defined above, applied simultaneously, any variable can be split into the four components, as (see [4–6] for details):

$$\begin{aligned}\varphi &= \langle\overline{\varphi}\rangle^{i} + \langle\varphi'\rangle^{i} + {}^{i}\overline{\varphi} + {}^{i}(\varphi') \\ &= \overline{\langle\varphi\rangle^{i}} + \langle\varphi\rangle^{i'} + \overline{{}^{i}\varphi} + ({}^{i}\varphi)'\end{aligned} \tag{14}$$

or

$$\varphi = \underbrace{\langle\overline{\varphi}\rangle^{i} + \langle\varphi'\rangle^{i}}_{\langle\varphi\rangle^{i}} + \overbrace{{}^{i}\overline{\varphi} + {}^{i}(\varphi')}^{{}^{i}\varphi} = \underbrace{\overline{\langle\varphi\rangle^{i}} + \overline{{}^{i}\varphi}}_{\overline{\varphi}} + \overbrace{\langle\varphi\rangle^{i'} + ({}^{i}\varphi)'}^{\varphi'}. \tag{15}$$

Equation (15) entails the double-decomposition concept and the significance of the four terms on the right-hand-side of Equation (14) can be reviewed as: (1) $\langle\overline{\varphi}\rangle^{i}$ is the intrinsic average of the time mean value of φ, (2) $\langle\varphi'\rangle^{i}$ is the volume average of the fluctuating component, (3) ${}^{i}\overline{\varphi}$ is the deviation of the time-averaged value and (4) ${}^{i}(\varphi')$ is the spatial deviation of the time fluctuation.

It is also noted that

$${}^{i}(\varphi') = ({}^{i}\varphi)';\ \langle{}^{i}\varphi'\rangle^{i} = \overline{{}^{i}\varphi'} = 0. \tag{16}$$

4. Momentum Equation

Applying both the volume and time averaging gives for the continuity Equation (11),

$$\nabla\cdot(\phi\left\langle\bar{\mathbf{u}}\right\rangle^{i}) = 0, \tag{17}$$

irrespective of the order of integration [5,6].

Also, regardless of the order of application of the average operations over the momentum equation, one has:

$$\begin{aligned}&\rho\left[\frac{\partial}{\partial t}(\phi\left\langle\bar{\mathbf{u}}\right\rangle^{i}) + \nabla\cdot(\phi\left\langle\bar{\mathbf{u}}\,\bar{\mathbf{u}}\right\rangle^{i})\right] = \\ &\quad -\nabla(\phi\langle p\rangle^{i}) + \mu\nabla^{2}(\phi\left\langle\bar{\mathbf{u}}\right\rangle^{i}) + (\phi\rho\left\langle\overline{\mathbf{u}'\mathbf{u}'}\right\rangle^{i}) + \phi\rho\,\mathbf{g} + \bar{\mathbf{R}}\end{aligned} \tag{18}$$

where

$$\bar{\mathbf{R}} = \frac{\mu}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\nabla\bar{\mathbf{u}})dS - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}\overline{p}dS, \tag{19}$$

is the time-averaged total drag force per unit volume.

Applying the double-decomposition idea seen before to the inertia term $\nabla \cdot (\phi \langle \bar{\mathbf{u}}\,\bar{\mathbf{u}} \rangle^i)$ will result in

$$\nabla \cdot \overline{[\phi(\langle \mathbf{u} \rangle^i \langle \mathbf{u} \rangle^i + \langle {}^i\mathbf{u}\,{}^i\mathbf{u} \rangle^i)]} = \nabla \cdot \{\phi\, [\underbrace{\langle \bar{\mathbf{u}} \rangle^i \langle \bar{\mathbf{u}} \rangle^i}_{\mathrm{I}} + \underbrace{\overline{\langle \mathbf{u}' \rangle^i \langle \mathbf{u}' \rangle^i}}_{\mathrm{II}} + \underbrace{\langle {}^i\bar{\mathbf{u}}\,{}^i\bar{\mathbf{u}} \rangle^i}_{\mathrm{III}} + \underbrace{\langle \overline{{}^i\mathbf{u}'\,{}^i\mathbf{u}'} \rangle^i}_{\mathrm{IV}}]\}. \tag{20}$$

The meaning of the four terms on the right-hand side of Equation (20) can be seen as: I—*convective* term of $\langle \bar{\mathbf{u}} \rangle^i$, II—*turbulent Reynolds stresses* over ρ due to $\langle \mathbf{u}' \rangle^i$, III—*dispersion* due to ${}^i\bar{\mathbf{u}}$, a term also present in laminar flow and IV—*turbulent dispersion* in a porous medium due to ${}^i\mathbf{u}'$.

5. Macroscopic Two-Equation Models

The turbulence kinetic energy can be defined in two ways depending on the order of integration in the time and volume domains. In Ref. [3], macroscopic k was defined as $k_m = \overline{\langle \mathbf{u}' \rangle^i \cdot \langle \mathbf{u}' \rangle^i}/2$. On the other hand, if one proceeds with time averaging first, one ends up with $\langle k \rangle^i = \langle \overline{\mathbf{u}' \cdot \mathbf{u}'} \rangle^i / 2$ [2]. The derivation of transport equations for k_m and $\langle k \rangle^i$ can be found in [4].

The modeled equations read,

$$\rho \frac{\partial(\phi\, k_m)}{\partial t} + \rho \nabla \cdot [\phi \langle \bar{\mathbf{u}} \rangle^i k_m] = \nabla \cdot \left[\mu + \frac{\mu_{t_m}}{\sigma_{k_m}} \nabla(\phi\, k_m)\right] + P_m - \rho\phi\varepsilon_m - D_m, \tag{21}$$

where D_m represents the dispersion of k_m, which reads [4]:

$$D_m = \rho \overline{\langle \mathbf{u}' \rangle^i \cdot \{\nabla \cdot [\phi\, (\langle {}^i\bar{\mathbf{u}}\,{}^i\mathbf{u}' \rangle^i + \langle {}^i\mathbf{u}'\,{}^i\bar{\mathbf{u}} \rangle^i + \langle {}^i\mathbf{u}'\,{}^i\mathbf{u}' \rangle^i)]\}}, \tag{22}$$

and involves correlations between deviations of velocity, ${}^i\bar{\mathbf{u}}$ and ${}^i\mathbf{u}'$, $P_m = -\rho\phi \overline{\langle \mathbf{u}' \rangle^i \langle \mathbf{u}' \rangle^i} : \nabla \langle \bar{\mathbf{u}} \rangle^i$ is the production rate of k_m due to the gradient of the macroscopic time-mean velocity $\langle \bar{\mathbf{u}} \rangle^i$ and,

$$\rho \left[\frac{\partial}{\partial t}(\phi \langle k \rangle^i) + \nabla \cdot (\bar{\mathbf{u}}_D \langle k \rangle^i)\right] = \nabla \cdot \left[(\mu + \frac{\mu_{t_\phi}}{\sigma_k}) \nabla(\phi \langle k \rangle^i)\right] + P_i + G_i - \rho\phi \langle \varepsilon \rangle^i, \tag{23}$$

where

$$P_i = -\rho \langle \overline{\mathbf{u}'\,\mathbf{u}'} \rangle^i : \nabla \bar{\mathbf{u}}_D,\ G_i = c_k \rho \phi \frac{\langle k \rangle^i \left|\bar{\mathbf{u}}_D\right|}{\sqrt{K}}, \tag{24}$$

are the production rate of $\langle k \rangle^i$ due to the mean gradients of $\bar{\mathbf{u}}_D$ and the generation rate of intrinsic k due the presence of the porous matrix, respectively, σ_k is a constant, K is the medium permeability and c_k is a constant to be commented below. The term G_i represents extra production of $\langle k \rangle^i$ due to the presence of solid material inside the integration volume and it is of null value for the limiting case of clear fluid flow when $\phi \to 1 => K \to \infty$.

5.1. Constant c_k for the Macroscopic Model

The constant c_k introduced in Equation (24) was determined for the closure of the mathematical model. For macroscopic fully developed unidimensional flow in isotropic and homogeneous media, the limiting values for $\langle k \rangle^i$ and $\langle \varepsilon \rangle^i$ are given the values k_ϕ and

ε_ϕ, respectively. In this limiting condition, transport terms are neglected and production and dissipation terms balance each other reducing the equations to:

$$\langle\varepsilon\rangle^i = c_k \frac{\langle k\rangle^i \left|\bar{\mathbf{u}}_D\right|}{\sqrt{K}}, \quad \frac{\langle\varepsilon\rangle^{i2}}{\langle k\rangle^i} = c_k \frac{\langle\varepsilon\rangle^i \left|\bar{\mathbf{u}}_D\right|}{\sqrt{K}}, \tag{25}$$

or

$$\varepsilon_\phi = c_k \frac{k_\phi \left|\bar{\mathbf{u}}_D\right|}{\sqrt{K}}, \quad \frac{{\varepsilon_\phi}^2}{k_\phi} = c_k \frac{\varepsilon_\phi \left|\bar{\mathbf{u}}_D\right|}{\sqrt{K}}. \tag{26}$$

Or say, for a steady-state fully developed one-dimensional macroscopic flow through an infinite porous medium ($\nabla\bar{\mathbf{u}}_D = 0$), transient and transport terms in Equations (23) and (31) vanish, in addition to $P_i = 0$. In this particular case, production due to the porous matrix, G_i, balances the dissipation of k, giving rise to Equation (25). Renaming k and ε for this particular condition, we have Equation (26). Therefore, k_ϕ and ε_ϕ are the values for $\langle k\rangle^i$ and $\langle\varepsilon\rangle^i$, respectively, when turbulence attains the equilibrium state, that is, when production and dissipation mechanisms solely dictate the levels of k and ε.

Using now the limiting cases k_ϕ and ε_ϕ, both Equations (26) can be combined into the non-dimensional forms:

$$\frac{\varepsilon_\phi\sqrt{K}}{\left|\bar{\mathbf{u}}_D\right|^3} = c_k \frac{k_\phi}{\left|\bar{\mathbf{u}}_D\right|^2}, \tag{27}$$

or

$$c_k = \frac{\varepsilon_\phi\sqrt{K}}{k_\phi\left|\bar{\mathbf{u}}_D\right|}. \tag{28}$$

In order to determine c_k from Equation (28), microscopic computations in unit cells for different porosity and permeabilities were used to calculate the corresponding limiting values k_ϕ and ε_ϕ (see Ref. [4] for details). Once these intrinsic values were obtained, they were plugged into Equation (27). The value of c_k equal to 0.28 was found by noting the collapse of all data into a straight line.

5.2. Proposals for Macroscopic k

If we compare the definitions of k_m and $\langle k\rangle^i$ considering the above, we get [4]:

$$\begin{aligned}\underbrace{\langle k\rangle^i}_{\text{following time} > \text{vol. int.}} &= \left\langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\right\rangle^i \Big/ 2 = \overline{\langle\mathbf{u}'\rangle^i\cdot\langle\mathbf{u}'\rangle^i} \Big/ 2 + \langle\overline{{}^i\mathbf{u}'\cdot{}^i\mathbf{u}'}\rangle^i \Big/ 2 \\ &= \underbrace{k_m}_{\text{following vol.} > \text{time int}} + \underbrace{\langle\overline{{}^i\mathbf{u}'\cdot{}^i\mathbf{u}'}\rangle^i/2}_{\text{extra turb. knetic energy}}\end{aligned} \tag{29}$$

where "vol. int." means volume integration. We see that not all the turbulent kinetic energy of the flow is accounted for by k_m. Further, if we expand the term P_i by means of Equation (7), a connection between the two generation rates can also be written as follows:

$$\begin{aligned}\underbrace{P_i}_{\text{following time} > \text{vol. int.}} &= -\rho\left\langle\overline{\mathbf{u}'\mathbf{u}'}\right\rangle^i : \nabla\bar{\mathbf{u}}_D \\ &= -\rho\left(\overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i} : \nabla\bar{\mathbf{u}}_D + \overline{\langle{}^i\mathbf{u}'\,{}^i\mathbf{u}'\rangle^i} : \nabla\bar{\mathbf{u}}_D\right) \\ &= \underbrace{P_m}_{\text{following vol.} > \text{time int.}} - \rho\overline{\langle{}^i\mathbf{u}'\,{}^i\mathbf{u}'\rangle^i} : \nabla\bar{\mathbf{u}}_D\end{aligned} \tag{30}$$

We also note here that the production rate of k_m accounts for only part of the general production rate responsible for the overall balance of $\langle k \rangle^i$.

5.3. Macroscopic Equations for ε

For the dissipation rate $\langle \varepsilon \rangle^i$, the modeled transport equation reads (see details in [4]),

$$\rho\left[\frac{\partial}{\partial t}(\phi\langle\varepsilon\rangle^i) + \nabla\cdot(\bar{\mathbf{u}}_D\langle\varepsilon\rangle^i)\right] = \nabla\cdot\left[(\mu + \frac{\mu_{t_\phi}}{\sigma_\varepsilon})\nabla(\phi\langle\varepsilon\rangle^i)\right] + \frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i}\left[c_1 P_i + c_2\left(G_i - \rho\,\phi\langle\varepsilon\rangle^i\right)\right], \tag{31}$$

where c_1, c_2 and σ_ε are constants and the terms from the left-hand-side of Equation (31) are the accumulation, convection, diffusion and source/sink terms.

For ε_m, a proposal for a final modeled equation reads (see [3,22]):

$$\rho\frac{\partial(\phi\,\varepsilon_m)}{\partial t} + \rho\nabla\cdot[\phi\left\langle\bar{\mathbf{u}}\right\rangle^i\varepsilon_m] = \nabla\cdot\left[\mu + \frac{\mu_{t_m}}{\sigma_{\varepsilon_m}}\nabla(\phi\varepsilon_m)\right] + \frac{\varepsilon_m}{k_m}[c_1 P_m - c_2(\rho\phi\varepsilon_m + D_m)], \tag{32}$$

The dissipation rates also carry a correspondence if we expand this as follows:

$$\begin{aligned}\langle\varepsilon\rangle^i = \nu\langle\overline{\nabla\mathbf{u}' : (\nabla\mathbf{u}')^T}\rangle^i &= \nu\overline{\langle\nabla\mathbf{u}'\rangle^i : [\langle\nabla\mathbf{u}'\rangle^i]^T} + \nu\langle\overline{{}^i(\nabla\mathbf{u}') : {}^i(\nabla\mathbf{u}')^T}\rangle^i \\ &= \frac{\nu}{\phi^2}\overline{\nabla(\phi\langle\mathbf{u}'\rangle^i) : [\nabla(\phi\langle\mathbf{u}'\rangle^i)]^T} + \nu\langle\overline{{}^i(\nabla\mathbf{u}') : {}^i(\nabla\mathbf{u}')^T}\rangle^i\end{aligned} \tag{33}$$

If the porosity ϕ is considered to be constant,

$$\langle\varepsilon\rangle^i = \varepsilon_m + \nu\left\langle\overline{{}^i(\nabla\mathbf{u}') : {}^i(\nabla\mathbf{u}')^T}\right\rangle^i, \tag{34}$$

indicating that an additional dissipation rate is necessary to fully account for the energy decay process inside the REV.

6. Concluding Remarks

In this paper we have revisited a methodology for the analysis of turbulent flow in permeable media, which was first published in the early 2000's. A novel concept, called the double-decomposition idea, is revisited, showing how a variable can be decomposed in both time and volume in order to simultaneously account for fluctuations (in time) and deviations (in space) around mean values. Transport equations for the mean and turbulence flow have been presented.

Since the introduction of the double-decomposition concept, several authors have worked on similar treatments for turbulence in porous media and at different levels of complexity, sometimes combining what was already understood and detailed, some other instances dividing the turbulence spectrum into bands, each of which was handled by its own transport equation. In most works, however, the time–volume or volume–time sequence of integration has always played a role in setting up the overall modeling strategy.

Further areas of study for upcoming researchers might include the numerical and experimental evaluations of individual terms in the double-averaged equations above. The evaluation of individual terms and comparison with experiments would shed more light on the proposals herein, ultimately helping to refine a general tool for the analysis of several import engineering and environmental flows.

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