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Local Regions with Expanding Extra Dimensions

Kirill A. Bronnikov [1,2,3] and Sergey G. Rubin [3,4,*]

[1] Center fo Gravitation and Fundamental Metrology, VNIIMS, Ozyornaya ul. 46, 119361 Moscow, Russia; kb20@yandex.ru
[2] Institute of Gravitation and Cosmology, RUDN University, ul. Miklukho-Maklaya 6, 117198 Moscow, Russia
[3] National Research Nuclear University "MEPhI", Kashirskoe Shosse 31, 115409 Moscow, Russia
[4] N.I. Lobachevsky Institute of Mathematics and Mechanics, Kazan Federal University, 18 Kremlyovskaya Street, 420008 Kazan, Russia
* Correspondence: sergeirubin@list.ru

Abstract: In this paper possible spatial domains, containing expanding extra dimensions, are studied. It is demonstrated that these domains are predicted in the framework of $f(R)$ gravity (where R is the scalar curvitture) and could appear due to quantum fluctuations during inflation. The interior of the domains is shown to be characterized by the multidimensional curvature ultimately tending to zero and a slowly growing size of the extra dimensions.

Keywords: multidimensional gravity; $f(R)$ gravity; inhomogeneous cosmology

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1. Introduction

It is usually assumed that the nucleation of our Universe is related to quantum processes at the Planck energy scale [1–3]. After nucleation, various manifolds evolve classically, forming a set of manifolds, one of which is our Universe. It is a considerable challenge to fix the Lagrangian parameters to satisfy the observations. The complexity of this problem is aggravated by the inclusion of extra dimensions. The latter is of particular interest because the concept of extra dimensions is widely used in modern research dealing with problems such as grand unification [4,5], the cosmological constant problem [6–8], etc. The assumption of extra space compactness immediately leads to the question: Why is a specific number of dimensions stable or slowly expanding [9–11]? Stabilizing factors could be, for example, scalar fields [9,12] or gauge fields [13]. A static solution can be obtained using the Casimir effect [14,15] or form fields [16]. A contradiction with observations can be avoided if the time-dependent extra-dimensional scale factor, $b(t)$, varies sufficiently slowly [17,18].

One of the ways to stabilize the extra dimensions is based on gravity with higher derivatives, which is widely used in modern research. One of the most promising models of inflation is the Starobinsky model using a purely gravitational action [19]. Research efforts to avoid the Ostrogradsky instabilities have been made [20], and extensions of the Einstein–Hilbert action have received much attention. The model, presented here, contains $f(R)$ gravity (where R is the scalar curvitture) with the addition of the Kretschmann invariant and the Ricci tensor squared. The corresponding action can be considered as a basis for an effective field theory [21,22]

The recent paper [23] studied the evolution of manifolds after creation of these manifolds on the basis of a pure gravitational Lagrangian with higher derivatives. The final metrics may differ in different spatial regions if the model admits several stationary states. This is precisely the case for the model discussed in [24] , in which one of the stationary states was studied. The Lagrangian parameters at low energies are chosen in such a way as to supply an (almost) Minkowski space for the present Universe and the stationarity of the extra-space metric, and to reproduce an inflationary stage of the expansion with the Hubble parameter of the order of $H \sim 10^{13}$ GeV. Here, another final state admitted by the same model is discused with a comparatively small curvature of the extra dimensions that can still be compatible with observations. Some 3-dimensional (3D) spatial regions in the Universe could be characterized by such a metric, and it is of interest to study this possibility.

2. Outlook

Self-stabilization of the extra dimensions is one of the necessary elements of models based on compact extra spaces. In [25,26], it has been shown that the models with higher derivatives could lead to stationary solutions. The analysis was based on the model where the initial action was taken in the form [27,28]

$$S = \frac{1}{2} m_D^{D-2} \int \sqrt{g_D}\, d^D x \left[f(R) + c_1 R^{AB} R_{AB} + c_2 R^{ABCD} R_{ABCD} + L_{\rm m} \right], \tag{1}$$

where capital Latin indices cover all D coordinates, $g_D = |\det(g_{MN})|$, g_{MN} is the D-dimensional space metric tensor, $f(R)$ is a smooth function of the D-dimensional scalar curvature R, c_1, c_2 are constants, $L_{\rm m}$ is a matter Lagrangian, and $m_D = 1/r_0$ is the D-dimensional Planck mass, so that r_0 is a fundamental length in this theory. Throughout this paper, the following conventions on the curvature tensor is used: $R^D_{ABC} = \partial_C \Gamma^D_{AB} - \partial_B \Gamma^D_{AC} + \Gamma^D_{EC}\Gamma^E_{BA} - \Gamma^D_{EB}\Gamma^E_{AC}$, where Γ^C_{AB} is the D-dimensional Christoffel symbol, $\partial_A \equiv \partial/\partial A$, and for the Ricci tensor, $R_{MN} = R^F_{MFN}$. The metric signature $(+ - - \cdots)$ and the units $\hbar = c = 1$, where $\hbar$ is the reduced Planck constant and c is the speed of light, are used. For $L_{\rm m}$, one can consider the Casimir energy density in space-time with the metric,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu - r_0^2\, e^{2\beta(x^\mu)} d\Omega_n^2, \tag{2}$$

where $g_{\mu\nu}$ is the 4-dimensional space-time metric tensor, the indices, denoted by Greek letters, take on the values 0 (time), 1, 2, 3 (space), x^μ are the observable four space-time coordinates, and $d\Omega_n^2$ is the metric on a unit sphere $\mathbb{S}^n$. Space-time is a direct product $\mathbb{M}_4 \times \mathbb{M}_n$. The function $f(R)$ is taken in a general quadratic form,

$$f(R) = a_2 R^2 + R - 2\Lambda_D, \tag{3}$$

where a_2 is a constant, and Λ_D is the cosmilogical constant in the D-dimensional space.

Assuming $L_{\rm m} = 0$, one obtains an effective scalar–tensor theory, with the potential presented in Figure 1. Here, the internal Ricci scalar R_n, in fact, plays the role of a scalar field ϕ; for details, see [24,25].

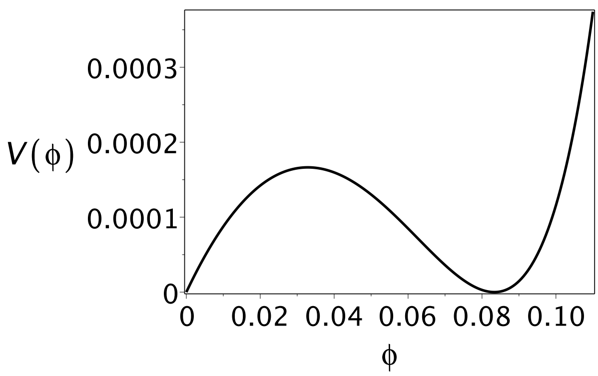

Figure 1. The effective potential for a viable version of the model (1). The minimum of the potential is at the point $\phi_{\rm min} \simeq 0.083$, the D-dimensional Planck mass, $m_D = 1$. In agreement with [24], $m_D \sim 0.1 M_{\rm Pl}$, where $M_{\rm Pl}$ is the Planck mass.

According to the experimental data, the scale of extra dimensions cannot be larger than 10^{-17}–10^{-18} cm. For example, the Ricci scalar $R_n = \phi \sim 0.01 M_{\rm Pl}^2$ from Figure 1 is very large, so that the inequality,

$$R_n \gg R_4, \tag{4}$$

appears natural. Here, R_n and R_4 are the Ricci scalars of the extra dimensions and of the 4D space-time, respectively. The inequality (4) was used in the derivation of the effective potential in Figure 1.

The scalar field could tend toward any minimum, depending on the initial conditions. Hence, different spatial regions could be filled with different states of ϕ, that with $R_n = \phi_{\rm min}$ or with a small Ricci scalar $R \to 0$ due to the metric fluctuations at high energies.

The metric evolution around the solution $R_n = \phi_{\rm min}$ could reproduce the inflationary scenario. One can adjust the model parameters to describe the observational properties of the inflationary stage. As usual, strong fine tuning is necessary to produce a small, almost zero 4D cosmological constant.

In this paper, the space-time metric that can emerge near the minimum $R \to 0$ is analyzed. Naively, an effective 4D cosmological constant should tend toward zero automatically in such regions of space. Is it applicable for life formation? The question is important even if the answer is negative. Even if we live at the standard minimum where $R_n = \phi_{\rm min}$, some neighboring regions could be filled with another minimum where $R \to 0$.

Notice that in this study one is interested in those regions where the Ricci scalar tends toward zero. In this case, the inequality (4) may seem questionable, but it should evidently hold if the extra dimensions are spherical and small enough to be invisible by modern instruments. However, beyond that, close to the limit $R \to 0$, one more inequality,

$$a_2 R^2 \ll R, \tag{5}$$

must hold, which strongly facilitates the analysis.

3. Field Equations

Under the assumption (5), one can simply let $F(R) = R - 2\Lambda_D$ and neglect all the curvature–nonlinear terms in the action (1). The action (1) is then reduced to four dimensions, in which the action takes the form inherent to a scalar–tensor theory specified in 4D space-time with the metric $g_{\mu\nu}$:

$$S = \frac{1}{2}\mathcal{V} m_D^2 \int \sqrt{g_4}\, d^4x\, \mathrm{e}^{n\beta}\Big[R_4 + \frac{n(n-1)}{r_0^2}\,\mathrm{e}^{-2\beta} + 2n\Box\beta + n(n+1)(\partial\beta)^2 - 2\Lambda_D\Big], \tag{6}$$

where $(\partial\beta)^2 = g^{\mu\nu}\beta_{,\mu}\beta_{,\nu}$, $\beta_{,\mu} \equiv \partial\beta/\partial x^\mu$, $\Box = \nabla_\mu \nabla^\mu$, and $\mathcal{V}(n) = 2\pi^{(n+1)/2}/\Gamma\left(\frac{n+1}{2}\right)$ is the volume of a unit sphere $\mathbb{S}^n$.

The standard transition to the Einstein frame with the 4D metric,

$$\overline{g}_{\mu\nu} = \mathrm{e}^{n\beta} g_{\mu\nu}, \tag{7}$$

brings the action to the form (up to a full divergence):

$$S = \frac{1}{2}\mathcal{V}(n) m_D^2 \int \sqrt{\overline{g}}\, d^4x \left[\overline{R}_4 + \frac{1}{2}n(n+2)(\overline{\partial}\beta)^2 + \frac{n(n-1)}{r_0^2}\,\mathrm{e}^{-(n+2)\beta} - 2\,\mathrm{e}^{-n\beta}\Lambda_D\right] + S_{\rm m}, \tag{8}$$

where overbars mark quantities obtained from or with $\overline{g}_{\mu\nu}$.

Now, if the action (8) is used to describe cosmological models with the Einstein frame metric,

$$ds_{\rm E}^2 = dt^2 - a^2(t) d\vec{x}^2, \tag{9}$$

and the effective scalar field $\beta(t)$, one gets the Einstein–scalar equations (only two of the equations are independent, and $V_\beta = dV/d\beta$):

$$\ddot{\beta} + 3\frac{\dot{a}}{a}\dot{\beta} + \frac{1}{n(n+2)}V_\beta = 0, \tag{10}$$

$$3\frac{\dot{a}^2}{a^2} = \frac{1}{2}n(n+2)\dot{\beta}^2 + V(\beta), \tag{11}$$

$$\frac{6}{a^2}\left(\dot{a}^2 + a\ddot{a}\right) = -n(n+2)\dot{\beta}^2 - 4V(\beta), \tag{12}$$

with the potential $V(\beta)$ given by

$$r_0^2 V(\beta) = \lambda\, \mathrm{e}^{-n\beta} - k_1\, \mathrm{e}^{-(n+2)\beta} + k_C\, \mathrm{e}^{-(2n+4)\beta},$$
$$\lambda = r_0^2 \Lambda_D, \qquad k_1 = \frac{1}{2}n(n-1). \tag{13}$$

The dot above the variables stays for the time derivative. The last term is related to the Casimir effect, with k_C of the order 10^{-3}–10^{-4}; see [27,28] and references therein.

As shown in Section 4, the first term in the potential (13) dominates.

4. Can We Live in a Region with $R \to 0$? Some Estimates

Consider the dimensionless version $W(x)$ of the potential $V(\beta)$, with $x = \mathrm{e}^{-\beta}$,

$$W(x) = \lambda x^n - k_1 x^{n+2} + k_C x^{2n+4}. \tag{14}$$

Let us determine the order of magnitude of the variable x such that the condition (5) is satisfied. This can be estimated by analogy with the known 4D models that employ $f(R)$ and, in particular, quadratic gravity. In these models (see, e.g., [19,29,30]), it is supposed that quadratic corrections to general relativity become important at energy scales of grand unification theories, which approximately corresponds to $a_2 \sim 10^{10} r_0^2$ in Equation (3), where r_0 is of the order of the Planck length. The same assumption was used in some models predicting a semiclassical bounce instead of a Schwarzschild singularity inside a black hole [31,32]. Now, assuming that the curvature R is of the order $R \sim \mathrm{e}^{-2\beta}/r_0^2 = x^2/r_0^2$ and substituting this into the condition (5), one obtains:

$$x^2 \ll r_0^2/a_2 = 10^{-10} \qquad \Longrightarrow \qquad x \ll 10^{-5}. \tag{15}$$

With such values of x and the above-mentioned values of k_C, it is clear that the Casimir contribution to the potential (14) is also quite negligible, and one can restrict $W(x)$ to the first two terms.

Other restrictions on admissible values of x follow from two evident conditions: (i) a classical space-time description requires that the size of the extra dimensions, $r = r_0\,\mathrm{e}^{\beta} = r_0/x$ should be much larger than the fundamental length $r_0 = 1/m_D$, hence $x \ll 1$; (ii) this size should be small enough for the extra dimensions to be invisible for modern instruments, that is, $r = r_0/x \lesssim 10^{-17}$ cm, approximately corresponding to the TeV energy scale. If one assumes that $m_D \sim m_4 \sim 10^{-5}$ g $\sim 10^{33}$ cm^{-1}, it follows $x \gtrsim 10^{-16}$. Admitting that m_D may be some orders of magnitude smaller than m_4 and recalling the condition (15), one can more or less safely assume:

$$10^{-13} \lesssim x \ll 10^{-5}. \tag{16}$$

For further study and estimation, it is necessary to make an assumption on a conformal frame in which the observations are treated. This choice ultimately depends on how fermions are included in a (so far unknown) underlying unification theory of all physical interactions. Two options are considered: the Jordan frame with the action (6), directly derived from the original D-dimensional theory, and the Einstein frame with the action (8).

4.1. The Einstein Frame

Assuming that the observed space-time corresponds to the Einstein frame, one finds Equations (11) and (12) for the scale factors $a(t)$ and $e^{\beta(t)}$. The Hubble parameter of the present Universe, $H(t) = \dot{a}/a$, is of the order $10^{-17}\,\mathrm{s}^{-1} \approx 10^{-61} r_0^{-2}$ if $r_0 = 1/m_4$; evidently, $\dot{\beta}$ should be at most of the same order; therefore, the same is required from $V(\beta)$. Then, it follows from Equation (11):

$$\lambda x^n - \frac{1}{2} n(n-1) x^{n+2} \lesssim 10^{-122}. \tag{17}$$

Assuming $x = e^{-\beta} \sim 10^{-10}$ (well in the range (16)), and using Equation (17), the following two options can be considered:

(i) $\lambda = 0, \quad n(n-1) \times 10^{-10n-2} \lesssim 10^{-122} \quad \Rightarrow \quad n \geq 12.$

(ii) $\lambda \neq 0$, then one can rewrite Equation (17) as

$$\lambda = \frac{1}{2} n(n-1) \times 10^{-20} + \mathcal{O}(10^{-122+10n}),$$

thus, the estimate of λ depends on n, which can now be smaller than 12, but a considerable fine tuning is necessary; for example,

$$n = 10 \quad \Rightarrow \quad \lambda = 45 \times 10^{-20} \pm \mathcal{O}(10^{-22}),$$
$$n = 5 \quad \Rightarrow \quad \lambda = 10^{-19} \pm \mathcal{O}(10^{-72}).$$

In all cases, Equations (11) and (12) should be solved numerically.

4.2. The Jordan Frame

If one assumes that the observed space-time metric is $ds_{\mathrm{J}}^2 = e^{-n\beta} ds_{\mathrm{E}}^2$, the cosmological metric can be expressed as

$$ds_{\mathrm{J}}^2 = e^{-n\beta}\left[dt^2 - a^2(t) d\vec{x}^2\right] = d\tau^2 - a_{\mathrm{J}}^2(\tau) d\vec{x}^2, \tag{18}$$

where τ is cosmological time such that $d\tau = e^{-n\beta/2} dt$, and $a_{\mathrm{J}}(\tau) = e^{-n\beta/2} a(t)$ is the Jordan-frame scale factor.

Now, the Hubble parameter is defined as

$$H = \frac{1}{a_{\mathrm{J}}} \frac{da_{\mathrm{J}}}{d\tau} = \left(\frac{\dot{a}}{a} - \frac{1}{2} n\dot{\beta}\right) e^{-n\beta/2}, \tag{19}$$

which leads to the estimate:

$$\frac{\dot{a}}{a} - \frac{1}{2} n\dot{\beta} \sim H x^{n/2}. \tag{20}$$

On the other hand, $\dot{\beta}$ is subject to the observational constraint on possible variations of the effective gravitational constant G (it is a true constant in the Einstein frame but varies in Jordan's proportionally to $e^{-n\beta}$, see Equation (6)): according to [33,34], one has to assume $(1/G)|dG/d\tau| \lesssim 10^{-3} H$, whence it follows that

$$|\dot{\beta}| \lesssim 10^{-3} H x^{n/2}. \tag{21}$$

It means that the second term in Equation (20) can be neglected and simply $\dot{a}/a \sim Hx^{n/2}$. Applying this to Equation (11) in the same manner as in the Einstein frame, one finds the relation:

$$\lambda - \frac{1}{2}n(n-1)x^2 \sim 10^{-122}, \tag{22}$$

which means, for any reasonable choice of x, an unnatural fine tuning of the value of λ. Recalling that $x = \mathrm{e}^{-\beta}$ varies with time while $\lambda = \text{const}$, one comes to the conclusion that the Jordan frame does not lead to a plausible cosmology in the present statement of the problem.

5. The Metric of a Region with $R \to 0$. Numerical Simulations

The main result of the previous discussion is the following: if an observer is inside such a region, then an unnaturally strong fine tuning of the model parameter λ is necessary (except for $n \geq 12$ in the Einstein frame). Furthermore, the first- and second-order derivatives of the potential are not defined at the point $\phi \equiv R_n \to 0$, so that there are no oscillations around such a minimum. However, such oscillations are necessary for a successful reheating just after inflation. Therefore, our Universe is hardly described by the metric discussed above.

Nevertheless, such regions could exist somewhere in the Universe, probably not too close to the Milky Way galaxy; otherwise, it could disturb too strongly the known observable picture. It is a separate problem how we, being external observers for such regions, could detect signals coming from there. Let us note that a similar problem was discussed some years ago concerning possible large-scale antimatter regions [35,36].

It seems worthwhile to analyze the possible metrics inside such regions. The first field equation for $\beta(t)$ (excluding $a(t)$ with Equation (11)) reads:

$$\ddot{\beta} + \dot{\beta}\sqrt{3\left[\frac{1}{2}n(n+2)\dot{\beta}^2 + V(\beta)\right]} + \frac{1}{n(n+2)}V_\beta = 0. \tag{23}$$

The second Equation (11) is necessary for obtaining the 4D scale factor.

There are no restrictions by the observational arguments if a region in question is much smaller than the whole Universe, but it should be assumed to be large and homogeneous enough so that the cosmological metric (9) could be applicable. On the other hand, no such severe fine tuning is needed there for the model parameters since it is not necessary to require an extreme smallness of the extra dimensions.

The field motion near a nonzero minimum of $V(\phi)$ in Figure 1 corresponds to the observable inflation for specific parameter values [24]. Only one of the parameters is needed, namely, $\lambda = \Lambda_D = 0.0125$. Furthermore, one consideres $V(\beta) = \Lambda_D e^{-n\beta}$, see Equation (13). A numerical solution to these equations is presented in Figure 2. One can see that the extra dimensions expand very slowly—the dynamics during the most interesting time interval from the sub-Planckian time scale to the post-inflationary period ($10^9 \sim 10^{-34}$ s in the units used here) is shown. The behavior of the curves remains roughly the same up to the present time. It is of interest that the expansion rate strongly depends on the number n of extra dimensions: the rate is inversely proportional to n.

This was the Einstein frame. A transition to the Jordan frame, written explicitly using Equation (18), gives no new results simply because of a very slow variation of the extra-space metric.

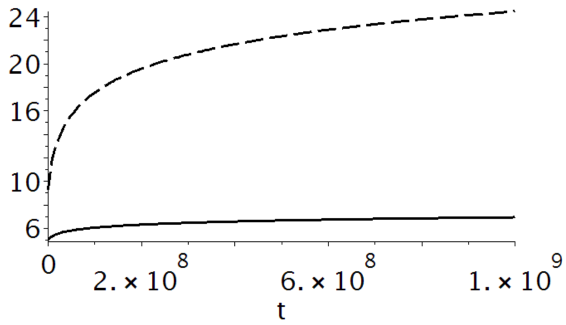

Figure 2. Time dependence of the size of extra space (solid line) and the main 4-dimensional space (dashed line) in a logarithmic scale; extra dimension, $n = 5$, the D-dimensional cosmological constant, $\Lambda_D = 0.0125$.

6. Discussion and Conclusions

The space domains containing the expanding extra dimensions such as those described above could exist in our Universe. It would be of interest to study light propagation with this observation in mind.

The compact extra-dimensional metric could play a crucial role in the physical parameter values such as particle masses and the relevant coupling constants [8]. The nuclear reaction rates inside such domains, and hence the final chemical content and temperature, could differ from the conditions in the surrounding space. The search for such regions is topical now [37]. It is worth mentioning that these regions could be connected with our space through wormholes [38–40]. Another kind of signal from an anomalous domain of space could be related to the so-called leakage phenomena studied in [41].

It is also of interest to study the stability of such regions from the point of view of an external observer: Do the regions expand or shrink? The answer to this question is not evident. A potential maximum separates two minima, as in Figure 1. In this case, a closed wall is formed [42], which could expand or shrink, sweeping out the internal domain. At first glance, it seems that a closed wall must quickly shrink due to the wall tension. It looks quite probable, but there is an argument against this conclusion. Indeed, the closed walls have a very complicated shape from the beginning. In this case, the walls fluctuate instead of shrinking [43] and produce gravitational waves. As it was shown in [44], the waves energy spectrum could explain the observable one. The wall motion through the surrounding media leads to the wall deceleration, which is quite effective if the wall tension σ is not very strong. Estimation gives:

$\sigma \sim \sqrt{V_{\max}}\phi_{\min} \sim 10^{-3} m_D^3 \sim 10^{-6} M_{\rm Pl}^3$. The numbers $V_{\max} \sim 0.0002$, $\phi_{\min} \sim 0.1$, and the relation for the units $m_D = 0.1 M_{\rm Pl}$ are taken from Figure 1. The tension appears to be quite strong, and therefore, wall fluctuations could last for a long time.

As was discussed in [26,45,46], shrinking walls could cause multiple formations of black holes in the Universe. This could solve the problem of primordial black hole formation [35,47].

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