

Article

Performance Evaluation of Displacement Estimation Methods for Early-Stage Breast Cancer Tumor Detection Using Strain Elastography

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Abstract

Displacement estimation methods in strain elastography use ultrasound signals to estimate displacements and strain in soft tissue. Although several methods exist, systematic comparisons under controlled simulation conditions for lesions smaller than 5 mm are limited. Evaluating axial accuracy for superficial and intermediate depths in small breast cancer lesions is clinically important, as early-stage detection with existing techniques remains challenging. In this study, speckle tracking, the Doppler method, and the combined autocorrelation method (CAM) were used to estimate axial displacements from simulated elastography signals. The performance of these methods was assessed using the root mean square error (RMSE) for displacement field, strain, and tumor size estimation in a three-layer model comprising healthy tissue–tumor–healthy tissue. An extended analysis considering anatomically realistic tissue and motion artifacts conditions for the case of smallest lesion is presented. Finally, the CAM method, which obtained the best results, was assessed varying SNR/strain values. Simulation results show that CAM outperforms the other methods in displacement estimation across early-stage tumor sizes at both superficial and intermediate depths in all performed tests.

Keywords: strain elastography; speckle tracking; Doppler method; combined autocorrelation method



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1. Introduction

Elastography is a technique that utilizes ultrasound to evaluate the mechanical properties of tissues, such as stiffness and elasticity. It provides complementary diagnostic information not offered by traditional ultrasound (B-mode). In the context of breast disease detection, this technology enables the differentiation of malignant lesions, such as cancer, due to their distinct mechanical properties, which manifest as significantly higher stiffness compared to healthy breast tissue [1].

At present, among the most widely used ultrasonic elastography techniques in commercial devices for the evaluation of breast pathologies are strain elastography (SE) and shear-wave elastography (SWE). One limitation of SE is its reduced ability to detect benign lesions, while SWE has limitations in detecting malignant lesions [2]. Complementing both techniques can address their individual limitations; therefore, they should be viewed as complementary rather than competing techniques [3]. These techniques have been

demonstrated to be effective in detecting a wide range of breast pathologies, including benign lesions (cystic lesions, fibrocystic changes, sclerosing lesions, papillary lesions, fibroadenomas) and malignant lesions (ductal carcinoma in situ, invasive ductal carcinoma, invasive lobular carcinoma, invasive papillary carcinoma) [1].

SE provides qualitative information based on the strain obtained by external manual compression or physiological body movement. SE has advantages that justify its current use, including the acquisition of images with high spatial resolution, greater availability in commercial ultrasound equipment compared to SWE, and lower equipment costs due to generally less complex and more accessible hardware. SWE quantifies tissue elasticity by estimating the propagation velocity of shear waves generated by an acoustic radiation force impulse (ARFI) [4,5]. The techniques identified in the World Federation for Ultrasound in Medicine and Biology Guidelines and Recommendations for Clinical Use of Ultrasound Elastography for measuring displacements in strain imaging include the speckle tracking method, the Doppler method, and the combined autocorrelation method. For SWE, time-of-flight (TOF) methods are standardized for estimating group velocity [6]. Other SWE approaches employ the calculation of shear-wave phase velocity using system identification methods [7].

A problem of great clinical importance is the detection of cancerous tumors at an early stage. Below, we discuss the limits of sensitivity, specificity, the area under the ROC curve (AUC), the size of detected tumors, and the ranges of Young's modulus reported in the current literature for SE and SWE.

The authors of [8] evaluated the diagnostic accuracy of SWE for differentiating malignant and benign breast tumors. The sensitivity of SWE for mean shear-wave velocity was 86%, the specificity was 87%, and the AUC was 0.93. The results confirm that SWE can be used as a complementary method to B-mode, demonstrating its ability to discriminate between cancerous and non-cancerous breast tumors. The authors also showed that SE alone or in combination with SWE yields better diagnostic performance than SWE alone.

The authors of [9] compared SE and SWE based on their performance in breast tumor diagnosis when tissue stiffness assessment was incorporated into tumor margins. The sensitivity of SE using the E/B ratio was 94.7%, the specificity was 81%, and the AUC was 0.878. The sensitivity of SWE was 73.7%, the specificity was 82.5%, and the AUC was 0.697. The AUC obtained using SE was higher than that of SWE. The results showed that SE with the E/B size ratio has greater sensitivity in breast cancer detection and better overall diagnostic performance than SWE. The accuracy of SWE also improves when edge tissue evaluation is incorporated. E/B is defined as the ratio of lesion size in the strain image to its size in the B-mode image.

Recent research has reported the ability to detect small breast cancer lesions (<5 mm) using SE techniques. In [10], the authors studied the impact of small tumors (3 to 22 mm in diameter) on stiffness measurements using SE and SWE in gelatin phantoms. Two stiffness levels with mean values of 56.1 kPa and 83.3 kPa were considered. The SWE results showed a significant underestimation of tumor size for lesions smaller than 16 mm, whereas SE produced reliable results independent of tumor size at both stiffness levels. The findings suggest the suitability of strain elastography for detecting tumors smaller than 16 mm.

Another study confirmed the impact of lesion size and stiffness in breast phantoms on the accuracy of small tumor detection using SWE [11]. The authors evaluated tumor diameters ranging from 2.5 mm to 18 mm across four stiffness levels. The results showed excellent stiffness estimation for tumors between 7 and 11 mm, whereas accuracy was low for tumors less than or equal to 4 mm in size.

The authors of [12] investigated recent advances in elastography with a focus on early breast cancer detection. Their main objective was to examine current elastography

techniques that improve the detection of small tumors at an early stage. Advanced techniques such as multi-frequency SWE improved spatial resolution, enabling the detection of 3 mm lesions and enhancing image visualization. These techniques allow stiffness to be measured throughout the entire breast volume, which is particularly useful in dense breasts. The development of new elastography methods aims to improve displacement estimation accuracy to achieve microscopic and cellular resolution. Consequently, the early detection of malignant lesions remains a priority for contemporary elastography techniques.

This paper presents a comparative analysis of the most common processing methods used in strain elastography, which has been shown in various studies to be a viable technique for identifying small lesions. Specifically, it evaluates the root mean square error obtained by each method for displacement and strain estimation in tumors with diameters smaller than 5 mm, as well as the accuracy of tumor size estimation. The motivation for this work is to enrich the available information on relatively less complex techniques, which are commonly associated with lower equipment costs, and their performance in supporting early breast cancer diagnosis. This work may provide a foundation for demonstrating the suitability of these methods as screening tools in developing countries.

2. Background

Various methods are used in elastography to estimate tissue displacements as indicators related to tissue stiffness. The choice of method depends on the type of elastography technique used. In strain elastography, displacements in the direction of the ultrasound beam are estimated to determine tissue strain, assuming that stiffer tissues, such as tumors, exhibit lower strain. Techniques based on strain elastography utilize information contained in the time and phase of the radio-frequency signal measured before and after compression.

Typical methods for estimating displacement in strain elastography are as follows [6]:

1. The spatial correlation method (speckle tracking method).
2. The phase difference detection method (Doppler method).
3. The combined autocorrelation method.

These methods have advantages and disadvantages that limit the assessment of small tumors at early stages of breast cancer.

The main advantage of the speckle tracking method is that it allows the estimation of displacements within soft tissues that are larger than one wavelength. However, a disadvantage is that the algorithm cannot be implemented in real time due to the high computational cost of calculating the correlation for each signal window moving in depth. In addition, its performance depends on the noise level of the elastography signal, leading to uncertainty when the amplitude of the ultrasonic echoes is relatively low.

The main advantages of the Doppler method include the possibility of real-time implementation and good robustness to noisy ultrasound signals. However, a key disadvantage is that displacement estimation within tissue is limited to half a wavelength due to phase-aliasing effects, regardless of whether phase-unwrapping techniques are used.

The combined autocorrelation method takes advantage of both the speckle tracking and Doppler methods to estimate displacements greater than one wavelength over a wide dynamic strain range from 0.05% to 5%, without errors due to phase aliasing [13].

It is therefore of great importance to evaluate the performance of these classical displacement estimation methods for strain elastography in detecting small tumors. Such evaluation allows the selection, adaptation, and modification of the most suitable algorithm to improve the detection of tumors smaller than 5 mm at superficial and intermediate depths.

The implementation of the displacement estimation method is a crucial step in tumor detection for elastography techniques due to the accuracy required to determine tumor size and position. Several methods are available for measuring displacement, and their use

depends on the manufacturer of the commercial ultrasound system, resulting in differences in spatial resolution.

2.1. Speckle Tracking Method

The speckle tracking method is based on tissue strain resulting from physiological or external forces. The method estimates the displacements undergone by acoustic scatterers in tissue as a function of depth. A reference signal is acquired in the tissue's resting state, and a second signal is measured after compression. A correlation window is moved over the two signals at different depths, and, at each depth, the delay that maximizes the correlation coefficient is estimated. This delay is associated with the displacement of the scatterers contained within that window [14,15]. As a result, tissue strain as a function of depth is obtained. This method is limited by its high computational cost due to the calculation of correlation throughout the tissue depth.

The simplest spatial correlation approach measures the one-dimensional displacement in the axial direction of the ultrasound beam using radio-frequency (RF) signals acquired before and after compression, as shown in Figure 1. In this figure, $x(t)$ and $y(t)$ represent the echo signals before and after compression, respectively, framed within two windows centered at depths z_1 and z_2 . The normalized correlation coefficient between the windowed segments is used to evaluate their degree of similarity before and after compression.

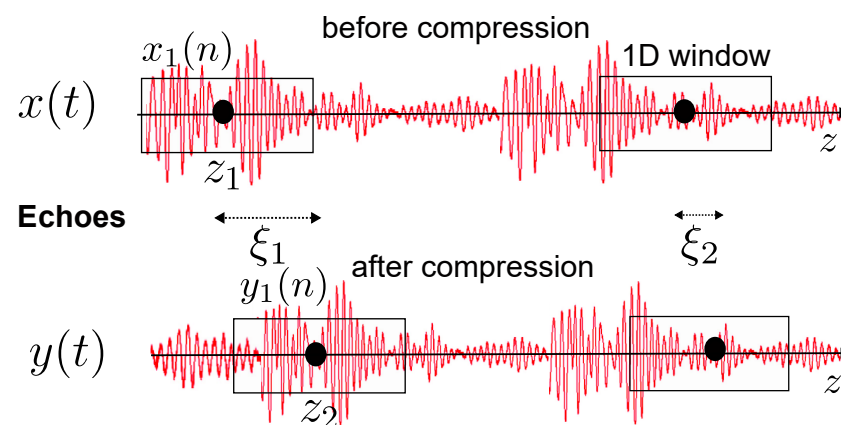


Figure 1. Correlation using the speckle tracking method. ξ_1 and ξ_2 are the displacements experienced by two speckle segments at different depths z due to compression. Since $\xi_1 > \xi_2$, the compression is not uniform in depth.

The first two discrete signal segments, represented by the windows $x_1(n)$ and $y_1(n)$ in Figure 1, are selected, and the cross-correlation function for windows of length N is given by

$$\hat{R}_{x_1 y_1}(m) = \sum_{n=0}^{N-m-1} x_1(n+m)y_1(n), \quad m \geq 0 \quad (1)$$

$$\hat{R}_{x_1 y_1}(m) = \hat{R}_{y_1 x_1}(-m), \quad m < 0 \quad (2)$$

where m denotes the lag between the signals.

The correlation sequence is normalized to obtain $\hat{R}_{x_1 y_1, \text{coeff}}$. The cross-correlation coefficient assumes a unit value when there is zero phase shift between the two signals:

$$\hat{R}_{x_1 y_1, \text{coeff}}(m) = \frac{1}{\sqrt{\hat{R}_{x_1 x_1}(0)\hat{R}_{y_1 y_1}(0)}} \hat{R}_{x_1 y_1}(m) \quad (3)$$

where $\hat{R}_{x_1 x_1}(0)$ and $\hat{R}_{y_1 y_1}(0)$ are the autocorrelation functions of the windows $x_1(n)$ and $y_1(n)$, respectively, at zero lag. The calculation of the cross-correlation coefficient is repeated

as the window is shifted in depth. The time delay between windows is defined as the lag m that maximizes the normalized correlation sequence [16].

2.2. Doppler Method

The Doppler method estimates tissue strain as a function of depth by measuring a signal before compression and a signal after compression. In this approach, scatterer displacement due to compression is estimated by calculating the phase shift of the ultrasonic echoes associated with the compression. To this end, a complex baseband representation of the signals is used. This representation allows displacement to be obtained by estimating the phase difference of the complex correlation function between two signal windows at a given depth. This process is repeated as the window is moved to different depths. A limitation of this method is that the maximum measurable compression is $\lambda/2$ due to phase aliasing.

The Doppler method calculates displacement based on the phase difference of the complex correlation function between two signal windows, $x_1(n)$ and $y_1(n)$, obtained before and after compression, respectively [17].

A sampled RF signal window with sampling period t_s and carrier frequency ω_0 is considered. The RF signal before compression is expressed as

$$x_1(n) = \Re\{A(n)e^{j\omega_0 n t_s}\}, \quad n \in \mathbb{Z} \quad (4)$$

where $A(n)$ is the complex envelope containing speckle amplitude and phase information, and $\Re\{\cdot\}$ denotes the real-part operator. The baseband representation, obtained via demodulation with $e^{-j\omega_0 n t_s}$ and low-pass filtering, is

$$\tilde{x}_1(n) = A(n) \quad (5)$$

Compression results in an axial shift of the window, represented in samples as $\Delta n = \frac{\tau}{t_s}$. Due to the time delay τ caused by compression, the post-compression signal window is approximated as

$$y_1(n) = \Re\{A(n - \Delta n)e^{j\omega_0(n t_s - \tau)}\}, \quad n \in \mathbb{Z} \quad (6)$$

Demodulation of $y_1(n)$ with $e^{-j\omega_0 n t_s}$ and low-pass filtering yields the baseband representation of the compressed signal $\tilde{y}_1(n)$:

$$\tilde{y}_1(n) = A(n - \Delta n)e^{-j\omega_0 \tau} \quad (7)$$

The signals $\tilde{x}_1(n)$ and $\tilde{y}_1(n)$ thus share the same shifted envelope and differ by a phase term $e^{-j\omega_0 \tau}$ due to the local delay [15].

The cross-correlation of the baseband signals is defined as

$$\tilde{R}_{\tilde{x}_1 \tilde{y}_1}(m) = \sum_{n=0}^{N-m-1} \tilde{x}_1(n+m) \tilde{y}_1^*(n), \quad m \geq 0 \quad (8)$$

$$\tilde{R}_{\tilde{x}_1 \tilde{y}_1}(m) = \tilde{R}_{\tilde{y}_1 \tilde{x}_1}^*(-m), \quad m < 0 \quad (9)$$

For zero lag $m = 0$, the simplified cross-correlation function is

$$\tilde{R}_{\tilde{x}_1 \tilde{y}_1}(0) = R_A(\Delta n)e^{-j\omega_0 \tau} \quad (10)$$

where $R_A(\Delta n)$ is the autocorrelation function of the signal envelope evaluated at Δn . The phase at zero lag is obtained as

$$\phi(0) = \tan^{-1} \left(\frac{\text{Im}(\tilde{R}_{\tilde{x}_1 \tilde{y}_1}(0))}{\text{Re}(\tilde{R}_{\tilde{x}_1 \tilde{y}_1}(0))} \right) \quad (11)$$

The phase value $\phi(0)$ represents the phase difference between the signals before and after compression. The corresponding time shift τ between the windows is given by

$$\tau = \frac{\phi(0)}{\omega_0} \quad (12)$$

Finally, the displacement estimate ξ is obtained as

$$\xi = \frac{v \cdot \tau}{2} \quad (13)$$

where v is the speed of sound.

Compared with other displacement estimation methods used in elastography, the Doppler method is a phase-based approach suitable for real-time imaging due to its low computational load under small compression conditions [18]. However, the method becomes ineffective when displacement exceeds half a wavelength due to phase discontinuities caused by aliasing.

2.3. Combined Autocorrelation Method

The combined autocorrelation method (CAM) was proposed as an alternative to overcome the limitations of the speckle tracking and Doppler methods. Like these methods, it requires a signal acquired both before and after compression. CAM estimates tissue displacement as a function of depth by converting the unwrapped phase into displacement. The autocorrelation functions of the signal at rest are obtained using the compressed and displaced signal shifted by $n\lambda/2$, where $n = 0, 1, 2, \dots$, resulting in n branches or autocorrelation outputs. The unwrapped phase is obtained from the wrapped phase selected in regions where phase aliasing does not occur. These non-aliasing regions are identified by selecting the integer n corresponding to the branch that maximizes the autocorrelation function. Once the non-aliasing branch n has been selected, the wrapped phase corresponding to that branch is used. The first branch does not consider the $\lambda/2$ shift of the compressed signal. The number of processing branches is determined based on the depth region of interest. This method can be applied to large compressions and reduces the computational cost of correlation.

CAM generates displacement estimates with high processing speed and precision. It achieves a wide range of strain estimates by combining two processing steps [19]:

1. Step 1: Obtain an approximate estimate of the maximum envelope correlation. A key feature of this step is that the search is performed only at points separated by $\lambda/2$ intervals in the axial direction, which improves processing speed. The objective is to detect a region without phase overlap over a wide dynamic range.
2. Step 2: Use the phase obtained in the first step for a refined estimation. In this step, the axial displacement is determined at a measurement point, and the axial strain distribution is obtained by differentiating the spatial displacement distribution. The phase difference between the two signals is directly converted into a displacement estimate to reduce the computation time.

In this technique, the tissue is compressed by an external force along a scan line (A-scan), and RF signals are measured before and after compression as $x(t)$ and $y(t)$. These signals can be modeled as complex functions:

$$x(t) = A(t) \cdot e^{-j\omega_0 t} \quad (14)$$

$$y(t) = A(t + \tau) \cdot e^{-j\omega_0(t+\tau)} \quad (15)$$

where $A(t)$ is the signal envelope, ω_0 is the angular frequency of the ultrasonic carrier, and τ is the time shift due to compression.

The complex correlation function $R_{xy}(t, n)$ between $x(t)$ and the signal $y(t - nT/2)$, generated by shifting $y(t)$ by integer multiples of the half period T (or, equivalently, integer multiples of $\lambda/2$), is defined as

$$R_{xy}(t, n) = \int_{-t_0/2}^{t_0/2} x(t + \zeta) y^*(t - nT/2 + \zeta) d\zeta \quad (16)$$

$$R_{xy}(t, n) = R_A(t, \tau - nT/2) \cdot e^{j\omega_0(\tau - nT/2)} \quad n = 0, 1, 2, \dots \quad (17)$$

where t denotes time, t_0 is the duration of the correlation window, ζ represents the integration variable within the window, n is an integer, T is the ultrasonic signal period, and $R_A(t, \tau)$ is the autocorrelation function of the envelope. For $n = 0$, Equation (16) reduces to [20]

$$R_{xy}(t, 0) = \int_{-t_0/2}^{t_0/2} x(t + \zeta) y^*(t + \zeta) d\zeta \quad (18)$$

$$R_{xy}(t, 0) = R_A(t, \tau) \cdot e^{j\omega_0 \tau} \quad (19)$$

Let $\psi(t, n)$ denote the unwrapped phase of $R_{xy}(t, n)$. Equation (17) can then be expressed as

$$R_{xy}(t, n) = R_A(t, \tau - nT/2) \cdot e^{j\psi(t, n)} \quad (20)$$

where

$$\psi(t, n) = \omega_0(\tau - nT/2) = \omega_0 \tau - n\pi \quad (21)$$

$$\psi(t, n) = \psi(t, 0) - n\pi \quad (22)$$

In the region of small displacement,

$$|\psi(t, 0)| = \omega_0 \tau < \pi \rightarrow \psi(t, 0) = \phi(t, 0) \quad (23)$$

where $\phi(t, n)$ is the wrapped phase of $R_{xy}(t, n)$, obtained for small displacements as

$$\phi(t, n) = \arctan\left(\frac{I}{Q}\right) \quad (24)$$

with $I = \Im\{R_{xy}(t, n)\}$ and $Q = \Re\{R_{xy}(t, n)\}$ denoting the imaginary and real parts of the cross-correlation function, respectively.

The normalized correlation coefficients $K_A(t, n)$ are defined as

$$K_A(t, n) = \frac{|R_{xy}(t, n)|}{\sqrt{R_{xx}(t, 0) \cdot R_{yy}(t, n)}} \quad (25)$$

where $R_{xx}(t, 0)$ is the autocorrelation function of the signal $x(t)$ at $n = 0$, and $R_{yy}(t, n)$ is the autocorrelation function of the signal $y(t)$ as a function of n .

Using Equations (24) and (25), a set of envelope correlation coefficients $\{K_A(t)\}$ and a set of wrapped phases $\{\phi(t)\}$ are obtained for each time point t :

$$\{K_A(t)\} = \{K_A^0, K_A^M, K_A^{M+1}, \dots, K_A^N\} \quad (26)$$

$$\{\phi(t)\} = \{\phi^0, \phi^M, \phi^{M+1}, \dots, \phi^N\} \quad (27)$$

where $K_A^n = K_A(t, n)$ and $\phi^n = \phi(t, n)$ is the phase of $R_{xy}(t, n)$.

To determine the optimal values of M and N , the minimum ψ_{min} and maximum ψ_{max} values of $\psi(t, 0)$ in Equation (23) are used:

$$M = \lfloor \psi_{min} / \pi \rfloor \quad (28)$$

$$N = \lfloor \psi_{max} / \pi \rfloor \quad (29)$$

where $\lfloor \cdot \rfloor$ denotes the integer-part operator. A value m is then defined to indicate the phase-aliasing-free region, satisfying $M \leq m \leq N$.

The coefficient K_A^n attains its maximum when $n = m$. Accordingly, the first step is to determine the value of n that maximizes K_A^n , after which the unwrapped phase displacement can be obtained using

$$m = \max_n \{K_A(t, n)\} \quad (30)$$

$$\psi(t, m) = \phi(t, m) \quad (31)$$

Once the region m is selected, the wrapped phase in that region is free of aliasing. Using Equations (22) and (31), the unwrapped phase is obtained as

$$\psi(t, 0) = \phi(t, m) + m\pi = \psi(t, m) + m\pi \quad (32)$$

Equation (32) shifts the wrapped phase by $m\pi$ in the selected region to obtain the unwrapped phase over the entire depth of analysis.

The displacement $\xi(z)$ is then calculated as

$$\xi(z) = \frac{\psi(z, 0) \cdot \lambda}{4\pi} \quad (33)$$

where λ is the ultrasonic wavelength, $z = v \cdot t$, and v is the speed of sound. Finally, the axial strain ϵ along the direction of the beam is obtained by differentiating the displacement with respect to depth:

$$\epsilon = \frac{d\xi}{dz} \quad (34)$$

Step 1 is similar to the *speckle tracking* method in that it uses envelope correlation. However, the first and second steps can be performed simultaneously, and the coefficient K_A^n is calculated only $N - M + 1$ times. Consequently, CAM achieves faster processing than the *speckle tracking* method, which requires repeated correlation calculations for varying values of τ .

In practice, the combined autocorrelation method can be implemented using the block diagram shown in Figure 2. The diagram consists of $N - M + 1$ cross-correlation processing units. The coefficients $\{K_A(t, n)\}$ are computed to identify regions without aliasing, after which the wrapped phase $\{\phi(t, n)\}$ of the selected region is chosen. The unwrapped phase $\psi(t, 0)$ is then obtained and converted into displacement.

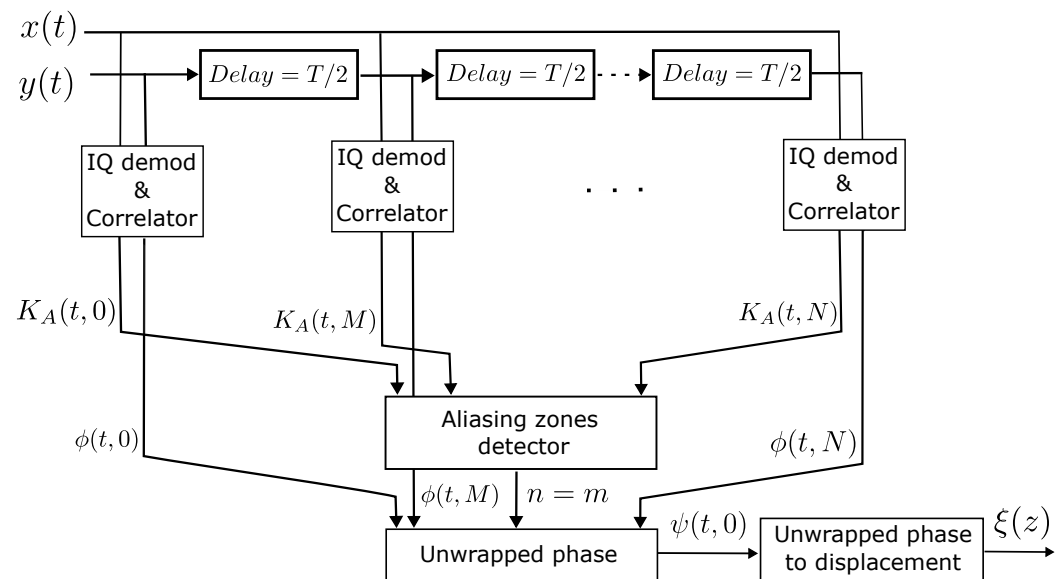


Figure 2. Schematic diagram of the signal processing steps in CAM.

3. Materials and Methods

3.1. Model Signal

A three-layer model known as the single-inclusion model (SIM) was used. This model is commonly used to analyze the performance of elastography methods. It was used to computationally simulate the behavior of DCIS tumor tissue embedded in healthy tissue.

A ductal carcinoma in situ (DCIS) tumor was modeled in layer 2 at depth d_1 with a Young's modulus $E_2 = 54$ kPa. This layer is stiffer than layers 1 and 3, which represent healthy soft tissue with Young's moduli E_1 and E_3 , respectively, where $E_1 = E_3 = 12$ kPa. Therefore, $E_2 > E_1$ and $E_2 > E_3$.

Determining the displacement field $\xi(z)$ of each tissue point as a function of depth is essential for simulating the compression of elastography signals. Due to the presence of stiffer regions, such as cancerous tumors, strain is not uniform in depth. Because tumors are stiffer than healthy tissue, they undergo less compression.

The displacement field at $z = 0$ (the surface of the breast tissue) under the applied surface force was denoted by δ , which represents the maximum tissue compression. The depths at which the layer interfaces are located are d_1 and d_2 . It was assumed that at depth d_3 there was no displacement due to surface compression. The Rayleigh–Ritz method was used to model compression of the ultrasound signals for strain elastography [21,22]. The displacement fields $\xi[mm]$ in each layer were approximated by linear polynomials, as shown in Figure 3.

Layer 1, consisting of healthy tissue, was assigned a thickness of 1 cm to model a superficial tumor and 3 cm to model a tumor at intermediate depth. The tumor layer was assigned thicknesses of 1 mm, 3 mm, and 5 mm, thereby simulating an early tumor stage. Layer 3 was assigned corresponding thicknesses such that the total depth d_3 reached 6 cm.

The displacement of tissue scatterers due to compression follows the depth-dependent displacement law $\xi(z)$ shown in Figure 3. Breast tissue was simulated by modeling scatterers following a gamma distribution with parameters $n = 40$ and $\bar{\tau} = 1.3 \mu s$, according to Landini [23]. The ultrasonic device was configured in pulse-echo mode.

Echo modeling based on the polynomial envelope method was used to simulate the transducer pulse shape [24]:

$$v(t) = -te^{-4 \cdot BW^2 \cdot t^2} \sin(2\pi f_0 t) \quad (35)$$

The approach was used to simulate the ultrasound pulse generated by the transducer model V326-SU (Olympus NDT, Inc., Waltham, MA, USA), which has a central frequency of $f_0 = 5$ MHz and a fractional bandwidth BW of 58%. A sampling frequency of $f_s = 80$ MHz was used. The speed of sound was assumed to be $v = 1540$ m/s, corresponding to a wavelength of $\lambda = v/f_0 = 0.3$ mm.

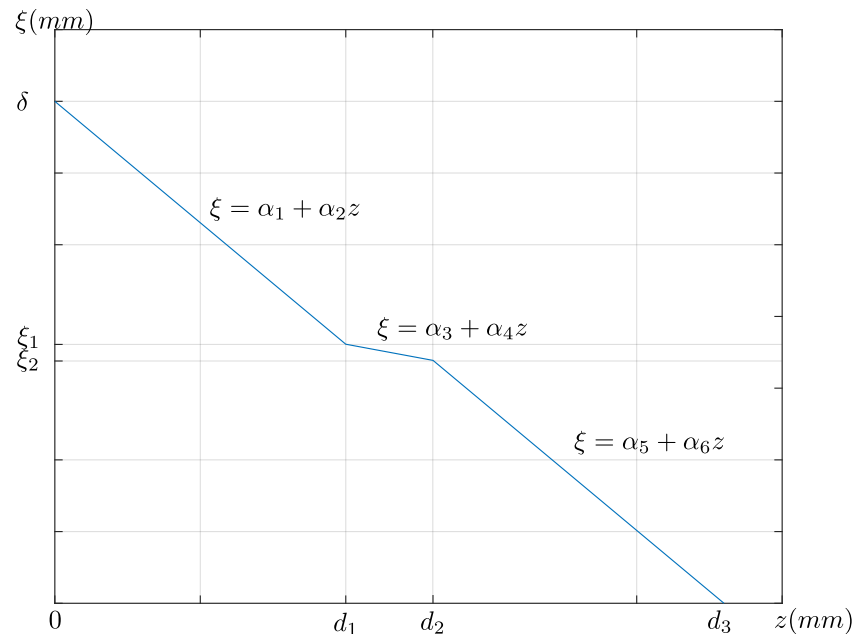


Figure 3. Tissue compression model for strain elastography using the Rayleigh–Ritz method. The z -axis represents tissue depth, and ξ denotes the tissue strain at each depth. d_1 and d_2 define the boundaries of the tumor layer, and d_3 is the maximum depth of the model.

The elastography signal results from the interaction between the ultrasonic pulse and the tissue and can be analytically expressed as the convolution of the pulse with the random positions of scatterers, represented using Dirac delta functions, in both the uncompressed and compressed states. A signal-to-noise ratio (SNR) of 35 dB was used to add noise to the simulated signals, which lies within the typical range for elastography applications. Representative SNR values for such applications can be found in [6,7,25,26].

Distortion effects due to ultrasound propagation were not considered in the simulated signals. This choice was made because the primary objective was to assess the performance of 3 strain elastography methods for estimating tumor size and position when the tumor is small (<5 mm).

As a special case, a generic evaluation of the performance of the methods under anatomically realistic tissue and motion artifact conditions was obtained. The effects of Young’s modulus variation in the healthy and tumor areas were considered, as well as a gradient boundary in the model. The effects of physiological motion and transducer slip were also added. The results for this special case under more realistic conditions are described in Section 4.4.

3.2. Simulated Experiments

A three-layer model, healthy tissue–tumor–healthy tissue, was implemented, in which the tumor was 4.5 times stiffer than the healthy tissue to simulate DCIS [1]. A background layer at a depth of 6 cm was assumed, where no scatterer mobility occurred due to surface deformation. For the evaluation of the speckle tracking method and the combined auto-correlation method, a deformation of 1.2λ was applied to the tissue. For the evaluation of the Doppler method, a maximum compression of $\lambda/2$ was considered, as this method

can only detect displacements below this value due to phase aliasing in the absence of phase-unwrapping techniques. Signals acquired in the uncompressed state were used as a reference for displacement estimation.

The estimation methods were implemented to evaluate the feasibility of detecting small tumors at superficial (1 cm) and intermediate (3 cm) depth using strain elastography. Three experiments were conducted to evaluate the accuracy of the displacement estimation methods: estimation of the displacement field as a function of tissue depth, estimation of strain, and estimation of the tumor size. The RMSE was used to quantify errors between the estimated values obtained with each method and the corresponding true values, which were computed previously.

Displacement estimates were obtained using time windows shifted along the depth direction. All implemented estimation methods used an 8 mm window length with a step size of 9.625 μm , corresponding to the distance covered by the pulse during a time interval of $t_s = 1/f_s$ in pulse-echo mode, to capture displacement variability in the sub-millimeter range. These parameters were kept identical across all methods for comparison purposes.

The evaluation cases were based on real clinical conditions reported in [1]. The cases included the detection of a small tumor with a diameter of 1 mm at a superficial depth of 1 cm, as well as of tumors with diameters of 1 mm, 3 mm, and 5 mm at an intermediate depth of 3 cm. The specific evaluation cases are listed in Table 1.

Table 1. Specific evaluation cases for tumor detection.

Case	Tumor Size	Depth
Case I	1 mm	1 cm
Case II	1 mm	3 cm
Case III	3 mm	3 cm
Case IV	5 mm	3 cm

3.3. Performance Evaluation

The root mean square error (RMSE) was used to evaluate the accuracy of the displacement estimation methods and is defined as [27]

$$\text{RMSE}(\hat{\zeta}) = \sqrt{E[(\hat{\zeta} - \zeta)^2]} \quad (36)$$

where ζ denotes the true value and $\hat{\zeta}$ denotes the estimated value of the variable ζ .

To apply the RMSE metric, 100 runs of the estimation process were performed for each estimation method. From a statistical perspective, these runs can represent different ultrasonic acquisitions from clinical cases with identical properties (tumor size and depth) because in each run, new elastography signals were generated using new random scatterer positions within the tissue.

A more in-depth analysis of the method that performed best based on the $\text{RMSE} \pm \text{SD}$ is described in Section 4.6. The susceptibility of the method under variable SNR/strain conditions, the convergence of the $\text{RMSE} \pm \text{SD}$ as a function of the number of realizations, and a failure rate map of the method for various compressions were determined.

4. Results

4.1. Root Mean Square Error of the Displacement Field Estimation

The final displacement field was estimated by averaging the displacement fields obtained across the 100 runs.

The Doppler method has a limitation of a maximum detectable displacement of $\lambda/2$ due to compression. However, the autocorrelation and combined autocorrelation methods

overcome this limitation by estimating displacements larger than a wavelength [15]. In our simulations, the speckle tracking method and the combined autocorrelation method were applied with a maximum surface compression of 1.2λ , whereas the Doppler method was applied with a maximum compression of $\lambda/2$.

The maximum tissue compression at the surface determines the number of signal processing units required for CAM. In our simulation, the maximum compression was $1.2\lambda = 0.37$ mm. Considering that the signal requires a round-trip time of $\tau = 0.48$ μ s to travel this distance at a speed of 1540 m/s, the maximum phase ψ_{max} is given by

$$\psi_{max} = \omega_0 \tau = 2\pi \cdot (5 \cdot 10^6) \cdot (0.48 \mu\text{s}) = 4.8\pi \quad (37)$$

Using Equation (29), the value of N was obtained as the integer part:

$$N = \lfloor \psi_{max} / \pi \rfloor = 4 \quad (38)$$

Because the deepest end of the model does not undergo compression due to superficial tissue strain, the minimum phase is zero $\psi_{min} = 0$. Therefore, using Equation (28), M takes the value

$$M = \lfloor \psi_{min} / \pi \rfloor = 0 \quad (39)$$

As a result, five processing units are required, with $0 \leq m \leq 4$.

The Doppler method uses a different surface compression from the other two methods. For comparison purposes, the displacement field estimated using the Doppler method was normalized by $\lambda/2$, whereas the displacement fields estimated using the other two methods were normalized by 1.2λ . This normalization establishes a common reference point across all methods despite the different surface compressions applied. By normalizing, the effect of compression magnitude is removed, allowing the graphical results to illustrate the relative behavior of the three methods. To obtain the actual displacement values from the graphs for a specific method, the values can be denormalized using the corresponding surface compression. The displacement field estimates obtained using the three methods are shown in Figure 4 for the case of a 3 mm tumor at a depth of 3 cm.

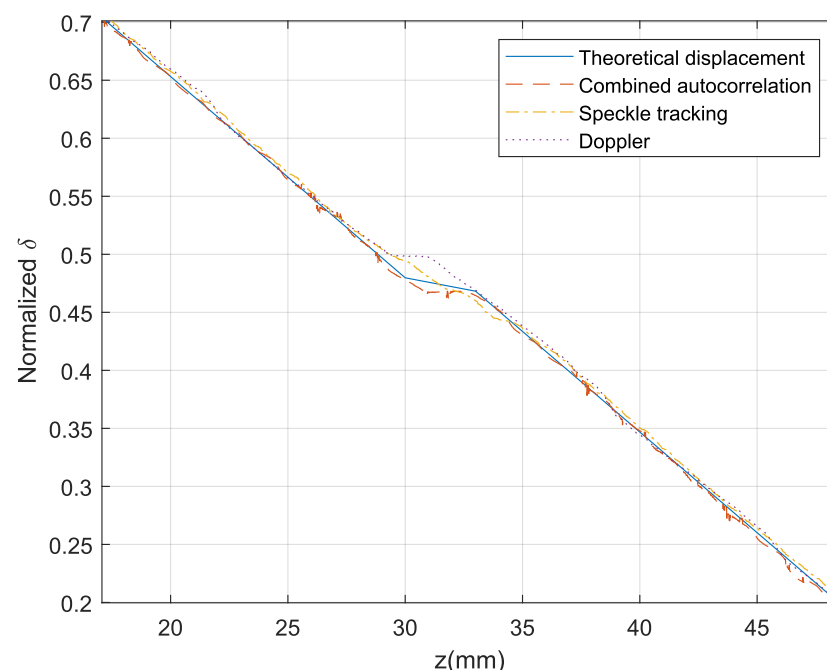


Figure 4. Comparison of the displacement field estimates obtained using the implemented methods.

The RMSE was calculated for the values estimated using the three methods, taking into account the theoretical reference values in the tumor region determined using the Rayleigh–Ritz method. To calculate the RMSE, the number of samples within the tumor was determined based on tumor size. For each millimeter of tumor thickness, 104 samples were used. One millimeter corresponds to a distance of 104 samples acquired at 80 MHz, assuming an ultrasound propagation velocity v in pulse-echo mode. The RMSE $\pm$ SD (mm) values for displacement field estimation in the tumor region using the three methods are reported in Table 2. Here, SD denotes the standard deviation.

Table 2. RMSE $\pm$ SD (mm) values for displacement field estimation in the tumor region using the three methods.

Case	Method	CAM	Speckle Tracking	Doppler
1 mm tumor, superficial depth		0.002610 $\pm$ 1.8700 $\times 10^{-4}$	0.02010 $\pm$ 6.5160 $\times 10^{-4}$	0.003795 $\pm$ 2.2429 $\times 10^{-4}$
1 mm tumor, intermediate depth		0.002579 $\pm$ 1.9228 $\times 10^{-4}$	0.01897 $\pm$ 4.4340 $\times 10^{-4}$	0.003409 $\pm$ 2.1350 $\times 10^{-4}$
3 mm tumor, intermediate depth		0.002221 $\pm$ 3.5433 $\times 10^{-4}$	0.01752 $\pm$ 1.1667 $\times 10^{-4}$	0.002498 $\pm$ 5.3951 $\times 10^{-4}$
5 mm tumor, intermediate depth		0.002082 $\pm$ 3.5488 $\times 10^{-4}$	0.01492 $\pm$ 1.3535 $\times 10^{-4}$	0.002084 $\pm$ 5.5979 $\times 10^{-4}$

The performance of the three methods in estimating the displacement field for the smallest tumor (1 mm) was analyzed at two depths: superficial (1 cm) and intermediate (3 cm). The RMSE of the displacement field estimation was higher when the tumor was closer to the surface. This increase is attributed to stronger signal decorrelation at shallower depths. Because tissue strain is greater near the surface, scatterers undergo larger displacements in superficial regions, resulting in greater changes in speckle signal patterns. The best displacement field estimates in the tumor region were obtained using CAM at both depths. For the largest tumor (5 mm), the RMSE values obtained with CAM and the Doppler method were identical. Overall, among the three implemented methods, CAM is best suited for detecting small tumors of 1 mm at both superficial and intermediate depths.

At an intermediate depth of 3 cm, the performance of the algorithms was further analyzed for different tumor sizes (1 mm, 3 mm, and 5 mm). For all three methods, tumor detectability improved with increasing tumor size, as reflected by decreasing RMSE values. CAM exhibited the highest accuracy at intermediate depth for all tumor sizes considered. The best displacement field estimation results obtained using CAM are highlighted in gray in Table 2. In addition, CAM consistently yielded lower SD values than the other methods.

4.2. Root Mean Square Error of the Strain Estimation

To obtain the strain ϵ from the displacement field shown in Figure 4, the variation of the displacement field as a function of depth was calculated using Equation (34).

The displacement field was divided into segments of 100 samples. Within each selected segment, the displacement and the corresponding depth were measured in pairs. The slope of the line fitted to the segment values was calculated and used as the strain value.

The strain was calculated over the entire analysis depth, up to $d_3 = 6$ cm, and a smoothing function was then applied to the strain values. To obtain the normalized strain for the cases described in Table 1, the strain for each method was determined using Equation (34) applied to the displacement fields estimated by each method. These strain values were then normalized with respect to the theoretical strain obtained using the same equation applied to the theoretical displacement field calculated with the Rayleigh–Ritz method. Figure 5 shows the normalized strain for the case of a tumor size of 3 mm at a depth of 3 cm.

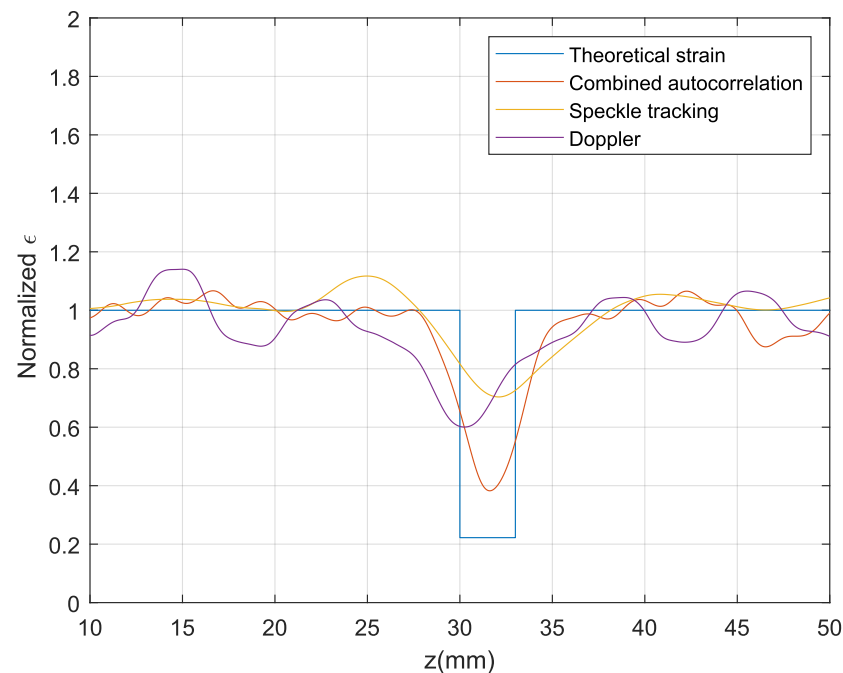


Figure 5. Tissue strain model and strain estimation using the three methods for the case of a tumor size of 3 mm at a depth of 3 cm.

The output samples of the smoothing function and the theoretical strain of the displacement field obtained using the Rayleigh–Ritz method, both within the tumor region, were used to calculate the RMSE. This RMSE calculation was performed for each displacement estimation method. The resulting RMSE values for strain estimation in the tumor region using the three methods are reported in Table 3.

Table 3. RMSE $\pm$ SD (mm) values for strain estimation in the tumor region using the three methods.

Cases	Method	CAM	Speckle Tracking	Doppler
1 mm tumor, superficial depth		0.002474 $\pm$ 2.1578 $\times 10^{-4}$	0.006441 $\pm$ 5.0166 $\times 10^{-4}$	0.003323 $\pm$ 7.5359 $\times 10^{-4}$
1 mm tumor, intermediate depth		0.002318 $\pm$ 4.2591 $\times 10^{-4}$	0.005381 $\pm$ 1.0666 $\times 10^{-4}$	0.003217 $\pm$ 3.3811 $\times 10^{-4}$
3 mm tumor, intermediate depth		0.002044 $\pm$ 1.715 $\times 10^{-4}$	0.004119 $\pm$ 2.4794 $\times 10^{-4}$	0.002924 $\pm$ 8.2077 $\times 10^{-4}$
5 mm tumor, intermediate depth		0.001225 $\pm$ 5.0427 $\times 10^{-4}$	0.003982 $\pm$ 6.1257 $\times 10^{-4}$	0.002335 $\pm$ 1.4857 $\times 10^{-4}$

The performance of the methods in estimating strain for small tumors of 1 mm at superficial and intermediate depth was analyzed. All methods showed improved strain estimation performance for small tumors at intermediate depth. CAM exhibited the best performance in detecting 1 mm tumors at both superficial and intermediate depths. To further analyze strain estimation performance at intermediate depth for different tumor sizes, all three methods were evaluated. Larger early-stage tumors provided a better characterization of the strain within the tumor region due to the larger number of samples available for analysis (104 samples per millimeter of tumor).

CAM performed best for all tumor sizes at intermediate depth. The best strain estimation results obtained with CAM are highlighted in gray in Table 3. In addition, CAM consistently yielded lower SD values than the other methods.

4.3. Tumor Size Estimation Error

Three threshold values were defined for estimating tumor size: 50%, 70%, and the full width at half maximum (FWHM). The 50% and 70% thresholds were arbitrarily selected to evaluate the effectiveness of the methods by estimating tumor size when the normalized strain

curves reached a minimum of 50% of the expected theoretical value (50% threshold) and 30% of the theoretical value (70% threshold). These thresholds were defined based on prior knowledge of the expected strain values in the tumor region and the surrounding healthy tissue.

The percentage thresholds were obtained by applying the threshold percentage to the difference between the theoretical strain in the tumor region and that in healthy tissue, after which the minimum normalized strain value was added. The intersection of these threshold levels with the strain curves in the tumor region yielded the estimated tumor width corresponding to each threshold. Selecting an appropriate threshold improves the accuracy of tumor size estimation. This procedure, illustrated for CAM in the case of a 3 mm tumor at a depth of 3 cm, is shown in Figure 6.

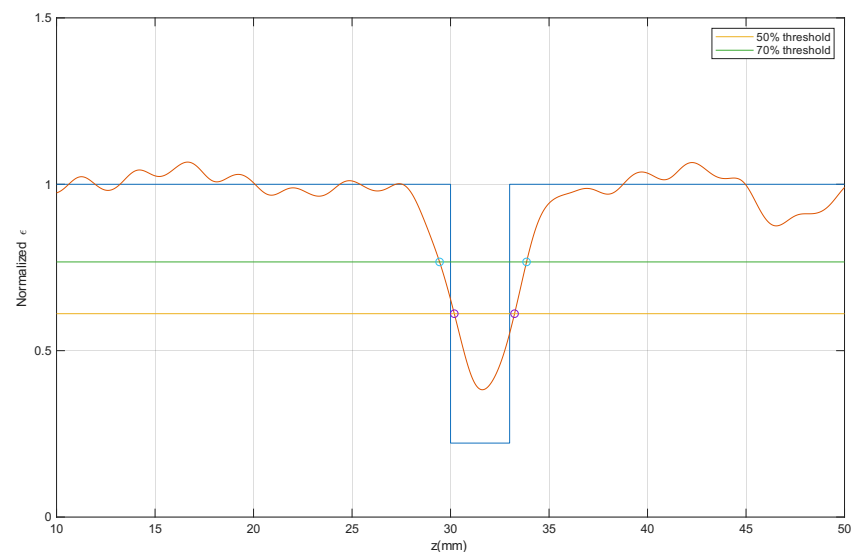


Figure 6. Different thresholds of 50% and 70% used for tumor size estimation with CAM. The blue line represents the normalized theoretical strain, and the red line represents the normalized strain obtained using the CAM.

Tumor size was estimated by subtracting the depth-axis (z) values of the two intersection points corresponding to the selected threshold. Different threshold values therefore yielded different tumor width estimates.

When theoretical strain values in the different regions were not available, the full width at half maximum (FWHM) criterion was used. The FWHM was defined as the distance along the depth axis between the two points at which the strain equals half of its maximum measured value. This distance was used as the tumor size estimate. The FWHM approach is less sensitive to small ripples in the strain curve.

For a quantitative analysis, tumor sizes were estimated using each threshold value. To compute bootstrap 95% confidence intervals for the mean size estimates, the number of iterations was increased to 500. The results are presented in Table 4. NS (no solution) indicates cases in which no intersection occurred between the selected threshold level and the normalized strain ϵ curve for a given method.

The RMSE values in Tables 2–4 were averaged over the depth samples. The best estimate was obtained using the 50% threshold for the combined autocorrelation method. The bootstrap 95% confidence intervals demonstrated the robustness of CAM, yielding narrower uncertainty margins in tumor size estimation compared with the speckle tracking and Doppler methods. Both the mean values, which were closer to the true tumor values, and the 95% confidence intervals, which showed smaller variations, confirmed the superior performance of CAM under these simulation conditions. These results are indicated by a gray background in Table 4.

Table 4. Tumor size estimates (mm) with bootstrap 95% confidence intervals.

Methods		CAM		Speckle Tracking			Doppler		
Case	50%	70%	FWHM	50%	70%	FWHM	50%	70%	FWHM
I	0.9559 [0.9445, 0.9673]	1.4845 [1.4668, 1.5022]	0.9920 [0.9822, 1.0018]	NS	2.4954 [2.4360, 2.5548]	2.3000 [2.2451, 2.3549]	NS	2.8544 [2.7977, 2.9111]	1.5000 [1.4762, 1.5238]
II	1.0198 [1.0077, 1.0319]	1.3279 [1.3121, 1.3437]	1.0030 [0.9931, 1.0129]	NS	2.2159 [2.1632, 2.2687]	2.0500 [2.0012, 2.0988]	0.8751 [0.8577, 0.8925]	1.4410 [1.4124, 1.4696]	0.9950 [0.9792, 1.0108]
III	2.9548 [2.9196, 2.9890]	3.4633 [3.4221, 3.5045]	2.9970 [2.9673, 3.0267]	NS	3.2629 [3.1853, 3.3406]	3.1000 [3.0270, 3.1730]	3.8165 [3.7408, 3.8922]	4.0907 [4.0096, 4.1718]	3.0500 [3.0016, 3.0984]
IV	5.0575 [4.9974, 5.1176]	5.3805 [5.3164, 5.4446]	5.0280 [4.9781, 5.0779]	NS	5.2280 [5.1036, 5.3524]	5.0000 [4.8810, 5.1190]	5.1730 [5.0703, 5.2757]	5.3858 [5.2790, 5.4926]	5.0300 [4.9502, 5.1098]

NS: No solution.

The tumor size estimation accuracy for the 1 mm tumor at intermediate depth was higher than that at superficial depth. At intermediate depth, as tumor size increased, the error in the estimated size decreased across all three methods. The combined autocorrelation method was the most accurate method for estimating small tumor sizes at superficial and intermediate depths, as well as for estimating different early-stage tumor sizes at intermediate depth.

The resolution of the evaluated methods, which depends on the window size and its overlap, is around 10 μm . Focusing on smaller errors, a 1 mm lesion exhibited an error of 44 μm at superficial depth and 10 μm at intermediate depth. This indicates a limitation in lesion size estimation accuracy, particularly for very small lesions ($\approx 1\text{ mm}$) at intermediate depths. This limitation may be exacerbated by the presence of artifacts such as body motion.

To make the results more comparable with the theoretical value, a binary map (tumor/no tumor) was obtained using the two thresholds (50%, and 70%), considering Cases II and III (1 mm and 3 mm tumors at a depth of 3 cm) for the three implemented methods. The selection criterion (tumor/no tumor) is determined by the presence of an intersection between the strain curves and the threshold defined in the tumor area. If there is an intersection, tumor is selected; otherwise, no tumor is selected. The binary map is shown in Table 5.

Table 5. Binary map (tumor/no tumor) across 100 iterations for Cases II and III.

Method	CAM	Speckle Tracking		Doppler			
Case	Threshold	50%	70%	50%	70%	50%	70%
1 mm tumor, intermediate depth		73/27 *	100/0	0/100	93/7	43/57	100/0
3 mm tumor, intermediate depth		98/2	100/0	0/100	100/0	86/14	98/2

* tumor/no tumor.

Table 5 illustrates the tumor–no tumor relationship across 100 process runs. The value 73/27, associated with the case of a 1 mm tumor at intermediate depth for a 50% threshold in CAM, is interpreted as follows: out of 100 instances in which the tumor was present, it was detected 73 times (tumor selected) and not detected 27 times (no tumor selected).

The table was constructed based on the principle that the strain curves intercept the defined thresholds of 50%, and 70%. The speckle tracking, Doppler, and CAM methods were applied to the selected cases. Among these methods, the 50% threshold was the most difficult to reach. However, this threshold provides better differentiation between the tumor region and the surrounding tissue because it produces a greater strain difference between the two regions. CAM allows for superior differentiation, achieving higher tumor-present/tumor-absent ratios for both lesion sizes. The speckle tracking method failed completely at the 50% threshold but performed favorably at the 70% threshold. The Doppler method exhibited poor performance in differentiating the smallest lesion at the 50% threshold, but performance improved as lesion size increased. Overall, the Doppler method demonstrated good performance; however, it was limited to a maximum strain of $\lambda/2$.

4.4. Evaluation Under Anatomically Realistic Tissue and Motion Artifact Conditions

After evaluating the methods' performance in estimating a small DCIS tumor under ideal conditions (single-inclusion model with sharp boundaries and uniform stiffness of 12 kPa vs. 54 kPa), we considered an anatomically informed case of a real tumor, characterized by irregular shapes, spatial stiffness gradients, and embedding in a fat background. We assumed a 20% stiffness variation within the tumor and a 30% variation in the background. The tumor's gradient boundary was defined based on [28], in which the

Young modulus of a real tumor decreases by 71.8% at 57% of the tumor diameter measured from the border. In our simulations, we implemented a transition zone between the tumor and healthy tissue of 0.62 mm, corresponding to a 77.7% decrease in Young's modulus at 62% of the tumor diameter. In addition to these anatomically informed conditions, motion artifacts were incorporated to evaluate method performance under more realistic scenarios typical of strain elastography applications. The main sources of error in clinical strain elastography are physiological motion (breathing, heartbeat) and transducer slip. To account for physiological movement, an elevation displacement of 0.1–0.3 mm between pre- and post-compression was applied. Transducer slip was simulated by adding 1% compression to the theoretical displacement field. Case I (1 mm tumor at a depth of 1 cm) was selected. The RMSE $\pm$ SD (mm) values for displacement field and strain estimation in the tumor region for the speckle tracking, Doppler, and CAM methods are presented in Table 6.

Table 6. RMSE $\pm$ SD (mm) values for displacement field and strain estimation in the tumor region using the three methods under motion artifacts and anatomically informed conditions.

Method	Displacement Field	Tumor Strain
Speckle tracking	$0.008142 \pm 7.1445 \times 10^{-4}$	$0.008893 \pm 5.8408 \times 10^{-4}$
Doppler	$0.007825 \pm 1.7512 \times 10^{-4}$	$0.005736 \pm 7.3772 \times 10^{-4}$
CAM	$0.003122 \pm 2.2514 \times 10^{-4}$	$0.002814 \pm 2.2765 \times 10^{-4}$

With these added effects, the RMSE inflation values were obtained using the theoretical RMSE values for the estimation of the displacement field and for the strain in the tumor area described in Tables 2 and 3 as a reference. An RMSE inflation of 10% was observed for the estimation of the displacement field and 13% for the strain for the CAM method. For the speckle tracking method, an RMSE inflation of 18% and 20% was obtained for the displacement field and strain, respectively. The Doppler method had an RMSE inflation of 12% and 13% for the displacement field and strain, respectively. The highest inflation was observed in the speckle tracking method. The CAM method is shown to be consistent and to perform better than the other methods in the presence of the main sources of error in clinical strain elastography.

Taking into account disturbances caused by motion artifacts and anatomically informed conditions, tumor size was estimated using the full width at half maximum (FWHM) method. All cases (I–V) were included in the evaluation. The results are presented in Table 7.

Table 7. Tumor size estimates (mm) using the FWHM method under motion artifacts and anatomically informed conditions with bootstrap 95% confidence intervals.

Case \ Method	CAM	Speckle Tracking	Doppler
1 mm tumor, superficial depth	1.162 [1.146, 1.178]	2.684 [2.596, 2.772]	1.750 [1.709, 1.791]
1 mm tumor, intermediate depth	1.173 [1.157, 1.189]	2.414 [2.335, 2.493]	1.215 [1.186, 1.243]
3 mm tumor, intermediate depth	3.267 [3.222, 3.311]	3.548 [3.432, 3.664]	3.393 [3.313, 3.473]
5 mm tumor, intermediate depth	5.399 [5.326, 5.473]	5.600 [5.417, 5.783]	5.492 [5.363, 5.621]

The FWHM for the CAM method offers the best results in estimating tumor sizes among the three methods. The tumor size values obtained are overestimated, but they offer a good approximation of the theoretical values considered for all cases.

4.5. Comparison of Computational Performance

To provide a reference for the computational performance of the evaluated methods, the time per frame was calculated based on the computational resources used in this work. The data presented in this section is for informational purposes only and does not represent realistic times for real-time imaging implementation.

The computational performance of the signal processing methods determines their feasibility for elastography applications. To meet the requirements of real-time image processing at 20 fps (frames per second), the processing time per A-scan, $t_{A\text{-scan}}$ (ms), must be minimized. A reference of $N_A = 128$ A-scans (lines) per frame was used [29]. One hundred runs were performed for Case I (1 mm tumor at a depth of 1 cm), and the results are reported as means $\pm$ SD. For each method, the execution time per frame, t_{frame} (ms), was calculated as

$$t_{frame} = N_A \cdot t_{A\text{-scan}} \quad (40)$$

The frame rate was determined using

$$\text{fps} = \frac{1000}{t_{frame}} \quad (41)$$

Table 8 presents a comparative analysis of the speckle tracking, Doppler, and CAM methods based on execution time tests using MATLAB™ version R2025a (MathWorks Inc., Natick, MA, USA). on modern hardware (DELL XPS 15 9530 with 13th Gen Intel(R) Core(TM) i7-13700H, 2.40 GHz CPU, and 16 GB of RAM).

Table 8. Performance comparison of the three methods based on execution time on a Dell XPS 15 9530 (Intel i7-13700H).

Method	$t_{A\text{-scan}}$ (ms)	t_{frame} (128 Lines) (ms)	fps
Speckle tracking	0.80 ± 0.05	102.4	9.8
Doppler	0.32 ± 0.03	41.0	24.4
CAM	0.37 ± 0.02	47.4	21.1

To achieve signal processing at ≥ 20 fps, $t_{frame} \leq 50$ ms is required. Both the Doppler and CAM methods meet this requirement, whereas the speckle tracking method does not. The Doppler method exhibited the shortest execution time, followed by CAM.

The reported runtimes include only the algorithm processing time. These results are specific to our simulation case and hardware. The feasibility of real-time execution on commercial hardware was not evaluated.

4.6. A Deeper Analysis of CAM Under SNR/Strain Variable Conditions

Given that CAM performed best in displacement field, strain, and tumor size estimation, a more in-depth analysis was conducted considering SNR variability and strain percentage.

First, the number of runs was justified for statistical relevance. In the simulations, 100 iterations of the parameter estimation process were performed. To demonstrate the convergence of the RMSE $\pm$ SD, Case I was selected, representing the smallest tumor (size of 1 mm at a depth of 1 cm). The RMSE $\pm$ SD of tumor strain estimation was obtained as a function of the number of Monte Carlo iterations, as shown in Figure 7.

It can be observed that by the 100th iteration, both the RMSE and the SD had stabilized. Therefore, 100 iterations were selected to ensure statistical stability. The mean RMSE converged with an acceptable SD ($<10\%$ RMSE).

The performance of CAM as a function of the failure rate is presented below. To obtain this, Case I (smallest tumor at superficial depth) was used, and two strain scenarios were applied: 0.5% and 5%. For each scenario, the SNR was varied from 10 to 40 dB in steps of 5 dB. The RMSE $\pm$ SD was obtained, as illustrated in Figure 8.

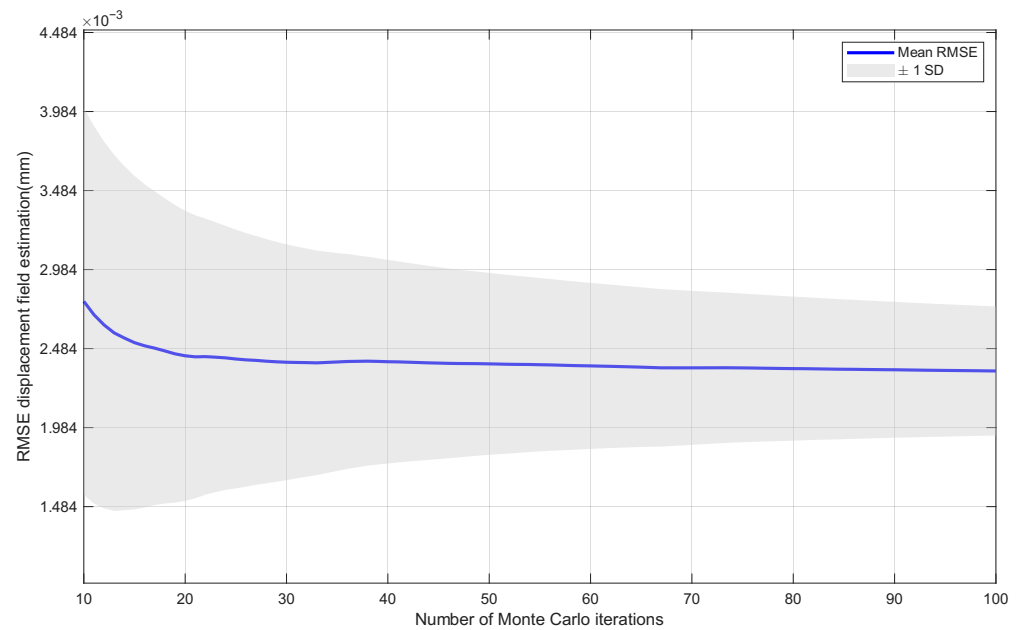


Figure 7. Convergence of RMSE $\pm$ SD as a function of the number of iterations for CAM applied to Case I.

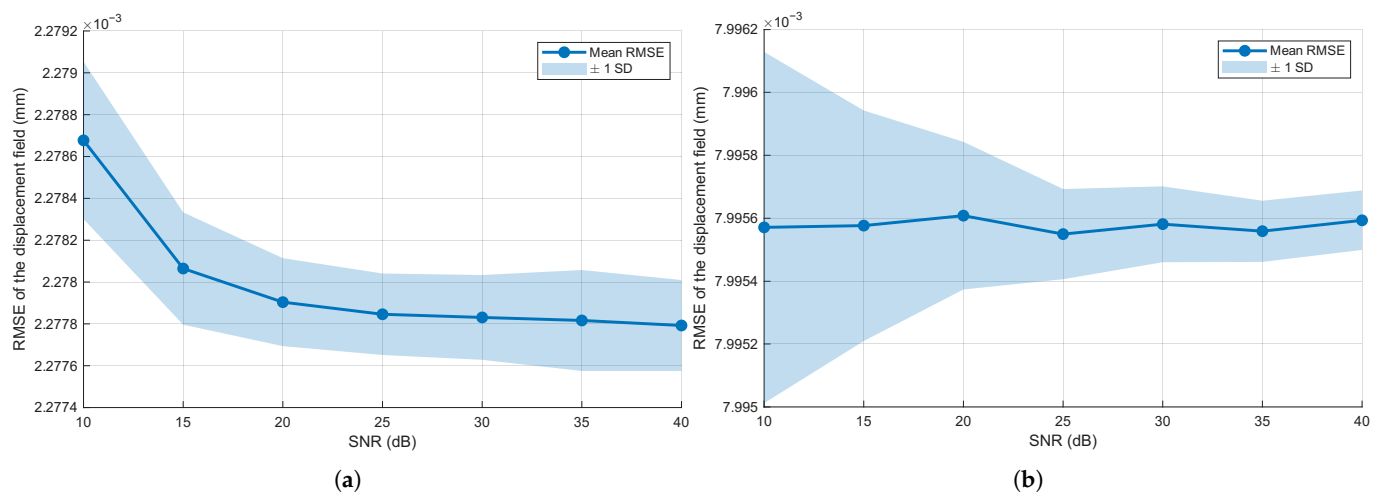


Figure 8. RMSE $\pm$ SD of displacement field estimation as a function of SNR for CAM applied to Case I: (a) low-strain scenario (0.5%); (b) high-strain scenario (5%).

For small strains (0.5%), the RMSE decreased with increasing SNR, and the variance remained stable across the SNR range. For higher strains (5%), the RMSE increased compared to 0.5% and fluctuated across the SNR range. Variance decreased favorably with increasing SNR.

To determine the failure rate, we applied the following criterion: if the standard deviation exceeded 10% of the RMSE, the result was considered a *failure*. From the results illustrated in Figure 8, SNR values of 10, 25, and 40 dB were selected. The resulting success–failure map is presented in Table 9.

Only in the scenario with 0.5% strain at SNR = 10 dB did the standard deviation exceed 10% of the RMSE. According to the adopted criteria, this is considered a *failure*; all other scenarios are considered *successes*.

Table 9. Success–failure map for CAM-based displacement estimation (RMSE $\pm$ SD, mm) for Case I under 0.5% and 5% strain. A scenario is classified as a *failure* if SD > 0.1 RMSE.

Strain	SNR = 10 dB	SNR = 25 dB	SNR = 40 dB
0.5%	Failure $2.279 \times 10^{-3} \pm 3.762 \times 10^{-4}$	Success $2.278 \times 10^{-3} \pm 1.947 \times 10^{-4}$	Success $2.278 \times 10^{-3} \pm 2.174 \times 10^{-4}$
5%	Success $7.996 \times 10^{-3} \pm 5.579 \times 10^{-4}$	Success $7.996 \times 10^{-3} \pm 1.437 \times 10^{-4}$	Success $7.996 \times 10^{-3} \pm 9.460 \times 10^{-5}$

5. Discussion

The implemented signal model, the single-inclusion model, considers a three-layer tissue structure (healthy tissue–tumor–healthy tissue) that has been commonly used in previous studies [19,30,31] to evaluate the performance of signal processing techniques for ultrasonic elastography. The advantages of this model are as follows: (1) It adequately approximates conditions of malignant tumor tissue embedded in healthy breast tissue: malignant tumors are usually firmly attached to the surrounding tissue by dense, rigid connective tissue [32], which can be represented by abrupt changes in elastic parameters at the tumor–healthy tissue interface. (2) It is computationally practical. (3) The results are graphically representative; however, since real tumors have irregular shapes, spatial stiffness gradients, and are embedded in heterogeneous fat/fibroglandular backgrounds, the effects of motion artifacts and anatomically realistic tissue simulation were included in the model.

In our model, a $4.5\times$ strain ratio (malignant lesion/healthy tissue) was selected. Young’s modulus values were extracted from clinical cases reported in [1], where this strain ratio is characteristic of DCIS relative to healthy tissue. In SE, a strain ratio greater than 4.5 suggests a malignant lesion. The average Young modulus of healthy breast tissue is 2.3 ± 0.8 kPa for fibroglandular tissue and 0.43 ± 0.07 kPa for fatty tissue [33], values that are similar and fall within the range (≤ 12 kPa) reported by several authors. In contrast, the average Young’s modulus of DCIS is 117.8 ± 54.72 kPa [34]. These differences allow discrimination between malignant lesions and surrounding healthy soft tissue. Simulating a layered model that separates fat and fibroglandular tissues is unlikely to provide additional insight for this analysis, which focuses on tumor depth and size.

The literature reports comparisons of displacement estimation methods in elastography, including the speckle tracking method and the combined autocorrelation method [35]. Experimental results validate a new method called the block-matching method, which improves estimation accuracy compared to the other two methods. A tissue model with a circular inclusion four times more rigid than the surrounding tissue was used. Experiments estimated the displacement field and strain, with the RMSE used to assess CAM performance for a 2 cm inclusion at a depth of 1 cm, resulting in an RMSE of 0.0798 mm. The block-matching method achieved an RMSE of 0.0657 mm. In our simulations, a consistent result was obtained for a 1 mm DCIS tumor at the same depth, yielding an RMSE of 0.0026 mm. Unlike the experimental results, which calculated the RMSE across the entire depth, our simulations estimated the RMSE only in the tumor region. Our results provide information on algorithm performance for small tumors (<5 mm) and include both the mean RMSE and standard deviation for displacement and strain estimation.

The results show that CAM outperformed the speckle tracking and Doppler methods for displacement field estimation, although the Doppler method obtains an RMSE value

almost equal to CAM for the cases of 3 mm and 5 mm tumors at intermediate depths. CAM exhibited greater stability against variations in ultrasonic speckle decorrelation when using simulated signals in a controlled environment. In strain elastography, an external force deforms the tissue surface. Scatterers close to the surface experience greater displacement than deeper scatterers, causing significant positional changes in near-surface scatterers and substantial variations in the speckle signal. CAM's superior ability to handle speckle variations resulted in lower RMSE values when estimating displacements in the tumor region at superficial and intermediate depths compared to other methods (see Table 2). CAM enabled reconstruction of smoother fields that more closely reflected physical reality. CAM maintained the best performance in estimating displacement fields for the smallest lesion, and this performance persisted even under strains exceeding half the wavelength, which is the displacement detection limit of the Doppler method.

CAM achieved the best balance of accuracy for strain and tumor size estimations (see Tables 3 and 4). The results indicate that CAM significantly improves stiffness accuracy and spatial resolution for all early-stage tumor sizes evaluated. CAM enables better delineation of tumor margins by providing greater separation between the estimated tumor strain and the healthy tissue strain threshold. It improves lesion size assessment by moving the threshold away from the noisy region of healthy tissue, allowing more accurate differentiation of small lesions in noisy backgrounds. This approach can detect cancers with Young's modulus closer to that of surrounding healthy tissue, where other methods lose accuracy (see Figure 5).

The simulation model assumes an SNR of 35 dB, a typical value for elastography RF signals, representing noise from common sources such as electronics, quantization, or the transducer. Given that CAM performed better in estimating the displacement field, strain and size, some experiments were conducted on Case I using an SNR sweep ($10 < \text{SNR} < 40$ dB) and two strain scenarios (low strain 0.5% and high strain 5%) to evaluate estimation performance. For the low strain scenario, the RMSE decreases as the SNR increases and the SD remains stable. For the high strain scenario, an RMSE inflation of 3.5 is obtained, fluctuating across the SNR range, and the SD decreases as the SNR increases, remaining stable at high SNRs. It is concluded that the CAM method demonstrates stable performance for the range of SNR values considered and under different strain conditions.

After evaluating the performance of the methods under ideal conditions (single-inclusion model with sharp boundaries and uniform stiffness), real conditions (gradient boundary, 30% stiffness variation in background, and 20% stiffness variation in the tumor) based on anatomically realistic tissue informed in the literature were added to the model. The effects of motion artifacts on the model were also considered. $\text{RMSE} \pm \text{SD}$ (mm) values for displacement field and strain estimation in the tumor region were obtained using the three methods under motion artifacts and anatomically informed conditions, quantitatively demonstrating that CAM yields the smallest RMSE and SD values. The Doppler method ranked second in this metric, and the speckle tracking method performed the worst. The CAM method demonstrates the best robustness against the presence of the main sources of error in strain elastography.

Under the same conditions of motion artifacts and anatomically informed conditions, tumor size estimates (mm) were obtained using the FWHM method. The CAM method obtained tumor size values closest to the theoretical values with bootstrap 95% confidence intervals followed by the Doppler method.

A comparison of computational performance was carried out for the three methods, obtaining the fps that each method is capable of processing on our specific hardware. The best performance was achieved by the Doppler method, followed by CAM. Using

MATLAB R2025a with the specific CPU described above, the Doppler and CAM methods meet the 20 fps requirement.

Clinically, two-dimensional processing provides valuable information; however, the aim of our work is to perform a quantitative, point-by-point assessment along an axial path of the medium under study, including small inclusions under controlled conditions. We seek to provide numerical errors obtained by three methods used to estimate displacements and strains. In technical analysis, quantifying the performance of each method and relating it to its one-dimensional processing capabilities is relevant for identifying the best-performing method. In one-dimensional processing, quantitative details reveal the advantages of one method over another, as shown in reference [19], where the authors present the axial behavior of strain to illustrate the central line profile in an inclusion and compare it with the true strain. A similar one-dimensional analysis is presented in [35], where displacement and strain are shown in one-dimensional graphs to compare method results with true values. In a future stage, we could expand the scope of the research to two-dimensional processing.

The results demonstrate CAM's potential for detecting early-stage tumors in the millimeter range, particularly for lesions smaller than 5 mm.

Other Error Sources Due to Real Conditions

Using a simulation methodology allows evaluation of method performance under controlled conditions and estimation of the impact of parameters of interest (e.g., tumor stiffness and size) independently of external phenomena that could produce undesirable effects. Without simulation, signals would contain uncontrollable parameters present in practice yielding results that deviate from the theoretical model and cannot be interpreted as errors associated solely with the method. In this work, we focus on evaluating the effectiveness of the most common strain elastography methods for estimating (a) displacements and strain associated with tissue compression in a small malignant lesion, and (b) lesion size, under typical strain elastography conditions ($\text{SNR} = 35 \text{ dB}$, $\% \text{ strain} \leq 1\%$, $f_0 = 5 \text{ MHz}$, $\text{BW} = 58\%$). Some extended analysis were performed to quantify the impact of factors (SNR and strain variations, anatomically realistic tissue and motion artifact conditions) on method performance, as the main objective was to determine the potential of each technique to support early detection of malignant lesions. Nevertheless, other factors present in real cases, such as phase aberration, ultrasound system and signal processing parameters, scatterer motion and excessive compression, could degrade results. To be aware their possible effects, this section describes some of these factors and their impact on parameter estimation.

Phase aberration (PA): PA arises from tissue structures interacting with the ultrasonic wave, producing undesired phase shifts in the wavefront. PA can induce errors in the correlation coefficient (CC) used in strain elastography to estimate compression-induced displacement. The effect of PA on the CC depends on the applied $\%$ strain. For example, a slight reduction in the CC vs. PA standard deviation curve is observed at 1% strain compared with 0% strain [30], whereas a strain of 4% produces a drastic decrease. In our simulation, a maximum strain of 1.76% was considered. Statistically similar phase errors in pre- and post-compression signals do not significantly affect CC estimation.

Ultrasound system and signal processing parameters: Pulse central frequency and bandwidth, window size, and overlap can affect strain estimation. Elastography resolution is sensitive to window size and overlap, which also influence the variance of time-delay estimation and, consequently, strain estimation variance.

Scatterer motion: Rotation or biaxial deformations in tissue structures during compression introduce errors in CC estimation and, hence, in strain estimation.

The variance in time-delay estimation due to these factors has been analyzed, and several metrics have been proposed to quantify their effects on strain estimation using elastography methods. Examples include the Cramer–Rao lower bound (CRLB), the Barankin bound, and the Ziv–Zakai lower bound (ZZLB), which estimate time-delay estimation variance for the cross-correlation method [36]. The CRLB is used under high SNRs, but at lower SNRs, variance exceeds the CRLB and the ZZLB is used.

To date, no metric exists in the literature to estimate time-delay variance for the CAM. Therefore, it is not possible to quantitatively assess the influence of the factors listed above on the measures presented in this work.

6. Conclusions

Detecting smaller tumors is increasingly challenging for signal processing algorithms, as the distribution of displacements within the tumor region becomes harder to resolve. Very small tumors, such as those in early stages (on the order of a few millimeters), provide few samples for displacement field and strain estimation. Early detection is critical to prevent tumor growth into more advanced stages. The results considered a range of tumor sizes from 1 mm to 5 mm based on real clinical cases.

The combined autocorrelation method (CAM) proved to be a highly effective approach for detecting early-stage breast tumors using strain elastography. CAM was able to detect tumors smaller than 5 mm at an intermediate depth of 3 cm, as well as superficial tumors of 1 mm, in a three-layer model: healthy tissue–tumor–healthy tissue.

The superior performance of CAM arises from its use of both amplitude and phase information in the speckle signal, which is not fully exploited by other methods. In this method, a coarse displacement estimate is first obtained using the correlation envelope, followed by a more precise adjustment using the phase difference between pre- and post-compression signals.

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Abbreviations

The following abbreviations are used in this manuscript:

RMSE	Root Mean Square Error
CAM	Combined Autocorrelation Method
DCIS	Ductal Carcinoma In Situ

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