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Distribution-Aware, Risk-Sensitive (DA-RS-FxNLMS) Active Noise Control for Non-Gaussian Acoustic Environments

Pushpraj Tanwar *, Ajay Somkuwar and Rakesh Kumar Gumasta

Department of Electronics and Communication Engineering, Maulana Azad National Institute of Technology, Bhopal 462003, India
* Correspondence: pushpraj2404@gmail.com

Abstract

Active noise control (ANC) in real-world acoustic environments frequently faces impulsive and heavy-tailed noise disturbances, which degrade the performance significantly and lead to slow convergence. This work proposes a dynamically adaptive distribution-aware risk-sensitive filtered-x normalized least mean square (DA-RS-FxNLMS) method for efficient ANC under a non-Gaussian and impulsive scenario. The proposed ANC framework employs a correntropy-based risk-sensitive exponential cost function, which incorporates higher-order statistics and adapts to the error distribution. Further, an adaptive and dynamically adjusted kernel width mechanism tracks the time-varying noise characteristics. The normalized filtered-x structure provides stability under secondary path uncertainty. Simulation is carried out by applying α-stable noise to create an impulsive noise environment, which is produced by the Chambers-Mallows-Stuck method. The outcomes of the proposed method are compared with the baseline methods, showing that the proposed method achieves a noise reduction of 7.02 dB, a significant 40% faster convergence, and improved robustness under strong impulsive noise conditions with $\alpha = 1.2$. The outcome confirms that the proposed method efficiently delivers a promising solution for the ANC system. To the best of our knowledge, for the first time, a unified ANC framework integrates distribution-aware, risk-sensitive learning, adaptive correntropy kernel estimation, and filtered-x normalization for non-Gaussian acoustic environments.

Keywords: active noise control; impulsive noise; non-Gaussian; correntropy; risk-sensitive adaptation

1. Introduction

Environmental noise is a matter of concern that affects the comfort of human beings, and the solution to this issue demands the mitigation of such noises [1,2]. Absorbers and barriers are used as an effective passive treatment for mid- and high-frequency noise but do not perform well for low-frequency disturbances due to material constraints. Active noise control (ANC) is a widely utilized, effective technique in headphones, automotive cabins, and HVAC systems to suppress unwanted, repetitive, low-frequency noise [3–5]. The ANC system suppresses the unwanted sound by generating an anti-noise signal with the same amplitude and opposite phases through destructive interference. The ANC is an effective method for low-frequency noise where the other basic methods are inefficient and bulky. The widely adopted feedforward ANC framework exploits a reference sensor to anticipate incoming disturbances in various applications, like headphones and vehicle cabins. Currently, adaptive filtering methods are a common route for ANC, especially the

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feedforward type filtered-x least mean square (FxLMS), which is becoming more popular due to its feasibility in real time, efficiency, and simplicity. The existing FxLMS methods assume a Gaussian distribution of noise and follow the mean square error (MSE) criteria to minimize the error between the residual noise and zero [6,7]. These methods assume Gaussian noise statistics and assign equal importance to all error samples. The FxLMS method minimizes the second-order error moment. i.e., $J(n) = E\{e^2(n)\}$, where $e(n)$ is the residual error signal and $E\{.\}$ denotes expectations. Here it is assumed that the error signal follows a Gaussian distribution. Feedforward-based FxLMS and the extended work serve as a baseline for ANC due to their improved tracking, faster convergence, and computational efficiency [8–11]. The other variant, filtered-x normalized least mean square (FxNLMS), addresses sensitivity to input power. It modifies the leakage and zero attraction to easily handle the sparsity and coefficient drift and reduce the complexity [12–14].

Further, the variable step size FxLMS-based approach adjusted the learning rate online to trade off convergence and steady-state error. VSS-FxLMS variants (including heuristic and data-driven schemes) improve adaptation under stationary conditions but retain the MSE objective, making them still vulnerable to impulsive outliers. Recent works propose improved step-size laws (arctangent, data-power normalization, and hybrid rules) and affine projection generalizations, which help in many scenarios but cannot fully eliminate distributional sensitivity [15–18]. Sparsity-aware FxLMS algorithms exploit structural properties of the impulse responses (e.g., zero-attracting or reweighted $\ell 1$ penalties) to accelerate convergence for sparse acoustic paths. Such approaches address structural, nonstatistical robustness issues, and thus they complement but do not replace distribution-aware strategies [19,20]. Robust adaptive filtering for impulsive environments has been extensively studied outside ANC, with LAD, Huber, and maximum correntropy criteria (MCC)-based methods showing superior outlier suppression [21–23]. In particular, correntropy and generalized correntropy (including mixtures and total generalized correntropy) have been developed to improve robustness and reduce steady-state bias; these methods have been adapted to system identification and state estimation with strong empirical performance. The application of correntropy/MCC to ANC is a recent and active direction. Generalized filtered-x MCC and maximum correntropy criterion-based FxLMS variants have been proposed to address non-Gaussian noise in ANC, but existing formulations often suffer from either high steady-state misalignment or from not addressing kernel adaptation and secondary-path issues jointly. Newer work proposes generalized or hybrid correntropy criteria and multi-objective designs, but systematic treatments that combine kernel adaptation, filtered-x correctness, and computational practicality remain limited [24–26]. Another popular route is based on information theoretic learning (ITL), mainly on correntropy and its maximum correntropy criteria (MCC) [27–29]. Correntropy is a localized similarity measure induced by a positive definite kernel that emphasizes small informative errors and suppresses the influence of large outliers. This property yields a robust adaptive filter for different purposes, like system identification, channel equalization, and echo cancellation in an impulsive noisy condition. However, in a real-world scenario, various noisy environments, such as industrial and heavy traffic, are often characterized by impulsive components, heavy-tailed noise distribution, and sudden transients. In such a condition, MSE-based adaptations are highly sensitive to the large error excursions, and the sporadic large error samples degrade the performance and stability of the MSE-based adaptive controller.

Motivated by this step, a distribution-aware, risk-sensitive, filtered-x NLMS (DA-RS-FxNLMS) method is proposed to replace the traditional MSE objective with a correntropy-based, risk-sensitive cost and combine it with an adaptive kernel width estimator and filtered reference compensation. For performance evaluation, the noise reduction, perceptual measurements, and convergence stability are determined and compared with the baseline methods under non-Gaussian conditions. The correntropy-based risk-sensitive learning framework is employed here, in which a Gaussian kernel measures the local similarity of the residual error signal. The kernel function suppresses the effect of impulse components. The adaptive kernel width adjustment dynamically tracks the time-varying noise statistics to support the additional institution-aware ANC method. For the evaluation of the performance, the parameters noise reduction, convergence behavior, excess error reduction level (ERLE), robustness against impulsive noise, and secondary path mismatch are analyzed.

2. Proposed Methodology

This section presents the distribution-aware, risk-sensitive, filtered-x NLMS algorithm to design a feedforward ANC framework for non-Gaussian and impulsive noise. The proposed framework integrates filtered-x normalization, correntropy-based risk-sensitive learning, and adaptive kernel width estimation to achieve robust performance. The adaptive update mechanism incorporates error distribution awareness. The methodology is sequenced according to the mathematical structure and follows the physical signal propagation of an ANC system. For implementational convenience, a step-by-step algorithm section (Algorithm S1) and a corresponding flow chart (Figure S6), in addition to the block diagram, have been incorporated in the Supplementary Materials. The flowchart illustrates the sequential signal processing mechanism to provide a strong understanding of the algorithmic workflow.

2.1. System Model

The simulation framework presents a feedforward ANC system under impulsive noise conditions. To accurately model real-world active noise control behavior, a widely accepted standard ANC architecture has been adopted. The corresponding system structure and the signal flow are depicted in Figure 1. The parameters were selected based on practical acoustic modeling considerations. The typical system consists of a reference microphone, a loudspeaker, an adaptive controller, a secondary acoustic path, and an error microphone. Let's consider that the reference microphone detects the reference noise signal $x(n)$, which is used as an input to the adaptive filter, with the acoustic path between the reference microphone and error microphone represented as $p(n)$, and the secondary path between the loudspeaker and the error microphone is as $s(n)$. The adaptive filter produces an anti-noise signal $y(n)$, which is emitted through the loudspeaker and propagates through the secondary path. The primary path allows the primary disturbance signal to reach the error microphone to produce $d(n)$. At the error microphone, the generated residual error signal is further used to update the adaptive coefficient. The reference signal is filtered through the estimated secondary path to produce the filtered reference signal to ensure the correct gradient adaptation. The adaptive update mechanism consists of distribution-aware, risk-sensitive weighting based on the residual error signal and an adaptive kernel width estimator. To determine the undesired primary noise at the error microphone, the equation can be represented as [30].

$$d(n) = x(n) * p(n) \tag{1}$$

where $x(n)$ is the reference noise signal, $p(n)$ is the primary path impulsive response, and $*$ is the linear convolution.

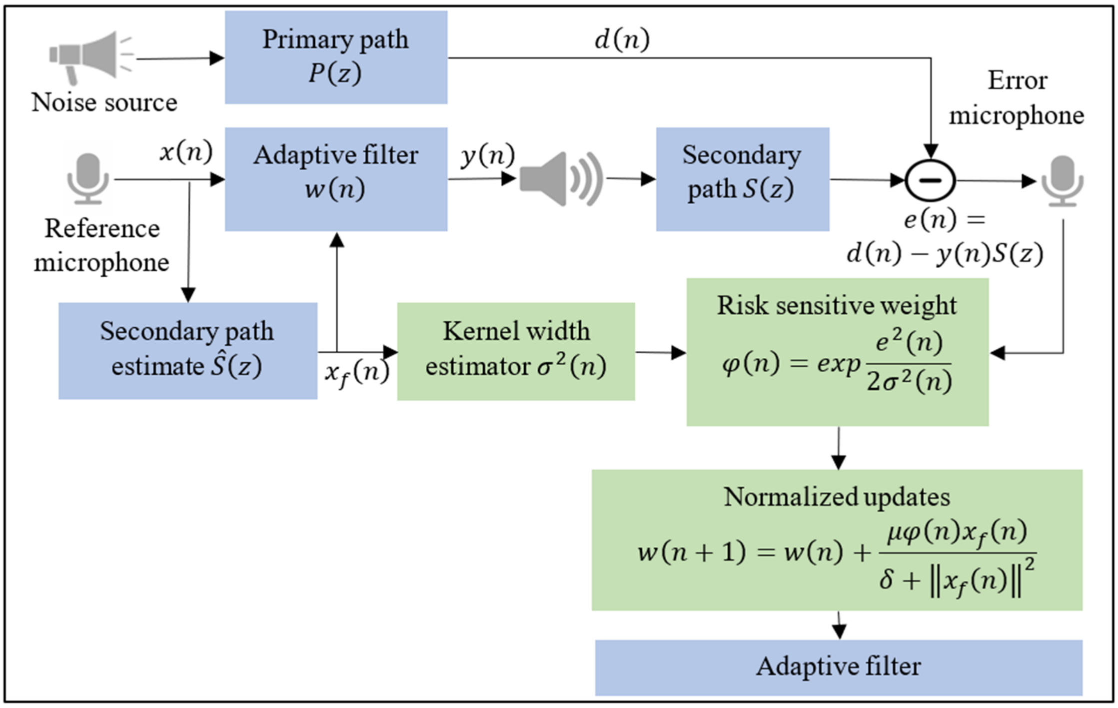

Figure 1. Block diagram of the proposed distribution-aware, risk-sensitive FxNLMS algorithm.

2.2. Initialization

To begin with the algorithm, the adaptive controller and the kernel parameters are initialized for an unbiased and stable start of the ANC process. A finite impulse response (FIR) filter of length L employed with an adaptive filter coefficient vector at time index n is expressed as

$$W(n) = [w_0(n),\ w_1(n),\ \ldots\ldots w_{L-1}(n)]^T \tag{2}$$

At initialization, the filter coefficients are set to $W(0) = 0$, which avoids initial unwanted anti-noise emission to confirm that there is no prior biasing in the control system. This confirms that there is no anti-noise signal generated before the start of the adaptation. The kernel width parameter of the correntropy function, $\sigma^2(n)$, controls the sensitivity of the risk-sensitive cost function to the magnitude of the residual error. A moderate initial value allows it to perform similarly to the FxLMS for the early adaptation and then gradually become robust to the impulsive disturbances to maintain both convergence and robustness. For this reason, the parameter is initialized as $\sigma(0) = \sigma_0$, where σ_0 is a moderate positive constant. The step size of the adaptation μ manages the convergence speed of the adaptive filter which should be needed to satisfy the normalized stability condition $0 < \mu < 2$. The δ is a positive regularization constant used at the denominator of the normalized update to prevent division by zero in normalization ($\delta > 0$) and to ensure numerical stability in low-power regions of the filtered reference signal. Ahead of this, the estimate of the secondary path impulse response is assumed to be available. It can be denoted as $\hat{s} = [\hat{s}_0, \hat{s}_1, \ldots\ldots\ldots\ldots \hat{s}_{M-1}]^T$. The estimates ensure compensation for the electroacoustic dynamics between the error microphone and the loudspeaker. Furthermore, to control the memory of adaptive kernel width estimation, a smoothing factor $0 < \beta < 1$ is adopted. Here, the larger β denotes smoother kernel updates and controls the memory of the kernel width adaptation. Here, the smaller β means faster tracking of the non-stationarity.

2.3. Reference Signal Acquisition

To facilitate adaptive filtering, a reference signal vector is represented as follows:

$$X(n) = [x(n),\ x(n-1)\ldots\ldots,\ x(n-L+1)]^T \tag{3}$$

where L is the most recent sample of the reference noise signal. The noise samples are fed as input to the adaptive control system that allows the anticipatory noise cancellation, which is required to suppress the low-frequency noise components.

2.4. Anti-Noise Signal Generation

The controller generates an anti-noise signal using the current estimates of the adaptive filter coefficients. The anti-noise signal has the same amplitude as the primary noise and the error microphone propagating through the secondary path. It can be represented as

$$y(n) = W^T(n)X(n) \tag{4}$$

where $W(n)$ is the adaptive filter coefficient vector and is the $X(n)$ reference input vector. The quality of the noise cancellation depends on the accuracy of the adaptive filter coefficient that models the inverse characteristics of the primary noise path.

2.5. Secondary Path Effect

During the ANC process, the loudspeaker generates the anti-noise signal, which gets modified due to the secondary path. The secondary path modification includes the digital-to-analog converter, loudspeaker dynamics, amplification, and acoustic propagation. For this purpose, the anti-noise signal is filtered through the estimated secondary path impulse response. Passing the $y(n)$ through a secondary path $\hat{s}(n)$ is determined as

$$y_s(n) = Y(n) * \hat{s}^T(n) \tag{5}$$

where $Y(n)$ is a vector containing the recent samples of $y(n)$. Secondary path compensation is critical; otherwise, the used gradient direction in the adaptive update would be inappropriate, which could lead to degraded performance.

2.6. Residual Error Calculation

After propagation through the secondary path, the error microphone measures the residual noise. The objective of the ANC process is the minimization of the error over time. The residual noise at the error microphone is determined as

$$e(n) = d(n) - y_s(n) \tag{6}$$

where $d(n)$ is the primary noise at the error microphone and $y_s(n)$ is the anti-noise signal after passing through the secondary path. Under a non-Gaussian and impulsive environment, the error signals may involve large outliers that can affect the MSE-based adaptation.

2.7. Secondary Path Compensation

The secondary path amends the correlation between the residual error signal and the adaptive filter coefficient. For gradient correctness and stability of the adaptation, the reference signal is passed through the estimate of the impulse response of the secondary path $\hat{s}(n)$. The filtered reference signal through the estimated secondary path can be derived in convolution form as

$$\hat{X}(n) = x(n) * \hat{s}(n) \tag{7}$$

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The corresponding filtered reference vector is expanded as

$$\hat{X}(n) = [\hat{x}(n),\ \hat{x}(n-1)\ldots\ldots,\ \hat{x}(n-L+1)]^T \tag{8}$$

The adaptive update accurately accounts for the secondary path dynamics and maintains the stability of the ANC system as a consequence of the filtered reference vector.

2.8. Correntropy-Based Risk-Sensitive Cost Function

The proposed algorithm reduces the correntropy-based risk-sensitive cost function, which can be derived as

$$J(e(n)) = exp\left(-\frac{e^2(n)}{2\sigma^2(n)}\right) \tag{9}$$

where $\sigma(n)$ is the adaptive kernel width parameter that controls sensitivity to error magnitude. The exponential term suppresses the influence of impulsive error samples. This emphasizes small and moderate errors that actually exhibit noise structure while downweighting the large deviation developed by impulsive components.

2.9. Adaptive Kernel Width Estimation

Earlier FxLMS methods minimize the mean square error (MSE) $E\{e^2(n)\}$, assuming Gaussian noise and depending on second-order statistics. While the actual acoustic environment could contain impulsive or heavy-tailed noise components. Hence, the large error samples dominate the adaptation process, causing inefficient and unstable coefficient updates. To overcome this issue, in the proposed method, the MSE criterion is replaced by a distribution-aware, risk-sensitive objective function that can attenuate the influence of outliers. To ensure the robust adaptation under both stationary and non-stationary scenarios, the mechanism of kernel width estimation supports the process to automatically adjust its sensitivity according to the recent error distribution. The kernel width $\sigma(n)$ is estimated using a recursive adaptive rule to accommodate time-varying statistics. The kernel width parameter update is determined as

$$\sigma^2(n) = \beta\sigma^2(n-1)(1-\beta)e^2(n) \tag{10}$$

where $\beta \dot{\in} (0,1)$ is the smoothing factor controlling memory. The kernel width of the correntropy function is adapted online, which makes the algorithm highly sensitive to the error distribution. The recursive update is an estimator of the error energy, which allows the algorithm to track time-varying noise statistics. For the attenuation of outliers, kernel width adapts accordingly for large impulsive errors. To avoid the kernel degeneration, the kernel width is maintained within a predefined range of $\sigma_{min} \leq \sigma(n) \leq \sigma_{max}$. To prevent the kernel from collapsing under extremely impulsive samples that are too small and avoid loss of robustness for values that are too large.

Effect of Adaptive Kernel Width

During the ANC process, the adaptive kernel width plays a crucial role in maintaining the convergence speed and provides robustness to the system. In an impulsive environment, large errors occur that demand an adjustment in the kernel width to support the exponential weighting, which becomes sharper to reduce the impact of outliers. However, under Gaussian or stationary conditions, the kernel width increase enables the process to work similarly to the FxLMS method. The correntropy-based update associates an influence function that is bounded for all values of e to improve the robustness under the variable noisy condition. The dynamic adaptation of the kernel width supports a self-regulating

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behavior that confirms a fast and stable ANC system under both Gaussian and non-Gaussian noisy environments.

2.10. Risk-Sensitive Weighting Factor

To reduce the effect of impulsive error, the correntropy-based exponential weighting is adopted. To derive the adaptive update rule, the stochastic gradient descent is applied to the risk-sensitive cost function, where the coefficient update equation can be determined by the filter coefficient using the risk-sensitive weighting factor as:

$$\rho(n) = exp\left(-\frac{e^2(n)}{2\sigma^2(n)}\right) \tag{11}$$

where $0 < \rho(n) \leq 1$. When small residual errors occur, $\rho(n)$ reaches unity, enabling fast adaptation that resembles the FxLMS algorithm; however, for large impulsive errors, $\rho(n)$ reaches zero, which effectively suppresses these errors.

2.11. DA-RS-FxNLMS Adaptive Update Rule

After normalization, the risk-sensitive rule updates the adaptive filter coefficients.

$$W(n+1) = W(n) + \mu\frac{\rho(n)e(n)X_f(n)}{\left\|X_f(n)\right\|^2 + \delta} \tag{12}$$

$$X_f(n) = \left[x_f(n),\ x_f(n-1), \ldots\ldots,\ x_f(n-L+1)\right]^T$$

where μ is the step size and $X_f(n)$ is the filtered reference signal obtained by passing $x(n)$ through the estimated secondary path and used for gradient correction. During the impulsive noise, the exponential term acts as a data-dependent weighting function and decreases the effective adaptation gain. The DA-RS-FxNLMS adaptive update rule incorporates three important mechanisms: normalization ensures the robustness to the variation in the reference signal power, risk-sensitive weighting mitigates the effect of impulsive noise, and the filtered-x compensation accounts for the secondary path effect. In addition, the data-dependent adaptive step size automatically increases under the stationary condition and decreases for impulsive components.

3. Simulation

The experimental work was carried out in a simulation environment using MATLAB (Version R2023b) under non-Gaussian conditions while maintaining linear computational complexity. The ANC model was represented using finite impulse response acoustic paths. The primary and secondary acoustic paths were assumed to be FIR filters. The reference noise was taken to emulate an industrial noise environment; considering Gaussian noise, it was combined with impulsive components. The sampling frequency of 8 KHz sufficiently captured the low-frequency components. The adaptive control filter length was chosen to be L = 128 taps, and the secondary path model length was 64 taps to provide adequate modeling capability for typical impulse responses encountered in DANC systems while maintaining computational complexity. The step size of $\mu = 0.3$, was chosen within the NLMS stability range to ensure mean convergence and provide a balance between convergence speed and steady-state misalignment. The regularization $\delta = 10^{-6}$ parameter avoided division by using a filtered reference signal power and enabled numerical stability. The kernel smoothing factor was chosen as β = 0.95, which is close to unity in recursive variance estimation to provide controlled memory of past error energy for a smooth kernel width evolution along with the ability to trace the non-stationary noise events. The initial kernel width was set to $\sigma(0) = 0.5$ to enable the proposed algorithm to initially behave similarly to FxNLMS under a non-Gaussian scenario and then gradually shift to perform robustly under impulsive events. Moreover, from a statistical perspective, the simulation was repeated and averaged following Monte Carlo trials justified by the Law of Large Numbers (LLN), which ensures that the mean and standard deviation converged to the true expected performance for the increased number of trials, validating a reliable evaluation and confirming robust performance. The mean allowed the estimation of confidence intervals, thereby quantifying the uncertainty and ensuring that the observed improvements were statistically reliable and reproducible.

4. Result and Discussion

The performance of the proposed DA-RS-FxNLMS method was evaluated after simulation under controlled non-Gaussian conditions and compared against the most widely adopted baseline methods like FxNLMS, VSS-FxNLMS, MCC-FxNLMS, and ZA-FxNLMS-based ANC models. The reference noise was modeled using a symmetric α-stable distribution produced through a Chambers-Mallos Stuck method for characterizing non-Gaussian impulsive disturbances. Table 1 shows the noise reduction achieved under an impulsive noise environment with $\alpha = 1.2$. The proposed method achieves the highest noise reduction of 7.02 dB, outperforming the baseline methods. The FxNLMS-based algorithm showed a degraded performance as it relies on the mean square error (MSE), in which all error samples are treated equally. The adaptable step size in VSS-FxNLMS improves the convergence speed, but it is still MSE-driven and sensitive against the heavy-tailed disturbances. The correntropy-based function was used in the MCC-FxNLMS method but still showed limited noise reduction due to the fixed kernel width, which restricted its performance to time-varying impulsive scenarios in the residual error signal. In contrast, the proposed method dynamically adopted the kernel width, as per the instantaneous error statistics. Under the Gaussian case, for the impulsiveness parameter $\alpha = 2$, the α-stable distribution was limited to the Gaussian noise. The residual error $e(n)$ followed a near-Gaussian distribution with a reduced possibility of large outliers. In this condition, the adaptive kernel width $\sigma^2(n)$, tends to increase due to the relatively stable error energy. Consequently, the exponential weighting term $\rho(n)$ approached unity, which caused the proposed DA-RS-FxNLMS algorithm to be equivalent to the conventional FxNLMS method. However, this slight improvement in the proposed method can be attributed to the normalization and adaptive kernel smoothing effect, which reduced gradient noise variance and stabilized the adaptation process.

Table 1. Noise reduction performance and convergence time comparison under impulsive noise.

Method	Noise Reduction (dB)	Convergence Sample
FxNLMS	5.21	9200
VSS-FxNLMS	5.11	8400
MCC-FxNLMS	0.04	15,000
ZA-FxNLMS	6.58	7800
DA-RS-FxNLMS (Proposed)	7.02	5200

Figure 2a shows the convergence behavior examined in terms of the evolution of the residual error power over α-stable impulsive noise. Table 1 summarizes the convergence performance using the number of samples required to reach 90% of the steady-state noise reduction. The proposed method converged in ~5200 samples, achieving 40% faster convergence as compared to the FxNLMS method, and demonstrated substantially faster

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convergence as compared to the baseline methods. Results exhibiting the uncompromised role of the risk-sensitive exponential weighting mechanism were able to selectively suppress the effect of impulsive error samples. Moreover, the proposed method maintained a smooth and stable convergence trajectory, whereas FxNLMS showed significant fluctuations due to large error excursions. It unveiled the fact that the correntropy-based update rule inherits the exponential error weighting. Furthermore, as illustrated in Figure 2b, under impulsive noise conditions, the outcomes of the excess error reduction level, indicating that the proposed method achieves the highest ERLE as compared to the baseline methods, exhibit superior residual noise suppression capability. Results (Figure S2) reveal that the DA-RS-FxNLMS not only performs faster convergence but also could achieve lower steady-state residual noise. This enhancement can be attributed to a direct consequence of the bounded influence function, which was introduced by correntropy-based risk-sensitive learning. It also prevents the sporadic impulsive samples from dominating the adaptation process and degrading the steady-state performance.

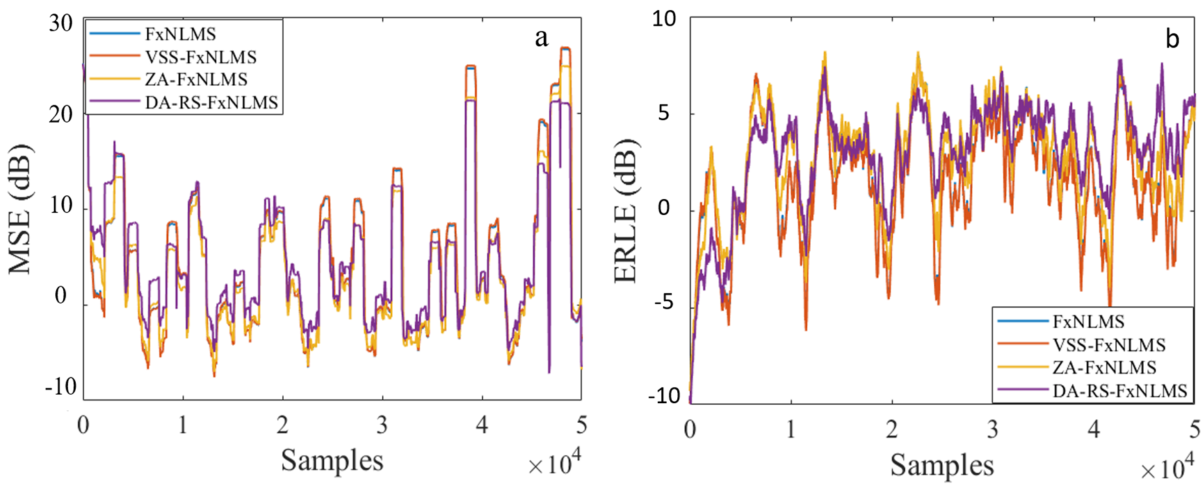

Figure 2. (**a**) Convergence behavior under α-stable impulsive noise, (**b**) ERLE performance comparison under impulsive noise condition of different ANC algorithms.

The assessment of secondary path uncertainty is done by enabling the online secondary path impulse response with an adaptive Gaussian model. Figure 3b illustrates the evaluation of the noise reduction under varying mismatch variances. Results indicate that the proposed method continued with stable progress and maintained performance under the effect of secondary path mismatch (σ = 0.10), while the FxNLMS demonstrated degraded performance due to inaccurate gradient estimation. This demonstrates that the bounded update nature of risk-sensitive adaptation reduces the impact of secondary path mismatch. It reveals that the normalized and bounded adaptive updates limit the effect of inaccurate gradient estimation caused by secondary path modeling errors. The DA-RS-FxNLMS simultaneously addresses both impulsive noise and the secondary path, whereas many other ANC systems primarily focus only on impulsive noise. Table 2 summarizes noise reduction under secondary path mismatch and confirms that the proposed framework is highly suitable for an ANC system in such conditions where environmental changes may alter the secondary path.

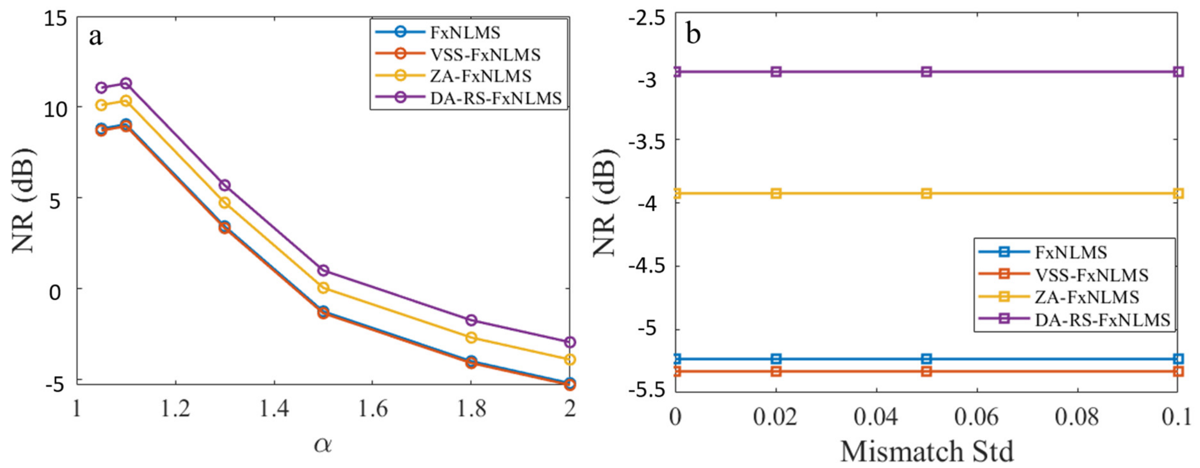

Figure 3. (**a**) Noise reduction vs. impulsiveness parameter α, (**b**) Noise reduction performance vs. secondary path mismatch variance for different ANC algorithms.

Table 2. Mean noise reduction under secondary path mismatch variance σ.

Method	σ = 0.02	σ = 0.05	σ = 0.10
FxNLMS	4.8	2.1	−0.8
MCC-FxNLMS	0.3	−1.2	−3.5
DA-RS-FxNLMS	6.9	5.4	3.2

The stability of the proposed algorithm can be analyzed through its effective step size, $\mu_{eff}(n) = \mu * \rho(n) = \mu * exp\left(-\frac{e^2(n)}{2\sigma^2(n)}\right)$ derived from the update rule (Equation (12)). As it is known that the exponential weighting factor satisfies $0 < \rho(n) \leq 1$, that inherently bounds the effective step size as $0 < \mu_{eff}(n) \leq \mu$. Thus, the proposed algorithm behaves as a normalized LMS with a data-dependent but bounded adaptive gain. The mean stability under independent assumptions and with a minor misalignment could satisfy $0 < \mu < 2$ which is similar to the FxNLMS stability bound. Therefore, the incorporation of the risk-sensitive weighting does not alter the fundamental stability region but provides a controlled and bounded adaptation mechanism.

The adaptive kernel width mechanism is important to dynamically adjust the sensitivity to the error variations. The self-regulating behavior automatically reduces the effective step size during impulsive events and prevents unstable coefficient updates. It means the kernel width decreases to sharpen the exponential weighting and suppress outliers under a highly impulsive scenario, while under the non-Gaussian condition, the kernel width increases and resembles FxNLMS. This adaptive mechanism of the kernel width proves its robustness as compared to the fixed-kernel correntropy and MSE-based approaches. The adaptive kernel width can be estimated recursively from Equation (10). Here, $\sigma^2(n-1)$ and $e^2(n)$ both remain non-negative, confirming positive and bounded $\sigma^2(n)$ for all time. For impulsive samples, the error magnitude $e(n)$ becomes large, which momentarily increases the $\sigma^2(n)$. In DA-RS-FxNLMS, the coefficient update is multiplied by the exponential weighting term. Thus, the exponential weighting term from the coefficient update equation decreases due to the faster growth of squared error as compared to the kernel width. It results in the form of a significantly decreased effective step size and reduced adaptation gain. Consequently, the attenuation of the coefficient update during impulsive events prevents large weight excursions. However, the kernel width appears in the exponential weighting term and is unavailable in the normalization denominator, so its adaptation cannot introduce instability. Unlike conventional FxNLMS, where large errors directly produce large coefficient jumps, in the proposed method, the kernel adaptation proposes a self-regulating mechanism that enhances resilience to the heavy-tailed noise without compromising the stability bounds while maintaining the stability region and enhancing the robustness.

For detailed analysis of the robustness, Figure 3a illustrates the noise reduction as a function of the impulsiveness parameter (α), where the decreasing value of α corresponds to stronger impulsive behavior and deteriorates the performance of the noise reduction capability. The results suggest that the proposed method consistently performs well, as compared to baseline methods across all values of α. The consistently higher performance of the DA-RS-FxNLMS exhibits that the proposed method follows true distribution awareness rather than parameter-specific tuning. Figure S1 illustrates the evolution of the l_2 norm of the adaptive filter coefficients, showing that the DA-RS-FxNLMS maintains bounded weight growth and excellent stability under impulsive noise. Moreover, to confirm statistical reliability, the performance analysis was repeated for 50 independent Monte Carlo trials under different α conditions. For comparative analysis, mean convergence curves for 50 Monte Carlo trials (Figure S3), convergence time (Figure S4), and noise reduction with a 95% confidence interval (Figure S5) have been demonstrated. The mean and standard deviation of the noise reduction and convergence time are illustrated in Table S1. The relatively small standard deviation and narrow confidence intervals exhibited by DA-RS-FxNLMS indicate stable and reproducible outcomes under impulsive scenarios, which validates that the performance evaluation is statistically significant and consistently outperforms the baseline methods, not in a single trial, but as per the Monte Carlo trials.

The overall results show that the DA-RS-FxNLMS method demonstrates excellent performance as compared to the baseline methods. The proposed method precisely targets noise components that are correlated with the reference input while minimizing its impact on the desired speech signal. In a feedforward ANC system, the reference microphone is positioned upstream of the noise propagation path to capture the noise disturbance signal before it reaches the cancelation zone. This configuration confirms that the reference signal primarily has the noise components and remains uncorrelated with the speech signal, which is presented at the error microphone. As these adaptive algorithms are employed, they update filter coefficients, which are based on the correlation between the reference signal and the residual error, which allows them to attenuate only those components that are correlated with the reference signal. Consequently, the adaptive controller does not cancel signals that are not correlated with the reference input, such as speech originating near the error microphone. This feature is preserved because the adaptive update still relies on a filtered reference signal. In addition, to improve the robustness, the correntropy-based exponential weighting mechanism suppresses large impulsive deviations in the error signals rather than steady or structured components. This bounded characteristic prevents abrupt coefficient updates that could distort the speech signals. The adaptive kernel width continuously tracks the error distribution, which allows the algorithm to behave normally like FxNLMS under Gaussian noise conditions while adopting robust correntropy-based functioning under impulsive conditions. Further, the risk-sensitive exponential wing prevents the unstable coefficient updates and ensures the smooth and fast convergence by bounding the influence of large error outliers. The filtered-x normalization confirms the gradient's correctness even in the presence of a secondary path mismatch. As compared to the previous method, the proposed algorithm demonstrates linear computational complexity per iteration and introduces negligible overhead. Altogether, the combined mechanism excellently performs with higher noise reduction and robustness under impulsive conditions.

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5. Conclusions

This work presents the dynamically adaptive distribution-aware risk-sensitive filtered-x normalized least mean square approach evaluated under an impulsive and non-Gaussian noisy environment for active noise control. The method incorporates a mechanism for adapting the kernel width within a risk-sensitive exponential cost function. Results indicate that the proposed method mitigates the adverse effect of the arising noise condition by achieving the best noise reduction of 7.02 dB, exhibits 40% faster convergence as compared to the FxNLMS, and improves ERLE, outperforming the baseline methods. The results confirm that the DA-RS-FxNLMS method exhibits robust performance for active noise control under a non-Gaussian noise scenario, and the correntropy-based exponential error weighting selectively suppresses the impulsive components. The combination of error distribution awareness through a risk-sensitive cost function addresses the limitation of MSE-based adaptation in non-Gaussian and impulsive noise environments. The method replaces the mean squared error function with a correntropy-based cost function, which provides an adaptive update rule. To the best of our knowledge, for the first time, a distribution-aware algorithm for ANC is developed under non-Gaussian environments. The outcomes are validated under controlled non-Gaussian conditions while maintaining the linear computation complexity; the work is well suited for real-time acoustic scenarios.

Supplementary Materials: The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/acoustics8020036/s1, Figure S1: Weight norm evolution comparison for the different ANC methods; Figure S2: Bar graph for steady-state ERLE comparison; Figure S3: Mean convergence curves for 50 Monte Carlo trials; Figure S4: Convergence time (95% confidence interval); Figure S5: Noise reduction (95% confidence interval); Figure S6: Flow chart of the proposed algorithm; Table S1: Monte Carlo Results; Algorithm S1. DA-RS-FxNLMS for Active Noise Control.

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Nomenclature

$x(n)$	Reference noise
$d(n)$	Primary noise at error mic
$e(n)$	Residual noise
$\omega(n)$	Adaptive filter
$y(n)$	Anti-noise signal
$P(z)$	primary acoustic path
$S(z)$	Secondary path
$\hat{S}(z)$	Secondary path estimate
$x_f(n)$	Filtered resistance
$\sigma^2(n)$	Adaptive kernel width

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