

Article

Event-Triggered Robust Fusion Estimation for Multi-Sensor Systems Under Random Packet Drops

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Abstract

This paper focuses on the design of robust fusion estimators for multi-sensor systems experiencing constrained communications, model uncertainties, and random packet dropouts. To mitigate the impact of modeling errors, a sensitivity-penalized robust state estimator is employed at each local estimator. At the local fusion estimators, a centralized robust fusion estimation algorithm is derived by improving the cost function of the sensitivity-penalized estimator. The implementation of an event-triggered strategy effectively alleviates the burden on the communication channels linking the sensors and the fusion center. Moreover, the fusion estimator is capable of handling packet drops caused by unreliable communication channels, and the pseudo cross-covariance matrix is accordingly formulated. Sufficient conditions are derived to ensure the uniform boundedness of the estimation error for the proposed robust fusion estimator. Finally, simulation experiments using a tractor-car system validate the performance and advantages of the presented algorithm.

Keywords: multi-sensor systems; robust fusion estimation; modeling errors; random packet dropouts; event-triggered

1. Introduction

The rapid progress in communication and control technologies has led to the extensive adoption of multi-sensor systems across diverse domains, including motor control [1], path planning [2], and trajectory tracking [3,4]. In multi-sensor systems, collaborative data acquisition by multiple sensors enhances the accuracy and stability of the overall system. However, sensor malfunctions or cyber-attacks in the communication channel may cause communication delays and stochastic packet drops. Communication delays may significantly affect information exchange and coordination in networked systems; for example, Shi analyzed the convergence rate of delayed consensus based on products of substochastic matrices [5]. Moreover, asynchronous interactions over cooperation–competition networks have also been investigated in recent consensus-tracking studies [6]. In a related line of research, networked control systems with remote and local controllers over unreliable communication channels have been studied to address the impact of channel unreliability on closed-loop performance [7]. Furthermore, decentralized output-feedback control with asymmetric one-step delayed information has been investigated to characterize how nonuniform delayed information affects decentralized controller design [8]. Therefore, how to effectively handle packet losses during data transmission and reduce the data transmission rate of sensor nodes has become a critical research topic.



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In multi-sensor systems, data processing is typically performed through information fusion. The fundamental fusion estimation methods can be broadly categorized into centralized [9,10] and distributed [11,12]. Centralized fusion estimation processes all sensor data within a central processing unit, allowing for access to global system state information and, in theory, providing optimal estimation performance. However, with more sensors, the centralized architecture places a significant burden on transmission bandwidth and computational resources. This can lead to limitations in the processing capacity of the central node, making it difficult to meet real-time and scalability requirements [13,14]. In contrast, distributed fusion estimation methods allocate the information processing tasks across multiple sensor nodes, where each node independently processes its local measurements and transmits the local estimates to a fusion center to form the global estimate. As a result, distributed schemes offer better stability and flexibility in resource-constrained or communication-limited environments. Common distributed fusion algorithms include the federated Kalman filter [15], the best linear unbiased estimator [16,17], and matrix-weighted fusion methods [18,19]. A two-layer distributed and clustered fusion estimation method was introduced in [20] for discrete-time linear systems with time-varying dynamics and correlated sensor noise. For clustered sensor networks with uncertain noise variances, ref. [21] proposed a two-level robust Kalman filtering method. Several studies, particularly [22,23], have developed distributed fusion estimation methods for multi-sensor systems affected by stochastic packet dropouts. In [24], a linear fusion method was adopted for fusion estimation in systems subject to packet loss.

Furthermore, fusion estimation often incurs high communication overhead. To balance estimation accuracy and communication costs, various transmission strategies have been proposed. For instance, in [25], the send-on-delta method triggers data transmission when the change exceeds a set threshold. In [26], a local data-driven transmission strategy is proposed and integrated with Kalman filtering to construct a fusion estimator. A threshold-based event-triggered strategy was developed in [27]. An adaptive event-triggered strategy was introduced in [28], dynamically adjusting its triggering threshold according to real-time changes in the system state. In [29], a distributed event-triggered strategy was introduced, where subsystems broadcast state information only when local state errors exceed predefined bounds. Overall, the event-triggered transmission strategy offers a more efficient solution for data transmission in multi-sensor systems, significantly improving closed-loop control performance and resource utilization efficiency.

State estimation constitutes a core research focus in the domain of industrial automation. Over the years, numerous estimation methods have been developed, including the Kalman filter, the Wiener filter, and related techniques. However, modeling errors remain unavoidable during system modeling. Therefore, the performance of the estimator should not exhibit abrupt degradation when system parameters deviate within a reasonable range from their nominal values. Estimators possessing this property are referred to as robust state estimators, and various related methods have been proposed in the literature [30–34]. A robust filtering framework based on regularized least squares was proposed in [30]. It is conceptually close to the classical Kalman filter in that it corrects model parameters around given nominal values. Nevertheless, this line of robust design typically emphasizes worst-case performance and may become conservative when the underlying uncertainties are relatively small. An alternative viewpoint in robust filtering employs a sensitivity-penalty mechanism with respect to variations in plant parameters. In [31], this idea was developed for single-input single-output systems in the frequency domain using transfer-function representations and spectral decomposition. The approach was later extended in [32] to multi-input multi-output time-varying systems within a state-space setting, where the plant parameters are affected by modeling errors in a fairly general manner. By exploit-

ing the connection between the Kalman filter and Regularized Least Squares, together with the sensitivity of the estimation error to parameter perturbations, one can derive an analytical expression for the optimal estimator, provided that the plant parameters are differentiable with respect to the modeling error. Furthermore, ref. [33] developed a robust estimation approach by minimizing the expected estimation error, while [34] proposed an implementation-friendly error estimator. These studies highlight the practical value of robust state estimation in multi-sensor systems. Representative robust estimation methods are summarized in Table 1.

Table 1. Comparison of representative robust estimation techniques.

Ref.	Technique	Main Features
[30]	Regularized robust state estimation	The effect of state estimation is conservative when the model error is relatively small.
[32]	Robust state estimation based on sensitivity penalty	There are only very weak restrictions on the ways in which uncertainty can affect plant parameters, and model uncertainty can take an arbitrary structure.
[33]	Robust state estimation based on minimizing estimation error variance	A robust state estimator derived by minimizing the estimation error variance, and the model uncertainty can have an arbitrary structure.
[34]	Implementation-friendly estimator	Recursive form convenient for coding and real-time implementation.

In addition to classical robust estimation and filtering techniques, recent studies have also explored robustness-enhancing mechanisms from fractional calculus and learning-oriented perspectives. Fractional-order modeling and filtering have been investigated to improve estimation robustness and capture memory effects, e.g., robust extended fractional Kalman filtering for nonlinear systems and related observability and estimation analysis for fractional-order dynamics [35,36]. Meanwhile, fractional-gradient-based learning has emerged as an alternative robustness mechanism, where tempered fractional gradient descent and stochastic fractional-order gradient methods have been reported to enhance convergence stability under noise and model mismatch [37,38]. Hence, robust state estimation remains a critical issue for multi-sensor systems operating under modeling uncertainties.

Robust fusion estimation is investigated for discrete-time linear multi-sensor systems in the presence of event-triggered transmission strategies, parameter uncertainties, and random packet dropouts. The main contributions of this work are summarized as follows:

- (1) A robust fusion estimation algorithm is proposed, which consists of a localized robust state estimation algorithm and the optimal information fusion criterion in the linear minimum variance sense, aiming to address the state estimation problem of multi-sensor systems subject to modeling uncertainties, communication constraints, and random packet dropouts.
- (2) To cope with event-triggered transmission strategies and data losses occurring in the communication channels between sensor nodes and the fusion center, the measurement channel model is reconstructed.
- (3) The pseudo cross-covariance matrix is updated to handle random packet dropouts in the measurement channel, based on which a novel global pseudo-covariance matrix is derived.
- (4) Under certain stabilizability and detectability conditions, the uniform boundedness of the estimation error of the fusion estimator is guaranteed.

The structure of the remainder of this paper is organized as follows: Section 2 discusses the relevant background and introduces the event-triggered transmission strategy.

Section 3 derives a robust fusion estimator for multi-sensor systems operating with an event-triggered transmission strategy and subject to random packet dropouts. Section 4 provides a boundedness analysis of the fusion estimator. Numerical simulations are provided in Section 5 to validate the effectiveness of the proposed approach, and this paper is concluded in Section 6.

The following symbols are used. The symbol $E[*]$ represents the mathematical expectation of a stochastic variable. $\|*\|_{\Omega}$ represents the Euclidean norm with weighting coefficient Ω . The m -dimensional Euclidean space is denoted by R^m . $f(*)$ represents a probability density function, and the Gaussian distribution, characterized by mean σ and covariance Ω , is represented as $N(\cdot; \sigma, \Omega)$. $col(*)$ denotes a column-stacked matrix. I_i represents the $i \times i$ identity matrix. The trace of the matrix is denoted by $tr(*)$.

2. Problem Formulation and Some Preliminaries

The state-space representation of the multi-sensor system can be expressed as follows:

$$\begin{cases} x_{k+1} = A_k(\varepsilon_k)x_k + B_k(\varepsilon_k)w_k \\ y_k^i = C_k^i(\varepsilon_k)x_k + v_k^i \end{cases}, i = 1, \dots, l \quad (1)$$

where k and i indicate the discrete-time and sensor indices, respectively. The system state is x_k , measurements are denoted by y_k^i , and w_k denotes process noise. v_k^i accounts for both measurement and communication noise. The variables x_0 , w_k , and v_k^i are independent. The elements of matrices $A_k(\varepsilon_k)$, $B_k(\varepsilon_k)$ and $C_k^i(\varepsilon_k)$ are all differentiable functions of the known model error ε_k . The model error ε_k consists of L real-valued scalar uncertainties $\varepsilon_{k,j}$, for $j = 1, 2, \dots, L$, and these L uncertainty components are assumed to be mutually independent.

$$\begin{aligned} E(w_k) = 0, E(v_k^i) = 0, \\ E\left(\begin{bmatrix} x_0 - E(x_0) \\ w_k \\ v_k^i \end{bmatrix} (*)^T\right) = \begin{bmatrix} \Pi_0 & & \\ & Q_k \delta_{kj} & \\ & & R_k^i \delta_{kj} \end{bmatrix}, \end{aligned}$$

where the matrices Π_0 , Q_k , and R_k^i are considered known and positive-definite. Here, $\delta_{kj} = \begin{cases} 1, k = j \\ 0, k \neq j \end{cases}$.

Assuming that each sensor node is capable of computing a robust state estimate, this paper proposes a fusion method that integrates local estimations gathered from several sensor nodes within transmission constraints. To reduce transmission congestion and energy consumption, an event-triggered transmission strategy similar to that in [22] is adopted. A brief introduction to the event-triggered transmission strategy is provided below.

A noisy measurement channel is considered, which can be mathematically represented as follows:

$$Y = HX + v, \quad (2)$$

where $Y \in R^m$ represents the measurement output, $H \in R^{m \times n}$ denotes the systems measurement matrix, and $X \in R^n$ represents the system state. The term $v \in R^m$ denotes the measurement noise. A binary variable c is introduced, where $c_k = 1$ indicates that the measurement Y is transmitted, and $c_k = 0$ indicates that it is not transmitted. The event-triggered transmission strategy can be described by:

$$c_k = \begin{cases} 0, & Y - \tilde{Y} \in \Xi \\ 1, & \text{others} \end{cases}, \quad (3)$$

where $\tilde{Y} \in R^m$ and $\Xi \subseteq R^m$ represent a measurable set. Define the triggering innovation as $e = Y - \tilde{Y}$. Under the measurement model (2), e is induced by the same measurement matrix H and noise v_k , while $\tilde{Y}_k$ serves as a reference measurement constructed from (2) by replacing the unknown state with its prior mean, i.e., $\tilde{Y} = H\bar{X}$. The triggering set Ξ in (3) is chosen as an origin-centered ellipsoid $\Xi = e^\top \Sigma_y^{-1} e$, i.e., $\int_{\Xi} \zeta d\zeta = 0$, where $\Sigma_y = \Sigma_v + H\Sigma_x H^\top$. Under the event-triggered transmission strategy, data transmission occurs only when the deviation between the measurement Y and a reference measurement $\tilde{Y}$ exceeds a predefined threshold Ξ .

The transmission rate α is utilized to quantify each sensor node, where $\alpha \in (0, 1)$ is derived through $\lim_{\kappa} \frac{1}{\kappa} \sum_{k=1}^{\kappa} E\{c_k\} = \alpha$. The virtual measurement $\tilde{Y} = HX + v - g$ is defined by Lemma 1 in [26], where g is a uniformly distributed random variable over Ξ , uncorrelated with both X and v . Assuming $f_X(x) = N(x; \bar{x}, \Sigma_x)$, $f_v(v) = N(v; 0, \Sigma_v)$ and $f_Y(y) = N(y; H\bar{x}, \Sigma_y)$, the optimal transmission strategy is given by:

$$\|Y - H\bar{x}\|_{\Sigma_y^{-1}}^2 \geq \Lambda, \quad (4)$$

where Σ_y denotes the covariance in the measurement Y under the Gaussian prior $X \sim \mathcal{N}(\bar{x}, \Sigma_x)$ and the measurement noise $v \sim \mathcal{N}(0, \Sigma_v)$. Accordingly, we set the threshold as $\Lambda = \rho_m^{-1}(1 - \alpha)$. The term $\|Y - H\bar{x}\|_{\Sigma_y^{-1}}^2$ follows a chi-squared distribution possessing m degrees of freedom, and ρ_m represents the cumulative distribution function corresponding to this chi-square distribution.

Remark 1. In the fusion framework illustrated in Figure 1, a local fusion estimation layer is introduced to effectively reduce the communication load and redundant data transmission among spatially correlated sensors. In this structure, sensors located within the same sub-area or exhibiting highly correlated measurements are grouped and jointly processed by a local fusion estimator. The estimator performs centralized fusion within the sub-area to generate a more compact local estimation result. This hierarchical architecture significantly alleviates the network communication burden, enhances system robustness under random packet dropout conditions, and improves the utilization efficiency of limited communication resources.

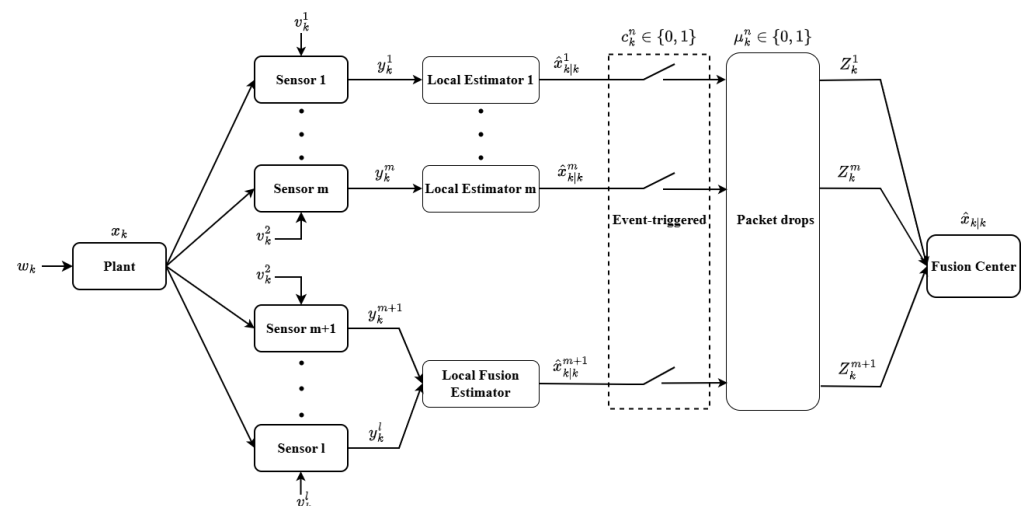


Figure 1. Robust fusion scheme.

3. Robust Fusion Estimation Procedure

Taking into account the influence of modeling errors on estimation accuracy, this paper employs the sensitivity-penalized robust state estimation method from [32] to obtain

both the local state estimates and the local fusion estimates. This robust state estimator is developed by combining the Kalman filter with regularized least squares, incorporating a penalty for the sensitivity of the estimation error to model uncertainties. It maintains the same recursive structure as the Kalman filter but incorporates a parameter correction procedure, resulting in a comparable computational complexity. In the following, the analytical expressions for the local state estimator and the local fusion estimator are derived.

3.1. Local Robust Estimation

Assume that the parameters of System (1) are accurately known during modeling, i.e., $\varepsilon_k = 0$. In this case, the state estimation problem for System (1) can be directly addressed using the conventional Kalman filter. As shown in [30], this filter may also be formulated as the solution to a regularized least-squares problem. It is now extended to multi-sensor systems as follows:

$$\begin{aligned} \hat{x}_{k+1|k+1}^n &= A_k(0)\hat{x}_{k|k+1}^n + B_k(0)\hat{w}_{k|k+1}^n, \\ \begin{pmatrix} \hat{x}_{k|k+1}^n \\ \hat{w}_{k|k+1}^n \end{pmatrix} &= \arg \min_{x_k, w_k} \begin{bmatrix} \|x_k - \hat{x}_{k|k}^n\|_{(P_{k|k}^n)^{-1}}^2 + \|w_k\|_{Q_k^{-1}}^2 \\ + \|y_{k+1}^n - C_{k+1}^n(0)x_{k+1}\|_{(R_{k+1}^n)^{-1}}^2 \end{bmatrix}, \end{aligned} \quad (5)$$

where n denotes the estimator index. $\hat{x}_{k|t}^n$ represents the state estimate obtained from the measurement sequence $y_g|_{g=0}^t$, and $P_{k|k}^n$ represents the covariance matrix of the estimation error.

Remark 2. For the first m sensors, each corresponds to one of the m local estimators. Since the sensor index and the estimator index have a one-to-one numerical correspondence, the notation in this section is unified by using the estimator index n for labeling, in order to simplify the expressions.

To simplify (5), define the following matrices:

$$\begin{aligned} \alpha_k^n &= \text{col} \{x_k - \hat{x}_{k|k}^n, w_k\}, \\ \Phi_k^n &= \text{diag} \left\{ (P_{k|k}^n)^{-1}, Q_k^{-1} \right\}, \\ H_k^n(\varepsilon_k, \varepsilon_{k+1}) &= C_{k+1}^n(\varepsilon_{k+1}) \begin{bmatrix} A_k(\varepsilon_k) & B_k(\varepsilon_k) \end{bmatrix}, \\ \eta_k^n(\varepsilon_k, \varepsilon_{k+1}) &= y_{k+1}^n - C_{k+1}^n(\varepsilon_{k+1})A_k(\varepsilon_k)\hat{x}_{k|k}^n. \end{aligned} \quad (6)$$

Substituting Equation (6) into Equation (5) allows the cost function in Equation (5) to be rewritten in the following form:

$$J(\alpha_k^n) = \|\alpha_k^n\|_{\Phi_k^n}^2 + \|H_k^n(0,0)\alpha_k^n - \eta_k^n(0,0)\|_{(R_{k+1}^n)^{-1}}^2. \quad (7)$$

When $\varepsilon_k \neq 0$, modeling errors can be addressed using the method from [32] to enhance estimation accuracy. The cost function in Equation (7) is then revised as follows:

$$\begin{aligned} J(\alpha_k^n) &= \gamma_k^n \left[\|\alpha_k^n\|_{\Phi_k^n}^2 + \|H_k^n(0,0)\alpha_k^n - \eta_k^n(0,0)\|_{(R_{k+1}^n)^{-1}}^2 \right] \\ &+ (1 - \gamma_k^n) \sum_{j=1}^L \left(\left\| \frac{\partial e_k^n(\varepsilon_k, \varepsilon_{k+1})}{\partial \varepsilon_{k,j}} \right\|^2 + \left\| \frac{\partial e_k^n(\varepsilon_k, \varepsilon_{k+1})}{\partial \varepsilon_{k+1,j}} \right\|^2 \right) \bigg|_{\substack{\varepsilon_k=0 \\ \varepsilon_{k+1}=0}}, \end{aligned} \quad (8)$$

where $e_k^n(\varepsilon_k, \varepsilon_{k+1}) = y_{k+1}^n - C_{k+1}^n(\varepsilon_{k+1})A_k(\varepsilon_k)\hat{x}_{k|k}^n - C_{k+1}^n(\varepsilon_{k+1})\begin{bmatrix} A_k(\varepsilon_k) & B_k(\varepsilon_k) \end{bmatrix}\alpha_k^n$ is referred to as the innovation process. Its deviation from the nominal innovation $y_{k+1}^n - C_{k+1}^n(0)[A_k(0)\hat{x}_{k|k}^n + B_k(0)\hat{w}_{k+1}^n]$ captures the contribution of modeling errors to the prediction error of y_{k+1}^n .

In the cost function (8), $\gamma_k^n \in (0, 1]$ is a nonnegative design scalar. The first term enforces nominal estimation performance, whereas the sensitivity-penalization term suppresses the degradation caused by model uncertainty by penalizing the sensitivity magnitude of the matching error $H_k^n(\varepsilon_k, \varepsilon_{k+1})\alpha_k^n - \eta_k^n(\varepsilon_k, \varepsilon_{k+1})$ with respect to the modeling-error components $(\varepsilon_{k,j}, \varepsilon_{k+1,j})_{j=1}^L$ around the nominal parameter values.

In practice, γ_k^n can be selected empirically and then tuned according to the relative magnitude of modeling errors: a larger modeling-error magnitude suggests using a smaller γ_k^n , which assigns more weight to robustness against model mismatch, consistent with intuition. When modeling errors are absent and $\gamma_k^n = 1$, the state estimator obtained by minimizing (8) reduces to the standard Kalman filter.

Next, we define two important matrices, S_k^n and T_k^n , which are essential for parameter correction, as described below:

$$\begin{aligned} S_k^n &= \left[\left(S_{k,1}^n(0,0) \right)^T, \dots, \left(S_{k,L}^n(0,0) \right)^T \right]^T, \\ T_k^n &= \left[\left(T_{k,1}^n(0,0) \right)^T, \dots, \left(T_{k,L}^n(0,0) \right)^T \right]^T, \end{aligned} \quad (9)$$

in which

$$\begin{aligned} S_{k,j}^n(\varepsilon_k, \varepsilon_{k+1}) &= \begin{bmatrix} \frac{\partial C_{k+1}^n(\varepsilon_{k+1})}{\partial \varepsilon_{k+1,j}} A_k(\varepsilon_k) \\ C_{k+1}^n(\varepsilon_{k+1}) \frac{\partial A_k(\varepsilon_k)}{\partial \varepsilon_{k,j}} \end{bmatrix}, \\ T_{k,j}^n(\varepsilon_k, \varepsilon_{k+1}) &= \begin{bmatrix} \frac{\partial C_{k+1}^n(\varepsilon_{k+1})}{\partial \varepsilon_{k+1,j}} B_k(\varepsilon_k) \\ C_{k+1}^n(\varepsilon_{k+1}) \frac{\partial B_k(\varepsilon_k)}{\partial \varepsilon_{k,j}} \end{bmatrix}, \\ j &= 1, 2, \dots, L. \end{aligned}$$

Remark 3. Compared with the Kalman filter, the methods in this subsection mitigate modelling errors effects by introducing parameter-correction terms into the recursive update. In particular, S_k^n and T_k^n are central to these corrections: they are designed to lower the sensitivity of the estimator's performance to modelling uncertainties while also streamlining the subsequent expansion of the cost function.

Given the properties of R_{k+1}^n and Φ_k^n , the function $J(\alpha_k^n)$ is strictly convex for $0 < \gamma_k^n \leq 1$. Setting $\partial J(\alpha_k^n) / \partial \alpha_k^n = 0$ yields the unique global minimizer of α_k^n , expressed as follows:

$$\begin{aligned} \frac{\partial J(\alpha_k^n)}{\partial \alpha_k^n} &= 2\gamma_k^n \Phi_k^n \alpha_k^n + 2\gamma_k^n (H_k^n(0,0))^T \\ &\quad \times (R_{k+1}^n)^{-1} H_k^n(0,0) \alpha_k^n \\ &\quad - 2\gamma_k^n (H_k^n(0,0))^T (R_{k+1}^n)^{-1} \eta_k^n(0,0) \\ &\quad + 2(1 - \gamma_k^n) \begin{bmatrix} S_k^n & T_k^n \end{bmatrix}^T \begin{bmatrix} S_k^n & T_k^n \end{bmatrix} \alpha_k^n \\ &\quad + 2(1 - \gamma_k^n) \begin{bmatrix} S_k^n & T_k^n \end{bmatrix}^T S_k^n \hat{x}_{k|k}^n. \end{aligned} \quad (10)$$

Define $\lambda_k^n = (1 - \gamma_k^n) / \gamma_k^n$. The local estimate $\hat{x}_{k|k}^n$ is then computed via the following recursive update. The detailed derivation is provided in Appendix A.

(1) Initialization. The matrix $P_{0|0}^n$ and the estimate $\hat{x}_{0|0}^n$ are expressed as follows:

$$\begin{aligned} P_{0|0}^n &= \left[(\hat{\Gamma}_0^n)^{-1} + (C_0^n(0))^T (R_0^n)^{-1} C_0^n(0) \right]^{-1}, \\ \hat{x}_{0|0}^n &= P_{0|0}^n (C_0^n(0))^T (R_0^n)^{-1} y_0^n, \end{aligned} \quad (11)$$

where

$$\hat{\Gamma}_0^n = \left(\Pi_0^{-1} + \lambda_0^n \sum_{j=1}^L \left(\frac{\partial (C_0^n(\varepsilon_0))}{\partial \varepsilon_{0,j}} \right)^T \left(\frac{\partial C_0^n(\varepsilon_0)}{\partial \varepsilon_{0,j}} \right) \Big|_{\varepsilon_0=0} \right).$$

(2) **Parameter Modification.** Define the matrices $\hat{T}_k^n$, $\hat{A}_k^n$, $\hat{B}_k^n$, $\hat{P}_{k|k}^n$ and $\hat{Q}_k^n$ as follows:

$$\begin{aligned} \hat{T}_k^n &= T_k^n - \lambda_k^n S_k^n \hat{P}_{k|k}^n (S_k^n)^T T_k^n, \\ \hat{A}_k^n(0) &= \left(A_k(0) - \lambda_k^n \hat{B}_k^n(0) \hat{Q}_k^n (T_k^n)^T S_k^n \right) \left(I - \lambda_k^n \hat{P}_{k|k}^n (S_k^n)^T S_k^n \right), \\ \hat{B}_k^n(0) &= B_k(0) - \lambda_k^n A_k(0) \hat{P}_{k|k}^n (S_k^n)^T T_k^n, \\ \hat{P}_{k|k}^n &= \left[\left(P_{k|k}^n \right)^{-1} + \lambda_k^n (S_k^n)^T S_k^n \right]^{-1}, \\ \hat{Q}_k^n &= \left[Q_k^{-1} + \lambda_k^n (T_k^n)^T \left(I + \lambda_k^n S_k^n P_{k|k}^n (S_k^n)^T \right)^{-1} T_k^n \right]^{-1}. \end{aligned} \quad (12)$$

(3) **State Estimate Updating.** Compute $\hat{x}_{k+1|k+1}^n$ and $P_{k+1|k+1}^n$ using the following equations:

$$\begin{aligned} P_{k+1|k}^n &= A_k(0) \hat{P}_{k|k}^n A_k^T(0) + \hat{B}_k^n(0) \hat{Q}_k^n (\hat{B}_k^n(0))^T, \\ R_{e,k+1}^n &= R_{k+1}^n + C_{k+1}^n(0) P_{k+1|k}^n (C_{k+1}^n(0))^T, \\ P_{k+1|k+1}^n &= P_{k+1|k}^n - P_{k+1|k}^n (C_{k+1}^n(0))^T \left(R_{e,k+1}^n \right)^{-1} C_{k+1}^n(0) P_{k+1|k}^n, \\ \hat{x}_{k+1|k+1}^n &= \hat{A}_k^n(0) \hat{x}_{k|k}^n + P_{k+1|k+1}^n (C_{k+1}^n(0))^T \left(R_{k+1}^n \right)^{-1} \\ &\quad \times \left(y_{k+1}^n - C_{k+1}^n(0) \hat{A}_k^n(0) \hat{x}_{k|k}^n \right). \end{aligned} \quad (13)$$

Remark 4. Although the recursive update of $P_{k+1|k+1}^n$ in (13) retains a Kalman-like form, it should be emphasized that the robustness enhancement is implicitly embedded through the modified system matrices $\hat{A}_k^n$ and $\hat{B}_k^n$. These matrices are not nominal system parameters, but are obtained via the sensitivity-penalized regularization and the parameter modification step in (12), which explicitly accounts for the sensitivity of the innovation error with respect to modeling uncertainties. As a result, the covariance propagation in (13) reflects the robustness modification through the use of $\hat{A}_k^n$ and $\hat{B}_k^n$. Therefore, while the update structure is Kalman-like, $P_{k+1|k+1}^n$ should be interpreted as the estimation error covariance associated with the modified model, rather than the nominal one.

3.2. Local Robust Fusion Estimation

To account for the impact of modeling errors on estimation performance and achieve improved fusion results, the cost function will be updated based on the sensitivity-penalized robust state estimation method presented in [32]. The local robust fusion estimator adopts the following updated cost function:

$$\begin{aligned} J(x_k - \hat{x}_{k|k}^{m+1}, w_k) &= \gamma_k^{m+1} \left[\left\| x_k - \hat{x}_{k|k}^{m+1} \right\|_{\left(P_{k|k}^{m+1} \right)^{-1}}^2 + \|w_k\|_{Q_k^{-1}}^2 + \right. \\ &\quad \left. \left\| z_{k+1} - D_{k+1}(0) [A_k(0)x_k + B_k(0)w_k] \right\|_{G_{k+1}^{-1}}^2 \right] \\ &\quad + \left(1 - \gamma_k^{m+1} \right) \sum_{h=1}^L \left(\left\| \frac{\partial e_k^{m+1}(\varepsilon_k, \varepsilon_{k+1})}{\partial \varepsilon_{k,h}} \right\|^2 + \left\| \frac{\partial e_{k+1}^{m+1}(\varepsilon_k, \varepsilon_{k+1})}{\partial \varepsilon_{k+1,h}} \right\|^2 \right)_{\substack{\varepsilon_k=0 \\ \varepsilon_{k+1}=0}} \end{aligned} \quad (14)$$

where

$$\begin{aligned} z_k &= \text{col}\left\{y_k^n\right\}_{n=m+1}^l, \\ G_k &= \text{diag}\left\{R_k^n\right\}_{n=m+1}^l, \\ D_k(\varepsilon_k) &= \text{col}\left(C_k^n(\varepsilon_k)\right)_{n=m+1}^l. \end{aligned}$$

To simplify Equation (14), define the following matrices:

$$\begin{aligned} \alpha_k^{m+1} &= \text{col}\left\{x_k - \hat{x}_{k|k}^{m+1}, w_k\right\}, \\ \Phi_k^{m+1} &= \text{diag}\left\{\left(P_{k|k}^{m+1}\right)^{-1}, Q_k^{-1}\right\}, \\ H_k^{m+1}(\varepsilon_k, \varepsilon_{k+1}) &= D_{k+1}(\varepsilon_{k+1})\left[A_k(\varepsilon_k) \quad B_k(\varepsilon_k)\right], \\ \eta_k^{m+1}(\varepsilon_k, \varepsilon_{k+1}) &= z_{k+1} - D_{k+1}(\varepsilon_{k+1})A_k(\varepsilon_k)\hat{x}_{k|k}^{m+1}. \end{aligned} \quad (15)$$

A simplified expression of Equation (14) is given below:

$$\begin{aligned} J\left(\alpha_k^{m+1}\right) &= \gamma_k^{m+1}\left[\left\|\alpha_k^{m+1}\right\|_{\Phi_k^{m+1}}^2 + \left\|H_k^{m+1}(0,0)\alpha_k^{m+1} - \eta_k^{m+1}(0,0)\right\|_{G_{k+1}^{-1}}^2\right] \\ &+ \left(1 - \gamma_k^{m+1}\right) \sum_{h=1}^L \left(\left\|\frac{\partial e_k(\varepsilon_k, \varepsilon_{k+1})}{\partial \varepsilon_{k,h}}\right\|^2 + \left\|\frac{\partial e_k(\varepsilon_k, \varepsilon_{k+1})}{\partial \varepsilon_{k+1,h}}\right\|^2\right)_{\substack{\varepsilon_k=0 \\ \varepsilon_{k+1}=0}}. \end{aligned} \quad (16)$$

Next, several important matrices S_k^{m+1} and T_k^{m+1} are defined, which serve an important function in parameter correction, given by:

$$\begin{aligned} S_k^{m+1} &= \left[\left(S_{k,1}^{m+1}(0,0)\right)^T, \dots, \left(S_{k,L}^{m+1}(0,0)\right)^T\right]^T, \\ T_k^{m+1} &= \left[\left(T_{k,1}^{m+1}(0,0)\right)^T, \dots, \left(T_{k,L}^{m+1}(0,0)\right)^T\right]^T, \end{aligned} \quad (17)$$

in which

$$\begin{aligned} S_{k,h}^{m+1}(\varepsilon_k, \varepsilon_{k+1}) &= \begin{bmatrix} \frac{\partial C_{k+1}(\varepsilon_{k+1})}{\partial \varepsilon_{k+1,h}} A_k(\varepsilon_k) \\ C_{k+1}(\varepsilon_{k+1}) \frac{\partial A_k(\varepsilon_k)}{\partial \varepsilon_{k,h}} \end{bmatrix}, \\ T_{k,h}^{m+1}(\varepsilon_k, \varepsilon_{k+1}) &= \begin{bmatrix} \frac{\partial C_{k+1}(\varepsilon_{k+1})}{\partial \varepsilon_{k+1,h}} B_k(\varepsilon_k) \\ C_{k+1}(\varepsilon_{k+1}) \frac{\partial B_k(\varepsilon_k)}{\partial \varepsilon_{k,h}} \end{bmatrix}, \\ j &= 1, 2, \dots, L. \end{aligned}$$

Given the properties of G_{k+1} and Φ_k^{m+1} , the function $J\left(\alpha_k^{m+1}\right)$ is strictly convex for $0 < \gamma_k^{m+1} \leq 1$. Setting $\partial J\left(\alpha_k^{m+1}\right) / \partial \alpha_k^{m+1} = 0$ yields the unique global minimizer of α_k^{m+1} , expressed as follows:

$$\begin{aligned} \frac{\partial J\left(\alpha_k^{m+1}\right)}{\partial \alpha_k^{m+1}} &= 2\gamma_k^{m+1}\Phi_k^{m+1}\alpha_k^{m+1} \\ &+ 2\gamma_k^{m+1}\left(H_k^{m+1}(0,0)\right)^T G_{k+1}^{-1} H_k^{m+1}(0,0)\alpha_k^{m+1} \\ &- 2\gamma_k^{m+1}\left(H_k^{m+1}(0,0)\right)^T G_{k+1}^{-1} \eta_k^{m+1}(0,0) \\ &+ 2\left(1 - \gamma_k^{m+1}\right)\left[S_k^{m+1} \quad T_k^{m+1}\right]^T \begin{bmatrix} S_k^{m+1} & T_k^{m+1} \end{bmatrix} \alpha_k^{m+1} \\ &+ 2\left(1 - \gamma_k^{m+1}\right)\left[S_k^{m+1} \quad T_k^{m+1}\right]^T S_k^{m+1} \hat{x}_{k|k}^{m+1}. \end{aligned} \quad (18)$$

Let $\lambda_k^{m+1} = (1 - \gamma_k^{m+1}) / \gamma_k^{m+1}$. The local fused state estimate $\hat{x}_{k|k}^{m+1}$ can then be obtained through the following recursive procedure. The detailed derivation is provided in Appendix B.

- (1) Initialization. The matrix $P_{0|0}^{m+1}$ and the estimate $\hat{x}_{0|0}^{m+1}$ are expressed as follows:

$$\begin{aligned} P_{0|0}^{m+1} &= \left[(\hat{\Pi}_0^{m+1})^{-1} + D_0^T(0) G_0^{-1} D_0(0) \right]^{-1}, \\ \hat{x}_{0|0}^{m+1} &= P_{0|0}^{m+1} D_0^T(0) G_0^{-1} z_0. \end{aligned} \quad (19)$$

where

$$\hat{\Pi}_0^{m+1} = \left(\Pi_0^{-1} + \lambda_0^{m+1} \sum_{j=1}^L \left(\frac{\partial D_0^T(\varepsilon_0)}{\partial \varepsilon_{0,j}} \right) \left(\frac{\partial D_0(\varepsilon_0)}{\partial \varepsilon_{0,j}} \right) \Big|_{\varepsilon_0=0} \right).$$

- (2) Parameter Modification. Define the matrices $\hat{T}_k^{m+1}$, $\hat{A}_k^{m+1}$, $\hat{B}_k^{m+1}$, $\hat{P}_{k|k}^{m+1}$ and $\hat{Q}_k^{m+1}$ as follows:

$$\begin{aligned} \hat{T}_k^{m+1} &= T_k^{m+1} - \lambda_k^{m+1} S_k^{m+1} \hat{P}_{k|k}^{m+1} (S_k^{m+1})^T T_k^{m+1}, \\ \hat{A}_k^{m+1}(0) &= \left[A_k(0) - \lambda_k^{m+1} \hat{B}_k^{m+1}(0) \hat{Q}_k^{m+1} (T_k^{m+1})^T S_k^{m+1} \right] \\ &\quad \times \left[I - \lambda_k^{m+1} \hat{P}_{k|k}^{m+1} (S_k^{m+1})^T S_k^{m+1} \right], \\ \hat{B}_k^{m+1}(0) &= B_k(0) - \lambda_k^{m+1} A_k(0) \hat{P}_{k|k}^{m+1} (S_k^{m+1})^T T_k^{m+1}, \\ \hat{P}_{k|k}^{m+1} &= \left[(P_{k|k}^{m+1})^{-1} + \lambda_k^{m+1} (S_k^{m+1})^T S_k^{m+1} \right]^{-1}, \\ \hat{Q}_k^{m+1} &= \left[Q_k^{-1} + \lambda_k^{m+1} (T_k^{m+1})^T \right. \\ &\quad \left. \times \left(I + \lambda_k^{m+1} S_k^{m+1} P_{k|k}^{m+1} (S_k^{m+1})^T \right)^{-1} T_k^{m+1} \right]^{-1}. \end{aligned} \quad (20)$$

- (3) State Estimate Updating. Compute $\hat{x}_{k+1|k+1}^{m+1}$ and $P_{k+1|k+1}^{m+1}$ using the following equations:

$$\begin{aligned} P_{k+1|k}^{m+1} &= A_k(0) \hat{P}_{k|k}^{m+1} A_k^T(0) + \hat{B}_k^{m+1}(0) \hat{Q}_k^{m+1} (\hat{B}_k^{m+1}(0))^T, \\ R_{e,k+1}^{m+1} &= G_{k+1} + D_{k+1}(0) P_{k+1|k}^{m+1} D_{k+1}^T(0), \\ P_{k+1|k+1}^{m+1} &= P_{k+1|k}^{m+1} - P_{k+1|k}^{m+1} D_{k+1}^T(0) (R_{e,k+1}^{m+1})^{-1} D_{k+1}(0) P_{k+1|k}^{m+1}, \\ \hat{x}_{k+1|k+1}^{m+1} &= \hat{A}_k^{m+1}(0) \hat{x}_{k|k}^{m+1} + P_{k+1|k+1}^{m+1} D_{k+1}^T(0) \\ &\quad \times G_{k+1}^{-1} (z_{k+1} - D_{k+1}(0) \hat{A}_k^{m+1}(0) \hat{x}_{k|k}^{m+1}). \end{aligned} \quad (21)$$

3.3. Transmission Strategy and Channel Reconstruction

A binary variable c_k^n dictates the transmission of local state estimates from sensor nodes to the fusion center. The aforementioned event-triggered transmission strategy can be formulated as follows:

$$c_k^n = \begin{cases} 0, & \|\hat{x}_{k|k}^n - \bar{x}_{k|k}^n\|_{\Omega_k^n}^2 \leq \delta^n, \\ 1, & \text{others} \end{cases}, \quad (22)$$

where $\bar{x}_{k|k}^n$ denotes the state vector, Ω_k^n is the positive-definite weighting coefficient, and δ^n is a positive real scalar. To achieve the desired transmission rate α , the parameters $\bar{x}_{k|k}^n$, Ω_k^n , and δ^n must be appropriately selected. $\hat{x}_{k|k}^n$ represents the local state estimate obtained by an individual sensor node. Each local estimate can be considered an observation of x_k , obtained through a virtual measurement channel.

Following the event-triggered transmission strategy in [26], the threshold δ^n is selected as a rate-controlling parameter to meet a desired average transmission rate α^n . Under the standard Gaussian assumption, the triggering quadratic statistic is chi-square distributed with m_n degrees of freedom, and thus δ^n can be set by the corresponding quantile, i.e., $\delta^n = F_{\chi_{m_n}^2}^{-1}(1 - \alpha^n)$. In practice, we initialize δ^n using this quantile and perform a short calibration to slightly adjust it such that the observed triggering rate matches α^n .

The associated measurement Z_k^n is defined by:

$$Z_k^n = \hat{x}_{k|k}^n = x_k + (\hat{x}_{k|k}^n - x_k), \quad (23)$$

where $\hat{x}_{k|k}^n - x_k$ may be viewed as a type of virtual measurement noise.

Under the transmission strategy, Equation (23) represents the scenario in which the measurement is successfully received by the fusion center from the sensor node, that is, when $c_k^n = 1$. If the sensor node does not transmit, Equation (23) is replaced by Equation (24).

$$Z_k^n = \tilde{x}_{k|k}^n = x_k + (\hat{x}_{k|k}^n - x_k) - g_k^n, \quad (24)$$

where g_k^n is uniformly distributed over the ellipsoid in Equation (22) and is independent of $\hat{x}_{k|k}^n - x_k$. In the event of packet drops between the estimator and the fusion center, the virtual measurement channel is adaptively modified based on the transmission strategy as described below:

$$\begin{aligned} Z_k^n &= c_k^n (\mu_k^n \hat{x}_{k|k}^n + (1 - \mu_k^n) \hat{x}_{k|k-1}^n) + (1 - c_k^n) \tilde{x}_{k|k}^n \\ &= c_k^n (x_k + \mu_k^n (\hat{x}_{k|k}^n - x_k) + (1 - \mu_k^n) (\hat{x}_{k|k-1}^n - x_k)) \\ &\quad + (1 - c_k^n) (x_k + (\hat{x}_{k|k}^n - x_k) - g_k^n), \end{aligned} \quad (25)$$

where μ_k^n indicates whether the fusion center successfully received the measurement value Z_k^n from the sensor node, with $\mu_k^n = \{0, 1\}$. When $c_k^n = 0$, the sensor node does not transmit its local state estimate; when $c_k^n = 1$, $\mu_k^n = 0$, a packet dropout occurs in the communication channel; and when $c_k^n = 1$ and $\mu_k^n = 1$, the channel communication is normal. Note that the case $c_k^n = 0$ and $\mu_k^n = 1$, does not occur; the admissible communication modes and the corresponding information available at the fusion center are summarized in Table 2. The variable $\hat{x}_{k|k-1}^n$ represents the predicted state of the sensor node at time k , computed as $\hat{x}_{k|k-1}^n = \hat{A}_{k-1}^n \hat{x}_{k-1|k-1}^n$. In the above formulation, $(c_k^n \mu_k^n + (1 - c_k^n)) (\hat{x}_{k|k}^n - x_k) + c_k^n (1 - \mu_k^n) (\hat{x}_{k|k}^n - x_k) - (1 - c_k^n) g_k^n$ denotes the virtual measurement noise of the virtual channel at time k , which is represented as $\phi_k^n = (c_k^n \mu_k^n + (1 - c_k^n)) (\hat{x}_{k|k}^n - x_k) + c_k^n (1 - \mu_k^n) (\hat{x}_{k|k}^n - x_k) - (1 - c_k^n) g_k^n$.

Table 2. Communication modes for node n at time k .

(c_k^n, μ_k^n)	Communication Status	Information Used at the Fusion Center
(0, -)	no transmission	$\tilde{x}_{k k}^n$
(1, 1)	successful transmission and reception	$\hat{x}_{k k}^n$
(1, 0)	transmission attempted but packet dropped	$\hat{x}_{k k-1}^n$

Next, we investigate the global fusion estimation under the combined effects of the event-triggered transmission strategy and random packet dropouts, and define the following matrices:

$$\begin{aligned} Z_k &= \text{col} \left\{ c_k^n \left(\mu_k^n \hat{x}_{k|k}^n + (1 - \mu_k^n) \hat{x}_{k|k-1}^n \right) + (1 - c_k^n) \tilde{x}_{k|k}^n \right\}_{n=1}^{m+1}, \\ \phi_k &= \text{col} \left\{ (c_k^n \mu_k^n + (1 - c_k^n)) \left(\hat{x}_{k|k}^n - x_k \right) \right\}_{n=1}^{m+1} \\ &\quad + \text{col} \left\{ c_k^n (1 - \mu_k^n) \left(\hat{x}_{k|k-1}^n - x_k \right) \right\}_{n=1}^{m+1} \\ &\quad + \text{col} \left\{ (1 - c_k^n) g_k^n \right\}_{n=1}^{m+1}, \\ H &= \text{col} \left\{ I_n \right\}_{n=1}^{m+1}. \end{aligned} \quad (26)$$

The fusion center acquires information via the virtual measurement channel:

$$Z_k = Hx_k + \phi_k. \quad (27)$$

where Z_k is a stacked virtual measurement vector for fusion at time k . The matrix H stacks $(m + 1)$ identity blocks so that each block corresponds to the same state x_k . The term ϕ_k denotes the stacked virtual measurement noise.

According to the optimal information fusion criterion in the linear minimum variance sense [16], the optimal matrix-weighted fusion estimator is formulated as follows:

$$\hat{x}_{k|k} = F_1 \hat{x}_{k|k}^1 + F_2 \hat{x}_{k|k}^2 + \dots + F_{m+1} \hat{x}_{k|k}^{m+1}. \quad (28)$$

Let the optimal matrix weights be denoted as $F = \text{col} \left\{ F_n \right\}_{n=1}^{m+1}$, subject to the constraint $F_1 + F_2 + \dots + F_{m+1} = I$. The optimal weights are computed according to the following formulation:

$$F = \tilde{P}_k^{-1} H \left(H^T \tilde{P}_k^{-1} H \right)^{-1}, \quad (29)$$

where $\tilde{P}_k$ represents the covariance matrix of the virtual measurement noise ϕ_k , reflecting the total estimation error covariance. Based on the formulation of ϕ_k in Equation (26), $\tilde{P}_k$ is calculated as follows:

$$\tilde{P}_k = \bar{P}_k + \text{diag} \left\{ (1 - c_k^n) \frac{\delta^n}{d + 2} (\Omega_k^n)^{-1} \right\}_{n=1}^{m+1}, \quad (30)$$

where $\bar{P}_k = \bar{P}_{k,1} + \bar{P}_{k,2} + \bar{P}_{k,2}^T + \bar{P}_{k,3}$, and the matrices $\bar{P}_{k,1}$, $\bar{P}_{k,2}$ and $\bar{P}_{k,3}$ can be expressed as follows:

$$\begin{aligned} \bar{P}_{k,1} &= \begin{bmatrix} (\Gamma_{1,k}^1)^2 P_{k|k}^{1,1} & \cdots & (\Gamma_{1,k}^1) (\Gamma_{1,k}^{m+1}) P_{k|k}^{1,m+1} \\ \vdots & \ddots & \vdots \\ (\Gamma_{1,k}^{m+1}) (\Gamma_{1,k}^1) P_{k|k}^{m+1,1} & \cdots & (\Gamma_{1,k}^{m+1})^2 P_{k|k}^{m+1,m+1} \end{bmatrix}, \\ \bar{P}_{k,2} &= \begin{bmatrix} 0 & \cdots & (\Gamma_{1,k}^1) (\Gamma_{2,k}^{m+1}) \hat{P}_{k|k}^{1,m+1} \\ \vdots & \ddots & \vdots \\ (\Gamma_{1,k}^{m+1}) (\Gamma_{2,k}^1) \hat{P}_{k|k}^{m+1,1} & \cdots & 0 \end{bmatrix}, \\ \bar{P}_{k,3} &= \begin{bmatrix} (\Gamma_{2,k}^1)^2 P_{k|k-1}^{1,1} & \cdots & (\Gamma_{2,k}^1) (\Gamma_{2,k}^{m+1}) P_{k|k-1}^{1,m+1} \\ \vdots & \ddots & \vdots \\ (\Gamma_{2,k}^{m+1}) (\Gamma_{2,k}^1) P_{k|k-1}^{m+1,1} & \cdots & (\Gamma_{2,k}^{m+1})^2 P_{k|k-1}^{m+1,m+1} \end{bmatrix}, \\ \Gamma_{1,k}^n &= c_k^n \mu_k^n + (1 - c_k^n), \Gamma_{2,k}^n = c_k^n (1 - \mu_k^n). \end{aligned} \quad (31)$$

Remark 5. With the definitions in (31), $\Gamma_{1,k}^n = c_k^n \mu_k^n + (1 - c_k^n)$ and $\Gamma_{2,k}^n = c_k^n (1 - \mu_k^n)$, $(\Gamma_{1,k}^n, \Gamma_{2,k}^n)$ forms a one-hot selector, i.e., $\Gamma_{1,k}^n, \Gamma_{2,k}^n \in \{0, 1\}$ and $\Gamma_{1,k}^n + \Gamma_{2,k}^n = 1$. Therefore, exactly one term is selected in the compact expression of ϕ_k^n in (26). Moreover, when $c_k^n = 0$, the dither g_k^n introduced in (26) contributes an extra block-diagonal covariance term, which explains the diagonal augmentation in (30).

The following expressions yield the fused estimate $\hat{x}_{k|k}$ and its associated error covariance P_k :

$$\begin{aligned}\hat{x}_{k|k} &= F^T Z_k = \left(\tilde{P}_k^{-1} H \left(H^T \tilde{P}_k^{-1} H \right)^{-1} \right)^T Z_k, \\ P_k &= \left(H^T \tilde{P}_k^{-1} H \right)^{-1}.\end{aligned}\quad (32)$$

The state estimation error of the dynamic system below is analyzed as follows:

$$\begin{cases} x_{k+1} = \hat{A}_k^n(\varepsilon_k) x_k + \hat{B}_k^n(\varepsilon_k) w_k, \\ y_k^i = C_k^i(\varepsilon_k) x_k + v_k^i, \end{cases} \quad i = 1 \dots l, n = 1 \dots m + 1. \quad (33)$$

First, the local estimator part is discussed, that is, $1 \leq i \leq m$ and $1 \leq n \leq m$. Then, the subsequent relation can be directly established:

$$\begin{aligned}x_{k+1} - \hat{x}_{k+1|k+1}^n &= \left[I + P_{k+1|k}^n (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} C_{k+1}^n(0) \right]^{-1} \left[\hat{A}_k^n(0) (x_k - \hat{x}_{k|k}^n) + \hat{B}_k^n(0) w_k \right] \\ &\quad - \left[\left(P_{k+1|k}^n \right)^{-1} + (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} C_{k+1}^n(0) \right]^{-1} \\ &\quad \times (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} v_{k+1}^n, \\ x_{k+1} - \hat{x}_{k+1|k}^n &= \hat{A}_k^n(0) (x_k - \hat{x}_{k|k}^n) + \hat{B}_k^n(0) w_k.\end{aligned}\quad (34)$$

The forms of the three pseudo cross-covariance matrices $P_{k+1|k+1}^{n,j}$, $\hat{P}_{k+1|k+1}^{n,j}$, and $P_{k+1|k}^{n,j}$ in Equation (31) are given by:

$$\begin{aligned}P_{k+1|k+1}^{n,j} &= E \left[\left(x_{k+1} - \hat{x}_{k+1|k+1}^n \right) \left(x_{k+1} - \hat{x}_{k+1|k+1}^j \right)^T \right] \\ &= \left(\left[I + P_{k+1|k}^{n,n} (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} C_{k+1}^n(0) \right]^{-1} \left[\hat{A}_k^n(0) (x_k - \hat{x}_{k|k}^n) + \hat{B}_k^n(0) w_k \right] \right. \\ &\quad \left. - \left[\left(P_{k+1|k}^{n,n} \right)^{-1} + (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} C_{k+1}^n(0) \right]^{-1} (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} v_{k+1}^n \right) \\ &\quad \times \left(\left[I + P_{k+1|k}^{j,j} (C_{k+1}^j(0))^T (R_{k+1}^j)^{-1} C_{k+1}^j(0) \right]^{-1} \left[\hat{A}_k^j(0) (x_k - \hat{x}_{k|k}^j) + \hat{B}_k^j(0) w_k \right] \right. \\ &\quad \left. - \left[\left(P_{k+1|k}^{j,j} \right)^{-1} + (C_{k+1}^j(0))^T (R_{k+1}^j)^{-1} C_{k+1}^j(0) \right]^{-1} (C_{k+1}^j(0))^T (R_{k+1}^j)^{-1} v_{k+1}^j \right)^T, \\ \hat{P}_{k+1|k}^{n,j} &= E \left[\left(x_{k+1} - \hat{x}_{k+1|k+1}^n \right) \left(x_{k+1} - \hat{x}_{k+1|k}^j \right)^T \right] \\ &= \left(\left[I + P_{k+1|k}^{n,n} (R_{k+1}^n)^{-1} C_{k+1}^n(0) \right]^{-1} \left[\hat{A}_k^n(0) (x_k - \hat{x}_{k|k}^n) + \hat{B}_k^n(0) w_k \right] \right. \\ &\quad \left. - \left[\left(P_{k+1|k}^{n,n} \right)^{-1} + (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} C_{k+1}^n(0) \right]^{-1} (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} v_{k+1}^n \right) \\ &\quad \times \left(\hat{A}_k^j(0) (x_k - \hat{x}_{k|k}^j) + \hat{B}_k^j(0) w_k \right)^T, \\ P_{k+1|k}^{n,j} &= E \left[\left(x_{k+1} - \hat{x}_{k+1|k}^n \right) \left(x_{k+1} - \hat{x}_{k+1|k}^j \right)^T \right] \\ &= \left(\hat{A}_k^n(0) (x_k - \hat{x}_{k|k}^n) + \hat{B}_k^n(0) w_k \right) \left(\hat{A}_k^j(0) (x_k - \hat{x}_{k|k}^j) + \hat{B}_k^j(0) w_k \right)^T.\end{aligned}\quad (35)$$

Since w_k and v_k^n are independent random variables, $P_{k+1|k+1}^{n,j}$, $\hat{P}_{k+1|k+1}^{n,j}$, and $P_{k+1|k}^{n,j}$ can be expressed as follows:

$$\begin{aligned} P_{k+1|k+1}^{n,j} &= \left(\left[I + P_{k+1|k}^{n,n} (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} C_{k+1}^n(0) \right]^{-1} \left[\hat{A}_k^n(0) (x_k - \hat{x}_{k|k}^n) + \hat{B}_k^n(0) w_k \right] \right. \\ &\quad \times \left. \left(\left[I + P_{k+1|k}^{j,j} (C_{k+1}^j(0))^T (R_{k+1}^j)^{-1} C_{k+1}^j(0) \right]^{-1} \left[\hat{A}_k^j(0) (x_k - \hat{x}_{k|k}^j) + \hat{B}_k^j(0) w_k \right] \right)^T, \\ \hat{P}_{k+1|k}^{n,j} &= \left(\left[I + P_{k+1|k}^{n,n} (R_{k+1}^n)^{-1} C_{k+1}^n(0) \right]^{-1} \left[\hat{A}_k^n(0) (x_k - \hat{x}_{k|k}^n) + \hat{B}_k^n(0) w_k \right] \right) \\ &\quad \times \left(\hat{A}_k^j(0) (x_k - \hat{x}_{k|k}^j) + \hat{B}_k^j(0) w_k \right)^T \\ &= \left[I + P_{k+1|k}^{n,n} (R_{k+1}^n)^{-1} C_{k+1}^n(0) \right]^{-1} \left[\hat{A}_k^n(0) P_{k|k}^{n,j} (\hat{A}_k^j(0))^T + \hat{B}_k^n(0) Q_k (\hat{B}_k^j(0))^T \right], \\ P_{k+1|k}^{n,j} &= \hat{A}_k^n(0) P_{k|k}^{n,j} (\hat{A}_k^j(0))^T + \hat{B}_k^n(0) Q_k (\hat{B}_k^j(0))^T. \end{aligned} \quad (36)$$

Based on the following matrix inversion theorem

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(DA^{-1}B + C^{-1})^{-1}DA^{-1},$$

we can then obtain:

$$\begin{aligned} P_{k+1|k+1}^{n,j} &= \left[I - P_{k+1|k}^{n,n} (C_{k+1}^n(0))^T (C_{k+1}^n(0) P_{k+1|k}^{n,n} (C_{k+1}^n(0))^T + R_{k+1}^n)^{-1} C_{k+1}^n(0) \right] \\ &\quad \times \left[\hat{A}_k^n(0) (x_k - \hat{x}_{k|k}^n) + \hat{B}_k^n(0) w_k \right] \left[\hat{A}_k^n(0) (x_k - \hat{x}_{k|k}^n) + \hat{B}_k^n(0) w_k \right]^T \\ &\quad \times \left[I - P_{k+1|k}^{j,j} (C_{k+1}^j(0))^T (C_{k+1}^j(0) P_{k+1|k}^{j,j} (C_{k+1}^j(0))^T + R_{k+1}^j)^{-1} C_{k+1}^j(0) \right]^T \\ &= \left[I - P_{k+1|k}^{n,n} (C_{k+1}^n(0))^T (C_{k+1}^n(0) P_{k+1|k}^{n,n} (C_{k+1}^n(0))^T + R_{k+1}^n)^{-1} C_{k+1}^n(0) \right] \\ &\quad \times \left[\hat{A}_k^n(0) P_{k|k}^{n,j} (\hat{A}_k^j(0))^T + \hat{B}_k^n(0) Q_k (\hat{B}_k^j(0))^T \right] \\ &\quad \times \left[I - P_{k+1|k}^{j,j} (C_{k+1}^j(0))^T (C_{k+1}^j(0) P_{k+1|k}^{j,j} (C_{k+1}^j(0))^T + R_{k+1}^j)^{-1} C_{k+1}^j(0) \right]^T. \end{aligned} \quad (37)$$

For the adopted Bernoulli dropout model in this paper, the pseudo cross-covariances in Equations (35)–(37) are obtained exactly from the corresponding error recursions. If packet losses are temporally correlated or state-dependent, these expressions should be regarded as an approximation and may deviate from the true cross-correlation.

Then, the local fusion estimator part is discussed, that is, $m+1 \leq i \leq l$ and $n = m+1$. The following relationship can be obtained:

$$\begin{aligned} x_{k+1} - \hat{x}_{k+1|k+1}^{m+1} &= \left[I + P_{k+1|k}^{m+1} D_{k+1}^T(0) G_{k+1}^{-1} D_{k+1}(0) \right]^{-1} \left[\hat{A}_k^{m+1}(0) (x_k - \hat{x}_{k|k}^{m+1}) + \hat{B}_k^{m+1}(0) w_k \right] \\ &\quad - \left[\left(P_{k+1|k}^{m+1} \right)^{-1} + D_{k+1}^T(0) G_{k+1}^{-1} D_{k+1}(0) \right]^{-1} \\ &\quad \times D_{k+1}^T(0) G_{k+1}^{-1} v_{k+1}^i, \\ x_{k+1} - \hat{x}_{k+1|k}^{m+1} &= \hat{A}_k^{m+1}(0) (x_k - \hat{x}_{k|k}^{m+1}) + \hat{B}_k^{m+1}(0) w_k. \end{aligned} \quad (38)$$

Derivation process of a pseudo-cross-covariance matrix analogous to a local estimator, $P_{k+1|k+1}^{n,m+1}$, $\hat{P}_{k+1|k+1}^{n,m+1}$, and $P_{k+1|k}^{n,m+1}$ can be expressed in the following forms:

$$\begin{aligned}
 P_{k+1|k+1}^{n,m+1} &= \left[I - P_{k+1|k}^{n,n} (C_{k+1}^n(0))^T (C_{k+1}^n(0) P_{k+1|k}^{n,n} (C_{k+1}^n(0))^T + R_{k+1}^n)^{-1} C_{k+1}^n(0) \right] \\
 &\quad \times \left[\hat{A}_k^n(0) P_{k|k}^{n,m+1} (\hat{A}_k^{m+1}(0))^T + \hat{B}_k^n(0) Q_k (\hat{B}_k^{m+1}(0))^T \right] \\
 &\quad \times \left[I - P_{k+1|k}^{m+1,m+1} D_{k+1}^T(0) (D_{k+1}(0) P_{k+1|k}^{m+1,m+1} D_{k+1}^T(0) + G_{k+1})^{-1} D_{k+1}(0) \right]^T \quad (39) \\
 \hat{P}_{k+1|k}^{n,m+1} &= \left[I + P_{k+1|k}^{n,n} (R_{k+1}^n)^{-1} C_{k+1}^n(0) \right]^{-1} \\
 &\quad \times \left[\hat{A}_k^n(0) P_{k|k}^{n,m+1} (\hat{A}_k^{m+1}(0))^T + \hat{B}_k^n(0) Q_k (\hat{B}_k^{m+1}(0))^T \right], \\
 P_{k+1|k}^{n,m+1} &= \hat{A}_k^n(0) P_{k|k}^{n,m+1} (\hat{A}_k^{m+1}(0))^T + \hat{B}_k^n(0) Q_k (\hat{B}_k^{m+1}(0))^T.
 \end{aligned}$$

Remark 6. For the standard Kalman filter, the computational complexity per iteration is $O[t^3]$, where t denotes the dimension of the state vector. The proposed robust state estimator has a computational complexity comparable to that of the Kalman filter. Therefore, although additional parameter corrections and the computation of global covariance differences are introduced at each time step, the overall complexity of the resulting robust estimator remains on the same order as that of the Kalman filter. The main difference from [11] lies in the fusion framework, since [11] does not account for packet drops caused by channel failures. The method in [11] has a per-iteration computational complexity of $O[(m+1)t^3]$, where $m+1$ is the number of estimators. Similarly, although our method incorporates channel reconstruction and updates the fusion framework to handle stochastic packet losses, its computational complexity remains comparable to that of [11].

4. Some Properties of the Fusion Estimator

This section analyzes key asymptotic characteristics of the proposed fusion estimator. To perform the asymptotic analysis, several assumptions are required. First, within the assumptions and demonstrations presented in this section, the model uncertainty $\varepsilon_{k,j}$ is assumed to lie within the set κ , where $\kappa = \{\varepsilon \mid |\varepsilon_{k,j}| \leq 1, j = 1, 2, \dots, l\}$. Furthermore, two additional assumptions are imposed:

- (a) $A_k(0), B_k(0), C_k^i(0), R_k^i, Q_k, S_k^n, T_k^n$ and γ_k^n are time-invariant. System (1) exhibits exponential stability in the sense of Lyapunov, and $A_k(\varepsilon_k), B_k(\varepsilon_k), C_k^i(\varepsilon_k), R_k^i, Q_k$ and Π_0 are bounded for all $k > 0$.
- (b) Each sensor node satisfies the following conditions: $(A_k(0), \bar{C}_k^i)$ is detectable, and

$$\left[\begin{array}{c} \left(A_k(0) - \lambda_k^n B_k(0) Q_k (T_k^n)^T (I + \lambda_k^n T_k^n Q_k (T_k^n)^T)^{-1} S_k^n \right)^T \\ \left(B_k(0) Q_k^{\frac{1}{2}} \left(I + \lambda_k^n Q_k^{\frac{1}{2}} (T_k^n)^T T_k^n Q_k^{\frac{1}{2}} \right)^{-\frac{1}{2}} \right)^T \end{array} \right] \text{ is stabilizable.}$$

For $0 \leq i \leq m$ and $0 \leq n \leq m$, the matrix $\bar{C}_k^n$ is defined as follows:

$$\bar{C}_k^n = \begin{bmatrix} (R_k^i)^{-\frac{1}{2}} C_k^i(0) \\ \sqrt{\lambda_k^n} S_k^n \end{bmatrix}.$$

For $m+1 \leq i \leq l$ and $n = m+1$, the matrix $\bar{C}_k^{m+1}$ is given by:

$$\bar{C}_k^{m+1} = \begin{bmatrix} J_k \\ \sqrt{\lambda_k^{m+1}} S_k^{m+1} \end{bmatrix}, J_k = \text{col} \left\{ (R_k^i)^{-\frac{1}{2}} C_k^i(0) \mid i=m+1 \right\}.$$

Theorem 1. Assuming Assumptions (a) and (b) are satisfied, each sensor sends the local estimate $\hat{x}_{k|k}^n$ following the event-triggered transmission strategy. If the weighting matrix Ω_k^n at each sensor node meets the condition below for each positive scalar ω^n :

$$\Omega_k^n \geq \omega^n I, \quad (40)$$

then $x_k - \hat{x}_{k|k}^n$ remains uniformly bounded for any admissible selection of $\{\hat{x}_{k|k}^n, k \in \mathbb{Z}_+\}$, implying that

$$\lim_{k \rightarrow \infty} \sup E \left\{ \|x_k - \hat{x}_{k|k}^n\|^2 \right\} < +\infty. \quad (41)$$

Proof of Theorem 1. Let $\bar{x}_{k|k}^n$ denote the state estimate at time k obtained by substituting $\bar{Z}_k$ for Z_k . The variable $\bar{Z}_k$ is defined as follows:

$$\bar{Z}_k = \text{col} \left\{ (c_k^n \mu_k^n + (1 - c_k^n)) \hat{x}_{k|k}^n + c_k^n (1 - \mu_k^n) \hat{x}_{k|k-1}^n \Big|_{n=1}^{m+1} \right\}. \quad (42)$$

The following can then be obtained:

$$\hat{x}_{k|k}^n = \bar{x}_{k|k}^n + \left(\bar{P}_k^{-1} H (H^T \bar{P}_k^{-1} H)^{-1} \right)^T (Z_k - \bar{Z}_k). \quad (43)$$

Therefore, the following holds:

$$\begin{aligned} E \left\{ \|x_k - \hat{x}_{k|k}^n\|^2 \right\} &\leq 2E \left\{ \|x_k - \bar{x}_{k|k}^n\|^2 \right\} \\ &+ 2 \left\| \left(\bar{P}_k^{-1} H (H^T \bar{P}_k^{-1} H)^{-1} \right)^T \right\|^2 E \left\{ \|Z_k - \bar{Z}_k\|^2 \right\}. \end{aligned} \quad (44)$$

Under Assumptions (a) and (b), the local pseudo-covariance matrix $P_{k|k}^{n,n}$ converges. Moreover, the matrices $\hat{P}_{k|k-1}^{n,j}$ and $P_{k|k-1}^{n,n}$ are guaranteed to converge as well. Consequently, the estimation error covariance matrix remains bounded at every time step k , indicating both the convergence of $\bar{P}_k$ and the stability of the estimation error variance.

Based on the inequality stated in Theorem 1, along with Assumption (a) and the remaining components of the matrix, the ensuing outcomes are obtained as follows:

$$\begin{aligned} \text{tr} \left((1 - c_k^n) \frac{\delta^n}{d+2} (\Omega_k^n)^{-1} \right) &= (1 - c_k^n) \frac{\delta^n}{d+2} \text{tr} \left((\Omega_k^n)^{-1} \right) \\ &\leq (1 - c_k^n) \frac{\delta^n}{(d+2)\omega^n}. \end{aligned} \quad (45)$$

Consequently, $E \left\{ \|\hat{x}_{k|k}^n - \bar{x}_{k|k}^n\|^2 \right\}$ is uniformly bounded. Furthermore, the second term on the right-hand side of the inequality is also uniformly bounded.

According to the inequality conditions in Theorem 1, The following expression can be derived:

$$\left\| \hat{x}_{k|k}^n - \bar{x}_{k|k}^n \right\|_{\Omega_k^n}^2 \geq \omega^n \left\| \hat{x}_{k|k}^n - \bar{x}_{k|k}^n \right\|^2. \quad (46)$$

When $c_k^n = 0$, it implies that $\left\| \hat{x}_{k|k}^n - \bar{x}_{k|k}^n \right\|_{\Omega_k^n}^2 \leq \delta^n$, which leads to $\left\| \hat{x}_{k|k}^n - \bar{x}_{k|k}^n \right\|^2 \leq \delta^n / \omega^n$. Therefore, $\left\| Z_k - \bar{Z}_{k|k} \right\|^2 \leq \sum_{i=1}^l \delta^n / \omega^n$, and the proof is complete. $\square$

Remark 7. Under Assumption a, the involved system and noise matrices are time-invariant and bounded. Therefore, the local pseudo-covariance recursion induced by the robustness-modified

model can be regarded as a time-invariant Riccati difference equation associated with the effective pair $(A_k(0), \bar{C}_k^n)$ and the corresponding process channel. Assumption b requires detectability of $(A_k(0), \bar{C}_k^n)$ and stabilizability of the robustness-adjusted state transition channel. These conditions guarantee the existence of a stabilizing solution to the associated Riccati equation and imply that the $P_{k|k}^{n,n}$ and $P_{k|k-1}^{n,n}$ are uniformly bounded and convergent. Consequently, the matrices constructed from them, including $\tilde{P}_k$ and the gain-related term in (43), are uniformly bounded. This justifies that the second term on the right-hand side of (44) is bounded whenever $E\{\|Z_k - \bar{Z}_k\|^2\}$ is bounded by the triggering condition.

To satisfy condition (40), it is reasonable to make the following selection for the vector $\bar{x}_{k|k}^i$ and the weighting matrix Ω_k^i :

$$\begin{aligned}\bar{x}_{k|k}^n &= \hat{x}_{k|k-1}^n = \hat{A}_{k-1}^n \hat{x}_{k-1|k-1}^n, \\ (\Omega_k^n)^{-1} &= \frac{1}{\text{tr}(\tilde{P}_{k|k-1}^n)} (\tilde{P}_{k|k-1}^n),\end{aligned}\quad (47)$$

where

$$\tilde{P}_{k|k-1}^n = A_{k-1} \begin{bmatrix} (c_k^n \gamma_k^n + (1 - c_k^n)) P_{k-1|k-1}^{n,n} \\ + c_k^n (1 - \gamma_k^n) P_{k-1|k-2}^{n,n} \\ + \left((1 - c_k^n) \frac{\delta^n}{d+2} (\Omega_{k-1}^n)^{-1} \right) \end{bmatrix} A_{k-1}^T + \hat{B}_{k-1}^n \hat{Q}_{k-1}^n (\hat{B}_{k-1}^n)^T \quad (48)$$

5. Numerical Simulations

All experiments in this study were conducted on a workstation running Windows 11 using MATLAB R2018a. The CPU is an Intel(R) Core(TM) i5-12600KF, and the GPU is an NVIDIA GeForce RTX 4070 Super with 12GB GDDR4 memory running at 3600 MHz.

This section presents simulations based on the tractor-car system (shown in Figure 2) as described in [39], and has subsequently been generalized to a multi-sensor system. Several examples illustrate the performance of the proposed fusion estimator. For each simulation scenario, 500 independent numerical experiments are performed, and each run consists of 200 discrete time steps, generating 200 corresponding input-output data pairs. During the simulation, the global average estimation error variance at each time step is computed using the expression $E\|x_k - \hat{x}_{k|k}\|^2 \approx \frac{1}{500} \sum_{j=1}^{500} \|x_k - \hat{x}_{k|k}^{(j)}\|^2$, and the actual state trajectories along with their corresponding estimated trajectories are illustrated.

The linearized representation of the tractor-car system depicted in Figure 2 can be formulated as follows:

$$\begin{cases} x_{k+1}^1 = \left(1.0000 - \frac{vt}{L}\right) x_k^1 + \frac{vt}{L} x_k^2 + \left(0.9804 + \frac{vt}{L}\right) w_k^1 \\ x_{k+1}^2 = \left(1.0000 - \frac{vt}{L}\right) x_k^2 + \left(0.9804 + \frac{vt}{L}\right) w_k^2 \end{cases}, \quad (49)$$

where the variables x_k^1 and x_k^2 represent the orientation angles of the trailer and tractor, respectively, with the system state vector defined as $x_k = \begin{bmatrix} x_k^1 & x_k^2 \end{bmatrix}$. The parameter L stands for tractor length, t indicates the sampling interval, and v signifies constant velocity. Considering the modeling errors caused by approximating the nonlinear system with a linear system, and representing these errors in the system model as a model perturbation ε_k , the matrix parameters can be obtained as follows:

$$\begin{aligned} A_k(\varepsilon_k) &= \begin{bmatrix} 1.0000 - \frac{vt}{D} & \frac{vt}{D} + \varepsilon_k \\ 0 & 1.0000 - \frac{vt}{D} \end{bmatrix} \\ B_k(\varepsilon_k) &= \begin{bmatrix} 0.9804 + \frac{vt}{D} & 0 \\ 0 & 0.9804 + \frac{vt}{D} \end{bmatrix}. \end{aligned} \quad (50)$$

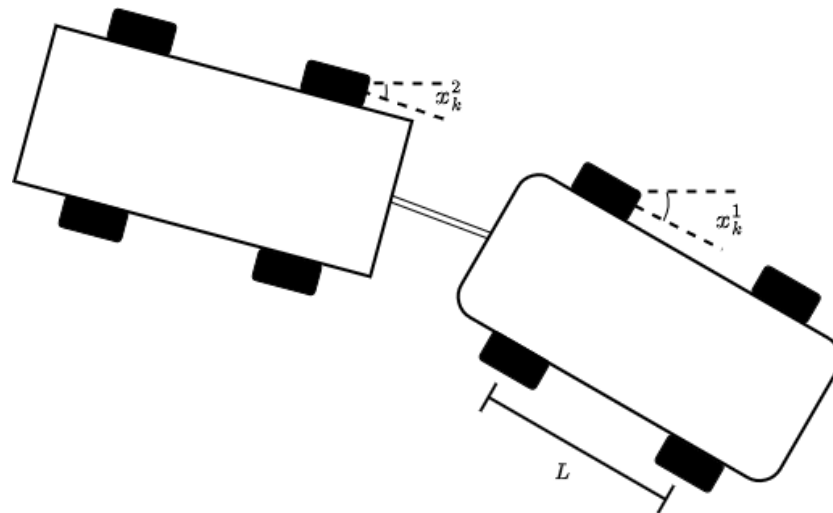


Figure 2. Schematic diagram of the tractor-car system.

Remark 8. In the tractor-car simulations, we adopt the same modeling assumptions as those used in the estimator design to ensure consistency between the theoretical derivations and the numerical validation and to facilitate reproducibility. Specifically, the initial state x_0 , process noise w_k , and measurement noise v_k^i are assumed to be mutually independent, zero-mean Gaussian white noises with positive-definite covariances Π_0 , Q_k , and R_k^i , where Q_k and R_k^i can be obtained from sensor specifications or offline calibration on the platform. This assumption is common in vehicle dynamics modeling, since the aggregated effect of multiple small disturbances, such as engine vibrations, road irregularities, and aerodynamic perturbations, can often be reasonably approximated by a Gaussian model, and the different physical origins of process and measurement noises also justify their statistical independence to a certain extent. Moreover, the modeling error ε_k is assumed to lie in a known bounded set, for example $|\varepsilon_{k,j}| \leq \rho$, and the system matrices are differentiable with respect to ε_k . This can be viewed as a lumped description of linearization errors and small parameter variations within an operating region, and ρ can be conservatively selected based on prior identification and validation results. Random measurement losses are modeled by a Bernoulli process via an arrival indicator that represents the random events of successful reception and packet dropout. In the tractor-car system, wireless communication may suffer from packet losses due to interference, signal fading, or bandwidth limitations, and the Bernoulli model provides a concise mathematical abstraction of such randomness. Finally, the boundedness analysis is established under the standard detectability and stabilizability conditions stated in Assumption (b). For the tractor-car example, the adopted linearized model and sensor configuration satisfy these conditions in the considered operating region. In addition, the fusion center is assumed to know the nominal model parameters and the uncertainty bound, the noise covariances, the event-triggering parameters, and the packet reception indicators, where the reception information can be obtained in practice from communication time-stamps and reception-confirmation messages, and the triggering thresholds δ^n are pre-specified design parameters for each sensor node and are known to the fusion center.

During the simulation analysis, the system is configured with the following parameter values: a constant velocity $v = 98$ cm/s, a sampling interval $t = 0.1$ s, and a trailer length $L = 500$ cm. Based on these settings, the corresponding system matrices are given by:

$$\begin{aligned} A_k(\varepsilon_k) &= \begin{bmatrix} 0.9804 & 0.0196 + 0.99\varepsilon_k \\ 0.0000 & 0.9804 \end{bmatrix}, \\ B_k(\varepsilon_k) &= \begin{bmatrix} 1.0000 & 0.0000 \\ 0.0000 & 1.0000 \end{bmatrix}, \\ C_k^1(\varepsilon_k) &= \begin{bmatrix} 1.0000 & -1.0000 \end{bmatrix}, \\ C_k^2 &= \begin{bmatrix} 1.0000 & -1.0000 \end{bmatrix}, C_k^3 = \begin{bmatrix} 0.4000 & -0.5000 \end{bmatrix}, \\ R_k^1 &= 1.0000, R_k^2 = 1.0000, R_k^3 = 1.0000, \\ Q_k &= \begin{bmatrix} 1.9608 & 0.0195 \\ 0.0195 & 1.9605 \end{bmatrix}, \Pi_0 = \begin{bmatrix} 1.0000 & 0.0000 \\ 0.0000 & 1.0000 \end{bmatrix}. \end{aligned}$$

5.1. Comparison of the Existing Works

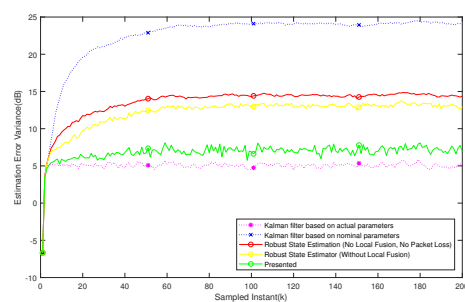
In Case 1, the modeling error is fixed at -0.8508 , the transmission rate is 0.8, and the packet dropout rate is 0.2. As shown in Figure 3a, in terms of the estimation error variance, the designed estimator not only outperforms the Kalman filter based on nominal parameters, but also achieves better performance than the estimator proposed in Ref. [11] (without packet dropouts and without local fusion) and the distributed robust state estimator without local fusion under packet dropouts. Figure 3b presents the state tracking results of different methods. It can be observed that the proposed method can stably track the true state, and its overall tracking error is smaller than that of the Kalman filter based on nominal parameters, the estimator proposed in Ref. [11], and the distributed robust state estimator without local fusion under packet dropouts. The proposed method performs slightly worse than the Kalman filter with actual parameters, which is expected since the Kalman filter is mean-square optimal for linear systems under Gaussian disturbances. Figure 3c,d validate the realized transmission rate and packet dropout rate, respectively. Note that in Figure 3c, the variable Z_k^n is inverted at each time step to explicitly reflect the execution of the event-triggered strategy.

In Case 2, the modeling error is independently sampled in each simulation run, with values uniformly distributed over $[-1, 1]$. The transmission rate is 0.8, and the packet dropout rate is 0.2. The results under this setting are shown in Figure 4. These results indicate that the proposed algorithm maintains robust performance even when the modeling error is time-varying. As shown in Figure 4a, the proposed method still achieves higher estimation accuracy than the other three methods. Similarly, Figure 4b shows that the proposed method also provides superior tracking performance. Figure 4c,d further confirm the realization of the transmission rate and the packet dropout rate.

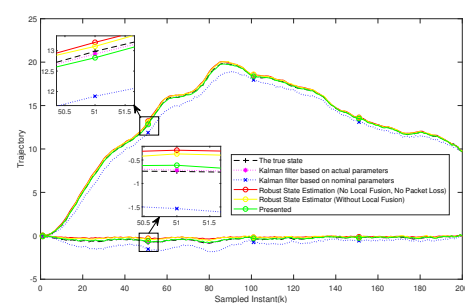
From Cases 1 and 2, under varying modeling error conditions, the proposed method exhibits superior performance. Table 3 lists the average estimation error covariance over 500 simulation runs for both scenarios. By combining the graphical and tabular data, it is evident that the proposed approach exhibits a significant advantage in estimation accuracy compared to the Kalman filter based on nominal parameters, as well as the estimator proposed in Ref. [11] and the distributed robust state estimator under the same framework.

Table 3. Estimation error variance means (dB).

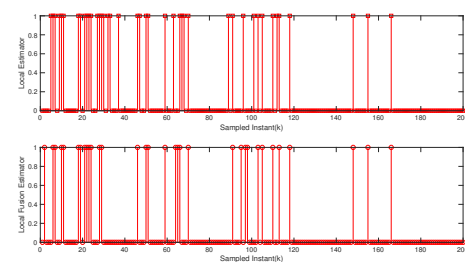
Algorithm	Modeling Errors	Case 1	Case 2
Kalman filter based on actual parameters		5.04	6.39
Presented		6.70	13.23
Robust state estimator (without local fusion)		11.79	15.08
[11]		13.23	16.33
Kalman filter based on nominal parameters		22.14	19.14



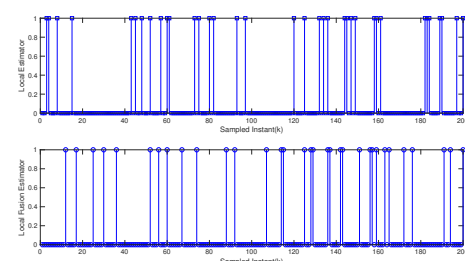
(a) Estimation error variance



(b) Trajectory of states and estimates

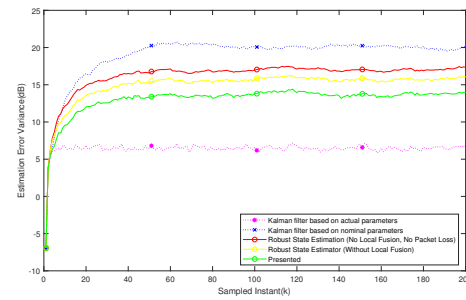


(c) Implementation of event-triggered

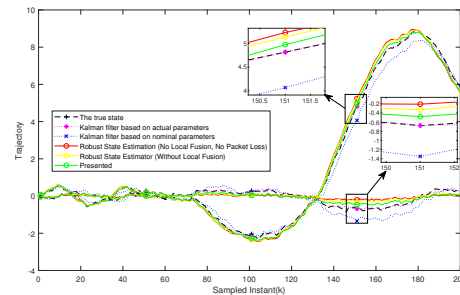


(d) Occurrence of packet drop

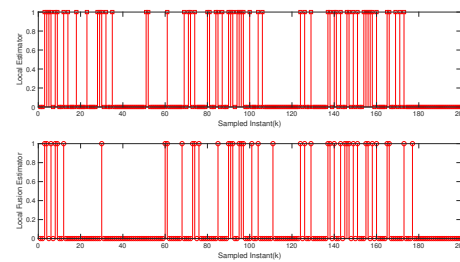
Figure 3. ε_k is fixed at -0.8508 .



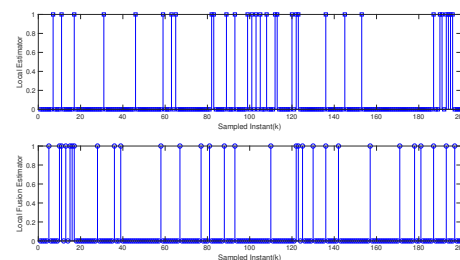
(a) Estimation error variance



(b) Trajectory of states and estimates



(c) Implementation of event-triggered



(d) Occurrence of packet drop

Figure 4. ε_k follows a uniform distribution in $[-1, 1]$.

5.2. Transmission Rate and Packet Drop Comparison

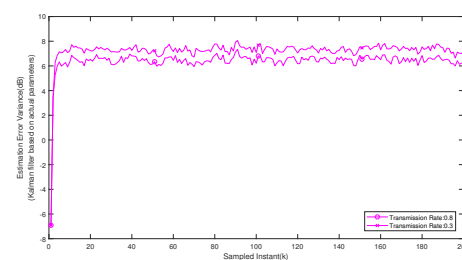
This subsection compares the estimation performance of various algorithms under different transmission rates. In this section, the model perturbations are randomly generated in each experiment and follow a uniform distribution within the interval $[-1, 1]$.

With the packet drop rate set at 0.2, the estimation error variances at transmission rates of 0.3 and 0.8 are compared, as shown in Figure 5. The results indicate that an increase in transmission rate corresponds to a decrease in estimation error variance, which aligns with expectations. To further investigate the impact of packet loss, with the transmission rate fixed at 0.8, the estimation error variances at packet loss rates of 0.3 and 0.8 are compared, as illustrated in Figure 6. In addition, Figure 7 reports the corresponding state trajectories

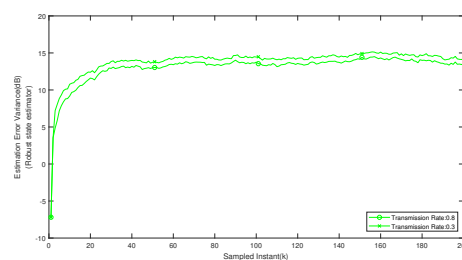
and estimates under different transmission rates, where the estimates obtained with a higher transmission rate exhibit visibly tighter tracking of the true state across all methods. The associated trajectory comparisons under different packet loss rates are further illustrated in Figure 8. The simulation results demonstrate that a lower packet loss rate improves accuracy, which is reasonable since the fusion center receives more information when the packet loss rate is low. Table 4 lists the averaged estimation error variances obtained from 500 simulation runs. The effectiveness of the proposed algorithm is further corroborated by these observations.

Table 4. Mean of Comparisons.

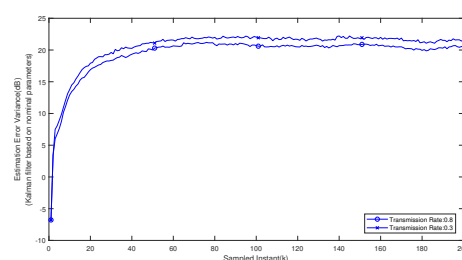
Algorithm	Transmission Rate	Packet Drop Rate	Estimation Error Covariance Mean
Kalman filter based on actual parameters	0.8	0.2	6.39
	0.3	0.2	7.24
	0.8	0.3	6.66
	0.8	0.8	7.85
Robust state estimator	0.8	0.2	13.02
	0.3	0.2	13.83
	0.8	0.3	13.53
	0.8	0.8	14.69
Kalman filter based on nominal parameters	0.8	0.2	19.43
	0.3	0.2	20.53
	0.8	0.3	19.15
	0.8	0.8	20.58



(a)



(b)



(c)

Figure 5. Comparison under various transmission rates (Estimation error variance). (a) Estimation error variance (Kalman filter based on actual parameters); (b) Estimation error variance (Robust state estimator); (c) Estimation error variance (Kalman filter based on nominal parameters).

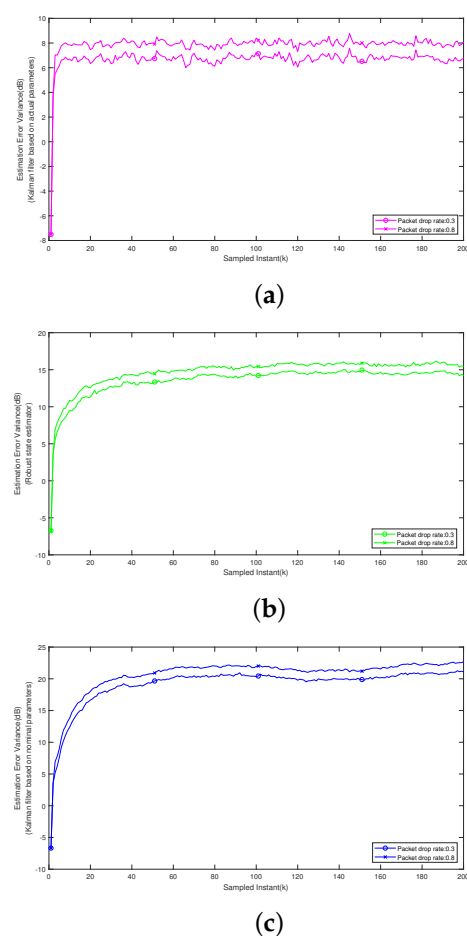


Figure 6. Comparison under various packet drop rates (Estimation error variance). (a) Estimation error variance (Kalman filter based on actual parameters); (b) Estimation error variance (Robust state estimator); (c) Estimation error variance (Kalman filter based on nominal parameters).

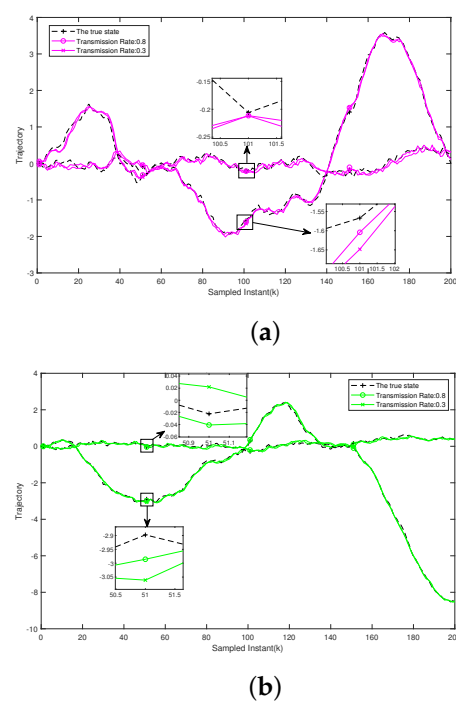


Figure 7. Cont.

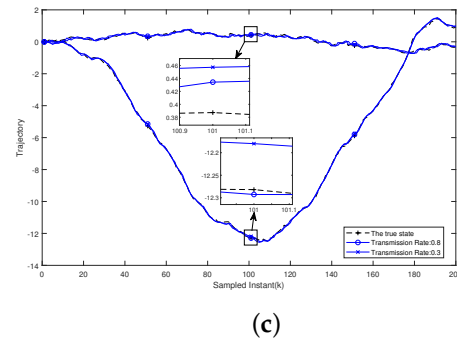


Figure 7. Comparison under various transmission rates (Trajectory of states and estimates). (a) Trajectory of states and estimates (Kalman filter based on actual parameters); (b) Trajectory of states and estimates (Robust state estimator); (c) Trajectory of states and estimates (Kalman filter based on nominal parameters).

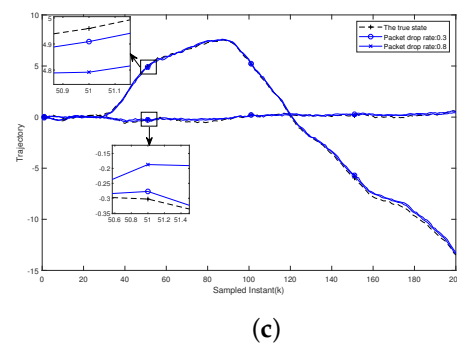
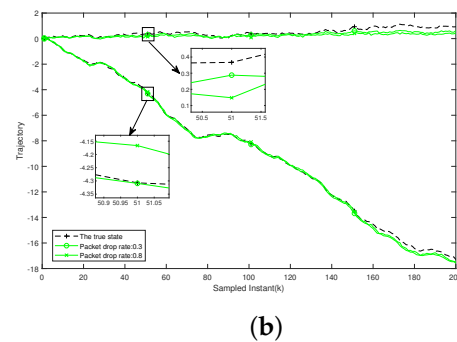
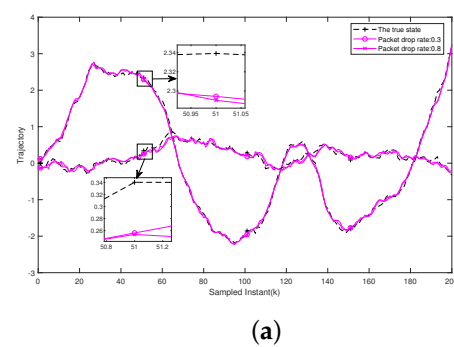


Figure 8. Comparison under various packet drop rates (Trajectory of states and estimates). (a) Trajectory of states and estimates (Kalman filter based on actual parameters); (b) Trajectory of states and estimates (Robust state estimator); (c) Trajectory of states and estimates (Kalman filter based on nominal parameters).

6. Conclusions

This paper addresses robust fusion estimation in multisensor systems under uncertainties, communication constraints, and random packet drops. For local state estimation, a sensitivity-penalized robust state estimation algorithm is employed in the local estimators.

In the local fusion estimators, a centralized robust state estimation algorithm is further constructed by modifying the cost function of the robust estimator. Taking into account the event-triggered transmission strategy and packet losses, we compute a pseudo cross-covariance matrix for the estimation error at the fusion center and employ a weighted linear minimum variance fusion strategy to derive the global state estimate. Under the conditions of system stabilizability and detectability, uniform boundedness of the estimation error at the fusion center is formally established. Finally, to validate the proposed method's effectiveness in state estimation, a simulation study on the attitude estimation of a tractor-car system is carried out. The simulation results indicate that the proposed estimator outperforms the nominal-parameter Kalman filter in estimation accuracy. Since the model uncertainties are not restricted to a specific structure, the proposed fusion estimator is broadly applicable.

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Appendix A. Derivation of Local Estimator

First, estimate the initial state x_0^n . Define $e_0^n(\varepsilon_0) = y_0^n - C_0^n(\varepsilon_0)x_0^n$. The associated cost function is given by:

$$J(\alpha_0^n) = \gamma_0^n \left[\|x_0^n\|_{\Pi_0^{-1}}^2 + \|y_0^n - C_0^n(\varepsilon_0)x_0^n\|_{(R_0^n)^{-1}}^2 \right] + (1 - \gamma_0^n) \sum_{j=1}^L \left(\left\| \frac{\partial e_0^n(\varepsilon_0)}{\partial \varepsilon_{0,j}} \right\|^2 \right)_{\varepsilon_0=0}. \quad (\text{A1})$$

By minimizing $J(\alpha_0^n)$, impose the first-order optimality condition $J(\alpha_0^n)/x_0 = 0$, which yields the initial estimate:

$$\begin{aligned} P_{0|0}^n &= \left[(\hat{\Pi}_0^n)^{-1} + (C_0^n(0))^T (R_0^n)^{-1} C_0^n(0) \right]^{-1}, \\ \hat{x}_{0|0}^n &= P_{0|0}^n (C_0^n(0))^T (R_0^n)^{-1} y_0^n, \end{aligned} \quad (\text{A2})$$

$$\text{where } \hat{\Pi}_0^n = \left(\Pi_0^{-1} + \lambda_0^n \sum_{j=1}^L \left(\frac{\partial (C_0^n(\varepsilon_0))^T}{\partial \varepsilon_{0,j}} \right) \left(\frac{\partial C_0^n(\varepsilon_0)}{\partial \varepsilon_{0,j}} \right) \Big|_{\varepsilon_0=0} \right).$$

According to the following algebraic relationship:

$$\begin{aligned} & \begin{bmatrix} (P_{k|k}^n)^{-1} & 0 \\ 0 & Q_k^{-1} \end{bmatrix} + \lambda_k^n \begin{bmatrix} S_k^n & T_k^n \end{bmatrix}^T \begin{bmatrix} S_k^n & T_k^n \end{bmatrix} \\ &= \begin{bmatrix} I & 0 \\ \lambda_k^n (T_k^n)^T S_k^n \hat{P}_{k|k}^n & I \end{bmatrix} \begin{bmatrix} (\hat{P}_{k|k}^n)^{-1} & 0 \\ 0 & (\hat{Q}_k^n)^{-1} \end{bmatrix} \begin{bmatrix} I & \lambda_k^n \hat{P}_{k|k}^n (S_k^n)^T T_k^n \\ 0 & I \end{bmatrix}. \end{aligned} \quad (\text{A3})$$

Left-multiplying (10) by $\begin{bmatrix} I & 0 \\ \lambda_k^n (T_k^n)^T S_k^n \hat{P}_{k|k}^n & I \end{bmatrix}^{-1}$ and substituting (A3) leads to:

$$\left(\begin{bmatrix} (\hat{P}_{k|k}^n)^{-1} & 0 \\ 0 & \hat{Q}_k^{-1} \end{bmatrix} \begin{bmatrix} I & \lambda_k^n \hat{P}_{k|k}^n (S_k^n)^T T_k^n \\ 0 & I \end{bmatrix} + \begin{bmatrix} I & 0 \\ -\lambda_k^n (T_k^n)^T S_k^n \hat{P}_{k|k}^n & I \end{bmatrix} \begin{bmatrix} A_k^T(0) \\ B_k^T(0) \end{bmatrix} \right) \begin{bmatrix} \hat{x}_{k|k+1}^n - \hat{x}_{k|k}^n \\ \hat{w}_{k|k+1} \end{bmatrix} \\ \times (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} C_{k+1}^n(0) \begin{bmatrix} A_k(0) & B_k(0) \end{bmatrix} \quad (A4)$$

$$= \begin{bmatrix} I & 0 \\ -\lambda_k^n (T_k^n)^T S_k^n \hat{P}_{k|k}^n & I \end{bmatrix} \left(\begin{bmatrix} A_k^T(0) \\ B_k^T(0) \end{bmatrix} (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} \right. \\ \left. \times (y_{k+1}^n - C_{k+1}^n(0) A_k(0) \hat{x}_{k|k}^n) - \lambda_k^n \begin{bmatrix} (S_k^n)^T \\ (T_k^n)^T \end{bmatrix} S_k^n \hat{x}_{k|k}^n \right) \\ \left(\begin{bmatrix} (\hat{P}_{k|k}^n)^{-1} & \lambda_k^n (S_k^n)^T T_k^n \\ 0 & \hat{Q}_k^{-1} \end{bmatrix} + \begin{bmatrix} A_k^T(0) \\ -\lambda_k^n (T_k^n)^T S_k^n \hat{P}_{k|k}^n A_k^T(0) + B_k^T(0) \end{bmatrix} (C_{k+1}^n(0))^T \right) \\ \times (R_{k+1}^n)^{-1} C_{k+1}^n(0) \begin{bmatrix} A_k(0) & B_k(0) \end{bmatrix} \\ \times \begin{bmatrix} I & -\lambda_k^n \hat{P}_{k|k}^n (S_k^n)^T T_k^n \\ 0 & I \end{bmatrix} \begin{bmatrix} I & \lambda_k^n \hat{P}_{k|k}^n (S_k^n)^T T_k^n \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k+1}^n - \hat{x}_{k|k}^n \\ \hat{w}_{k|k+1} \end{bmatrix} \quad (A5)$$

$$= \begin{bmatrix} A_k^T(0) \\ -\lambda_k^n (T_k^n)^T S_k^n \hat{P}_{k|k}^n A_k^T(0) + B_k^T(0) \end{bmatrix} (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} (y_{k+1}^n - C_{k+1}^n(0) A_k(0) \hat{x}_{k|k}^n) \\ - \lambda_k^n \begin{bmatrix} (S_k^n)^T \\ -\lambda_k^n (T_k^n)^T S_k^n \hat{P}_{k|k}^n (S_k^n)^T + (T_k^n)^T \end{bmatrix} S_k^n \hat{x}_{k|k}^n, \\ \left(\begin{bmatrix} (\hat{P}_{k|k}^n)^{-1} & 0 \\ 0 & \hat{Q}_k^{-1} \end{bmatrix} + \begin{bmatrix} A_k^T(0) \\ -\lambda_k^n (T_k^n)^T S_k^n \hat{P}_{k|k}^n A_k^T(0) + B_k^T(0) \end{bmatrix} (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} C_{k+1}^n(0) \right) \\ \times \begin{bmatrix} A_k(0) & -\lambda_k^n A_k(0) \hat{P}_{k|k}^n (S_k^n)^T T_k^n + B_k(0) \end{bmatrix} \\ \times \begin{bmatrix} \hat{x}_{k|k+1}^n + \lambda_k^n \hat{P}_{k|k}^n (S_k^n)^T T_k^n \hat{w}_{k|k+1} - \hat{x}_{k|k}^n \\ \hat{w}_{k|k+1} \end{bmatrix} \quad (A6) \\ = \begin{bmatrix} A_k^T(0) \\ -\lambda_k^n (T_k^n)^T S_k^n \hat{P}_{k|k}^n A_k^T(0) + B_k^T(0) \end{bmatrix} (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} (y_{k+1}^n - C_{k+1}^n(0) A_k(0) \hat{x}_{k|k}^n) \\ - \lambda_k^n \begin{bmatrix} (S_k^n)^T \\ -\lambda_k^n (T_k^n)^T S_k^n \hat{P}_{k|k}^n (S_k^n)^T + (T_k^n)^T \end{bmatrix} S_k^n \hat{x}_{k|k}^n.$$

To facilitate the simplification of Equation (A6), the following matrices are defined:

$$\begin{aligned} \hat{B}_k^n(0) &= B_k(0) - \lambda_k^n (T_k^n)^T S_k^n \hat{P}_{k|k}^n A_k^T(0), \\ \hat{H}_k^n &= C_{k+1}^n \begin{bmatrix} A_k^n(0) & \hat{B}_k^n(0) \end{bmatrix}, \\ \hat{T}_k^n &= T_k^n - \lambda_k^n S_k^n \hat{P}_{k|k}^n (S_k^n)^T T_k^n, \\ \hat{x}_{k|k+1}^n &= \hat{x}_{k|k+1}^n + \lambda_k^n \hat{P}_{k|k}^n (S_k^n)^T T_k^n \hat{w}_{k|k+1}, \\ (\hat{P}_{k|k}^n)^{-1} &= (P_{k|k}^n)^{-1} + \lambda_k^n (S_k^n)^T S_k^n, \\ (\hat{Q}_k^n)^{-1} &= Q_k^{-1} + \lambda_k^n (T_k^n)^T (I + \lambda_k^n S_k^n P_{k|k}^n (S_k^n)^T)^{-1} T_k^n. \end{aligned} \quad (A7)$$

Thus, it follows that:

$$\begin{aligned} & \left(\begin{bmatrix} (\hat{P}_{k|k}^n)^{-1} & 0 \\ 0 & \hat{Q}_k^{-1} \end{bmatrix} + (\hat{H}_k^n)^T (R_{k+1}^n)^{-1} \hat{H}_k^n \right) \begin{bmatrix} \tilde{x}_{k|k+1}^n - \hat{x}_{k|k}^n \\ \hat{w}_{k|k+1} \end{bmatrix} \\ &= (\hat{H}_k^n)^T (R_{k+1}^n)^{-1} (y_{k+1}^n - C_{k+1}^n(0) A_k(0) \hat{x}_{k|k}^n) - \lambda_k^n \begin{bmatrix} (S_k^n)^T \\ (\hat{T}_k^n)^T \end{bmatrix} S_k^n \hat{x}_{k|k}^n. \end{aligned} \quad (A8)$$

It can be further derived that:

$$\begin{cases} \tilde{x}_{k|k+1}^n = \hat{x}_{k|k}^n + \hat{P}_{k|k}^n A_k^T(0) (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} \\ \quad \times [y_{k+1}^n - C_{k+1}^n(0) (A_k(0) \hat{x}_{k|k+1}^n + \hat{B}_k^n(0) \hat{w}_{k|k+1})] - \lambda_k^n \hat{P}_{k|k}^n (S_k^n)^T S_k^n \hat{x}_{k|k}^n, \\ \hat{w}_{k|k+1} = \hat{Q}_k^n (\hat{B}_k^n(0))^T (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} \\ \quad \times [y_{k+1}^n - C_{k+1}^n(0) (A_k(0) \hat{x}_{k|k+1}^n + \hat{B}_k^n(0) \hat{w}_{k|k+1})] - \lambda_k^n \hat{Q}_k^n (\hat{T}_k^n)^T S_k^n \hat{x}_{k|k}^n. \end{cases} \quad (A9)$$

Define the variable $\tilde{x}_{k+1|k+1}^n = A_k(0) \tilde{x}_{k|k+1}^n + \hat{B}_k^n(0) \hat{w}_{k|k+1}$. Substituting Equation (A9) into the expression for $\tilde{x}_{k+1|k+1}^n$, the following expression can be derived:

$$\begin{aligned} \tilde{x}_{k+1|k+1}^n &= A_k(0) \hat{x}_{k|k}^n + (A_k(0) \hat{P}_{k|k}^n A_k^T(0) + \hat{B}_k^n(0) \hat{Q}_k^n (\hat{B}_k^n(0))^T) \\ &\quad \times (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} (y_{k+1}^n - C_{k+1}^n(0) \hat{x}_{k+1|k+1}^n) \\ &\quad - \lambda_k^n (A_k(0) \hat{P}_{k|k}^n (S_k^n)^T + \hat{B}_k^n(0) \hat{Q}_k^n (\hat{T}_k^n)^T) S_k^n \hat{x}_{k|k}^n. \end{aligned} \quad (A10)$$

By defining $P_{k+1|k}^n = A_k(0) \hat{P}_{k|k}^n A_k^T(0) + \hat{B}_k^n(0) \hat{Q}_k^n (\hat{B}_k^n(0))^T$, Equation (A10) is transformed into the structure of Equation (A11):

$$\begin{aligned} & (I + P_{k+1|k}^n (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} C_{k+1}^n(0)) \tilde{x}_{k+1|k+1}^n \\ &= A_k(0) \hat{x}_{k|k}^n + P_{k+1|k}^n (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} y_{k+1}^n \\ &\quad - \lambda_k^n (A_k(0) \hat{P}_{k|k}^n (S_k^n)^T + \hat{B}_k^n(0) \hat{Q}_k^n (\hat{T}_k^n)^T) S_k^n \hat{x}_{k|k}^n. \end{aligned} \quad (A11)$$

According to the matrix inversion lemma applied to the relation $(A + BCD)^{-1} = A^{-1} - A^{-1}B(DA^{-1}B + C^{-1})^{-1}DA^{-1}$, the result below is derived:

$$P_{k+1|k+1}^n = P_{k+1|k}^n - P_{k+1|k}^n (C_{k+1}^n(0))^T (R_{k+1}^n + C_{k+1}^n(0) P_{k+1|k}^n (C_{k+1}^n(0))^T)^{-1} C_{k+1}^n(0) P_{k+1|k}^n. \quad (A12)$$

(A12) can be changed to

$$\begin{aligned} & (I + P_{k+1|k}^n (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} C_{k+1}^n(0)) \tilde{x}_{k+1|k+1}^n \\ &= [A_k(0) - \lambda_k^n (A_k(0) \hat{P}_{k|k}^n (S_k^n)^T + \hat{B}_k^n(0) \hat{Q}_k^n (\hat{T}_k^n)^T) S_k^n] \hat{x}_{k|k}^n + P_{k+1|k}^n (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} y_{k+1}^n. \end{aligned} \quad (A13)$$

$\hat{A}_k^n(0)$ is defined as follows:

$$\hat{A}_k^n(0) = (A_k(0) - \lambda_k^n \hat{B}_k^n(0) \hat{Q}_k^n (\hat{T}_k^n)^T S_k^n) (I - \lambda_k^n \hat{P}_{k|k}^n (S_k^n)^T S_k^n). \quad (A14)$$

Therefore, (A10) can be written in the following form:

$$\tilde{x}_{k+1|k+1}^n = \hat{A}_k^n(0) \hat{x}_{k|k}^n + P_{k+1|k+1}^n (C_{k+1}^n(0))^T (R_{k+1}^n)^{-1} (y_{k+1}^n - C_{k+1}^n(0) \hat{A}_k^n(0) \hat{x}_{k|k}^n). \quad (A15)$$

(A15) has a form similar to that in [32]; therefore, $\tilde{x}_{k+1|k+1}^n$ can be chosen as $\hat{x}_{k+1|k+1}^n$. The derivation is complete.

Appendix B. Derivation of Local Fusion Estimator

First, estimate the initial state x_0^{m+1} . Define $e_0^{m+1}(\varepsilon_0) = z_0^{m+1} - D_0(\varepsilon_0)x_0^{m+1}$. The cost function at the initial time is given by:

$$J(\alpha_0^{m+1}) = \gamma_0^{m+1} [\|x_0\|_{\Pi_0^{m+1}}^2 + \|z_0 - D_0(0)x_0\|_{G_0^{-1}}^2] + (1 - \gamma_0^{m+1}) \sum_{h=1}^L \left(\left\| \frac{\partial e_0(\varepsilon_0)}{\partial \varepsilon_{0,h}} \right\|^2 \right)_{\varepsilon_0=0}. \quad (\text{A16})$$

Enforcing $\partial J(\alpha_0^{m+1}) / \partial x_0 = 0$ yields the initial state estimate as follows:

$$\begin{aligned} P_{0|0}^{m+1} &= \left[(\hat{\Pi}_0^{m+1})^{-1} + D_0^T(0)G_0^{-1}D_0(0) \right]^{-1}, \\ \hat{x}_{0|0}^{m+1} &= P_{0|0}^{m+1}D_0^T(0)G_0^{-1}z_0. \end{aligned} \quad (\text{A17})$$

where

$$\hat{\Pi}_0^{m+1} = \left(\Pi_0^{-1} + \lambda_0^{m+1} \sum_{j=1}^L \left(\frac{\partial D_0^T(\varepsilon_0)}{\partial \varepsilon_{0,j}} \right) \left(\frac{\partial D_0(\varepsilon_0)}{\partial \varepsilon_{0,j}} \right) \Big|_{\varepsilon_0=0} \right).$$

Define the following matrices:

$$\begin{aligned} \tilde{x}_{k|k+1}^{m+1} &= \hat{x}_{k|k+1}^{m+1} + \lambda_k^{m+1} \hat{P}_{k|k}^{m+1} (S_k^{m+1})^T T_k^{m+1} \hat{w}_{k|k+1}, \\ \hat{T}_k^{m+1} &= T_k^{m+1} - \lambda_k^{m+1} S_k^{m+1} \hat{P}_{k|k}^{m+1} (S_k^{m+1})^T T_k^{m+1}, \\ \hat{B}_k^{m+1}(0) &= B_k(0) - \lambda_k^{m+1} A_k(0) \hat{P}_{k|k}^{m+1} (S_k^{m+1})^T T_k^{m+1}, \\ \hat{P}_{k|k}^{m+1} &= \left[(P_{k|k}^{m+1})^{-1} + \lambda_k^{m+1} (S_k^{m+1})^T S_k^{m+1} \right]^{-1}, \\ \hat{Q}_k^{m+1} &= \left[Q_k^{-1} + \lambda_k^{m+1} (T_k^{m+1})^T \left(I + \lambda_k^{m+1} S_k^{m+1} P_{k|k}^{m+1} (S_k^{m+1})^T \right)^{-1} T_k^{m+1} \right]^{-1} \\ \hat{H}_k^{m+1}(\varepsilon_k, \varepsilon_{k+1}) &= D_{k+1}(\varepsilon_{k+1}) \begin{bmatrix} A_k(\varepsilon_k) & \hat{B}_k^{m+1}(\varepsilon_k) \end{bmatrix}. \end{aligned} \quad (\text{A18})$$

According to the following algebraic relationship:

$$\begin{aligned} &\begin{bmatrix} (P_{k|k}^{m+1})^{-1} & 0 \\ 0 & Q_k^{-1} \end{bmatrix} + \lambda_k^{m+1} \begin{bmatrix} S_k^{m+1} & T_k^{m+1} \end{bmatrix}^T \begin{bmatrix} S_k^{m+1} & T_k^{m+1} \end{bmatrix} \\ &= \begin{bmatrix} I & 0 \\ \lambda_k^{m+1} (T_k^{m+1})^T S_k^{m+1} \hat{P}_{k|k}^{m+1} & I \end{bmatrix} \begin{bmatrix} (\hat{P}_{k|k}^{m+1})^{-1} & 0 \\ 0 & (\hat{Q}_k^{m+1})^{-1} \end{bmatrix} \begin{bmatrix} I & \lambda_k^{m+1} \hat{P}_{k|k}^{m+1} (S_k^{m+1})^T T_k^{m+1} \\ 0 & I \end{bmatrix}. \end{aligned} \quad (\text{A19})$$

Substituting Equation (A19) into Equation (18) yields:

$$\begin{aligned} &\left(\begin{bmatrix} (\hat{P}_{k|k}^{m+1})^{-1} & 0 \\ 0 & \hat{Q}_k^{-1} \end{bmatrix} + (\hat{H}_k^{m+1})^T G_{k+1}^{-1} \hat{H}_k^{m+1} \right) \times \begin{bmatrix} \tilde{x}_{k|k+1}^{m+1} - \hat{x}_{k|k}^{m+1} \\ \hat{w}_{k|k+1} \end{bmatrix} \\ &= (\hat{H}_k^{m+1})^T G_{k+1}^{-1} (z_{k+1} - D_{k+1}(0)A_k(0)\hat{x}_{k|k}^{m+1}) - \lambda_k^{m+1} \begin{bmatrix} (S_k^{m+1})^T \\ (\hat{T}_k^{m+1})^T \end{bmatrix} S_k^{m+1} \hat{x}_{k|k}^{m+1}. \end{aligned} \quad (\text{A20})$$

Define $\tilde{x}_{k+1|k+1}^{m+1} = A_k(0)\tilde{x}_{k|k+1}^{m+1} + \hat{B}_k^{m+1}(0)\hat{w}_{k|k+1}$. Then, the above equation can be written in the following form:

$$\begin{cases} \hat{x}_{k|k+1}^{m+1} = \hat{x}_{k|k}^{m+1} + \hat{P}_{k|k}^{m+1} A_k^T(0) D_{k+1}^T(0) G_{k+1}^{-1} \\ \quad \times \left[z_{k+1} - D_{k+1}(0) \left(A_k(0) \hat{x}_{k|k+1}^{m+1} + \hat{B}_k^{m+1}(0) \hat{w}_{k|k+1} \right) \right] - \lambda_k^{m+1} \hat{P}_{k|k}^{m+1} \left(S_k^{m+1} \right)^T S_k^{m+1} \hat{x}_{k|k}^{m+1}, \\ \hat{w}_{k|k+1} = \hat{Q}_k^{m+1} \left(\hat{B}_k^{m+1}(0) \right)^T D_{k+1}^T(0) G_{k+1}^{-1} \\ \quad \times \left[z_{k+1} - D_{k+1} \left(A_k(0) \hat{x}_{k|k+1}^{m+1} + \hat{B}_k^{m+1}(0) \hat{w}_{k|k+1} \right) \right] - \lambda_k^{m+1} \hat{Q}_k^{m+1} \left(\hat{T}_k^{m+1} \right)^T S_k^{m+1} \hat{x}_{k|k}^{m+1}. \end{cases} \quad (\text{A21})$$

This further leads to:

$$\begin{aligned} \hat{x}_{k+1|k+1}^{m+1} &= A_k(0) \hat{x}_{k|k}^{m+1} + \left(A_k(0) \hat{P}_{k|k}^{m+1} A_k^T(0) + \hat{B}_k^{m+1}(0) \hat{Q}_k^{m+1} \left(\hat{B}_k^{m+1}(0) \right)^T \right) \\ &\quad \times D_{k+1}^T(0) G_{k+1}^{-1} \left(z_{k+1} - D_{k+1}(0) \hat{x}_{k+1|k+1}^{m+1} \right) \\ &\quad - \lambda_k^{m+1} \left(A_k(0) \hat{P}_{k|k}^{m+1} \left(S_k^{m+1} \right)^T + \hat{B}_k^{m+1}(0) \hat{Q}_k^{m+1} \left(\hat{T}_k^{m+1} \right)^T \right) S_k^{m+1} \hat{x}_{k|k}^{m+1}. \end{aligned} \quad (\text{A22})$$

Define $P_{k+1|k}^{m+1} = A_k(0) \hat{P}_{k|k}^{m+1} A_k^T(0) + \hat{B}_k^{m+1}(0) \hat{Q}_k^{m+1} \left(\hat{B}_k^{m+1}(0) \right)^T$ and $\hat{A}_k^{m+1}(0) = \left(A_k(0) - \lambda_k^{m+1} \hat{B}_k^{m+1}(0) \hat{Q}_k^{m+1} \left(\hat{T}_k^{m+1} \right)^T S_k^{m+1} \right) \left(I - \lambda_k^{m+1} \hat{P}_{k|k}^{m+1} \left(S_k^{m+1} \right)^T S_k^{m+1} \right)$. Substituting them into Equation (A22) yields:

$$\begin{aligned} \hat{x}_{k+1|k+1}^{m+1} &= \hat{A}_k^{m+1}(0) \hat{x}_{k|k}^{m+1} + \left(I + P_{k+1|k}^{m+1} D_{k+1}^T(0) G_{k+1}^{-1} \right)^{-1} P_{k+1|k}^{m+1} \\ &\quad \times D_{k+1}^T(0) G_{k+1}^{-1} \left(z_{k+1} - D_{k+1}(0) \hat{A}_k^{m+1}(0) \hat{x}_{k|k}^{m+1} \right). \end{aligned} \quad (\text{A23})$$

Define $P_{k+1|k+1}^{m+1} = \left(I + P_{k+1|k}^{m+1} D_{k+1}^T(0) G_{k+1}^{-1} D_{k+1}(0) \right)^{-1} P_{k+1|k}^{m+1}$. By the matrix inversion lemma, it follows that:

$$P_{k+1|k+1}^{m+1} = P_{k+1|k}^{m+1} - P_{k+1|k}^{m+1} (D_{k+1}(0))^T \left(R_{e,k+1}^{m+1} \right)^{-1} D_{k+1}(0) P_{k+1|k}^{m+1}. \quad (\text{A24})$$

where

$$R_{e,k+1}^{m+1} = G_{k+1} + D_{k+1}(0) \left(P_{k+1|k}^{m+1} \right)^{-1} (D_{k+1}(0))^T.$$

Therefore, it follows that:

$$\hat{x}_{k+1|k+1}^{m+1} = \hat{A}_k^{m+1}(0) \hat{x}_{k|k}^{m+1} + P_{k+1|k+1}^{m+1} D_{k+1}^T(0) G_{k+1}^{-1} \left(z_{k+1} - D_{k+1}(0) \hat{A}_k^{m+1}(0) \hat{x}_{k|k}^{m+1} \right). \quad (\text{A25})$$

(A25) has a form similar to that in [32]; therefore, $\hat{x}_{k+1|k+1}^{m+1}$ can be chosen as $\hat{x}_{k+1|k+1}^{m+1}$. The derivation is complete.

References

1. Bai, H.; Yu, B.; Gu, W. Research on Position Sensorless Control of RDT Motor Based on Improved SMO with Continuous Hyperbolic Tangent Function and Improved Feedforward PLL. *J. Mar. Sci. Eng.* **2023**, *11*, 642. [\[CrossRef\]](#)
2. Meng, W.; Zhang, X.; Zhou, L.; Guo, H.; Hu, X. Advances in UAV Path Planning: A Comprehensive Review of Methods, Challenges, and Future Directions. *Drones* **2025**, *5*, 376. [\[CrossRef\]](#)
3. Tran, Q.-K.; Ryoo, Y.-J. Multi-Sensor Fusion Framework for Reliable Localization and Trajectory Tracking of Mobile Robot by Integrating UWB, Odometry, and AHRS. *Biomimetics* **2025**, *10*, 478. [\[CrossRef\]](#)
4. Qin, J.; Du, J.; Li, J. Adaptive Finite-Time Trajectory Tracking Event-Triggered Control Scheme for Underactuated Surface Vessels Subject to Input Saturation. *IEEE Trans. Intell. Transp. Syst.* **2023**, *24*, 8809–8819. [\[CrossRef\]](#)
5. Shi, L.; Yan, S.; Li, W. Consensus and Products of Substochastic Matrices: Convergence Rate With Communication Delays. *IEEE Trans. Syst. Man, Cybern. Syst.* **2025**, *55*, 4752–4761. [\[CrossRef\]](#)
6. Li, W.; Yan, S.; Shi, L.; Yue, J.; Shi, M.; Lin, B.; Qin, K. Multiagent Consensus Tracking Control Over Asynchronous Cooperation–Competition Networks. *IEEE Trans. Cybern.* **2025**, *55*, 4347–4360. [\[CrossRef\]](#) [\[PubMed\]](#)

7. Liang, X.; Xu, J. Control for Networked Control Systems with Remote and Local Controllers over Unreliable Communication Channel. *Automatica* **2018**, *98*, 86–94. [\[CrossRef\]](#)
8. Liang, X.; Xu, J.; Wang, H.; Zhang, H. Decentralized Output-Feedback Control With Asymmetric One-Step Delayed Information. *IEEE Trans. Autom. Control* **2023**, *68*, 7871–7878. [\[CrossRef\]](#)
9. Yang, J.; Zhang, W.-A.; Chen, B. Computation-Efficient Centralized Fusion Estimation with Packet Dropouts. In Proceedings of the 2018 37th Chinese Control Conference (CCC), Wuhan, China, 25–27 July 2018; pp. 6196–6200.
10. Shen, Q.; Liu, J.; Zhou, X.; Qin, W.; Wang, L.; Wang, Q. Centralized Fusion Methods for Multi-Sensor System With Bounded Disturbances. *IEEE Access* **2019**, *7*, 141612–141626. [\[CrossRef\]](#)
11. Liu, H.; Wang, D. Robust state estimation for wireless sensor networks with data-driven communication. *Int. J. Robust Nonlinear Control* **2017**, *27*, 4622–4632. [\[CrossRef\]](#)
12. Peng, Z.; Zhang, Y.; Wen, G.; Yang, S.; Huang, T. Optimal Distributed Filter for State and Unknown Input Estimations in Multi-Sensor Networks. *IEEE Trans. Circuits Syst. II Express Briefs* **2023**, *70*, 4128–4132. [\[CrossRef\]](#)
13. Caballero-Águila, R.; Hermoso-Carazo, A.; Linares-Pérez, J. Networked Distributed Fusion Estimation under Uncertain Outputs with Random Transmission Delays, Packet Losses and Multi-Packet Processing. *Signal Process.* **2019**, *156*, 71–83. [\[CrossRef\]](#)
14. Li, L.; Fan, M.; Xia, Y.; Geng, Q. Dynamic Event-Triggered Feedback Fusion Estimation for Nonlinear Multi-Sensor Systems With Auto/Cross-Correlated Noises. *IEEE Trans. Signal Inf. Process. Over Netw.* **2022**, *8*, 868–882. [\[CrossRef\]](#)
15. Carlson, N.A. Federated Square Root Filter for Decentralized Parallel Processors. *IEEE Trans. Aerosp. Electron. Syst.* **1990**, *26*, 517–525. [\[CrossRef\]](#)
16. Li, X.R.; Zhu, Y.; Wang, J.; Han, C. Optimal Linear Estimation Fusion. I. Unified Fusion Rules. *IEEE Trans. Inf. Theory* **2003**, *49*, 2192–2208. [\[CrossRef\]](#)
17. Niu, D.; Zhang, L.; Song, E.; Zhu, Y. The Equivalence Between Distributed and Centralized Best Linear Unbiased Estimation Fusion. In Proceedings of the 2018 21st International Conference on Information Fusion (FUSION), Cambridge, UK, 10–13 July 2018; pp. 1–7.
18. Sun, S.-L.; Deng, Z.-L. Multi-Sensor Optimal Information Fusion Kalman Filter. *Automatica* **2004**, *40*, 1017–1023. [\[CrossRef\]](#)
19. Yang, J.; Wang, S.; Na, X.; Yang, Y. A Weighted Data Fusion Method in Distributed Multi-Sensors Measurement and Control System. In Proceedings of the 2019 6th International Conference on Information Science and Control Engineering (ICISCE), Shanghai, China, 20–22 December 2019; pp. 654–658.
20. Li, L.; Li, P. Distributed Cluster Fusion Estimation Under Correlated Noise Conditions. In Proceedings of the 2021 5th Chinese Conference on Swarm Intelligence and Cooperative Control, Shenzhen, China, 19–22 November 2021; Lecture Notes in Electrical Engineering, pp. 1431–1441.
21. Zhang, P.; Qi, W.; Deng, Z. Two-Level Robust Measurement Fusion Kalman Filter for Clustering Sensor Networks. *Acta Autom. Sin.* **2014**, *40*, 2585–2594. [\[CrossRef\]](#)
22. Caballero-Águila, R.; Hermoso-Carazo, A.; Linares-Pérez, J. Fusion Estimation Using Measured Outputs with Random Parameter Matrices Subject to Random Delays and Packet Dropouts. *Signal Process.* **2016**, *127*, 12–23. [\[CrossRef\]](#)
23. Caballero-Águila, R.; Hermoso-Carazo, A.; Linares-Pérez, J. Distributed Fusion Filters from Uncertain Measured Outputs in Sensor Networks with Random Packet Losses. *Inf. Fusion* **2017**, *34*, 70–79. [\[CrossRef\]](#)
24. Chen, B.; Yu, L.; Zhang, W.-A.; Liu, A. Robust Information Fusion Estimator for Multiple Delay-Tolerant Sensors With Different Failure Rates. *IEEE Trans. Circuits Syst. I Regul. Pap.* **2013**, *60*, 401–414. [\[CrossRef\]](#)
25. Suh, Y.S.; Nguyen, V.H.; Ro, Y.S. Modified Kalman Filter for Networked Monitoring Systems Employing a Send-on-Delta Method. *Automatica* **2007**, *43*, 332–338. [\[CrossRef\]](#)
26. Battistelli, G.; Benavoli, A.; Chisci, L. Data-Driven Communication for State Estimation With Sensor Networks. *Automatica* **2012**, *48*, 926–935. [\[CrossRef\]](#)
27. Maity, D.; Baras, J.S. Optimal Event-Triggered Control of Nondeterministic Linear Systems. *IEEE Trans. Autom. Control* **2020**, *65*, 604–619. [\[CrossRef\]](#)
28. Qin, C.; Lin, W. Adaptive Event-Triggered Fault-Tolerant Control for Markov Jump Nonlinear Systems With Time-Varying Delays and Multiple Faults. *Commun. Nonlinear Sci. Numer. Simul.* **2024**, *128*, 107655. [\[CrossRef\]](#)
29. Wang, X.; Lemmon, M.D. Event-Triggering in Distributed Networked Control Systems. *IEEE Trans. Autom. Control* **2011**, *56*, 586–601. [\[CrossRef\]](#)
30. Sayed, A.H. A Framework for State-Space Estimation With Uncertain Models. *IEEE Trans. Autom. Control* **2001**, *46*, 998–1013. [\[CrossRef\]](#)
31. Neveux, P.; Blanco, E.; Thomas, G. Robust Filtering for Linear Time-Invariant Continuous Systems. *IEEE Trans. Signal Process.* **2007**, *55*, 4752–4757. [\[CrossRef\]](#)
32. Zhou, T. Sensitivity Penalization Based Robust State Estimation for Uncertain Linear Systems. *IEEE Trans. Autom. Control* **2010**, *55*, 1018–1024. [\[CrossRef\]](#)
33. Liu, H.; Zhou, T. Robust State Estimation for Uncertain Linear Systems With Random Parametric Uncertainties. *Sci. China Inf. Sci.* **2016**, *60*, 1869–1919. [\[CrossRef\]](#)

34. Zienkiewicz, O.C.; Zhu, J.Z. A Simple Error Estimator and Adaptive Procedure for Practical Engineering Analysis. *Int. J. Numer. Methods Eng.* **1987**, *24*, 337–357. [[CrossRef](#)]
35. Sun, Y.; Wang, Y.; Wu, X.; Hu, Y. Robust Extended Fractional Kalman Filter for Nonlinear Fractional System with Missing Measurements. *J. Frankl. Inst.* **2018**, *355*, 361–380. [[CrossRef](#)]
36. Sabatier, J.; Farges, C.; Merveillaut, M.; Feneteau, L. On Observability and Pseudo State Estimation of Fractional Order Systems. *Eur. J. Control* **2012**, *18*, 260–271. [[CrossRef](#)]
37. Naifar, O. Tempered Fractional Gradient Descent: Theory, Algorithms, and Robust Learning Applications. *Neural Netw.* **2026**, *193*, 108005. [[CrossRef](#)] [[PubMed](#)]
38. Yang, Y.; Mo, L.; Hu, Y.; Long, F. The Improved Stochastic Fractional Order Gradient Descent Algorithm. *Fractal Fract.* **2023**, *7*, 631. [[CrossRef](#)]
39. Zhang, Z.; Mao, Y.; Gao, J.; Liu, H. State Estimation of Discrete-Time T–S Fuzzy Systems Based on Robustness Ideas. *Int. J. Fuzzy Syst.* **2023**, *25*, 2007–2019. [[CrossRef](#)]

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