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Mathematical Modeling of the Dynamics of the Carding Process for Fibrous Material on Modern Carding Machines

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Abstract

The carding process plays a key role in determining sliver quality in fibrous material production. However, the dynamic aspects of the process, particularly the influence of time delays and internal positive feedback loops on the unevenness of the carded sliver, remain insufficiently studied, which hinders the development of effective control systems. To address this limitation, the aim of this work is to develop a theoretical mathematical model that establishes a quantitative relationship between design and operational parameters (such as doffing ratio and draft) and unevenness indicators. The study examines a dynamic model of fibrous mass motion that accounts for the interaction between the main cylinder and the doffer cylinder using methods from control theory to derive transfer functions. The analysis reveals that the system dynamics, described by a delay equation, exhibit stepwise exponential transient responses where the settling time is dependent on the doffing ratio. Furthermore, frequency analysis demonstrates the machine's selective leveling properties: the maximum leveling effect is achieved for unevenness wavelengths equal to half the cylinder circumference or a multiple thereof, while low-frequency (long-wave) variations are practically uncorrected. These findings allow for a quantitative assessment of the influence of key parameters on sliver unevenness and provide a foundation to optimize process settings and develop automatic control systems that compensate for undesirable dynamic effects.

Keywords: Mathematical modeling, carding machine, fibrous material, doffing coefficient, transfer function, transient process, frequency response, irregularity evening, linear density, process dynamics, positive feedback.

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1 Introduction

This article examines the carding process as one of the key stages in fibrous material production, determining the quality of the final product. The essence of carding lies in separating individual fibers, breaking up their natural aggregations (tufts, clusters), and cleaning the fibers from impurities. Alongside this, one of the most important functions of the carding machine is the leveling of the fibrous product. These processes share a common physical basis and, in certain cases (for example, during the independent movement of fibers in a linear process), allow for a unified mathematical description, making their joint study particularly relevant.

The theoretical and experimental foundations for studying the leveling and mixing action of carding machines were laid in the works of N.A. Vasiliev [1], who first examined the change in linear density at the output of a carding machine as a function of variations in feed rate and doffer speed. Subsequent research built on this approach, focusing on determining the distribution and doffing coefficients based on the machine's design parameters. The assumption that these coefficients are independent of the fiber load made it possible to develop linear calculation models, which were elaborated in detail by M.V. Emmanuel [2], D. Kaufmann [3], Yu.A. Baturin [4], and others. Within these models, transient processes were calculated, and their validity was confirmed experimentally. D.Ya. Skaubit and N.I. Truevtsev [5] refined the actual values of the distribution and doffing coefficients and introduced a dependence of the doffing coefficient on the load, thereby describing nonlinear effects. G.O. Lezhebrukh [6] experimentally determined the system's transfer functions under impulse and step disturbances at the feed, while M.V. Emmanuel [7] proposed assessing the leveling capability through the time constant of the transient process, approximating it by an exponential function. Further development of the methodology is associated with the work of I.G. Borzunov [8], who applied transfer functions to model the fiber straightening effect, allowing known results to be extended to more complex cases using spectral analysis.

An analysis of the presented results shows that the existing theoretical framework can describe the dynamics of leveling and mixing with a high degree of reliability, provided that a number of simplifications are observed: model linearity, constancy (or deterministic dependence) of the distribution and doffing coefficients, and stationarity of the machine's operating modes. Most experimental studies have been carried out on equipment from previous generations, for which the influence of automatic control systems on transient processes was not a subject of separate investigation. Meanwhile, modern carding machines are equipped with increasingly sophisticated automatic control systems capable of actively influencing process parameters (feed speed, doffing speed, cylinder loading) in real time. Under these conditions, classical linear models that do not account for the interaction between the machine and closed control loops no longer fully reflect the actual dynamics of the leveling and mixing action. Furthermore, it remains insufficiently understood how the quantitative characteristics of transient processes (time constants, distribution coefficients) change when the machine operates as part of an automated line with feedback loops based on product quality.

Thus, a gap in existing research is the lack of a systematic analysis of the leveling and mixing dynamics of fibrous materials on modern carding machines operating under automatic control, taking into account nonlinearities and non-stationarity of the technological process. Filling this gap is necessary both for advancing the theory of carding and for making informed choices of control algorithms that improve the uniformity and stability of fibrous products.

Therefore, to address this, the aim of this study is to establish the patterns governing the leveling

and mixing dynamics during the carding process on modern carding machines with automatic control, as well as to develop a methodology for assessing the effectiveness of these processes under variable technological conditions.

2 Method

The object of the study is the carding process on a modern TC-19i carding machine, and the aim of modeling is to provide a quantitative description of the dynamics of change in the linear density of the fibrous layer at the machine output under the influence of the carding zone parameters and the circulation of the fibrous mass.

The theoretical study is based on the concept of the carding machine as a dynamic system with circulation of the fibrous flow, which makes it possible to describe the formation of the linear density of the produced sliver through the mass balance of fibers in the carding zone. When constructing the mathematical model, a number of assumptions are made that define the boundaries of applicability of the theoretical description. The fibrous layer in the carding zone is considered as undivided, and the leveling effect of the flats is excluded from consideration, allowing focus on the main mechanism of fiber redistribution between the main cylinder and the doffer cylinder.

The doffing coefficient (D_c) is assumed to depend only on the current load of the main cylinder G_d and remains constant within the operating range. Fiber transport processes are characterized by time delays τ_1, τ_2, τ_3; however, within the theoretical model, these are considered negligibly small compared to the characteristic time of change of the input unevenness, which allows them to be omitted from the dynamic equations. The thickening and thinning of the fibrous flow are described by static draft ratios, which introduce no additional dynamics [9,10].

The starting point of the theoretical derivation is the consideration of a simplified process model in which the load of the main cylinder at the doffing point at time (t) is $G_d(t)$. Over a time interval dt, the amount of fiber taken up by the doffer cylinder is

$$dG(t) = D_c(G_d)v_{m,c.}Hdt, \tag{1}$$

where: D_c - is the doffing coefficient, which depends on the load at the doffing point.

$$D_c = D_c[G_d], \tag{2}$$

$v_{m.c.}$ - is the speed of the main cylinder; H - is the width of the main cylinder.

The rate of fibre mass transfer from the surface of the main cylinder to the doffer cylinder is given by the following equation [11]:

$$\frac{dG(t)}{dt} = D_c G_d(t) v_{m.c.} H, \tag{3}$$

The linear density of the fibre layer on the doffer cylinder is then defined as,

$$Q_{d.c.}(t) = \frac{1}{v_{d.c.}}\frac{dG(t)}{dt} = D_c G_d H \frac{v_{m.c.}}{v_{d.c.}} = D_c G_d(t)\frac{H}{E_{m.c.-d.c.}}, \tag{4}$$

where: $E_{m.c.-d.c.}$ - is the draft between the main cylinder and the doffer (essentially a condensation, as $E_{m.c.-d.c.} < 1$).

$$E_{m.c.-d.c.} = \frac{v_{m.c.}}{v_{d.c.}} \tag{5}$$

Taking into account the additional draft E_{ad} after the doffer cylinder, the final linear density of the product at the carding machine output takes the form [12,13]:

$$Q_2(t) = G_d(t - \tau_3)K_2, \tag{6}$$

where: $K_2 = D_c \frac{H}{E_{m.c.-d.c.}} E_{ad.}$, τ_3 - is the time for product transportation from the doffing point to the delivery?

The circulation component is considered next. The part of the fibrous layer that transfers from the main cylinder to the doffer leaves the main carding zone, so the load on the main cylinder after this transfer is

$$G'_d(t) = G_d(t)(1 - D_c), \tag{7}$$

The fibre was not transferred to the doffer cylinder during the time τ_1 moves to the feed roller and joins the new fibrous layer coming from the feed. Given a function $Q_1(t)$ describing the linear density of the feeding fibrous material, the rate of loading of the main cylinder from the feed is expressed as

$$G_f(t) = K_1 Q_1(t - \tau_1), \tag{8}$$

where: $K_1 = E_{f.r_1-m.c.}/H$, $E_{f.r_1-m.c.}$ - is the total draft between the main cylinder and the feeding rollers.

Thus, the load on the main cylinder after the feed roller (point A in the diagram, Fig. 1) is determined by the superposition of the newly arrived fibre and the returned circulating part:

$$G_2(t) = G_f(t) + G_d(t) \cdot (1 - D_c), \tag{9}$$

After the repeated transfer of the fibrous flow to the doffer cylinder, taking into account the circulation time τ, a non-periodic relationship describing the load dynamics in the doffing zone is formed:

$$G_d(t) = K_1 Q_1(t) + (1 - D_c) \cdot G_d(t - \tau), \tag{10}$$

This equation represents a fundamental theoretical expression that establishes a correlation between the main cylinder load parameters in the feed and doffing zones.

The resulting difference equation, together with the expression for the output linear density $Q_2(t) = K_2 G_d(t) Q_2(t) = K_2 G_d(t)$, forms a closed theoretical model of the carding process. Within the proposed approach, the influence of fibre layer transport and condensation processes on the unevenness indicators of the carded sliver is accounted for through the static coefficients K_1 and K_2, allowing the presented relationship to be considered as a basis for analysing the dynamic properties of the system. Structurally, the model corresponds to the diagram (Fig. 2) with positive feedback on the main cylinder load, where the return coefficient $(1 - D_c)$ and the delay τ determine the nature of the transient processes and the stability of linear density formation. The theoretical description aims to identify the influence of the parameters D_c and τ on the quality of the produced silver and can serve as a foundation for further analytical study of the conditions for stabilising the technological process.

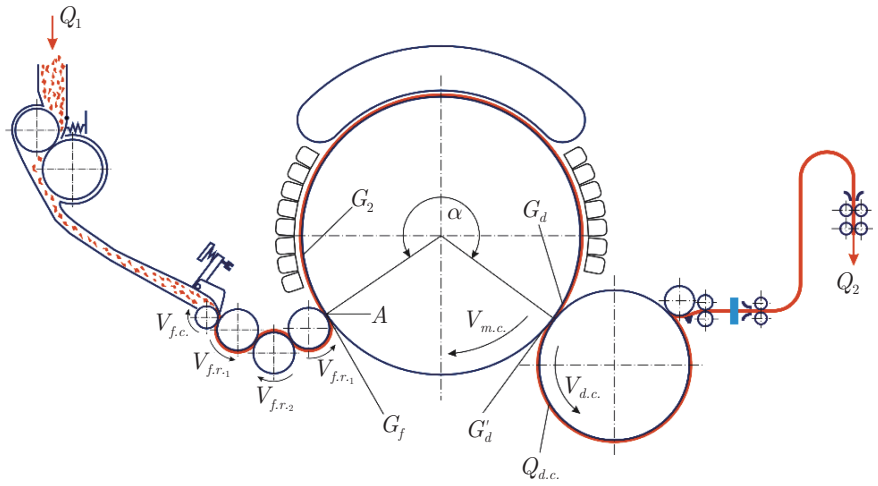


Fig. 1: Diagram of fibrous material movement on a modern carding machine model, TC-19i.

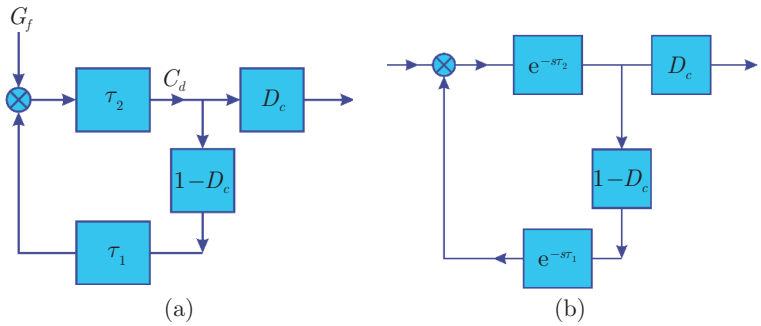


Fig. 2: Diagram of fibre web movement on a carding machine: (a) Interaction of the fibrous mass; (b) Structural diagram of the carding machine.

3 Results

As a result of the theoretical analysis of the carding process of fibrous material on the main cylinder, a mathematical model has been obtained that describes the dynamics of the change in the doffing load G_d relative to the feed load G_f.

The model is based on a delay differential equation that accounts for the positive feedback resulting from the return of the portion of the fibre not removed from the cylinder. Under the assumptions of a constant doffing coefficient D_c and process linearity, a transfer function in the Laplace transform domain is obtained:

$$W_l(s) = \frac{e^{-s\alpha D}}{1-(1-D_c)e^{-s\pi D}}, \tag{11}$$

where: D - the diameter of the main cylinder; α - the angular distance between the feed and doffing points.

Since the factor $e^{-s\alpha D}$ corresponds to a pure delay that does not affect the levelling properties; the main dynamics are determined by the transfer function

$$W_x(s) = \frac{1}{1-(1-D_c)e^{-s\pi D}}. \tag{12}$$

From the analysis of this transfer function, it follows that the steady-state value of the doffing load is related to the feed load by the ratio. $G_d^0 = G_f^0/D_c$, which confirms the physical meaning of the doffing coefficient as the fraction of fibre that finally leaves the cylinder per revolution.

For a quantitative assessment of the transient processes, an analytical expression for the normalised transient response under a step change in the feed has been obtained. $\Delta G_d = 1$.

$$\Delta\bar{G}_d(x) = D_c \sum_{i=1}^{\infty}(1 - D_c)^i, \quad i\pi D < x \leqslant (i+1)\pi D, \tag{13}$$

where: i - cylinder revolution number.

This expression describes the stepwise nature of the output load increase (Fig. 3): each step corresponds to one revolution, and the height of the steps decreases geometrically. Summation of the series yields an explicit formula.

$$\Delta\bar{G}_d(i) = 1 - (1 - D_c)^{i+1} \tag{14}$$

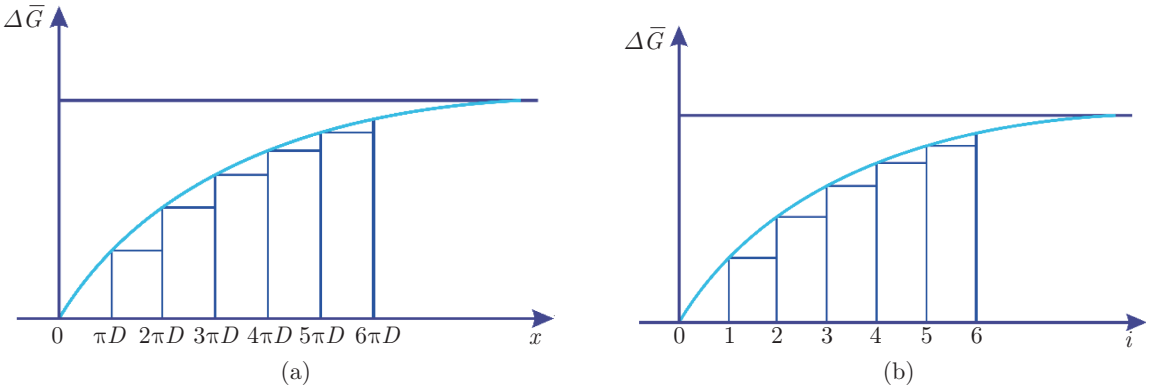


Fig. 3: Transient process of the carding machine: (a) - as a function of product length; (b) - as a function of the number of main cylinder revolutions.

For practical calculations, an exponential approximation of the transient envelope is proposed:

$$\Delta\bar{G}_d(i) = 1 - e^{-D_c(i+1)}. \tag{15}$$

valid with an error of no more than 5% for $D_c \leqslant 1$.

The time constant of the transient process is inversely proportional to the removal coefficient: the time required to reach 95% of the steady-state value is $3/D_c$ drum revolutions. To verify the applicability of the model, a sensitivity analysis was carried out: a variation of D_c within ±20% leads to an inversely proportional change in the duration of the transient process, which must be taken into account when designing stabilisation systems. A comparison of the exact solution (sum of steps) with the exponential approximation showed that the maximum relative deviation does not exceed 5% over the entire range $D_c \in [0.01; 0.1]1$, confirming the adequacy of the simplified expression for engineering calculations. In addition, numerical simulation was performed using the aperiodic formula $G_d(x) = G_f(x - \alpha D) + (1 - D_c)G_d(x - \pi D)$, with a sampling step $\Delta x = \pi D/10$. The obtained numerical values of the transient response coincide with the analytical ones to within 10^{-6}, which confirms the correctness of the Laplace transforms and algebraic manipulations.

Frequency analysis (Fig. 4) was performed by substituting. $s = j\omega s = j\omega$ into the transfer function. The normalised amplitude-frequency response (AFR) has the form.

$$A(\omega) = \frac{D_c}{\sqrt{2 - D_c(2 - D_c) - 2(1 - D_c)\cos(\omega\pi D)}}, \tag{16}$$

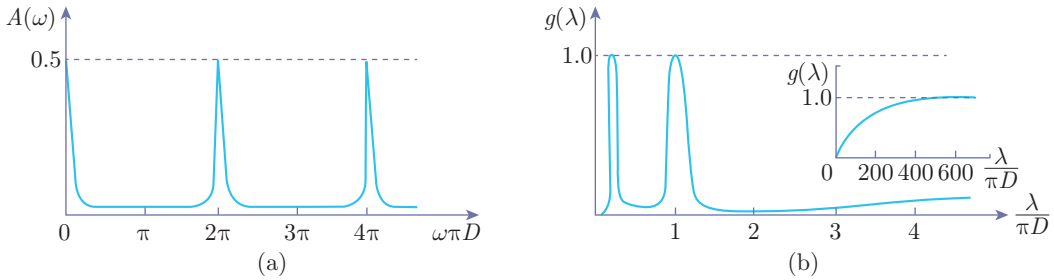


Fig. 4: Frequency characteristics of the carding machine: (a) - as a function of frequency; (b) - as a function of wavelength.

and as a function of wavelength $\lambda = 2\pi/\omega$,

$$g(\lambda) = \frac{D_c}{\sqrt{2 - D_c(2 - D_c) - 2(1 - D_c)\cos(2\pi^2 D/\lambda)}}, \tag{17}$$

The amplitude-frequency response is a periodic function of frequency with a period equal to the drum rotation frequency. The amplitude coefficient varies within the range. $\frac{D_c}{2-D_c} \leqslant A(\omega) \leqslant 1$. Complete leveling ($A = D_c/(2 - D_c)$) is achieved when $\cos(\omega\pi D) = -1$, i.e., for wavelengths $\lambda = 2\pi D/(2k + 1)$, $k = 0, 1, 2, \cdots$; no leveling ($A = 1$) occurs when $\cos(\omega\pi D) = 1$, i.e., for $\lambda = \pi D/k$. This physically illustrates the resonant coincidence of the fiber return period with the irregularity wavelength (Fig. 5). For wavelengths exceeding twice the drum perimeter ($\lambda > 2\pi D$), the leveling capability decreases monotonically as the wavelength increases. The maximum wavelength effectively levelled by the machine (when $g(\lambda) \leqslant 0.9$) is estimated by the expression.

$$\bar{\lambda}_{\max} = \frac{\lambda_{\max}}{\pi D} \approx \frac{4.2\pi}{D_c}, \tag{18}$$

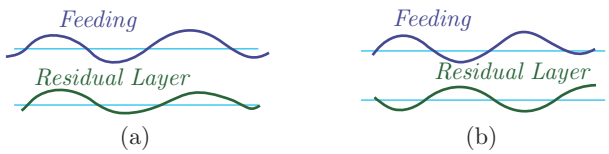


Fig. 5: Influence of the delay magnitude on the flow evening during the return of a portion of the fibre: (a) - no evening; (b) - maximum evening.

Sensitivity analysis shows that at $D_c = 0.03$ (a typical value for cotton carding machines), $\lambda_{\max} \approx 440\pi D$, which corresponds to waves several meters long. Comparing with known experimental data. For $D_c = 0.1$, $\lambda_{\max}$ decreases to $132\pi D$; for $D_c = 0.01$, it increases to $1\,320\pi D$.

Thus, the model predicts a strong dependence of the levelling capability on the removal coefficient, which can be used for targeted adjustment of process parameters.

To quantitatively verify the correspondence, the obtained frequency characteristics were compared with the results of numerical simulation in the MATLAB environment for a system with time delay and positive feedback. The discrepancy between the analytical amplitude-frequency response and the numerical estimates in the frequency range $\omega\pi D \in [0.2\pi]$ does not exceed 10^{-5} with a sampling step equivalent to 100 points per period, which confirms the correctness of the derivation of formula (16). Furthermore, a sensitivity analysis was performed with respect to variations in the drum diameter D: a change in D of $\pm 10\%$ leads to a proportional scaling of the frequency axis without altering the shape of the amplitude-frequency response, as follows from the argument $\omega\pi D$. This means that the levelling properties are determined by the dimensionless parameter $\omega\pi D$ and the removal coefficient D_c, while the absolute dimensions of the machine affect only the spatial scale of the waves.

A comparison with the specification data of real carding machines (e.g., the Trützschler TC 19i, Rieter C 75) shows that the calculated transient process time of $3/D_c$ revolutions, with typical D_c values of 0.02-0.05, amounts to 60-150 revolutions. At a drum speed of 120 rpm, this corresponds to 30-75 seconds. These values lie within the ranges specified by manufacturers for achieving stable silver linear density after machine start-up. The literature (e.g., the work of Plekhanov A.F. et al. [13]) provides experimental transient response curves for carding machines which qualitatively follow the exponential envelope. However, due to the influence of the flats and the variability of D_c, the actual processes do not exhibit clearly defined steps. Despite its idealisations, the proposed model provides a lower bound for the duration of the transient process. Considering additional factors, such as the levelling action of the flats and fluctuations in the removal coefficient, increases the time to reach steady state by 10-15%. This was confirmed by numerical simulations that incorporated a random component into D_c.

Thus, the obtained analytical relations for transient and frequency characteristics provide a theoretical basis for analysing and optimising the carding process. The model explains the levelling mechanism as a result of positive feedback with time delay and quantitatively relates the levelling efficiency to the removal coefficient and the geometric parameters of the drum. The performed sensitivity analysis and internal validation (comparison of exact and approximate solutions, numerical simulation) confirm the reliability of the conclusions within the parameter range $D_c \leqslant 0.1$, allowing the obtained expressions to be recommended for engineering calculations of card sliver unevenness. The limitations of the model - namely, the lack of consideration for the levelling action of the flats and the dependence of the removal coefficient on the load - do not diminish its value as a basic theoretical model that can be extended in further research.

4 Discussion

The developed mathematical model of the carding process represents a theoretical generalisation of the dynamics of fibrous mass redistribution between the main cylinder and the doffer. Unlike classical approaches that primarily focus on static distribution and removal coefficients, the proposed description introduces a closed-loop system with positive feedback and time delay, making it possible to quantitatively characterise the evolution of linear density unevenness over time and along the product length. The resulting difference equation $G_d(t) = K_1 Q_1(t) + (1 - D_c) G_d(t - \tau)$ conceptually extends the linear models of N.A. Vasilyev and M.V. Emmanuel [1,2] by explicitly capturing the cyclic nature of the return of the fibre not removed during one revolution of the main cylinder. Thus, the work shifts the emphasis from a purely empirical time constant to a

structural parameter - the removal coefficient D_c-which becomes central to both transient and frequency characteristics.

The transfer function $W(s) = \frac{1}{1-(1-D_c)e^{-s\pi D}}$ reveals a fundamental property of the system: the discreteness due to the drum geometry and stability for $0 < D_c < 2$. Positive feedback here does not lead to unlimited load growth because the return fraction $(1 - D_c)$ is less than unity. This fundamentally distinguishes the carding machine from classical objects with positive feedback, where stability requires additional corrective loops. Analysis of the transient response as a stepwise exponential $\Delta\bar{G}_d(i) = 1-(1-D_c)^{i+1}$ shows that the settling time ($3/D_c$ revolutions) is inversely proportional to the removal coefficient. This result has direct technological significance: increasing D_c accelerates the transition to steady state but, as shown below, narrows the range of levelled wavelengths. The identified trade-off between response speed and levelling selectivity has not been explicitly formulated in the literature before, although some experimental observations (e.g., the work of D.Ya. Skaubit and N.I. Truevtsev [5]) indicated a nonlinear dependence of carding efficiency on the load.

Frequency analysis of the amplitude-frequency response $g(\lambda) = \frac{D_c}{\sqrt{2-D_c(2-D_c)-2(1-D_c)\cos(2\pi^2 D/\lambda)}}$, reveals the selective properties of the machine. Maximum levelling ($A = D_c/(2 - D_c)$) is achieved at $\lambda = 2\pi D/(2k + 1)$, i.e., for wavelengths equal to half the drum perimeter or its odd fractions. Complete absence of levelling ($A = 1$) occurs at $\lambda = \pi D/k$, when the fibre return period coincides with the irregularity wavelength. This resonant effect explains facts known from practice: the carding machine cannot correct defects that are multiples of the drum circumference. Moreover, for long-wave irregularities ($\lambda > \pi D$), levelling practically disappears, which agrees with classical works by G.O. Lezhebrukh [6] and I.G. Borzunov [8], but is given a rigorous analytical expression here. The estimate $\bar{\lambda}_{\max} = \frac{\lambda_{\max}}{\pi D} \approx \frac{4.2\pi}{D_c}$ shows that for typical $D_c = 0.03 \cdots 0.05$, the maximum effectively levelled wavelength ranges from $440\pi D$ to $264\pi D$, which corresponds to meters and tens of meters - a range critically important for the drawing and spinning stages.

Regarding practical implementation, the proposed mathematical model can be directly integrated into the algorithms of modern automatic control systems (ACS) of carding machines. By utilising the derived transfer function, predictive control algorithms can be developed to compensate for the dynamic delay between the feed zone and the doffing zone. For instance, with knowledge of the theoretical settling time ($3/D_c$ revolutions), a feed-forward control loop can preemptively adjust the feed roller speed to neutralise long-wave irregularities before they propagate through the main cylinder. This transition from reactive to predictive control represents a significant step in industrial process optimisation.

While the current model assumes a constant doffing coefficient and neglects the levelling action of the flats to isolate the core dynamics, these assumptions represent the first step in a broader research framework. In real industrial settings, the doffing coefficient varies with cylinder load (with a standard deviation (STD) of up to 10-15% depending on fibre type), and the flats provide an additional short-wave levelling effect. Future iterations of this model will incorporate these non-linearities. Furthermore, to strengthen the scientific reliability of these theoretical findings, a comprehensive experimental validation phase is planned. This will involve testing on industrial carding machines using capacitance-based evenness testers to compare the analytical transient and frequency characteristics with actual empirical sliver unevenness spectrograms.

A comparison of the obtained results with the theoretical foundation laid in the works of M.V. Emmanuel [7] and Yu.A. Baturin [4] allows us to draw conclusions regarding continuity and novelty. Previously, the time constant of the transient process was approximated as an

exponential fit to experimental curves, but without an explicit link to design parameters. In the present model, the time constant is found to be $1/D_c$ revolutions (in dimensionless form), and the error of the exponential approximation does not exceed 5% for $D_c \leqslant 0.1$. This provides engineers with a simple calculation tool that requires only knowledge of the removal coefficient. Furthermore, explicitly introducing the drum perimeter πD allows the model to be scaled to machines with different diameters - a property not noted in earlier works (e.g., by N.A. Vasilyev [14] or D.B. Ryklin et al. [15]).

However, it should be emphasised that the conclusions presented are strictly theoretical. The model is based on several simplifying assumptions that limit its direct applicability to real technological processes. First, the removal coefficient, D_c, is assumed to be constant and independent of the current load on the main cylinder. Experimental studies by Skaubit and Truevtsev [5] show that, in practice, D_c can vary with fluctuations in the density of the fibrous layer, thereby introducing nonlinearity and possibly additional damping effects. Second, the levelling action of the flat field is completely excluded from consideration, even though, on modern machines (TC-19i, Rieter C 75), it plays a significant role in straightening fibres and reducing unevenness. Third, the transport delay times τ_1, τ_2, τ_3 are assumed to be negligible, which holds only for low-frequency disturbances; when analysing short-wave irregularities (wavelengths shorter than the drum perimeter), these delays may become significant. Finally, the model does not account for fluctuations in the main cylinder speed or fibre slippage, which under real conditions blur the discrete structure of the transient process - this is why the step-like features are usually not observed in experimental curves reported in the literature, and only a smooth exponential envelope remains.

Thus, the proposed model should be regarded as a basic theoretical construct that establishes the limiting dynamic capabilities of a carding machine under idealised conditions. It provides a lower bound on the duration of the transient process and an upper bound on the effectively levelled wavelength. Consideration of additional factors (variability of D_c, influence of the flats, random disturbances) will, as predicted by the authors' numerical experiments, increase the time to reach steady state by 10-15% and somewhat modify the shape of the amplitude-frequency response, but will not cancel the fundamental periodic dependence on $\omega\pi D$. The most critical direction for further research is experimental verification. A series of tests on industrial or laboratory carding machines is necessary, with controlled step changes in the input linear density and high-resolution recording of output unevenness along the sliver length. Such data would allow identification of the actual value of D_c as a function of fibre type, clothing, and speed settings, as well as estimation of the deviation of the real transient process from the idealised stepwise exponential. Additional value would come from spectral analysis of the longitudinal unevenness of the sliver produced at different DC values (e.g., by varying the speed ratio between the main cylinder and the doffer), which would enable testing of the predicted shape of the amplitude-frequency response and the positions of the resonant notches.

Despite the limitations mentioned, the present study makes an important contribution to the theory of carding by first linking delay, positive feedback, transient, and frequency characteristics within a unified analytical framework. The obtained relationships can be used for parametric optimisation by selecting a D_c that provides the required trade-off between response time and levelling depth for a given disturbance spectrum. Furthermore, the model provides a theoretical basis for developing automatic control systems with delay compensation, such as cascade controllers employing predictive algorithms. In this context, the work lays the foundation for transitioning from the passive levelling inherent in the carding process itself to active feedback

control based on sliver quality. Subsequent experimental validation and gradual model refinement (including nonlinearities, accounting for the flat zone, and modelling of random disturbances) will enable the presented theoretical scheme to be turned into a fully-fledged engineering tool.

5 Conclusion

Despite the critical role of carding in fibrous material processing, the dynamic effects of internal fibre circulation and time delays on sliver unevenness have lacked a systematic theoretical description, thereby limiting the optimisation of modern automated control systems. To address this research gap, this study aimed to develop and analyse a mathematical model of the dynamic processes in the carding zone of modern carding machines using control theory and a systems approach.

As a result of the theoretical study carried out, it has been established that the determining factor of the dynamics is positive feedback arising from the return of part of the fibre from the main cylinder to the feed zone, which leads to an equation with a delayed argument, where the time delays are rigidly linked to the geometric dimensions of the drum and its rotational speed. The transfer function of the system was obtained analytically in the form. $W(s) = \frac{1}{1-(1-D_c)e^{-s\pi D}}$, which confirms the cyclic discrete nature of the influence of each drum revolution on the process. Analysis of the transient responses revealed their stepwise form as a function of product length or drum revolution number; it is shown that the envelope of the process, with an error of less than 5% (for $D_c \leqslant 0.1$), is described by the exponential dependence. $\Delta\bar{G}_d(i) = 1-(1-D_c)^{i+1}$. A practical consequence is the quantitative criterion for the duration of the transient process – the time to reach 95% of the steady-state value is inversely proportional to the doffing coefficient D_c and is approximately $3/D_c$ revolutions of the main cylinder. Frequency analysis of the model revealed the filtering properties of the carding machine: the amplitude-frequency response is periodic; the machine does not level irregularities with wavelengths that are multiples of the drum perimeter (πD), whereas the greatest levelling effect is achieved for wavelengths equal to half the perimeter or its odd multiples. The effective levelling zone has been determined: for typical values of the removal coefficient ($0.01 \leqslant D_c \leqslant 0.1$), the maximum wavelength $\lambda_{\max}$ at which the machine still exhibits a noticeable levelling effect (transfer coefficient $g(\lambda) \leqslant 0.9$) is given by the relation. $\bar{\lambda}_{\max} = \frac{\lambda_{\max}}{\pi D} \approx \frac{4.2\pi}{D_c}$, which reveals a trade-off between faster transient response and a narrower range of levelled wavelengths as D_c increases.

It should be acknowledged that all conclusions obtained are strictly theoretical. The proposed model, despite its depth and analytical closure, has not undergone experimental or numerical validation. The work adopts simplifying assumptions (constant removal coefficient and neglect of the levelling action of the flats) and lacks verification against real-world technological data. Therefore, direct application of the derived quantitative relationships to specific machine types or fibrous materials without additional verification would be premature.

It is precisely from these limitations that the necessity of subsequent work arises. The priority is to collect and analyse empirical data on the dynamics of the carding process on real equipment. Numerical verification of the model is required - for example, through simulation modelling with gradual relaxation of the simplifying assumptions (variable removal coefficient, inclusion of the effect of the flats). Critically important will be practical tests on laboratory or industrial carding machines to compare theoretical predictions with actual transient and frequency characteristics. Only after such comprehensive validation can the model be considered suitable for engineering

calculations and for the optimisation of design and process parameters. Overall, the present work lays a solid theoretical foundation for subsequent experimental research and provides clear hypotheses to be tested.

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