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Computational Testing Procedure for the Overall Lifetime Performance Index of Multi-Component Exponentially Distributed Products

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Abstract

In addition to products with a single component, this study examines products composed of multiple components whose lifetimes follow a one-parameter exponential distribution. An overall lifetime performance index is developed to assess products under the progressive type I interval censoring scheme. This study establishes the relationship between the overall and individual lifetime performance indices and derives the corresponding maximum likelihood estimators along with their asymptotic distributions. Based on the asymptotic distributions, the lower confidence bounds for all indices are also established. Furthermore, a hypothesis testing procedure is formulated to evaluate whether the overall lifetime performance index achieves the specified target level, utilizing the maximum likelihood estimator as the test statistic under a progressive type I interval censored sample. Moreover, a power analysis is carried out, and two numerical examples are presented to demonstrate the practical implementation for the overall lifetime performance index. This research can be applied to the fields of life testing and reliability analysis.

Keywords: exponential distribution; progressive type I interval censored sample; maximum likelihood estimation method; overall lifetime performance index; testing procedure algorithm; Confidence lower bounds

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1. Introduction

Because the lifetime of each product component possesses a larger-the-better characteristic, this study adopts the lifetime performance index with a unilateral tolerance limit proposed by Montgomery [1]. It is also noted that several process capability indices (PCIs) commonly assume that the quality characteristics follow a normal distribution for quality characteristics. However, product lifetimes are frequently modeled by continuous probability distributions such as exponential, gamma, or Weibull distributions (see Epstein and Sobel [2], Meyer [3], Anderson et al. [4], Keller et al. [5]). This study focuses on exponential distribution. This distribution is a commonly applied continuous probability model for describing the time intervals between independent events that occur at a constant average rate. One of its key characteristics is the memoryless property, which implies that the likelihood of a future event is unaffected by any previous occurrences. This feature makes the distribution particularly appropriate for modeling the lifetimes of components that neither age nor degrade over time, such as electronic devices operating under stable conditions. In fields such as reliability engineering and survival analysis, it is frequently

employed to represent failure or waiting times under the assumption of a constant hazard rate. Owing to its mathematical tractability and clear interpretability, the exponential distribution continues to serve as a foundational model in reliability studies, queueing systems, and stochastic process analysis. For this distribution, Zelen [6] applied exponential models to address various problems in cancer research. Lawless and Singhal [7] analyzed life-test experiment data assuming an exponential model. Yakovenko and Barkley [8] discussed how exponential distributions appear in modeling income or wealth for the majority of the population. Liang et al. [9] conducted an empirical study showing that taxi displacement and waiting times follow exponential laws in urban settings. Douglas et al. [10] utilized the exponential distribution to enhance environmental quality at a company in southern Rio de Janeiro State. Hu et al. [11] explored methods for estimating the mean exponential survival time in the presence of sequential censoring. Adeyeye et al. [12] conducted a study on time-domain sampling of exponential distributions employing superparamagnetic tunnel junctions. Balakrishnan [13] presented numerous application areas for exponential distribution in a comprehensive book. For the inferences on the lifetime performance index, Tong et al. [14] developed a hypothesis testing procedure using the uniformly minimum variance unbiased estimator (UMVUE) of this index for lifetimes of products following one-parameter exponential distribution. However, practical constraints, such as limited time, resources (including financial and material limitations), human error in recording, and operational or experimental difficulties, may prevent the experimenter from observing the lifetimes of all tested items. As a result, incomplete data, such as progressive censoring data, may be obtained (see Balakrishnan and Aggarwala [15], Hong et al. [16], Wu et al. [17]). Lee et al. [18] evaluated the lifetime performance index for exponential products under step-stress accelerated life-testing conditions. For the exponential distribution, Wu [19] utilized the upper record value data to propose an interval estimation based on the Bayesian approach. The focus of this research is the progressive type I interval censoring scheme.

The progressive type I interval censoring process is elaborated as follows: Suppose that there are n units of product are put on a life test at time 0. Let $(t_1, \ldots, t_m)$ be the predetermined inspection times while $t_m = T$ is the planned time to terminate the experiment. For the products with k components with lifetime following a one-parameter exponential distribution, for the ith component, we observe the number of failures as X_{ij} in the jth time interval $(t_{j-1}, t_j]$ and then R_{ij} unites are removed with removal rate p_i for $j = 1, \ldots, m-1$. At the end, at the termination time point $t_m = T$, the number of failures as X_{im} is observed and then the remaining units are removed and the experiment is terminated. We focus on this type of censored sample. Please see Figure 1 to illustrate the concept of exponential products with multiple components for this type of censoring, where X_{ij} represents the number of failures for the ith component in the jth time interval $(t_{j-1}, t_j]$, $i = 1, \ldots, k$, $j = 1, \ldots, m$.

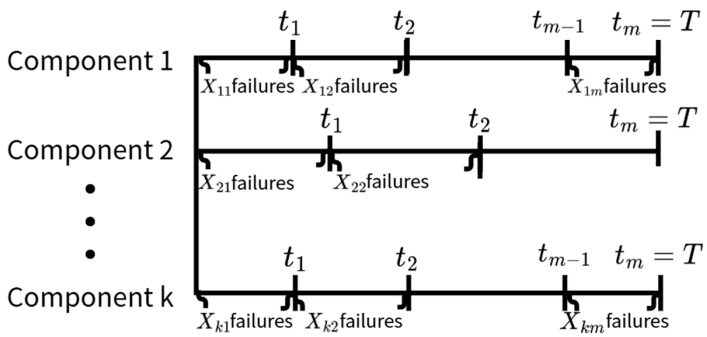


Figure 1. The concept of exponential products with multiple components for the progressive type I interval censored sample.

For a single production line or products with a single component, Wu and Lin [20] proposed the testing procedure for lifetimes of product following a one-parameter exponential distribution using the progressive type I interval censored sample. Wu et al. [21] proposed an experimental design, based on the testing procedure described by Wu and Lin [20], that optimizes either the desired test power or the overall experimental cost. When the products possess multiple components whose lifetimes follow a one-parameter exponential distribution, the overall lifetime performance index is proposed for multiple components in this study. This study extends the analysis from the single-component case to the multi-component case. We explore how the overall lifetime performance index is related to each individual lifetime performance index. The maximum likelihood estimator and the asymptotic distribution for individual lifetime performance index and overall lifetime performance index are investigated. Referring to maximum likelihood estimation, Aletti et al. [22] studied the existence, geometry, and efficiency of the maximum likelihood estimation under the Emax model. Kneib et al. [23] revisited the maximum log-likelihood parameter estimation for two-parameter Weibull distributions. Using the maximum likelihood estimator of the index as the test statistic to test if the overall lifetime performance index adhere the desired target level for the progressive type I interval censored sample, the hypothesis testing procedure for all individual lifetime performance indices is developed.

The rest of this paper is organized as follows: In Section 2.1, the overall lifetime performance index is proposed for products with multiple components and the relationship among the conforming rate, the overall lifetime performance index and the individual lifetime performance index are presented. The maximum likelihood estimator and the corresponding asymptotic distribution for the overall lifetime performance index and individual lifetime performance indices based on the progressive type I interval censored sample under the assumption of exponential distribution are obtained in Section 2.2. In this section, the lower confidence bounds for all indices are derived using their corresponding asymptotic distributions. In Section 3.1, we develop a new testing algorithmic procedure for the overall lifetime performance index and find the test power function for this test. We provide two numerical examples to demonstrate our proposed testing procedure given in Section 3.2. The power analysis is carried out and the main findings are listed and discussed in Section 4. The final conclusions are presented in Section 5.

2. Materials and Methods

In Section 2.1, we propose the overall performance index and explore the relationship for this index with the individual indices. The maximum likelihood estimators and the corresponding asymptotic distributions for the estimators are derived in Section 2.2. In this section, we also found the lower confidence bounds for the individual and overall lifetime performance indices based on the asymptotic distributions for the estimators.

2.1. The Overall Lifetime Performance Index and Overall Conforming Rate

Suppose that the lifetime (Y_i) of the ith component of products follows a one-parameter exponential distribution and the related probability density function (pdf) and the cumulative distribution function (cdf) are given as follows:

$$f_{Y_i}(y) = \frac{1}{\lambda_i} exp\left\{-\frac{y}{\lambda_i}\right\},\ y \geq 0,\ \lambda_i > 0, \tag{1}$$

And

$$F_{Y_i}(y) = 1 - exp\left\{-\frac{y}{\lambda_i}\right\},\ y \geq 0,\ \lambda_i > 0, \tag{2}$$

where λ_i is the scale parameter. The failure rate function is defined as

$$r(y) = \frac{f_{Y_i}(y)}{1 - F_{Y_i}(y)} = \frac{1}{\lambda_i},\ \lambda_i > 0. \tag{3}$$

The mean and standard deviation of the lifetime of products following the distribution defined in Equation (1) are given by $\mu_i = E(Y_i) = \lambda_i$ and $\sigma_i = \sqrt{Var(Y_i)} = \lambda_i$. The lifetime is a larger-the-better type quality characteristic because a longer lifetime implies a better quality of products. To achieve economic profitability and fulfill customer expectations, the product lifetime is typically expected to exceed L_i units, where Li is a specified lower specification limit. Using the lifetime performance index proposed in Montgomery to assess the performance of lifetime of products, the lifetime performance index for the ith component is C_{Li} expressed as

$$C_{Li} = \frac{\mu_i - L_i}{\sigma_i} = \frac{\lambda_i - L_i}{\lambda_i} = 1 - \frac{L_i}{\lambda_i},\ -\infty < C_{Li} < 1. \tag{4}$$

We observe a decreasing function of the failure rate $\frac{1}{\lambda_i}$ and an increasing function of λ_i. That is, the smaller the failure rate $\frac{1}{\lambda_i}$, the larger the lifetime performance index C_{Li}. Hence, the lifetime performance index C_{Li} serves as an effective and accurate indicator for evaluating the product lifetime performance of products.

When all the lifetimes of products exceed the lower specification limit (i.e., $Y_i \geq L_i,\ i = 1, \ldots k$), all components of products are regarded as conforming. Hence, the overall conforming rate is given by

$$PrP_r = P(Y_i \geq L_i,\ i = 1, ..., k) = exp\left\{-\sum_{i=1}^{k} \frac{L_i}{\lambda_i}\right\} = exp\left\{\sum_{i=1}^{k} C_{Li} - k\right\},\ -\infty < C_{Li} < 1. \tag{5}$$

The setup of the overall lifetime performance index C_T is designed to satisfy

$$P_r = exp\left\{\sum_{i=1}^{k} C_{Li} - k\right\} = exp\{C_T - 1\},\ -\infty < C_T < 1 \tag{6}$$

It is noted that C_T increases as P_r increases. From Equation (6), we can explore the relationship between the overall lifetime performance index with all individual lifetime performance indices as follows:

$$C_T = \sum_{i=1}^{k} C_{Li} - (k-1),\ -\infty < C_T < 1 \tag{7}$$

Consider the setting in which the individual lifetime performance indices are assumed to be equally important as $C_{L1} = ... = C_{Lk} = C_L$, then

$$C_L = (C_T + k - 1)/k,\ -\infty < C_T < 1 \tag{8}$$

Setting the desired overall conforming rate, the desired level of overall lifetime performance index C_T can be determined from Equation (6). The desired individual lifetime performance index C_L for each component can be determined by Equation (8). For example if the experimenter wants the overall conforming rate to be P_r = 0.951229, the desired value of C_T = 0.95 from Equation (6) and the corresponding desired level for each individual lifetime performance index C_L is C_L = 0.975, 0.98333, 0.9875, 0.99, 0.99166, 0.99285, 0.99375, 0.99444, 0.995 when k = 2, 3, . . ., 10 from Equation (8).

2.2. Maximum Likelihood Estimation

For the *i*th component of product consisting of k components, the progressive type I interval censored data as $X_{i1}, \ldots, X_{im}$ at the inspection times $t_1, \ldots, t_m$, $i = 1, \ldots, k$. For this sample $X_{11}, \ldots, X_{km}$, the likelihood function for k components is

$$\begin{aligned} L^*(\lambda_1, \ldots, \lambda_k) &\propto \prod_{i=1}^{k} \prod_{j=1}^{m} \left(F(t_j) - F(t_{j-1})\right)^{X_{ij}} \left(1 - F(t_j)\right)^{R_{ij}} \\ &\propto \prod_{i=1}^{k} \prod_{j=1}^{m} exp\left\{-\frac{X_{ij}t_{j-1} + R_{ij}t_j}{\lambda_i}\right\} \left(1 - exp\left\{-\frac{t_j - t_{j-1}}{\lambda_i}\right\}\right)^{X_{ij}} \end{aligned}$$

The likelihood function for the *i*th component is

$$\begin{aligned} L(\lambda_i) &\propto \prod_{j=1}^{m} \left(F(t_j) - F(t_{j-1})\right)^{X_{ij}} \left(1 - F(t_j)\right)^{R_{ij}} \\ &\propto \prod_{j=1}^{m} exp\left\{-\frac{X_{ij}t_{j-1} + R_{ij}t_j}{\lambda_i}\right\} \left(1 - exp\left\{-\frac{t_j - t_{j-1}}{\lambda_i}\right\}\right)^{X_{ij}} \end{aligned} \tag{9}$$

and the log-likelihood function for the *i*th component is

$$logL(\lambda_i) \propto \sum_{j=1}^{m} \left(X_{ij} log\left(1 - exp\{-a_{ij}\}\right) - B_{ij}\right), \tag{10}$$

where $a_{ij} = \frac{t_j - t_{j-1}}{\lambda_i}$ and $B_{ij} = \frac{X_{ij}t_{j-1} + R_{ij}t_j}{\lambda_i}$, $j = 1, \ldots, m$, $i = 1, \ldots, k$.

Differentiating with respect to λ_i and setting the result to zero gives the log-likelihood equation

$$\frac{d}{d\lambda_i} logL(\lambda_i) = \sum_{j=1}^{m} \left(X_{ij} \frac{exp\{-a_{ij}\} \frac{d}{d\lambda_i} a_{ij}}{1 - exp\{-a_{ij}\}} - \frac{d}{d\lambda_i} B_{ij} \right) = 0. \tag{11}$$

Solving Equation (11), we can find the maximum likelihood estimator (MLE) for λ_i denoted by $\hat{\lambda}_i$, where $\frac{d}{d\lambda_i} a_{ij} = -\frac{t_j - t_{j-1}}{\lambda_i^2}$ and $\frac{d}{d\lambda_i} B_{ij} = -\frac{X_{ij}t_{j-1} + R_{ij}t_j}{\lambda_i^2}$, $j = 1, \ldots, m$, $i = 1, \ldots, k$.

The MLE follows an asymptotic normal distribution, which can be derived in the usual manner. The corresponding Fisher information is $I(\lambda_i) = -\frac{E d^2 logL(\lambda_i)}{d\lambda_i{}^2}$. From Equation (10), we have

$$\begin{aligned} &\frac{d^2 logL(\lambda_i)}{d\lambda_i{}^2} \\ &= \sum_{j=1}^{m} \left[-X_{ij} \left(exp\{a_{ij}\} - 1\right)^{-2} exp\{a_{ij}\} \left(\frac{da_{ij}}{d\lambda_i}\right)^2 + X_{ij} \left(exp\{a_{ij}\} - 1\right)^{-1} \frac{d^2 a_{ij}}{d\lambda_i{}^2} - \frac{d^2 B_{ij}}{d\lambda_i{}^2} \right], \end{aligned} \tag{12}$$

where $\frac{d^2 a_{ij}}{d\lambda_i^2} = \frac{2\left(t_j - t_{j-1}\right)}{\lambda_i^3}$ and $\frac{d^2 B_{ij}}{d\lambda_i^2} = \frac{2\left(X_{ij}t_{j-1} + R_{ij}t_j\right)}{\lambda_i^3}$, $j = 1, \ldots, m$, $i = 1, \ldots, k$.

It is observed that

$$X_{ij} \big| X_{i,j-1}, \ldots, X_{i1} \sim \text{Binomial}\left(n - \sum_{l=1}^{j-1} X_{il}, q_{ij}\right), \tag{13}$$

where $q_{ij} = \frac{F(t_j) - F(t_{j-1})}{1 - F(t_{j-1})} = 1 - exp\left\{-\frac{t_j - t_{j-1}}{\lambda_i}\right\}$ is the failure probability in the *j*th inspection time interval, $j = 1, \ldots, m$.

Referring to Wu and Lin [20], the conditional expectation of X_{ij} given $X_{i,j-1}, \ldots, X_{i1}, R_{i,j-1}, \ldots, R_{i1}$ is formulated as

$$E\left(X_{ij} \big| X_{i,j-1}, \ldots, X_{i1}, R_{i,j-1}, \ldots, R_{i1}\right) = n q_{ij} \prod_{l=1}^{j-1} (1 - p_l)(1 - q_{il}), j = 1, \ldots, m\,. \tag{14}$$

Using Equation (14), Fisher's information number becomes

$$I(\lambda_i) = \frac{n}{\lambda_i^3}\sum_{j=1}^{m}\left\{2t_{j-1} - (1-q_{ij})\left[2t_j(1-p_j) - \frac{(t_j - t_{j-1})^2}{\lambda_i q_{ij}}\right]\right\}\prod_{l=1}^{j-1}(1-p_l)(1-q_{il}) \,. \quad (15)$$

Then, we have $\hat{\lambda}_i \xrightarrow[m\to\infty]{d} N(\lambda_i, I^{-1}(\lambda_i))$.

Considering the special case in which all interval lengths are equal, equivalently $t_j - t_{j-1} = t$ and $q_i = 1 - exp\left\{-\frac{t}{\lambda_i}\right\}$, $j = 1, \ldots, m$. Thus, the monitoring and censoring processes are performed periodically with a constant time interval t. We also set $p_j = p$, $j = 1, \ldots, m-1$.

Equation (11) can be simplified as

$$\frac{d}{d\lambda_i} logL(\lambda_i) = \frac{t}{\lambda_i^2}\sum_{i=1}^{m}\left[(j-1)X_{ij} + jR_{ij} - \left(exp\left\{\frac{t}{\lambda_i}\right\} - 1\right)^{-1} X_{ij}\right] = 0 \quad (16)$$

Solving Equation (16) for λ_i numerically, we can obtain the MLE for λ_i denoted by $\hat{\lambda}_i$.

The asymptotic variance of $\hat{\lambda}_i$ is given by the following expression:

$$\begin{aligned} V(\hat{\lambda}_i) = I^{-1}(\hat{\lambda}_i) = \frac{\hat{\lambda}_i^3}{nt}\Big(\sum_{j=1}^{m-1}(1-p)\{2(j-1) - (1-q_i)[2j(1-p) - \\ \frac{t}{\lambda_i q_i}\Big]\Big\}[(1-q_i)]^{j-1} + (1-p)\Big[2(m-1) + (1-q_i)\frac{t}{\lambda_i q_i}\Big][(1-q_i)]^{m-1}\Big)^{-1}. \end{aligned} \quad (17)$$

Utilizing the invariance property of MLE, the MLE of C_{Li} derived as follows

$$\hat{C}_{Li} = 1 - \frac{L}{\hat{\lambda}_i}. \quad (18)$$

Since we have $\hat{\lambda}_i \xrightarrow[n\to\infty]{d} N(\lambda_i, V(\hat{\lambda}_i))$, we can show that $\frac{1}{\hat{\lambda}_i} \xrightarrow[n\to\infty]{d} N\Big(\frac{1}{\lambda_i}, \frac{1}{\lambda_i^4}V(\hat{\lambda}_i)\Big)$. In addition, we can show that

$$\hat{C}_{Li} \xrightarrow[n\to\infty]{d} N(C_{Li}, V(\hat{C}_{Li})) \,, \quad (19)$$

where $V(\hat{C}_{Li}) = \frac{L^2 V(\hat{\lambda}_i)}{\lambda_i^4}$. Therefore, the MLE $\hat{C}_{Li}$ serves as an asymptotically unbiased estimator of C_{Li}. The asymptotic variance of $\hat{C}_{Li}$ is estimated by substituting λ_i with $\hat{\lambda}_i$ in the corresponding expression, as shown below:

$$\hat{V}(\hat{C}_{Li}) = \frac{L^2 V(\hat{\lambda}_i)}{{\lambda_i}^4} = \frac{L^2 I^{-1}(\hat{\lambda}_i)}{\hat{\lambda}_i^4}. \quad (20)$$

Based on the distribution for $\hat{C}_{Li}$ given in Equation (19), we can formulate the $100(1-\alpha)^{1/k}$% lower confidence bound for C_{Li} as

$$\hat{C}_{Li} - Z_{1-(1-\alpha)^{\frac{1}{k}}}\sqrt{\hat{V}(\hat{C}_{Li})}, \quad (21)$$

where $Z_{1-(1-\alpha)^{\frac{1}{k}}}$ denotes the $100 \times (1-\alpha)^{\frac{1}{k}}$th percentile of the standard normal distribution.

Therefore, we can develop the $100 \times (1-\alpha)$% simultaneous confidence lower bounds for $C_{Li}, i = 1, \ldots, k$ as

$$\hat{C}_{Li} - Z_{1-(1-\alpha)^{\frac{1}{k}}}\sqrt{\hat{V}(\hat{C}_{Li})}, i = 1, \ldots, k \quad (22)$$

The coverage probability of the simultaneous confidence lower bounds for C_{Li}, i = 1, . . . , k in Equation (22) is developed as

$$P\left(C_{Li} \geq \hat{C}_{Li} - Z_{1-(1-\alpha)^{\frac{1}{k}}} \sqrt{\hat{V}(\hat{C}_{Li})}, i = 1, \ldots, k \right) = 1 - \alpha. \tag{23}$$

This coverage probability assures that the overall confidence level for all indices C_{Li}, $i = 1, \ldots, k$ attains $1 - \alpha$.

By the property of the invariance of MLE, the MLE of $C_T = \sum_{i=1}^{k} C_{Li} - (k-1)$ is

$$\hat{C}_T = \sum_{i=1}^{k} \hat{C}_{Li} - (k-1). \tag{24}$$

Apparently, the asymptotic variance of $\hat{C}_T$ is obtained as $\sum_{i=1}^{k} V(\hat{C}_{Li})$ and its estimate is $\sum_{i=1}^{k} \hat{V}(\hat{C}_{Li})$.

In addition, we can show that

$$\hat{C}_T \underset{n\to\infty}{\overset{d}{\to}} N\left(C_{Li}, \sum_{i=1}^{k} \hat{V}(\hat{C}_{Li})\right). \tag{25}$$

Based on the distribution for $\hat{C}_T$ given in Equation (25), we obtain the lower bound for C_T with confidence coefficient $1 - \alpha$ as

$$\hat{C}_T - Z_\alpha \sqrt{\sum_{i=1}^{k} \hat{V}(\hat{C}_{Li})}, \tag{26}$$

where Z_α denotes the $100 \times (1-\alpha)$ th percentile of the standard normal distribution.

3. Results

In Section 3.1, we propose the testing procedure to test whether the overall lifetime performance index reaches the target level. In Section 3.2, two numerical examples are provided to demonstrate the testing procedure for the overall lifetime performance index.

3.1. The Proposed Testing Procedure for the Overall Performance Index

In this section, we construct a statistical testing procedure to assess whether the overall lifetime performance index reaches the required target level c_0. The process is capable if the overall lifetime performance index C_T is greater than c_0 , i.e., the alternative hypothesis is expressed as $H_a: C_T = \sum_{i=1}^{k} C_{Li} - (k-1) > c_0$. Considering the condition of $C_{Li} = C_L$, $i = 1, \ldots, k$, the alternative hypothesis is equivalent to $H_0 : C_{Li} \leq c_0^*$ for all $i = 1, ..., k$, where $c_0^* = \frac{c_0 + k - 1}{k}$. The hypotheses for the statistical test are specified as follows:

$H_0 : C_{Li} \leq c_0^*$ for some i (the production process for products with k components is not capable) v.s. $H_a : C_{Li} > c_0^*, i = 1, ..., k$ (the production process for products with k components is capable). This type of procedure is commonly referred to as the Intersection–Union test (IUT). For the ith test of $H_{0i} : C_{Li} \leq c_0^*$ and $H_{ai} : C_{Li} > c_0^*$, using the MLE of C_{Li} given by $\hat{C}_{Li} = 1 - \frac{L}{\hat{\lambda}_i}$ as the test statistic. Based on the work of Wu and Lin [20], the critical value is determined as $C_{Li}^0 = 1 - \frac{L_i}{\lambda_{i0} + Z_{\alpha'} \sqrt{I^{-1}(\lambda_{i0})}}$ with the test size of $\alpha' = \alpha^{1/k}$. The critical region for the ith test is $R_i = \left\{ \hat{C}_{Li} \middle| \hat{C}_{Li} > C_{Li}^0 \right\}$. For testing $H_0 : C_{Li} \leq c_0^*$ for some i v.s. $H_a : C_{Li} > c_0^*, i = 1, ..., k$, the overall critical region is given by $R = \bigcap_{i=1}^{k} R_i = \bigcap_{i=1}^{k} \{ \hat{C}_{Li} | \hat{C}_{Li} > C_{Li}^0 \}$ ensuring that the overall test maintains a significance level of α.

The algorithmic framework for the proposed test concerning C_T is developed as follows:

Algorithm 1: The testing steps for the overall lifetime performance index

Step 1: With the lower specification limit L_i given, the progressive type I interval censored sample $X_{i1}, \ldots, X_{im}$ is obtained at designated inspection times $t_1, \ldots, t_m$ according to the censoring scheme $R_1, \ldots, R_m$ $R_1, \ldots, R_m$ from a one-parameter exponential distribution.

Step 2: For a prespecified conforming rate P_r, determine the required target level c_0 for C_T from Equation (6) and then determine the required target level $c_0^* = \frac{c_0+k-1}{k}$ for C_L from Equation (8). Then we can construct the testing null hypothesis $H_0 : C_{Li} \leq c_0^*$ for some i v.s. $H_a : C_{Li} > c_0^*, i = 1, ..., k$.

Step 3: Compute the value of the test statistic $\hat{C}_{Li} = 1 - \frac{L_i}{\hat{\lambda}_i}$, where $\hat{\lambda}_i$ is defined in Equation (16).

Step 4: For the given level of significance of α, we can compute the critical value $C_{Li}^0 = 1 - \frac{L_i}{\lambda_{i0} + Z_{\alpha'}\sqrt{I^{-1}(\lambda_{i0})}}$, where $\alpha' = \alpha^{1/k}$, $\lambda_{i0} = \frac{L_i}{1-c_0^*}$ and $I^{-1}(\lambda_{i0})$ is defined in Equation (17).

Step 5: We conclude that the overall lifetime performance index of the products attains the prescribed level provided that $\hat{C}_{Li} > C_L^0$ for all $i = 1, \ldots, k$.

Theorem 1: *The power of this statistical test at the point of* $C_T = c_1 > c_0$ *in the alternative hypothesis is*

$$h(c_1) = \prod_{i=1}^{k} \left(1 - \Phi\left(\frac{\lambda_{i0} - \lambda_{i1} + Z_{\alpha'}\sqrt{I^{-1}(\lambda_{i0})}}{\sqrt{I^{-1}(\lambda_{i1})}} \middle| \lambda_{i1} = \frac{L_i}{1-c_1^*}, \lambda_{i0} = \frac{L_i}{1-c_0^*} \right) \right),$$

where $\Phi(\cdot)$ *is the cdf for the standard normal distribution,* $Z_{\alpha'}$ *is the* $100 \times (1-\alpha')$ *percentile for the standard normal distribution,* $\alpha' = \alpha^{1/k}$, $\lambda_{i1} = \frac{L_i}{1-c_1^*}, \lambda_{i0} = \frac{L_i}{1-c_0^*}, i = 1, \ldots, k$.

Proof. For the point of $C_T = kC_L - (k-1) = c_1$ in the alternative hyppothesis, we solve this sequation to yield $C_L = \frac{c_1+k-1}{k} = c_1^*$. The power $h(c_1)$ of the test at the point of $c_1^* = 1 - \frac{L_i}{\lambda_{i1}}$

$$\begin{aligned}
h(c_1) &= P\Big(\hat{C}_{Li} > C_{Li}^0, i = 1, \ldots, k \big| c_1^* = 1 - \tfrac{L_i}{\lambda_{i1}}, i = 1, \ldots, k \Big) \\
&= P\left(1 - \frac{L_i}{\hat{\lambda}_i} > 1 - \frac{L_i}{\lambda_{i0} + Z_{\alpha'}\sqrt{I^{-1}(\lambda_{i0})}}, i = 1, \ldots, k \middle| \lambda_{i1} = \frac{L_i}{1-c_1^*}, \lambda_{i0} = \frac{L_i}{1-c_0^*}, i = 1, \ldots, k \right) \\
&= P\Big(\hat{\lambda}_i > \lambda_{i0} + Z_{\alpha'}\sqrt{I^{-1}(\lambda_{i0})}, i = 1, \ldots, k \Big| \lambda_{i1} = \tfrac{L_i}{1-c_1^*}, \lambda_{i0} = \tfrac{L_i}{1-c_0^*}, i = 1, \ldots, k \Big) \\
&= \textstyle\prod_{i=1}^{k} P\Big(\hat{\lambda}_i > \lambda_{i0} + Z_{\alpha'}\sqrt{I^{-1}(\lambda_{i0})} \Big| \lambda_{i1} = \tfrac{L_i}{1-c_1^*}, \lambda_{i0} = \tfrac{L_i}{1-c_0^*} \Big) \\
&= \textstyle\prod_{i=1}^{k} P\left(\frac{\hat{\lambda}_i - \lambda_{i1}}{\sqrt{I^{-1}(\lambda_{i1})}} > \frac{\lambda_{i0} - \lambda_{i1} + Z_{\alpha'}\sqrt{I^{-1}(\lambda_{i0})}}{\sqrt{I^{-1}(\lambda_{i1})}} \middle| \lambda_{i1} = \frac{L_i}{1-c_1^*}, \lambda_{i0} = \frac{L_i}{1-c_0^*} \right) \\
&= \textstyle\prod_{i=1}^{k} \left(1 - \Phi\left(\frac{\lambda_{i0} - \lambda_{i1} + Z_{\alpha'}\sqrt{I^{-1}(\lambda_{i0})}}{\sqrt{I^{-1}(\lambda_{i1})}} \middle| \lambda_{i1} = \frac{L_i}{1-c_1^*}, \lambda_{i0} = \frac{L_i}{1-c_0^*} \right) \right).
\end{aligned}$$

The proof is established. □

Assuming $L_1 = \ldots = L_k = L$, the power function in Theorem 1 can be simplified as

$$h(c_1) = \left(1 - \Phi\left(\frac{\lambda_{i0} - \lambda_{i1} + Z_{\alpha'}\sqrt{I^{-1}(\lambda_{i0})}}{\sqrt{I^{-1}(\lambda_{i1})}} \middle| \lambda_{i1} = \frac{L_i}{1-c_1^*}, \lambda_{i0} = \frac{L_i}{1-c_0^*} \right) \right)^k \quad (27)$$

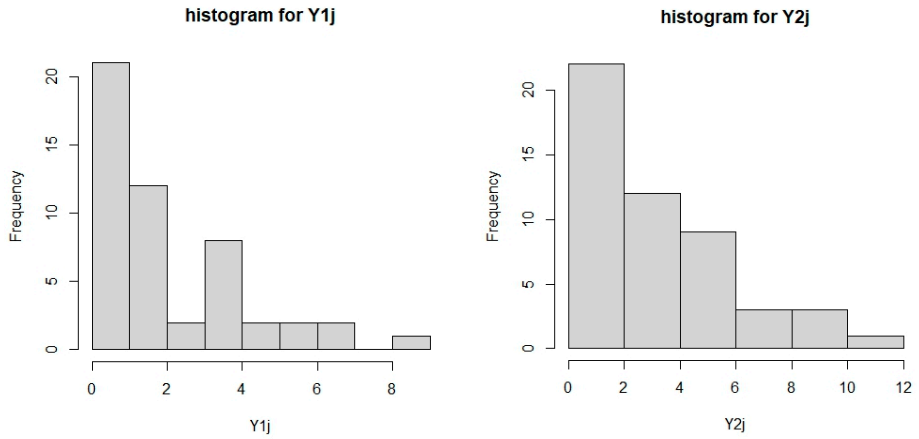
3.2. Two Numerical Examples

To exemplify the algorithmic testing procedure outlined above, we will focus on two specific datasets.

Example 1. *The data in Table 1 is simulated from two exponential distributions with scale parameters* $\lambda_1 = 2$ *and* $\lambda_2 = 3$ *representing the failure times (in years) for two components of n = 50 electrical appliances. The histograms for failure times for component 1 and component 2 are displayed in Figure 2a,b.*

Table 1. Failure times of two components for Example 1.

Failure Times for Component 1 (Y_{1j})					**Failure Times for Component 2 (Y_{2j})**				
0.0080	0.0132	0.0157	0.0182	0.0292	0.0195	0.0695	0.2533	0.3460	0.3905
0.0342	0.0580	0.0635	0.0768	0.1771	0.3958	0.4754	0.4880	0.5056	0.5231
0.1799	0.1977	0.2583	0.4052	0.4873	0.5805	0.5865	0.6114	0.6464	0.7810
0.4935	0.6740	0.7084	0.7671	0.7744	0.7905	0.8238	0.9727	1.0454	1.2530
0.8691	1.0387	1.4568	1.4886	1.5209	1.4496	1.5437	2.1598	2.2307	2.2562
1.6284	1.6482	1.6761	1.7539	1.8511	2.3209	2.4425	2.6818	2.7416	2.7939
1.8638	1.8809	1.9514	2.0245	2.4604	2.8218	2.9721	2.9828	2.9876	4.1560
3.1922	3.2206	3.3173	3.4855	3.5014	4.2173	4.2780	4.3176	4.3686	4.3768
3.6701	3.7602	3.9926	4.1792	4.4006	4.8804	4.9996	5.8671	6.0596	7.1819
5.0035	5.0376	6.1049	6.8506	8.6491	7.7860	8.0752	8.2921	8.7216	11.0411



(**a**) (**b**)

Figure 2. (**a**) Histogram for failure times of component 1 for Example 1. (**b**) Histogram for failure times of component 2 for Example 1.

We initiate the construction of a progressive Type I interval-censored sample for the failure times of two components. Consider the termination time $T = 2.0$ with the number of inspections $m = 8$, the equal inspection interval length $t = 0.25$ (thousand cycles), and the fixed prespecified removal percentages as $(p_1, p_2, \ldots, p_8) = (0.05, 0.05, \ldots, 1.0)$. We now proceed to conduct the hypothesis testing using Algorithm 1 for $H_0 : C_T \leq 0.75$ vs. $H_a : C_T > 0.75$ as follows:

Step 1: With the lower specification limit L1 = L2 = 0.01 given, we obtain the progressive type I interval censored samples (X11, X12, X13, X14, X15, X16, X17, X18)= (12, 4, 2, 3, 1, 1, 1, 3) and (X21, X22, X23, X24, X25, X26, X27, X28) = (2, 6, 5, 3, 1, 2, 0, 0) for two components at the pre-set inspection times (t1, t2,. . ., t8) = (0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0) with censoring schemes of (R11, R12, R13, R14, R15, R16, R17,

R18) = (2, 2, 2, 2, 1, 1, 1, 12) and (R11, R12, R13, R14, R15, R16, R17, R18)= (3, 2, 2, 2, 2, 1, 1, 18).

Step 2: For a prespecified conforming rate $P_r = 0.779$, we can determine the required target level $c_0 = 0.75$ for C_T and then we can determine the required target level $c_0^* = \frac{c_0+k-1}{k} = \frac{0.75+2-1}{2} = 0.875$ for C_L. Then, we can construct the testing null hypothesis $H_0 : C_{Li} \leq c_0^*$ for some i vs. $H_a : C_{Li} > c_0^*, i = 1, 2$

Step 3: We can find the maximum likelihood estimators $\hat{\lambda}_1$ =1.8537 and $\hat{\lambda}_2$ =3.1102 for two components. Compute the values of test statistic $\hat{C}_{L1} = 1 - \frac{L}{\hat{\lambda}_1} = 1 - \frac{0.01}{1.8537} = 0.9946$ and $\hat{C}_{L2} = 1 - \frac{L}{\hat{\lambda}_2} = 1 - \frac{0.01}{3.1102} = 0.9968$.

Step 4: For the given level of significance of $\alpha = 0.05$, we yield $\alpha' = 0.05^{1/2} = 0.2236$ and $\lambda_{i0} = \frac{L_i}{1-c_0^*} = 0.08$. Then, we can compute the critical values $C_{Li}^0 = 1 - \frac{L_i}{\lambda_{i0}+Z_\alpha\sqrt{I^{-1}(\lambda_{i0})}} = 1 - \frac{0.01}{0.08+(-0.7601)0.013} = 0.8573$ for both components.

Step 5: Since $\hat{C}_{L1} = 0.9946 > C_{L1}^0 = 0.8573$ and $\hat{C}_{L2} = 0.9968 > C_{L2}^0 = 0.8573$, we can infer that two individual lifetime performance indices have reached the prespecified target values, ensuring that the overall lifetime performance index also meets the required standard.

Example 2. *The dataset in Lawless [24] comprises the failure times (in number of cycles) for n = 36 electrical appliances subjected to an automatic life-testing experiment. The products with k = 2 components and two random samples of n = 20 for two components are listed in Table 2.*

Table 2. Failure times of two components for Example 2.

Failure Times for Component 1 (Y_{1j})					**Failure Times for Component 2 (Y_{2j})**				
35	49	170	329	381	34	59	69	142	574
708	958	1062	2223	2327	1064	1174	1578	1702	1893
2400	2565	2702	2761	3034	2292	2785	2811	2886	2993
3112	3214	3504	6976	7846	3122	3790	3857	5267	9701

The histograms for failure times for component 1 and component 2 are displayed in Figure 3a,b. The G test showed that the values of Gini statistics were 0.4843 and 0.4836 for each component and both fell into the acceptance region listed by Gail and Gastwirth ([25], p. 352). This suggests that both data align well with exponential distributions. Furthermore, we create a progressive type I interval censored sample for the failure times of two components. Consider the termination time T = 3.0 with number of inspections m = 5, the equal inspection interval length t = 0.6 (thousand cycles), and the prespecified removal percentages as $(p_1, p_2, p_3, p_4, p_5) = (0.05, 0.05, 0.05, 0.05, 1.0)$. We now proceed to conduct the hypothesis testing using Algorithm 1 for $H_0 : C_T \leq 0.8$ vs. $H_a : C_T > 0.8$ as follows:

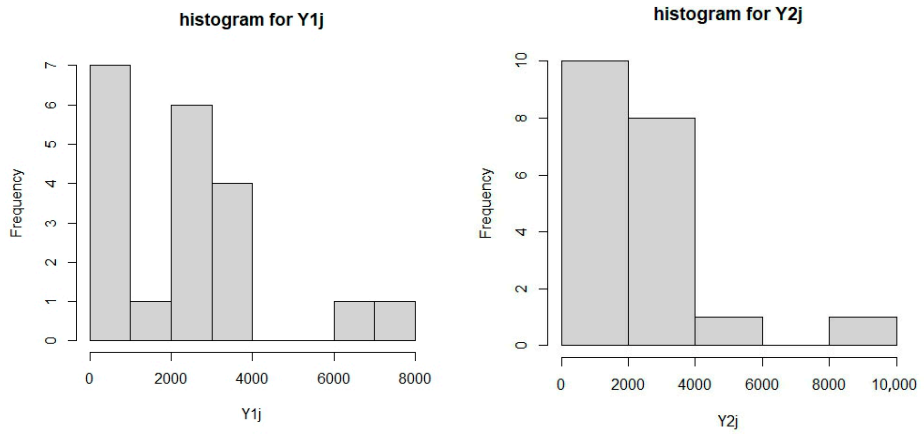
Step 1: With the lower specification limit $L_1 = L_2 = 0.2$ given, we can obtain the progressive type I interval censored samples (X_{11}, X_{12}, X_{13}, X_{14}, X_{15}) = (5, 3, 0, 1, 3) and (X_{21}, X_{22}, X_{23}, X_{24}, X_{25}) = (5,2,1,0,4) for two components at the pre-set inspection times ($t_1, \ldots, t_5$ = (0.6, 1.2, 1.8, 2.4, 3.0) with censoring schemes of (R_{11}, R_{12}, R_{13}, R_{14}, R_{15}) = (1, 1, 1, 1, 4) and (R_{11}, R_{12}, R_{13}, R_{14}, R_{15}) = (5, 2, 1, 0, 4).

Step 2: For a prespecified conforming rate $P_r = 0.8187$, we can determine the required target level $c_0 = 0.8$ for C_T and then we can determine the required target level $c_0^* = \frac{c_0+k-1}{k} = \frac{0.8+2-1}{2} = 0.9$ for C_L. Then, we can construct the testing null hypothesis $H_0 : C_{Li} \leq c_0^*$ for some i vs. $H_a : C_{Li} > c_0^*, i = 1, 2$.

Step 3: Find the maximum likelihood estimators $\hat{\lambda}_1$ =2.6889 and $\hat{\lambda}_2$ =2.7893 for two components. Compute the values of test statistic $\hat{C}_{L1} = 1 - \frac{L}{\hat{\lambda}_1} = 1 - \frac{0.2}{2.6889} = 0.9256$ and $\hat{C}_{L2} = 1 - \frac{L}{\hat{\lambda}_2} = 1 - \frac{0.2}{2.7893} = 0.9283$.

Step 4: For the given level of significance of $\alpha = 0.1$, we yield $\alpha' = 0.1^{1/2} = 0.3162$ and $\lambda_{i0} = \frac{L_i}{1-c_0^*} = 2$. Then, we can compute the critical values $C_{Li}^0 = 1 - \frac{L_i}{\lambda_{i0} + Z_\alpha \sqrt{I^{-1}(\lambda_{i0})}} = 1 - \frac{0.2}{2+(-0.4783)0.2603} = 0.8934$ for both components.

Step 5: Since $\hat{C}_{L1} = 0.9256 > C_{L1}^0 = 0.8934$ and $\hat{C}_{L2} = 0.9283 > C_{L2}^0 = 0.8934$, we can infer that two individual lifetime performance indices have reached the prespecified target values so that the overall lifetime performance index also meets the required level.



(**a**) (**b**)

Figure 3. (**a**) Histogram for failure times of component 1 for Example 2. (**b**) Histogram for failure times of component 2 for Example 2.

4. Discussion

Power analysis and the related discussion are carried out in this section. Using Theorem 1, the powers $h(c_1)$ for testing $H_0 : C_L \leq 0.75$ are calculated and tabulated in Tables A1–A9 at $\alpha = 0.01,\ 0.05,\ 0.1$, respectively, for $c_1 = 0.75, 0.775, 0.8, 0.825, 0.85, 0.875$, $m = 6, 7, 8$, $n = 30, 50, 70$ and $p = 0.05, 0.075, 0.1$ under $L = 0.05$, $T = 0.5$ in Appendix A. The powers are also displayed in Figures 4–8 for some typical cases. We have the following key findings from Tables A1–A9 and Figures 4–8. (1) As shown in Figure 4, the power rises with increasing n when $k = 2$, $\alpha = 0.05$, $m = 5$ and $p = 0.05$ (the same trend is also observed for other configurations of k, m, p and α). (2) As shown in Figure 5, the power rises with increasing m when $k = 2$, $n = 30$, $p = 0.05$, and $\alpha = 0.05$ (the same trend is also observed for other configurations of k, n, p and α). (3) Figure 6 illustrates that the test power improves as the value of p increases when $k = 2$, $n = 30$, $m = 5$, and $\alpha = 0.05$ (the same trend appears for other settings of k, n, m and α). (4) As shown in Figure 7, the power rises with increasing α when $k = 2$, $n = 30$, $m = 5$, and $p = 0.05$ (similar patterns are observed for other configurations of k, n, m and p). (5) Figure 8 illustrates that the test power increases as the value of k decreases when $n = 30$, $m = 5$, $p = 0.05$, and $\alpha = 0.05$ (the same trend occurs for other parameter combinations of n, m, p and α). (6) From Figures 4–8, the power grows as the value of c_1 increases for any combinations of k, n, m, p, and α. Through these six key findings, we can see how different parameter settings, especially the number of components k, affect the test power.

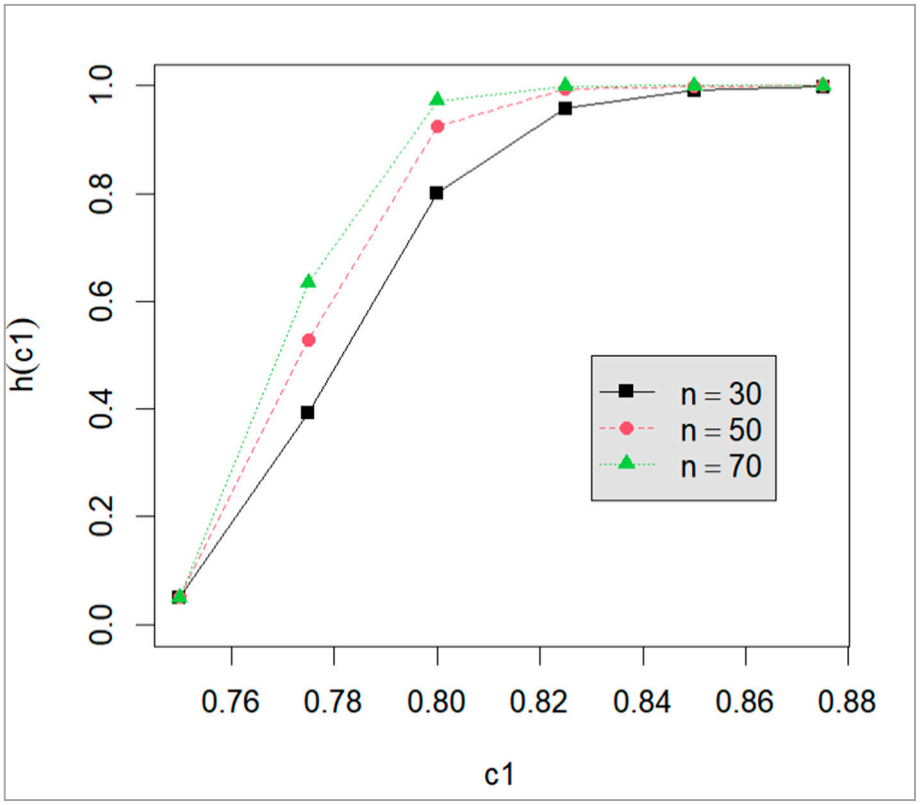


Figure 4. The power curve of $h(c_1)$ for the configuration of $k = 2$, $\alpha = 0.05$, $m = 5$, and $p = 0.05$.

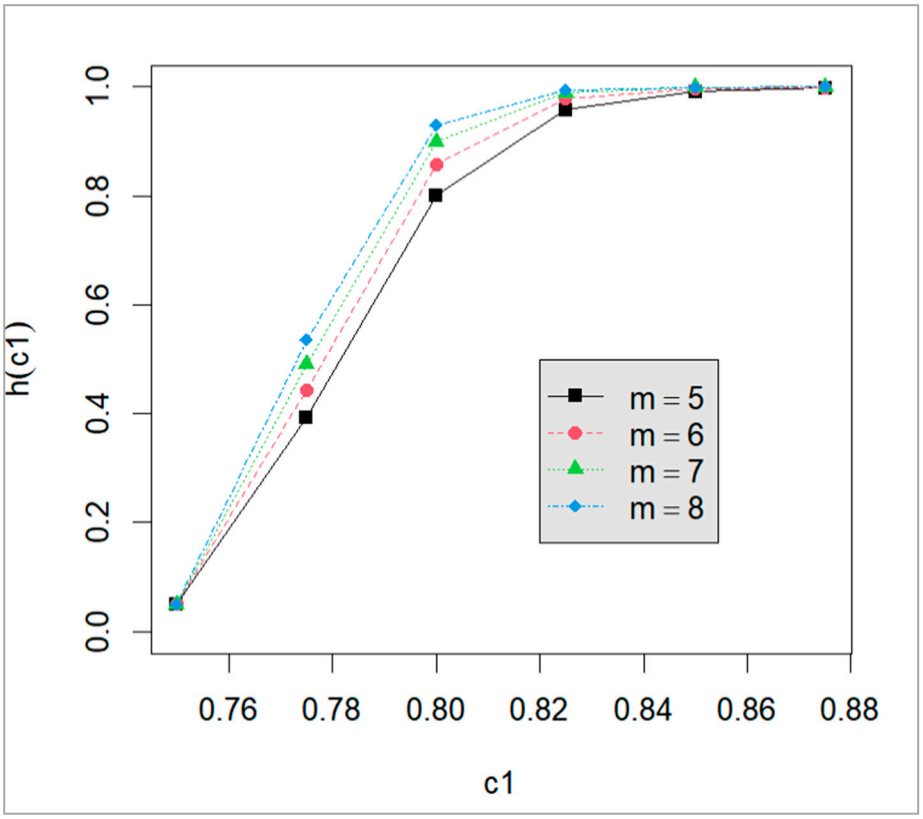


Figure 5. The power curve of $h(c_1)$ for the configuration of $k = 2$, $\alpha = 0.05$, $n = 30$, and $p = 0.05$.

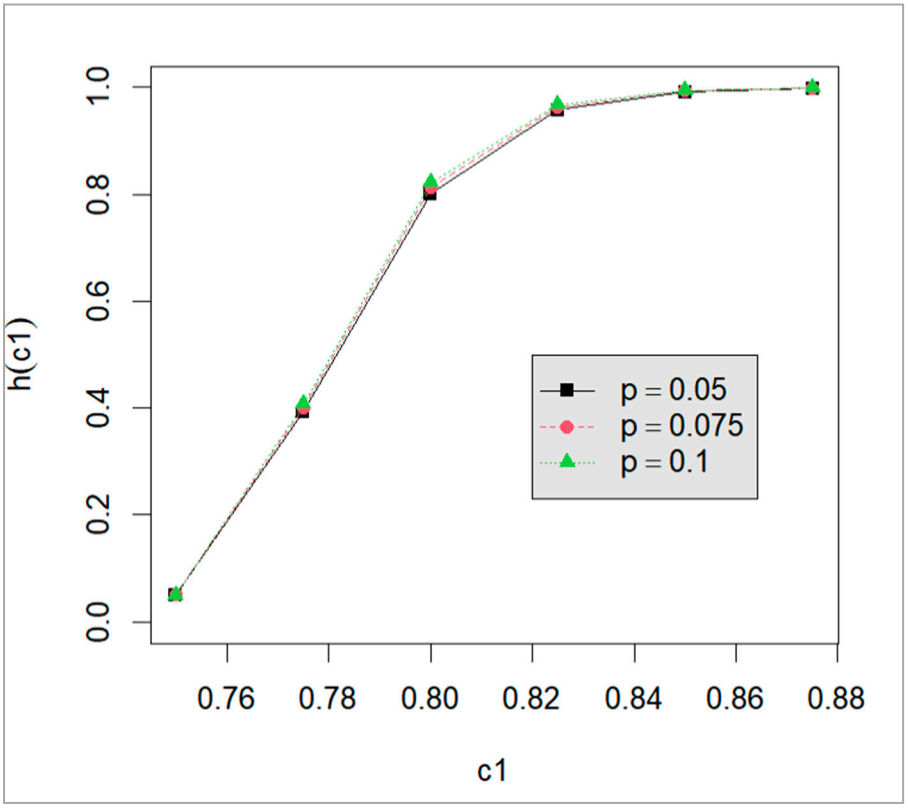


Figure 6. The power curve of $h(c_1)$ for the configuration of $k = 2$, $\alpha = 0.05$, $n = 30$, and $m = 5$.

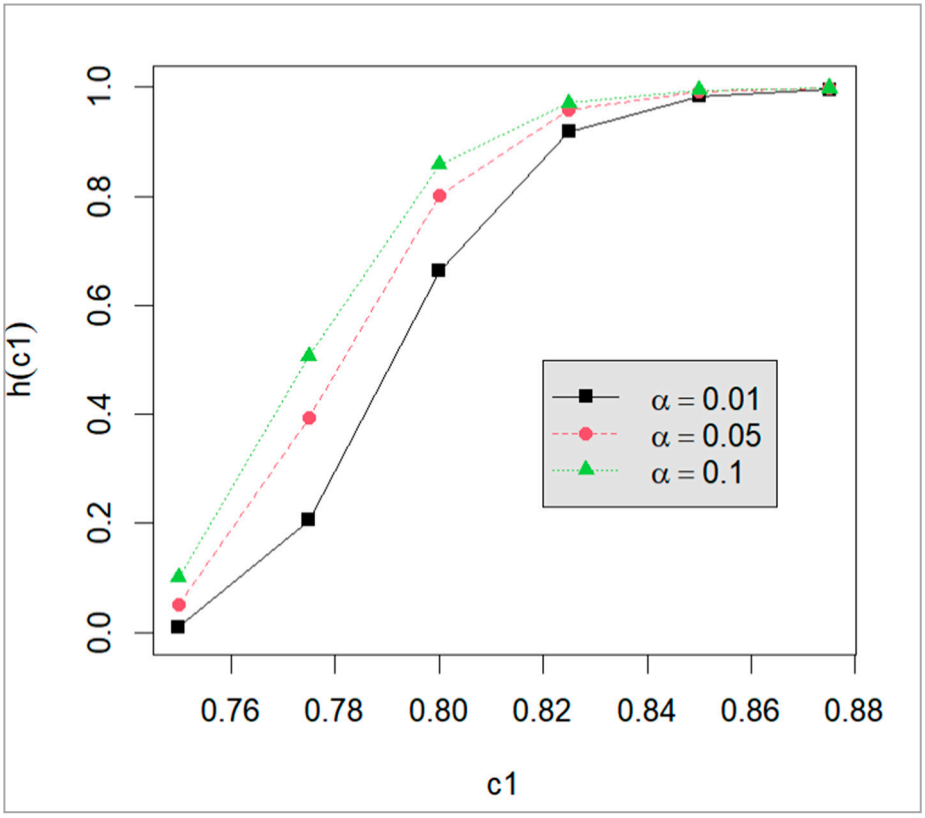


Figure 7. The power curve of $h(c_1)$ for the configuration of $k = 2$, $n = 30$, $m = 5$, and $p = 0.05$.

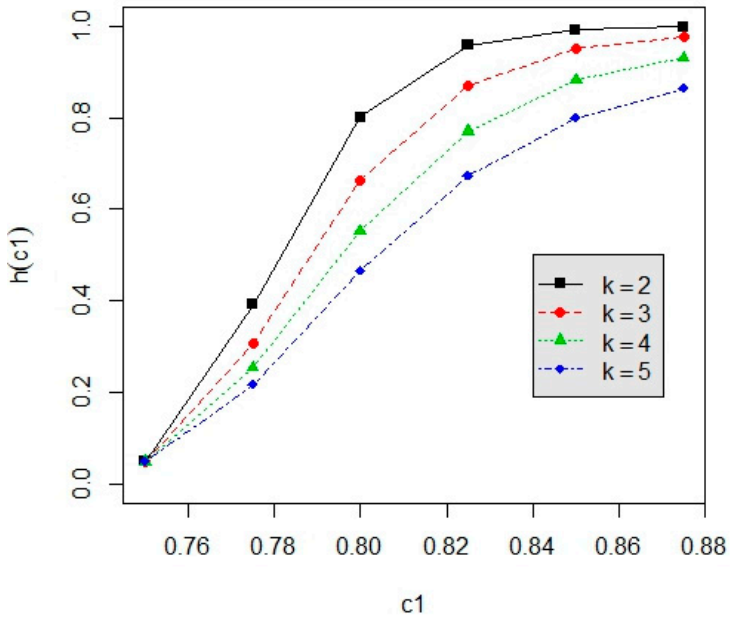


Figure 8. The power curve of $h(c_1)$ for the configuration of $n = 30$, $m = 5$, $p = 0.05$, and $\alpha = 0.05$.

5. Conclusions

For products made up of multiple components with lifetimes following a one-parameter exponential distribution, we propose an overall lifetime performance index for these multiple components. We study its relationship with the individual lifetime performance index and derive the maximum likelihood estimators and their asymptotic distributions for both the individual and overall lifetime performance indices using the progressive Type I interval-censored sample. A hypothesis testing procedure is developed to check whether the overall lifetime performance index meets the desired target level, using the maximum likelihood estimator as the test statistic. We also analyze how different parameter settings, especially the number of components, affect the test power through graphical comparisons. Finally, two numerical examples are provided to illustrate how to apply the proposed testing algorithm to the overall lifetime performance index. This research can be applied to the fields of life testing and reliability analysis. In future research, we will investigate the Bayesian estimation for all individual and overall lifetime performance indices. We will also explore alternative forms of the lifetime performance index, such as replacing the mean and standard deviation with the median and the range divided by four in the index C_{Li}. We will also investigate other types of censoring schemes, such as adaptive type II progressive censoring, balanced joint type-II progressive censoring, and block adaptive type-II progressive hybrid censoring. We will also consider the case of production of products with k-dependent components by proposing another performance index to ensure the overall testing procedure will achieve the given level of significance.

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Data Availability Statement: The data are available via a publicly accessible repository: Lawless [24].

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Table A1. The power $h(c_1)$ at $k = 2$, $\alpha = 0.01$, $\boldsymbol{c_0} = 0.75$ and $\boldsymbol{c_1}$= 0.75, 0.775, 0.8, 0.825, 0.85, 0.875.

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
6	30	0.05	0.01	0.245780	0.741612	0.954608	0.993248	0.998657
		0.075	0.01	0.254485	0.758842	0.961799	0.995037	0.999151
		0.1	0.01	0.262560	0.773897	0.967455	0.996268	0.999444
	50	0.05	0.01	0.381475	0.905200	0.994622	0.999744	0.999979
		0.075	0.01	0.395156	0.916324	0.996008	0.999848	0.999990
		0.1	0.01	0.407715	0.925510	0.996972	0.999906	0.999995
	70	0.05	0.01	0.503027	0.966670	0.999380	0.999990	1.000000
		0.075	0.01	0.519858	0.972211	0.999595	0.999995	1.000000
		0.1	0.01	0.535117	0.976535	0.999727	0.999998	1.000000
7	30	0.05	0.01	0.286201	0.805455	0.975384	0.997363	0.999595
		0.075	0.01	0.298036	0.823563	0.980639	0.998278	0.999785
		0.1	0.01	0.308787	0.838690	0.984448	0.998831	0.999879
	50	0.05	0.01	0.442293	0.942528	0.998093	0.999947	0.999997
		0.075	0.01	0.460000	0.951759	0.998739	0.999974	0.999999
		0.1	0.01	0.475837	0.958899	0.999135	0.999987	1.000000
	70	0.05	0.01	0.575298	0.983765	0.999857	0.999999	1.000000
		0.075	0.01	0.595835	0.987404	0.999921	1.000000	1.000000
		0.1	0.01	0.613881	0.990010	0.999954	1.000000	1.000000
8	30	0.05	0.01	0.327066	0.855881	0.987033	0.999017	0.999885
		0.075	0.01	0.342167	0.873467	0.990542	0.999437	0.999950
		0.1	0.01	0.355589	0.887468	0.992871	0.999658	0.999976
	50	0.05	0.01	0.500969	0.966034	0.999356	0.999990	1.000000
		0.075	0.01	0.522341	0.973050	0.999626	0.999996	1.000000
		0.1	0.01	0.540937	0.978109	0.999770	0.999998	1.000000
	70	0.05	0.01	0.641169	0.992359	0.999969	1.000000	1.000000
		0.075	0.01	0.664423	0.994530	0.999986	1.000000	1.000000
		0.1	0.01	0.684181	0.995947	0.999993	1.000000	1.000000

Table A2. The power $h(c_1)$ at $k = 2$, α= 0.05, $\boldsymbol{c_0} = 0.75$ and c_1= 0.75, 0.775, 0.8, 0.825, 0.85, 0.875.

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
6	30	0.05	0.05	0.442664	0.857150	0.977769	0.996628	0.999252
		0.075	0.05	0.453753	0.869227	0.981773	0.997595	0.999542
		0.1	0.05	0.463861	0.879516	0.984831	0.998239	0.999708
	50	0.05	0.05	0.592016	0.956942	0.997869	0.999895	0.999990
		0.075	0.05	0.605954	0.962939	0.998471	0.999940	0.999996
		0.1	0.05	0.618484	0.967748	0.998875	0.999964	0.999998
	70	0.05	0.05	0.703306	0.987068	0.999793	0.999997	1.000000
		0.075	0.05	0.717775	0.989534	0.999870	0.999998	1.000000
		0.1	0.05	0.730591	0.991397	0.999916	0.999999	1.000000
7	30	0.05	0.05	0.490493	0.898946	0.988722	0.998762	0.999786
		0.075	0.05	0.504496	0.910561	0.991417	0.999222	0.999890
		0.1	0.05	0.516950	0.919976	0.993302	0.999489	0.999940
	50	0.05	0.05	0.649930	0.975885	0.999307	0.999980	0.999999
		0.075	0.05	0.666446	0.980388	0.999561	0.999991	1.000000
		0.1	0.05	0.680851	0.983750	0.999710	0.999995	1.000000
	70	0.05	0.05	0.761537	0.994260	0.999957	1.000000	1.000000
		0.075	0.05	0.777544	0.995710	0.999977	1.000000	1.000000
		0.1	0.05	0.791219	0.996709	0.999987	1.000000	1.000000

Table A2. *Cont.*

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
8	30	0.05	0.05	0.535840	0.929481	0.994431	0.999566	0.999942
		0.075	0.05	0.552520	0.939878	0.996096	0.999762	0.999976
		0.1	0.05	0.566985	0.947873	0.997156	0.999861	0.999989
	50	0.05	0.05	0.701576	0.986789	0.999785	0.999996	1.000000
		0.075	0.05	0.719946	0.989908	0.999881	0.999999	1.000000
		0.1	0.05	0.735471	0.992067	0.999930	0.999999	1.000000
	70	0.05	0.05	0.810120	0.997529	0.999992	1.000000	1.000000
		0.075	0.05	0.826678	0.998309	0.999996	1.000000	1.000000
		0.1	0.05	0.840299	0.998796	0.999998	1.000000	1.000000

Table A3. The power $h(c_1)$ at $k = 2$, $\alpha = 0.1$, $c_0 = 0.75$ and $c_1 = 0.75, 0.775, 0.8, 0.825, 0.85, 0.875$.

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
6	30	0.05	0.1	0.557285	0.901505	0.985365	0.997727	0.999461
		0.075	0.1	0.568444	0.910813	0.988175	0.998406	0.999675
		0.1	0.1	0.578520	0.918637	0.990288	0.998850	0.999796
	50	0.05	0.1	0.696956	0.973283	0.998749	0.999937	0.999994
		0.075	0.1	0.709477	0.977324	0.999119	0.999965	0.999997
		0.1	0.1	0.720608	0.980515	0.999363	0.999979	0.999999
	70	0.05	0.1	0.791711	0.992627	0.999890	0.999998	1.000000
		0.075	0.1	0.803641	0.994132	0.999932	0.999999	1.000000
		0.1	0.1	0.814079	0.995248	0.999957	1.000000	1.000000
7	30	0.05	0.1	0.603633	0.932619	0.992838	0.999193	0.999850
		0.075	0.1	0.617204	0.941176	0.994648	0.999503	0.999925
		0.1	0.1	0.629139	0.948000	0.995890	0.999680	0.999960
	50	0.05	0.1	0.747222	0.985660	0.999612	0.999988	0.999999
		0.075	0.1	0.761411	0.988541	0.999760	0.999995	1.000000
		0.1	0.1	0.773624	0.990651	0.999845	0.999997	1.000000
	70	0.05	0.1	0.838173	0.996887	0.999978	1.000000	1.000000
		0.075	0.1	0.850736	0.997722	0.999989	1.000000	1.000000
		0.1	0.1	0.861312	0.998284	0.999994	1.000000	1.000000
8	30	0.05	0.1	0.646163	0.954463	0.996585	0.999726	0.999961
		0.075	0.1	0.661772	0.961809	0.997658	0.999854	0.999984
		0.1	0.1	0.675132	0.967354	0.998326	0.999917	0.999993
	50	0.05	0.1	0.790301	0.992458	0.999885	0.999998	1.000000
		0.075	0.1	0.805445	0.994359	0.999938	0.999999	1.000000
		0.1	0.1	0.818050	0.995646	0.999965	1.000000	1.000000
	70	0.05	0.1	0.875254	0.998724	0.999996	1.000000	1.000000
		0.075	0.1	0.887672	0.999148	0.999998	1.000000	1.000000
		0.1	0.1	0.897716	0.999406	0.999999	1.000000	1.000000

Table A4. The power $h(c_1)$ at $k = 3$, $\alpha = 0.01$, $c_0 = 0.75$ and $c_1 = 0.75, 0.775, 0.8, 0.825, 0.85, 0.875$.

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
6	30	0.05	0.01	0.176311	0.576375	0.854788	0.954827	0.983120
		0.075	0.01	0.185801	0.602866	0.875682	0.965346	0.988424
		0.1	0.01	0.194605	0.626139	0.892376	0.972873	0.991821
	50	0.05	0.01	0.276703	0.777328	0.962699	0.994238	0.998757
		0.075	0.01	0.292406	0.801849	0.971396	0.996297	0.999333
		0.1	0.01	0.306844	0.822255	0.977625	0.997538	0.999625
	70	0.05	0.01	0.372965	0.886450	0.990492	0.999253	0.999906
		0.075	0.01	0.393666	0.904130	0.993458	0.999598	0.999960
		0.1	0.01	0.412482	0.918075	0.995369	0.999773	0.999982

Table A4. *Cont.*

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
7	30	0.05	0.01	0.205378	0.646151	0.901366	0.974929	0.991984
		0.075	0.01	0.218487	0.677155	0.920559	0.982611	0.995228
		0.1	0.01	0.230400	0.703225	0.934764	0.987515	0.997013
	50	0.05	0.01	0.323632	0.838260	0.980562	0.997821	0.999633
		0.075	0.01	0.344697	0.862711	0.986532	0.998815	0.999845
		0.1	0.01	0.363579	0.881806	0.990347	0.999317	0.999928
	70	0.05	0.01	0.433618	0.928271	0.996180	0.999807	0.999983
		0.075	0.01	0.460348	0.943346	0.997719	0.999918	0.999995
		0.1	0.01	0.483901	0.954302	0.998571	0.999962	0.999998
8	30	0.05	0.01	0.235748	0.708551	0.934673	0.986597	0.996383
		0.075	0.01	0.252768	0.742122	0.950817	0.991677	0.998156
		0.1	0.01	0.267907	0.769050	0.961828	0.994550	0.998985
	50	0.05	0.01	0.371338	0.885079	0.990257	0.999225	0.999901
		0.075	0.01	0.397746	0.907448	0.993963	0.999649	0.999967
		0.1	0.01	0.420778	0.923770	0.996059	0.999827	0.999988
	70	0.05	0.01	0.493061	0.955971	0.998545	0.999954	0.999997
		0.075	0.01	0.525159	0.967696	0.999257	0.999985	0.999999
		0.1	0.01	0.552510	0.975505	0.999592	0.999994	1.000000

Table A5. The power $h(c_1)$ at $k = 3$, $\alpha = 0.05$, $c_0 = 0.75$ and $c_1 = 0.75, 0.775, 0.8, 0.825, 0.85, 0.875$.

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
6	30	0.05	0.05	0.346868	0.726135	0.913632	0.972590	0.988892
		0.075	0.05	0.360344	0.748365	0.927890	0.979526	0.992575
		0.1	0.05	0.372570	0.767325	0.938941	0.984344	0.994871
	50	0.05	0.05	0.472748	0.875584	0.980894	0.996944	0.999267
		0.075	0.05	0.491084	0.892078	0.985796	0.998101	0.999620
		0.1	0.05	0.507510	0.905363	0.989190	0.998775	0.999792
	70	0.05	0.05	0.575575	0.943399	0.995666	0.999645	0.999949
		0.075	0.05	0.596437	0.953593	0.997124	0.999816	0.999979
		0.1	0.05	0.614858	0.961360	0.998029	0.999900	0.999991
7	30	0.05	0.05	0.385732	0.781541	0.944056	0.985413	0.994900
		0.075	0.05	0.403174	0.805661	0.956322	0.990213	0.997060
		0.1	0.05	0.418609	0.825246	0.965080	0.993172	0.998210
	50	0.05	0.05	0.524649	0.914665	0.990603	0.998904	0.999793
		0.075	0.05	0.547427	0.929823	0.993737	0.999429	0.999916
		0.1	0.05	0.567215	0.941216	0.995659	0.999683	0.999963
	70	0.05	0.05	0.633623	0.966495	0.998372	0.999914	0.999991
		0.075	0.05	0.658372	0.974479	0.999072	0.999965	0.999997
		0.1	0.05	0.679440	0.980050	0.999441	0.999985	0.999999
8	30	0.05	0.05	0.424135	0.827946	0.964615	0.992520	0.997776
		0.075	0.05	0.445436	0.852298	0.974323	0.995536	0.998909
		0.1	0.05	0.463807	0.871073	0.980680	0.997173	0.999419
	50	0.05	0.05	0.573951	0.942611	0.995550	0.999630	0.999946
		0.075	0.05	0.600521	0.955480	0.997368	0.999841	0.999983
		0.1	0.05	0.622875	0.964473	0.998348	0.999925	0.999994
	70	0.05	0.05	0.686375	0.980677	0.999420	0.999981	0.999999
		0.075	0.05	0.713840	0.986420	0.999720	0.999994	1.000000
		0.1	0.05	0.736342	0.990070	0.999853	0.999998	1.000000

Table A6. The power $h(c_1)$ at $k = 3$, $\alpha = 0.1$, $c_0 = 0.75$ and $c_1 = 0.75, 0.775, 0.8, 0.825, 0.85, 0.875$.

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
6	30	0.05	0.1	0.456389	0.793378	0.936645	0.979441	0.991230
		0.075	0.1	0.470800	0.812291	0.947835	0.984869	0.994221
		0.1	0.1	0.483723	0.828173	0.956367	0.988579	0.996057
	50	0.05	0.1	0.582893	0.913166	0.987062	0.997869	0.999455
		0.075	0.1	0.600668	0.925730	0.990546	0.998701	0.999722
		0.1	0.1	0.616375	0.935678	0.992914	0.999175	0.999851
	70	0.05	0.1	0.678237	0.962816	0.997245	0.999766	0.999964
		0.075	0.1	0.697031	0.969999	0.998209	0.999882	0.999986
		0.1	0.1	0.713382	0.975374	0.998794	0.999937	0.999994
7	30	0.05	0.1	0.496685	0.839208	0.959988	0.989304	0.996047
		0.075	0.1	0.514738	0.858967	0.969294	0.992955	0.997761
		0.1	0.1	0.530490	0.874719	0.975812	0.995162	0.998658
	50	0.05	0.1	0.631831	0.942222	0.993833	0.999257	0.999850
		0.075	0.1	0.653083	0.953304	0.995977	0.999622	0.999940
		0.1	0.1	0.671247	0.961468	0.997263	0.999794	0.999974
	70	0.05	0.1	0.729186	0.978736	0.999003	0.999945	0.999994
		0.075	0.1	0.750568	0.984123	0.999446	0.999978	0.999998
		0.1	0.1	0.768452	0.987802	0.999673	0.999991	0.999999
8	30	0.05	0.1	0.535407	0.876358	0.975308	0.994637	0.998308
		0.075	0.1	0.556783	0.895620	0.982445	0.996870	0.999188
		0.1	0.1	0.574923	0.910167	0.987016	0.998054	0.999575
	50	0.05	0.1	0.676782	0.962261	0.997168	0.999757	0.999962
		0.075	0.1	0.700699	0.971319	0.998368	0.999898	0.999989
		0.1	0.1	0.720451	0.977507	0.998998	0.999953	0.999996
	70	0.05	0.1	0.773796	0.988140	0.999658	0.999988	0.999999
		0.075	0.1	0.796620	0.991860	0.999840	0.999996	1.000000
		0.1	0.1	0.814948	0.994167	0.999918	0.999999	1.000000

Table A7. The power $h(c_1)$ at $k = 4$, $\alpha = 0.01$, $c_0 = 0.75$ and $c_1 = 0.75, 0.775, 0.8, 0.825, 0.85, 0.875$.

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
6	30	0.05	0.01	0.136133	0.454428	0.739675	0.884514	0.941875
		0.075	0.01	0.145385	0.484937	0.772966	0.908217	0.957791
		0.1	0.01	0.153974	0.511988	0.800090	0.925769	0.968555
	50	0.05	0.01	0.214356	0.656968	0.903239	0.974291	0.991319
		0.075	0.01	0.230069	0.691154	0.923544	0.982469	0.994869
		0.1	0.01	0.244562	0.719977	0.938505	0.987677	0.996835
	70	0.05	0.01	0.292079	0.790216	0.964460	0.994199	0.998662
		0.075	0.01	0.313564	0.819999	0.974476	0.996591	0.999354
		0.1	0.01	0.333182	0.843878	0.981191	0.997910	0.999669
7	30	0.05	0.01	0.158378	0.520048	0.802876	0.924539	0.966091
		0.075	0.01	0.171259	0.557888	0.837247	0.945084	0.978164
		0.1	0.01	0.182985	0.590082	0.863320	0.958810	0.985316
	50	0.05	0.01	0.251612	0.727780	0.939760	0.987283	0.996402
		0.075	0.01	0.273096	0.765274	0.956515	0.992486	0.998251
		0.1	0.01	0.292434	0.795038	0.967631	0.995326	0.999087
	70	0.05	0.01	0.342413	0.849926	0.981681	0.997810	0.999604
		0.075	0.01	0.371003	0.878858	0.988386	0.998945	0.999854
		0.1	0.01	0.396346	0.900325	0.992308	0.999454	0.999941
8	30	0.05	0.01	0.182122	0.583008	0.854187	0.952327	0.981070
		0.075	0.01	0.199001	0.626475	0.886685	0.968490	0.989306
		0.1	0.01	0.214052	0.661833	0.909527	0.978174	0.993544
	50	0.05	0.01	0.290705	0.788288	0.963784	0.994029	0.998611
		0.075	0.01	0.318198	0.826040	0.976318	0.996983	0.999455

Table A7. *Cont.*

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
		0.1	0.01	0.342302	0.854133	0.983759	0.998352	0.999762
	70	0.05	0.01	0.393881	0.895436	0.990983	0.999231	0.999894
		0.075	0.01	0.429322	0.921089	0.995013	0.999702	0.999971
		0.1	0.01	0.459742	0.938618	0.997050	0.999871	0.999991

Table A8. The power $h(c_1)$ at $k = 4$, $\alpha = 0.05$, $c_0 = 0.75$ and $c_1 = 0.75, 0.775, 0.8, 0.825, 0.85, 0.875$.

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
6	30	0.05	0.05	0.286332	0.615083	0.828284	0.922640	0.958445
		0.075	0.05	0.300516	0.643808	0.853858	0.940023	0.970503
		0.1	0.05	0.313358	0.66846	0.874032	0.952523	0.978454
	50	0.05	0.05	0.393624	0.785674	0.943626	0.984549	0.994312
		0.075	0.05	0.413742	0.811896	0.956742	0.989776	0.996731
		0.1	0.05	0.431758	0.833232	0.966082	0.992998	0.998032
	70	0.05	0.05	0.485608	0.880833	0.981142	0.996794	0.999182
		0.075	0.05	0.509629	0.900671	0.986898	0.998181	0.999618
		0.1	0.05	0.530860	0.916002	0.990621	0.998918	0.999810
7	30	0.05	0.05	0.318716	0.673988	0.875014	0.951152	0.976403
		0.075	0.05	0.337315	0.707196	0.899867	0.965519	0.985227
		0.1	0.05	0.353743	0.734381	0.918027	0.974794	0.990299
	50	0.05	0.05	0.439163	0.837816	0.966496	0.992664	0.997720
		0.075	0.05	0.464788	0.864497	0.976670	0.995825	0.998930
		0.1	0.05	0.487054	0.884821	0.983157	0.997484	0.999458
	70	0.05	0.05	0.539472	0.919177	0.990774	0.998846	0.999767
		0.075	0.05	0.569037	0.937032	0.994387	0.999468	0.999918
		0.1	0.05	0.594252	0.949730	0.996411	0.999735	0.999968
8	30	0.05	0.05	0.351564	0.727186	0.911042	0.970166	0.987181
		0.075	0.05	0.374580	0.762909	0.933264	0.980975	0.992996
		0.1	0.05	0.394401	0.790736	0.948247	0.987206	0.995887
	50	0.05	0.05	0.484074	0.879542	0.980751	0.996694	0.999149
		0.075	0.05	0.514736	0.904619	0.987935	0.998403	0.999680
		0.1	0.05	0.540556	0.922444	0.992012	0.999159	0.999865
	70	0.05	0.05	0.590874	0.946499	0.995688	0.999614	0.999940
		0.075	0.05	0.624933	0.961265	0.997729	0.999858	0.999984
		0.1	0.05	0.652911	0.970889	0.998711	0.999941	0.999995

Table A9. The power $h(c_1)$ at $k = 4$, $\alpha = 0.1$, $c_0 = 0.75$ and $c_1 = 0.75, 0.775, 0.8, 0.825, 0.85, 0.875$.

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
6	30	0.05	0.1	0.389512	0.694706	0.866661	0.938763	0.965686
		0.075	0.1	0.405324	0.720687	0.888038	0.953168	0.975949
		0.1	0.1	0.419450	0.742608	0.904618	0.963365	0.982623
	50	0.05	0.1	0.502645	0.840758	0.959018	0.988464	0.995521
		0.075	0.1	0.523141	0.862183	0.969056	0.992493	0.997466
		0.1	0.1	0.541219	0.879300	0.976075	0.994933	0.998495
	70	0.05	0.1	0.592752	0.915798	0.986961	0.997713	0.999380
		0.075	0.1	0.615637	0.930911	0.991106	0.998727	0.999715
		0.1	0.1	0.635534	0.942370	0.993734	0.999256	0.999861
7	30	0.05	0.1	0.424660	0.746698	0.904959	0.962034	0.980797
		0.075	0.1	0.444779	0.775702	0.925107	0.973649	0.988165
		0.1	0.1	0.462272	0.798979	0.939541	0.981009	0.992330

Table A9. *Cont.*

m	*n*	*p*	c_1					
			0.75	**0.775**	**0.8**	**0.825**	**0.85**	**0.875**
	50	0.05	0.1	0.547952	0.882485	0.976240	0.994643	0.998237
		0.075	0.1	0.573159	0.90349	0.983781	0.997014	0.999189
		0.1	0.1	0.594670	0.919155	0.988486	0.998233	0.999596
	70	0.05	0.1	0.642945	0.944471	0.993799	0.999198	0.999827
		0.075	0.1	0.670055	0.957564	0.996313	0.999639	0.999940
		0.1	0.1	0.692728	0.966671	0.997688	0.999824	0.999977
8	30	0.05	0.1	0.459418	0.792270	0.933726	0.977231	0.989722
		0.075	0.1	0.483612	0.822470	0.951236	0.985766	0.994488
		0.1	0.1	0.504074	0.845477	0.962783	0.990584	0.996812
	50	0.05	0.1	0.591291	0.914812	0.986679	0.997640	0.999354
		0.075	0.1	0.620469	0.933890	0.991845	0.998888	0.999763
		0.1	0.1	0.644538	0.947132	0.994705	0.999427	0.999902
	70	0.05	0.1	0.689254	0.964225	0.997182	0.999739	0.999956
		0.075	0.1	0.719383	0.974677	0.998556	0.999907	0.999989
		0.1	0.1	0.743591	0.981323	0.999199	0.999962	0.999997

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