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# Detailed Command vs. Mission Command: A Cancer-Stage Model of Institutional Decision-Making

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**Abstract:** Those accustomed to acting within 'normal' bureaucracies will have experienced the degradation, distortion, and stunting imposed by inordinate levels of hierarchical 'decision structure', particularly under the critical time constraints so fondly exploited by John Boyd and his followers. Here, via an approach based on the asymptotic limit theorems of information and control theories, we explore this dynamic in detail, abducting ideas from the theory of carcinogenesis. The resulting probability models can, with some effort, be converted into new statistical tools for analysis of real time, real world data involving cognitive phenomena and their dysfunctions across a considerable range of scales and levels of organization.

**Keywords:** cognition; control theory; dysfunction; information theory; organized conflict; phase transition; statistical tools

***A favorable situation will never be exploited if commanders wait for orders. The highest commander and the youngest soldier must always be conscious of the fact that omission and inactivity are worse than resorting to the wrong expedient.***

— Von Moltke the Elder [1].

***Gentlemen, I demand that your divisions completely cross the German borders, completely cross the Belgian borders and completely cross the river Meuse. I don't care how you do it, that's completely up to you.***

— Kurt Zeitzler, Chief of Staff, Panzergruppe Kleist, 13 May 1940.

***The Wehrmacht's ability to generate ad hoc battlegroups from fragments of units, to counterattack strongly even before the precise nature of a threat had been identified, and the tempo its units often demonstrated all indicate a loose, flexible and decentralized command philosophy.***

— Storr [2].

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## 1. Introduction

Western 'Management Strategy', as differentiated from the 'Institutional Decision-Making' of the title, has been the focus of two contrasting, but perhaps complementary, schools of thought, characterized by the works of Mintzberg [3,4] and Porter [5,6].

Mintzberg centers on the 'roles' and 'activities' of managers in their day-to-day work. He identified ten managerial roles grouped into three categories: interpersonal (e.g., leader, liaison), informational (e.g., monitor, disseminator), and decisional (e.g., entrepreneur,

negotiator). His work emphasizes the multifaceted and dynamic nature of management, highlighting that managers must balance multiple roles simultaneously to adapt to organizational needs.

His framework is practical for managers seeking to improve their effectiveness by understanding and balancing their roles. It is particularly useful for leadership development and organizational behavior studies.

Mintzberg challenges traditional views of management as a linear process of planning, organizing, leading, and controlling. Instead, he portrays management as a dynamic interplay of roles that require adaptability to complex situations.

Porter focuses on 'strategic management' and achieving competitive advantage. His frameworks, such as the Five Forces Model and value chain analysis, help organizations analyze industry dynamics and develop strategies to outperform competitors. Porter emphasizes long-term strategic positioning through cost leadership, differentiation, or focus strategies.

Porter's models are primarily used by organizations to analyze industries, evaluate competition, and formulate strategies for market positioning. They are widely applied in business strategy and competitive analysis.

Porter adopts an economic perspective, focusing on how firms can create value by leveraging competitive forces. His work is rooted in the idea that strategic choices determine a firm's success in its environment.

Military enterprise constitutes a form of organized conflict that can differ materially from current neoliberal visions of Schumpeterian market dynamics. Indeed, the long history of various imperial defeats in Afghanistan, and the egregious failures of the USA in the Korean and and Vietnam Wars, suggest that God may not, indeed, always be on the side of the bigger—or more technologically advanced—batallions. The term 'doctrine' intrudes as a kind of Lamarckian 'genetic' determinant of tactical and operational dynamics. Strategic matters are left to higher, usually grossly incompetent, authority often both driven and constrained by narrow ideology. Russia's debacle in the Ukraine, as the Section Appendix explores, provides a contemporary case history. The experience of the Wehrmacht in the USSR supplies historical perspective.

This being said, both Mintzberg's emphasis on adaptability to complex situations, and Porter's focus on the importance of strategic choice, resonate with what we will attempt here, the development of new probability models of institutional cognition and its dysfunctions that might be converted into robust and validated statistical tools for the study, and in some measure, the control and remediation, of institutional process.

However, institutional decision-making—essentially institutional cognition in the sense of Atlan and Cohen [7]—as presented here, unlike Mintzberg's and Porter's work, does not elevate 'management' as a particular salient object central to understanding institutional dynamics. Rather, such a perspective places 'managers' soundly within the confines of doctrine, culturally-confined path-dependent historical trajectory, and the constraint that 'we know this but we can't say this'.

In short, we attempt to move beyond the classic Western cognitive pattern of overfocus on 'salient points' at the expense of context and contextual dynamics [8]. The role of 'managers' is, we assert, precisely one of those overfocused points. Corporate culture, as a whole, remains an all-powerful determinant of institutional—as opposed to individual—cognitive dynamics, in partnership with the 'enemy that has a vote' as Sun Tzu remarks. Indeed, a cardinal rule for advancement in an institutional context is the admonition 'to get along, go along'. What individual 'managers' actually think often has little effect on what they ultimately do. Humans are, after all, the naked mole rats of hominid species.

What comes to mind is the line officers ordering soldiers to advance against barbed-wire protected machine gun emplacements at the battle of the Somme.

As Louis Gerstner Jr., former CEO of IBM put it [9], "Culture is everything", and he should know, although the 'Auftragstaktic' that defeated France so handily in 1940 did not ultimately fare well a year later in the Wehrmacht's reprise of Napoleonic grandiosity in Russia, circumstances in which such 'mission command' provides decisive advantage continue to abound at the tactical and operational scales of organized conflict [2]. More recently, Matzenbacher [10] characterizes the approach according to the schematic of Figure 1.

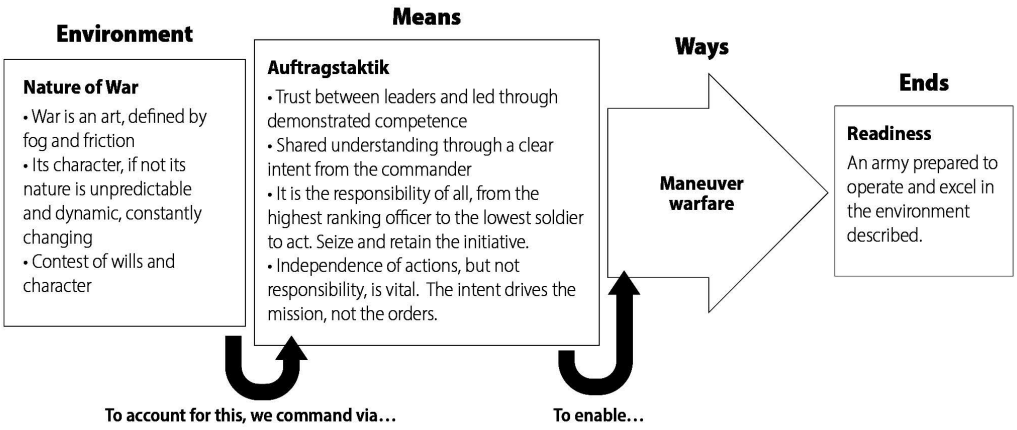



**Figure 1.** Adapted from Matzenbacher [10]. German Army *Auftragstaktik* Logic Map.

Here, we provide a formal analysis of this and other, more convoluted, decision processes, using newly-developed models of cognitive dynamics based on the asymptotic limit theorems of information and control theories, exploring a surprising parallel between hierarchical decision-making and the staged nature of cancer progression. Those whose careers have been deeply embedded in conventional Western hierarchical organizations are likely to experience a visceral recognition of the analogy.

We begin with some considerable methodological boilerplate, aimed at characterizing institutional cognitive dynamics. The reader should, however, be fully cognizant that the translation of any such probability models into robust statistical tools for data analysis is always an arduous enterprise that, in this case, remains to be done.

### *1.1. The Asymptotic Limit Theorems*

A surprisingly limited set of asymptotic limit theorems from information and control theories constrain all possible cognitive phenomena [11–13]. The construction of a new set of new statistical tools for the analysis of institutional cognitive dynamics and dysfunctions must be based on these theorems, much as the development of regression models, *t*-tests, and the like was based on the Central Limit Theorem and Friends. The derivation of robust statistical tools from any asymptotic limit theorems of probability theory, however, is not for the faint-of-heart.

The limit theorems of interest to us here are:

- The Shannon Coding Theorem that prescribes how to code a message in 'typical' sequences for maximum error-free transmission through a particular channel at a maximum 'channel capacity' (and the 'tuning theorem' variant in which the message formally 'transmits' the channel).
- The Source Coding Theorem that splits a set of sequences emitted by an information source into two subsets, a relatively small one that is 'meaningful'—the sequences

are consonant with an underlying 'grammar' and 'syntax'—and a much larger 'meaningless' one 'of measure zero'. Only for stationary, ergodic systems is it possible to characterize 'information' and 'information source uncertainty' in terms of a Shannon entropy-analog. For nonergodic systems, each (infinite) signal sequence has it's own scalar source uncertainty that cannot be expressed in 'Shannon entropy' form [11]. It is here that methods must be abducted from statistical physics, introducing as well equivalence-classes of sequences forming a groupoid, leading to groupoid symmetry-breaking phase transitions characterizing 'cognitive' phase transitions, quite unlike those of 'conventional' physical systems.

- The Rate Distortion Theorem (RDT) and an associated Rate Distortion Control Theory that, in a sense, 'tunes' the channel capacity around a maximum-permissible scalar measure of average distortion (which finesses the Friston 'free energy minimization' approach).
- The Data Rate Theorem (DRT) stating that, for stability of an inherently unstable control system, 'control information' must be provided at a rate greater than the system produces it's own 'topological information'. A version of this is, somewhat surprisingly, 'easily' derived from a time and 'free energy rate' optimization based on the Hartley-Shannon Theorem.

Further development leads to the following:

- Cognition implies choice—of a small set of actions from a larger embedding set of what is possible to the system of interest. Such choice reduces uncertainty in a formal manner, implying the existence of an information source 'dual' to the cognitive process studied, at and across the varied scales and levels of organization of the living state [7,14,15].
- Real-world, real-time cognition is always inherently unstable under the impact of the 'topological information' imposed by embedding environmental 'roadways'. Invocation of the DRT then implies that *cognition must always be paired with regulation for system stability and long-term function under selection pressures*. Further, *it can be argued that the degree of pairing between cognition and regulation is, in fact, a good measure of a system's 'self awareness', a cardinal criterion in many contemporary definitions of consciousness*.
- Here, we further assume an adiabatically, piecewise, stationary (APS) 'Born Oppenheimer' model in which the system of interest remains 'close enough' to stationary during the period(s) of interest so that the asymptotic limit theorems are effectively 'anytime algorithms' [16]. Between 'pieces' for which this assumption is sufficiently true, we invoke (groupoid) symmetry-breaking and/or Fisher Zero [17] phase transitions.
- The ergodic restriction is 'easily' lifted, at the cost of imposing methods abducted from statistical physics, with dynamics approximated via an analog to the first (and higher) order Onsager treatment of nonequilibrium thermodynamics, recognizing, however, that, due to the failure of information source microreversibility—e.g., 'eht' does not have the same probability as 'the' in English—there are no 'Onsager reciprocal relations' in high order approximations.
- For purely physical phenomena, free energy is often defined in terms of Boltzmann distribution-derived partition functions. In contrast, free energy analogs associated with the dynamics of cognitive phenomena can, and may well, be characterized by 'fat-tailed' distributions that do not have well-defined first or second moments [18,19].

### *1.2. Resource Rates*

Embodiment recognizes that cognitive systems—biological, institutional, mechanical, or composite—are, in reality, ultimately embedded in an environment that includes

themselves. There will typically be (at least) three resource streams necessary for their function:

- The first is the rate at which elements of the system can communicate with each other, instantiated by some channel capacity $\mathcal{C}$.
- The second is the rate at which 'sensory information' is available from the embedding circumstance, say according to a channel capacity $\mathcal{H}$.
- Third, there is the rate at which 'material/materiel' resources can be provided, a rate $\mathcal{M}$. For an organism, this might be measured by the rate at which metabolic free energy is provided. Machine/institutional systems will have a different measure.

These three rates will cross-correlate: one is confronting a $3 \times 3$ (or larger) matrix $\mathbf{Z}$. An $n \times n$ matrix will have will have $n$ scalar invariants determined by the polynomial relation

$$p(\gamma) = \det[\mathbf{Z} - \gamma \mathbf{I}] = (-1)^n \gamma^n + (-1)^{n-1} r_1 \gamma^{n-1} + \cdots - r_{n-1}\gamma + r_n \quad (1)$$

$\mathbf{I}$ is the appropriate identity matrix, 'det' is the determinant, and $\gamma$ is a real-valued parameter. The first invariant is the matrix trace, and the last the determinant. We seek to construct a scalar resource rate measure from these invariants.

The most direct such index might be $Z = \mathcal{C} \times \mathcal{H} \times \mathcal{M}$, but our worlds are not likely so simple.

We postulate—in first-order—that a single scalar index $Z(r_1, \ldots, r_n)$ can be constructed that sufficiently represents information and material resource rates. This approach is analogous to, but different from, the more usual principal component analysis based on a correlation matrix. Wallace [20] outlines the complications associated with generalization to more than a single such index.

### *1.3. Rate Distortion Control Theory*

A directly 'embodied' formalism emerges via a 'Rate Distortion Control Theory' based on the Rate Distortion Theorem. As described, the RDT addresses transmission under conditions of noise over an information channel. There will be, for that channel and a particular scalar measure of average distortion $D$ between what is sent and what is received, a minimum necessary channel capacity $R(D)$.

The theorem asks, in effect, what is the 'best' channel for transmission of a message with the least possible average distortion. $R(D)$ can be defined for nonergodic information sources via a limit argument based on the ergodic decomposition of a nonergodic source into a 'sum' of ergodic sources. The RDT can be reconfigured in a control theory context, if we envision a system's topological information from the Data Rate Theorem as another form of noise, adding to the average distortion $D$. See Figure 2.

The punctuation implied by the Data Rate Theorem [13] happens if there is a critical maximum average distortion beyond which control fails.

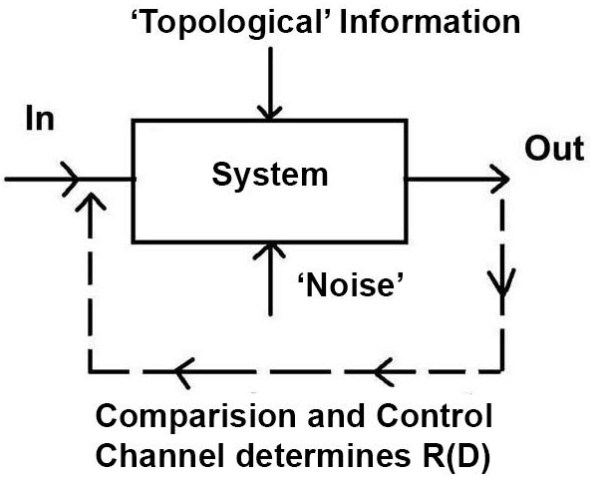



**Figure 2.** The Rate Distortion Control Theory model. An inherently unstable system's 'topological information', according to the Data Rate Theorem [13], is added to the effects of 'noise' to determine the minimum channel capacity $R(D)$ needed to assure transmission with average distortion $D$. The punctuation implied by the DRT emerges here if there is a critical maximum average distortion beyond which control fails.

## 2. The Model

We provide a basic approach, applying and abducting standard formalism from statistical mechanics [21]—given a proper definition of 'temperature'—for a cognitive system.

Consider the full ensemble of high probability developmental trajectories available to the system, written as $Y_j$, $j = 1, 2, \ldots$. Each trajectory is associated with a Rate Distortion Function-defined minimum necessary channel capacity $R_j$ for a particular maximum average scalar distortion $D_j$. Then, assuming some basic underlying probability model having distribution $\rho_R(C_R, x)$, where $C_R$ is a parameter set, we can define a pseudoprobability for a meaningful 'message' $Y_j$ sent into the system of Figure 2 as

$$dP = \frac{\rho_R(C_R, R/g(Z))dR}{\int_0^\infty \rho_R(C_R, R/g(Z))dR} = \frac{\rho_R(C_R, R_j/g(Z))dR}{g(Z)} \tag{2}$$

since, by definition, $\rho_R(C_R, x)$ is a probability density function that integrates to one on the interval $x = 0 \to \infty$. Here, for simplicity, we only treat the continuous case. For discrete or finite-state systems, the integral is a sum and the partition function denominator becomes some more general function $h(g(Z))$.

To reiterate, $Y_j$ is some particular trajectory, while the (generalized) integral is over all possible 'high probability' paths.

We thus, at least implicitly, impose the Shannon-McMillan Source Coding Theorem [12]: the set of possible system trajectories can be divided in two. These are, first, a very large set of measure zero—vanishingly low probability—that is not consistent with the underlying grammar and syntax of some basic information source, and a much smaller consistent set [11].

Again, $R_j$ is the RDT channel capacity keeping average distortion less than a given limit $D_j$ for message $Y_j$. $g(Z)$ is the temperature analog from above, depending on the scalar resource rate $Z$. $\rho(C, x)$, for physical systems, is usually taken as the Boltzmann distribution: $\rho(x) = \exp[-x]$. We suggest that, for cognitive phenomena—from the living and institutional to the machine and composite—it is necessary to move beyond analogs with physical systems. That is, it becomes necessary explore the influence of a variety of

different probability distributions, including those with 'fat tails' [8,22], on the dynamics of cognition/regulation stability.

Again, standard methodology from statistical physics [21] identifies the denominator of Equation (2) as a partition function, giving an iterated free energy-analog $F$ as

$$\rho_F(C_F, F/g(Z)) \equiv \int_0^\infty \rho_R(C_R, R/g(Z))dR = g(Z)$$
$$g(Z) = \text{RootOf}(X - \rho_F(C_F, F/X)) \quad (3)$$

where 'RootOf' means setting the expression to zero and solving for $X$ and $\rho_F$ is an appropriate 'free energy' distribution that must be determined case-by-case for cognitive, as opposed to simple physical, phenomena. For physical phenomena, $\rho_F$ can often be taken as the Boltzmann distribution. Again, $C_X$ is simply an associated parameter set.

From chemical physics [23], we can define a cognition rate for the system as the reaction rate analog

$$L \equiv \frac{\int_{R_0}^\infty \rho_R(C_R, R/g(Z))dR}{\int_0^\infty \rho_R(C_R, R/g(Z))dR} = \frac{1}{g(Z)} \int_{R_0}^\infty \rho_R(C_R, R/g(Z))dR \quad (4)$$

$R_0$ is the minimum channel capacity needed to keep the average distortion below a critical value $D_0$ for the full system of Figure 2.

The basic underlying probability model of Equation (2)—via the partition function argument of Equation (3)—determines system dynamics, but not system structure, and cannot be associated with a particular underlying network conformation, although these are related [24,25]. More specifically, a set of markedly different networks may all be mapped on to any single given dynamic behavior pattern and, indeed, vice-versa, The same static network may display a spectrum of behaviors. We focus on dynamics rather than network topology, a shift from much current work.

Feynman [26]—following Bennett [27]—identifies 'information' as a form of free energy, using Bennett's ideal machine that turns a message directly into work. We are concerned here with an iterated, rather than a direct, construction.

$F$, as a free energy, is subject to symmetry-breaking transitions as $g(Z)$ varies [28]. These symmetry changes, however, are not as associated with physical phase transitions as represented by standard group algebras. Such symmetry changes represent transitions from playing one 'game' to playing another. For example, a cognitive system may engage in foraging behaviors that trigger a predatory attack by another system. Then the game changes from 'foraging' to 'escape'.

Thus 'cognitive phase changes' involve shifts between equivalence classes of high probability developmental/behavioral pathways that are represented as *groupoids*, a generalization of the group concept such that a product is not necessarily defined for every possible element pair, although multiple products with multiple identity elements are defined [29,30].

Dynamics emerge via a first-order Onsager approximation akin to that of nonequilibrium thermodynamics [31] in the gradient of an entropy measure $S$ constructed from the 'iterated free energy' $F$ of Equation (3) by means of a Legendre transform that should be familiar from the Black-Scholes approximation [21,28].

Recall from the (first-order) Onsager approximation of nonequilibrium thermodynamics that $\partial Z/\partial t \propto \partial S/\partial Z$. The full—again, standard, first-order—development is

$$\begin{aligned} S(Z) &\equiv -F(Z) + ZdF(Z)/dZ \\ \partial Z/\partial t &\approx dS/dZ = f(Z) \\ f(Z) &= Zd^2F/dZ^2 \\ F(Z) &= \int\int \frac{f(Z)}{Z} dZdZ + C_1 Z + C_2 \\ g(Z) &= \text{RootOf}(X - \rho_F(C_F, F(Z)/X)) \end{aligned} \tag{5}$$

An expression for $g(Z)$ follows from the second expression of Equation (3). Note that the 'diffusion coefficient' has been implicitly set equal to 1 in the second part of Equation (5).

Higher order approximations are in the same realm as higher order regression models, modulo dimensional consistency. Full treatment may involve generalization of the Legendre transform via formal power series methods [32].

There are several important points:

- Since 'RootOf' may have complex number solutions, the temperature analog $g(Z)$ enters the realm of the 'Fisher Zeros' characterizing phase transition in physical systems [17,33,34]. Such phase transitions lie at the base of the punctuated accession to consciousness in the Baars, and related, Global Workspace Models. Indeed, the 'RootOf' construction in Equation (3) actually generalizes the Lambert W-function [35,36].
- To reiterate, information sources are not microreversible, that is, palindromes are highly improbable, for example the sequence 'ti' has far lower probability than 'it' in English. Thus, there are no 'Onsager Reciprocal Relations' in higher dimensional systems. The imposition of groupoid symmetries on cognitive phenomena appears to follow upon this directed homotopy.
- Further, there will always be a delay in the rate of provision of $Z$, so that, in Equation (5), for example, $f(Z) = \beta - \alpha Z(t)$—an exponential model having $Z(t) = (\beta/\alpha)(1 - \exp[-\alpha t])$ —where $Z \to \beta/\alpha$ at a rate determined by $\alpha$. Other dynamics are possible, such as the 'Arrhenius', $Z(t) = \beta \exp[-\alpha/t]$, with $f(Z) = (Z/\alpha)(\log(Z/\beta))^2$. However $Z$ is constructed from the components $\mathcal{C}, \mathcal{H}$ and $\mathcal{M}$, so here, it is the scalar resource rate $Z$ itself that counts.

## 3. Dispersed and Hierarchical Cognition Rates

### *3.1. The Basic Idea*

We assume an exponential model for $dZ/dt = f(Z) = \beta - \alpha Z$ in Equation (5), so that $Z(t) \to \beta/\alpha$ and

$$F = \beta Z(\ln(Z) - 1) - \frac{\alpha Z^2}{2} + C_1 Z + C_2 \tag{6}$$

where $C_1$ and $C_2$ are important boundary conditions.

We consider a direct, single-stage, 'Auftragstaktik' mission command system for which the basic underlying probability distribution is the Boltzmann. After some simple calculation, Equations (3) and (4) become

$$\begin{aligned} \exp[-F/g] &= g \\ g &= \frac{-F}{W(n, -F)} \\ L &= \exp[-R_0/g] \end{aligned} \tag{7}$$

$W(n, x)$ is the famous Lambert W-function of order $n$ that solves the relation

$$W(n, x) \exp[W(n, x)] = x.$$

It is real-valued only for $n = 0, -1$ and $x > -\exp[-1]$.

Setting $n = 0$, $C_1 = -1$, $C_2 = 3$, we fix $R_0 = 1$, $\alpha = 1$ and let $\beta$ vary as the index of 'arousal' in a Yerkes-Dodson model [19,37] of cognition rate $L$ in Figure 3. The real-value inverse-U of the Y-D Effect is bracketed on the left by non-zero imaginary-valued component instabilities representing 'hallucination' at low, and 'panic' at high arousal.

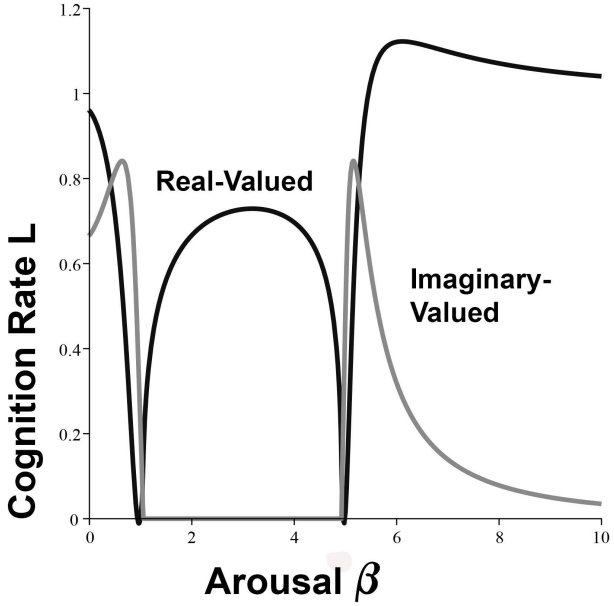



**Figure 3.** Cognition rate vs. arousal for a single-stage Auftragstaktik/mission command model follows a classic Yerkes-Dodson inverse-U pattern for the given boundary conditions. The real-value inverse-U is bracketed by nonzero imaginary-valued components on the left as 'hallucination' at low, and as 'panic' at high arousal.

A standard model of carcinogenesis [38] involves multiple triggering events that individually take place according to 'random' exponential distributions having the same rate constants. This leads to the Erlang Distribution. To show this, one can compute the moment generating function of the sum as a power of the moment generating functions of the exponentials, separately compute the moment generating function of the Erlang, and note they are the same.

Given $k$ sequential incidents, the basic relations are

$$\rho(k, \lambda, x) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$$
$$\rho(k, \lambda, F/g) = g \tag{8}$$

If, however, $k \geq 0$ is not an integer, then the denominator $(k-1)!$ is replaced by the Gamma Function $\Gamma(k)$, and the Erlang becomes the Gamma Distribution.

The relations corresponding to Equation (7) for $k = 2$—a 'simple' two-step process of hierarchical cognition—are then

$$g = -\frac{\lambda F}{2W\left(n, -\frac{\sqrt{F}}{2}\right)}$$
$$L = \frac{e^{-\frac{\lambda R_0}{g}}(\lambda R_0 + g)}{g} \tag{9}$$

Setting $R_0 = 1$, $\lambda = 1$, $n = 0$ and imposing the same boundary conditions on $F$ as in Figure 3, we see a remarkably different pattern of cognition rate vs. arousal. Only for two very narrow inverse-U spikes is the imaginary-valued component actually zero:

A 'best case' variation of the boundary condition on $F$, however, does lead to the coalescence of the two narrow inverse-U spikes as in Figure 4. Here, $C1 = 1$, $C_2 = 33/20$ in the definition of $F$. Other parameters are as in Figure 5. Note that, even in this example, cognition is badly stunted in comparison with the one-stage example of Figure 3.

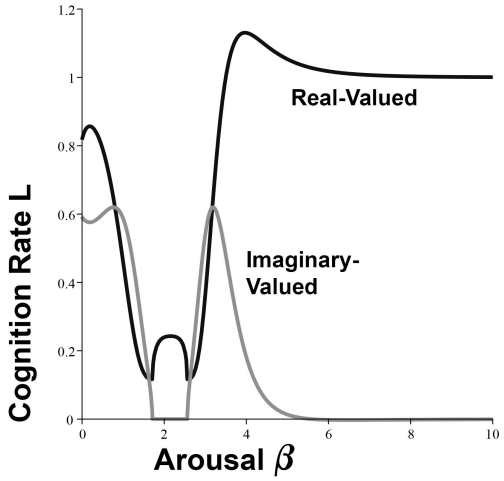



**Figure 4.** A 'best case' treatment of the two-stage cognition process of Figure 5, merging the two narrow real-valued peaks by taking $C_1 = 1$, $C_2 = 33/20$ in $F$. Even here, the cognition rate is badly stunted in comparison with the one-stage model of Figure 3.

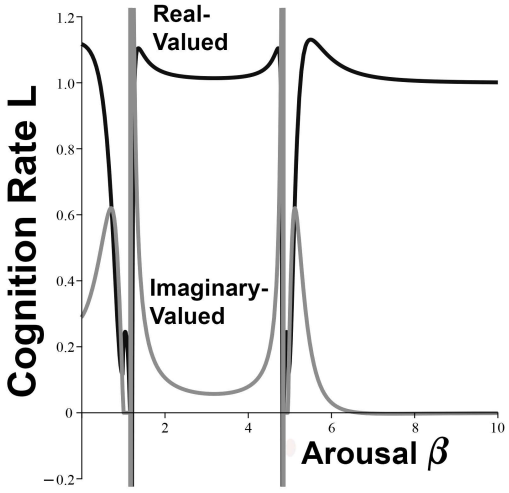



**Figure 5.** Cognition rate vs. arousal for a dual-stage hierarchical system in which two layers of authority must be traversed before a decision is reached. The boundary conditions on $F$ are the same as in Figure 3. Only for two very narrow inverse-U spikes is the imaginary-valued component zero.

The interested reader can pursue matters to higher, or non-integer, order in $k$—representing added or 'smeared' levels of deliberation—where the cognitive dynamics become even more abusive.

The intermediate case in $k$ for the Gamma Distribution is exactly solvable, after a little work, generating an analog to Figure 5. Even 'some' micromanagement can be catastrophic. The relevant relations, taking $R_0 = 1$, are

$$g = -\frac{\lambda F}{kW\left(n, -\frac{\left(\Gamma(k)F\lambda^{-k}\right)^{\frac{1}{k}}\lambda}{k}\right)}$$

$$L = \frac{(\lambda/g)^k \exp[-\lambda/g]}{\Gamma(k+2)} \times \left(\frac{e^{\frac{\lambda}{g}}\Gamma(k+2)}{\left(\frac{\lambda}{g}\right)^k} - e^{\frac{\lambda}{2g}}\left(\frac{\lambda}{g}\right)^{-\frac{k}{2}} M_{\frac{k}{2},\frac{k}{2}+\frac{1}{2}}\left(\frac{\lambda}{g}\right) - k - 1\right) \quad (10)$$

where $W$ is the Lambert W-function and $M$ the Whittaker M-function.

The Mathematical Appendix reconsiders the Boltzmann vs. second-order Erlang model in discrete, rather than continuous circumstances, when there are only two possible states, $R_\pm = R_0 \pm \delta$, $\delta \ll R_0$. $R_+$ is taken as the cognitively 'on' state.

In addition, the Appendix reexamines the 'best case' of Figure 4 for the Gamma Distribution, assuming variation in $k$.

### *3.2. Stochastic Extension*

Under stochastic burden, it is possible to derive $F$ from the assumption of stability imposed by evolutionary selection pressure on some particular function $Q(Z)$, i.e., first assuming $< dQ_t >= 0$ and then calculating $F$ from the relation

$$dZ_t = (Z_t d^2F/dZ^2)dt + \sigma h(Z_t)dB_t \quad (11)$$

via the Ito Chain Rule applied to $Q$. Then, after some development,

$$F(Z) = \int\left(\int -\frac{\sigma^2 h(Z)^2\left(\frac{d^2}{dZ^2}Q(Z)\right)}{2Z\left(\frac{d}{dZ}Q(Z)\right)}dZ\right)dZ + C_1 Z + C_2 \quad (12)$$

with $g$ and $L$ determined from the basic underlying probability distribution as in Equations (7) and (9).

We make one detailed application, assuming that institutional signal detection follows something like the classic Weber-Fechner Law of sensory perception [39], that is, 'perceived sensation' is scaled as the logarithm of the of the signal energy, in 'decibels' of incoming and internal signal strength.

We are then interested in the nonequilibrium steady state for $Q(Z) \equiv \ln(1+Z)$, calculating $< d\ln(1+Z_t) >= 0$ for ordinary volatility, $h(Z) = Z$, in Equation (12). Then

$$F(Z) = \frac{\sigma^2\left(\frac{Z^2}{2} - \ln(1+Z)(1+Z) + 1 + Z\right)}{2} + C_1 Z + C_2 \quad (13)$$

We again take $C_1 = -1$, $C_2 = 3$, $n = 0$ and calculate the cognition rates for varying $\sigma$ and $Z$ for the single-stage Boltzmann and two-stage Erlang distributions in Figure 6. As in Figures 3 and 5, while the Boltzmann distribution, Figure 6a, shows an even progression from the 'easy' to the 'hard' Yerkes-Dodson pattern with increasing 'noise' $\sigma$, the two-fold

hierarchy of Figure 6b displays stunted cognitive dynamics. Both, however, show collapse of cognition under sufficient noise burden.

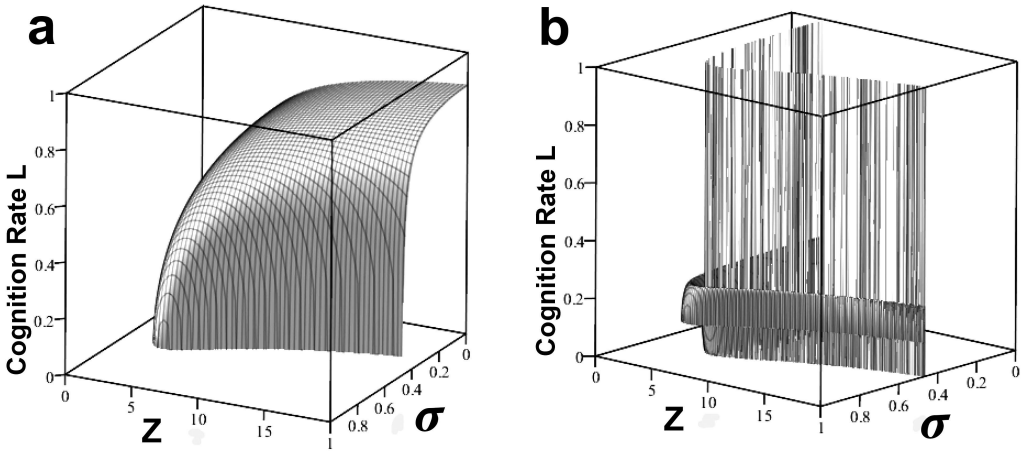



**Figure 6.** Joint variation of cognition rate with $\sigma$ and $Z$ for **a** Boltzmann and **b** Second-order Erlang distributions. As in Figures 3 and 5, while the one-stage Boltzmann distribution shows an even progression from the 'easy' to the 'hard' Yerkes-Dodson pattern with increasing 'noise' $\sigma$, the two-stage hierarchy displays severely stunted cognitive dynamics.

Taking a Pieron's Law perspective [39]—that the system detects signals at a rate $\propto Z^{-m}$, $m > 0$—then the nonequilibrium steady state condition $< dZ_t^{-m} >= 0$, again for ordinary volatility under the Ito Chain Rule, leads to

$$F(Z) = \frac{(m+1)\sigma^2 Z^2}{4} + C_1 Z + C_2 \tag{14}$$

Setting $m = 2$, calculations of cognition rate for the two distributions essentially reprises Figure 6.

### *3.3. Time-and-Resource Constraints*

More typically, one usually has a large, multicomponent, interlinked structure of $i = 1, ..., n$ subcomponents that must act together under constraints of both resource rates $Z_i$ and available length of time $T_i$ under which a component must act. The resource and time constraints are $Z = \sum_i Z_i$ and $T = \sum_i T_i$, and one seeks to optimize some collective performance index $\sum_i Q_i(Z_i, T_i)$ under those constraints, leading to the Lagrangian system of equations

$$\begin{aligned} \mathcal{L} = \sum_i Q_i(Z_i, T_i) + \lambda\left(Z - \sum_i Z_i\right) + \mu\left(T - \sum_i T_i\right) \\ \partial\mathcal{L}/\partial Z_i = \partial Q_i/\partial Z_i - \lambda = 0 \\ \partial\mathcal{L}/\partial T_i = \partial Q_i/\partial T_i - \mu = 0 \\ \partial\mathcal{L}/\partial Z = \lambda,\ \partial\mathcal{L}/\partial T = \mu \end{aligned} \tag{15}$$

where $\lambda$ and $\mu$ are interpreted as *environmentally-imposed shadow price constraints* in the economic sense [40,41].

Mild abuse of formalism from the last line of Equation (15) leads to $dZ/dT \sim \mu/\lambda \equiv \omega$ for the *total* resource rate and the *total* available time. This permits definition of an underlying base differential equation for the full multicomponent system as

$$dZ/dt \sim \omega = Fd^2F/dZ^2$$
$$F(Z) = \omega Z(\ln(Z) - 1) + C_1 Z + C_2 \tag{16}$$

This characterization of $F$, as above, permits calculation of the cognition rates for the Boltzmann and Erlang order 2 distributions under the conditions of Figures 3, 5 and 6, leading to Figure 7. Somewhat surprisingly, there is a bifurcation according to the sign of the shadow price ratio, with the single-stage Boltzmann distribution still far more regular than the two-stage Erlang system, which again seems stunted and incoherent.

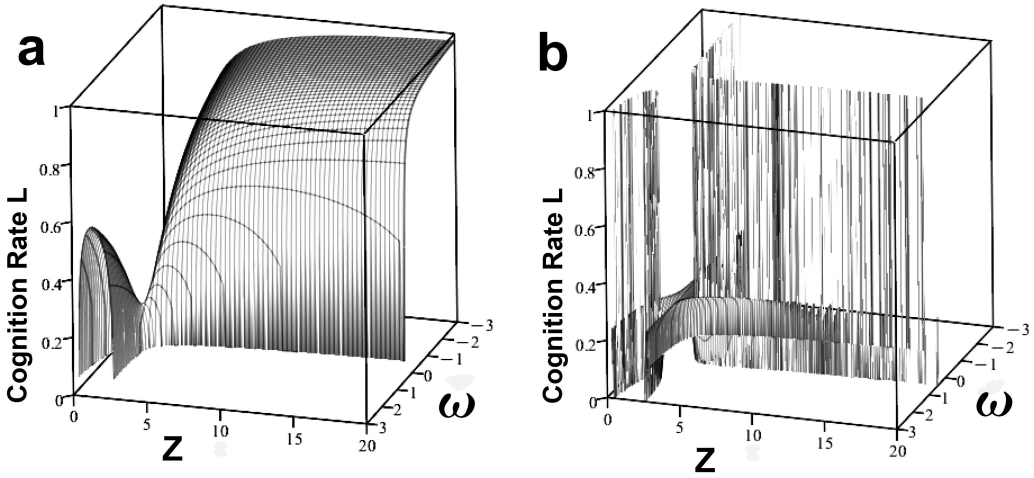



**Figure 7.** Analog to Figure 6 for the Lagrange optimization under time and resource rate constraints. The Figure displays the joint variation of cognition rate with shadow price ratio $\omega$ and total resource rate $Z$. Here, as above, $C_1 = -1$, $C_2 = 3$, likewise assigning Equations (7) and (9). There is a bifurcation according to the sign of the shadow price ratio, with the single-stage Boltzmann distribution (**a**) more regular than the two-stage Erlang system (**b**), which, again, is stunted and incoherent.

It is possible, if algebraically exasperating, to iterate the argument a further step, exploring the effect of 'noise' via the SDE

$$dZ_t = \omega dt + \sigma Z_t dB_t \tag{17}$$

imposing, under Equation (16) and the Ito Chain Rule, the nonequilibrium steady state condition $< dL_t >= 0$ on the single and double-stage expressions for cognition rate from Equations (7) and (9). Solving for $\sigma^2(Z, \omega)$, one again finds coherence for the one-stage Boltzmann, and incoherence for the two-stage Erlang model. Details are left as an exercise.

## 4. The Dynamics of Delay

We have, in this one- vs. two-level analysis, implicitly assumed a best-case model for which there is no significant added time lag in the ability to respond to environmental perturbations, i.e., no 'OODA Loop burden' [42]. As a general matter, however, for all examples, imposing higher order Erlang distributions—adding more supervisory levels—enters the fatal realm that follows from having an opponent 'get inside your command cycle'.

Even the most elementary one-stage model can be driven into instability by sufficient delay. Consider an 'exponential' system

$$\begin{aligned} dZ/dt &= \beta - \alpha Z(t) \\ Z(t) &= \frac{\beta}{\alpha}(1 - \exp[-\alpha t]) \rightarrow \frac{\beta}{\alpha} \end{aligned} \tag{18}$$

Imposing a fixed delay, say $\tau$, so that $dZ/dt = \beta - \alpha Z(t - \tau)$, leads to the relations

$$\begin{aligned} Z(t) &= \frac{\beta}{\alpha}(1 - \exp[St]) \\ S &= \frac{W(n, -\alpha\tau)}{\tau} \\ dZ/dt &= S\left(Z - \frac{\beta}{\alpha}\right) = Zd^2F/dZ^2 \\ F(Z) &= \frac{S\,Z^2}{2} - \frac{S\beta(Z\ln(Z) - Z)}{\alpha} + C_1 Z + C_2 \\ dZ_t &= S\left(Z_t - \frac{\beta}{\alpha}\right)dt + \sigma Z_t dB_t \end{aligned} \tag{19}$$

where, again, $W(n, X)$ is the Lambert W-function of orders $n = 0, -1$, real-valued only for $X \geq -\exp[-1]$. Sufficiently large $\alpha\tau$, in the expression for $S$, will drive even this system into thrashing instability. If the imaginary component of $S$ is zero, the real component is always negative, leading to stochastic instabilities with rising $\sigma$ in the stochastic differential equation calculations.

Figure 8 reprises Figures 3 and 5, the one-step and two-step models, under fixed delays, taking $n = 0$, $\alpha = 1$, $C_1 = -1$, $C_2 = 3$. Note the imaginary-onset disjunctions at $\tau = \exp[-1]$ for both models. The two-step system is again stunted and incoherent compared to the single step.

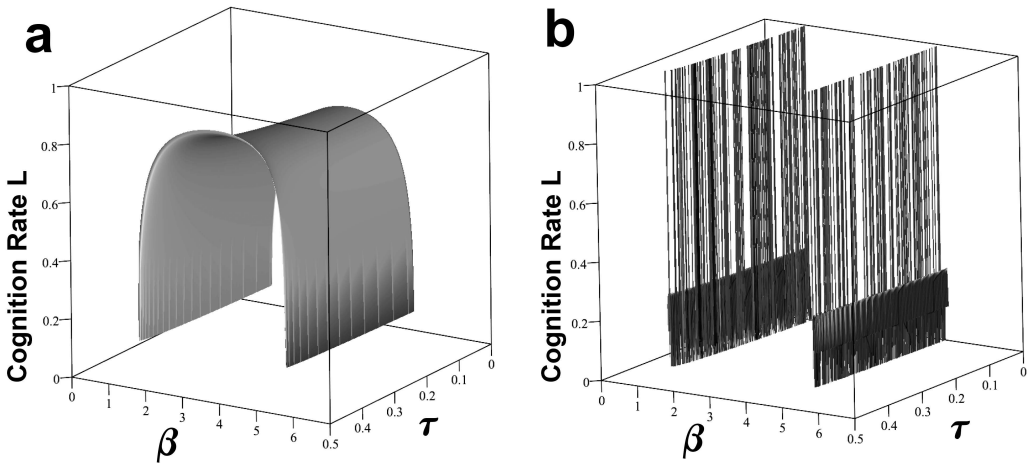



**Figure 8.** Reprising the models of Figures 3 and 5 under fixed delay. (**a**) The one-step model under the Boltzmann Distribution. (**b**) The two-step Erlang Distribution system. Here, $n = 0$, $\alpha = 1$, $C_1 = -1$, $C_2 = 3$. Note the non-zero imaginary part disjunctions in both cases at $\tau = \exp[-1]$. The two-step system is again stunted and incoherent compared to the single step.

Assuming first and second-order Erlang Distributions, even simple distributed delay relations such as

$$dZ/dt = \beta - \alpha \int_0^t Z(t-\tau)\lambda \exp[-\lambda\tau]d\tau$$
$$dZ/dt = \beta - \alpha \int_0^t Z(t-\tau)\lambda^2\tau \exp[-\lambda\tau]d\tau \quad (20)$$

which can be directly solved via the Laplace Transform method, are found to drive thrashing instabilities, depending on complicated interactions between $\alpha$ and $\lambda$.

Higher order Erlang systems will, by their sequential staging, suffer progressively longer delays than lower, since, from Equation (8), $< \tau >= k/\lambda$.

We can carry the discussion of delay a step further, again invoking Pieron's Law of system response time, that $T \propto Z^{-m}$ for $m > 0$ [39]. The last relation of Equation (19) allows calculation of the nonequilibrium steady state $< dZ_t^{-m} >= 0$ according to the Ito Chain Rule, leading to

$$Z_{nss} = \frac{S(n, \alpha, \tau)(\beta/\alpha)}{S(n, \alpha, \tau) - \frac{(m+1)\sigma^2}{2}}$$
$$T_{nss} \sim Z_{nss}^{-m} \quad (21)$$

For Figure 9, $T = Z^{-2}$, $\alpha = 1$, $\beta = 3$ and for $S$, $n = -1$, $\alpha = 1$. $T$ increases synergistically with both $\sigma$ and $\tau$ until the critical point $\alpha\tau = \exp[-1] \approx 0.37$, where thrashing sets in.

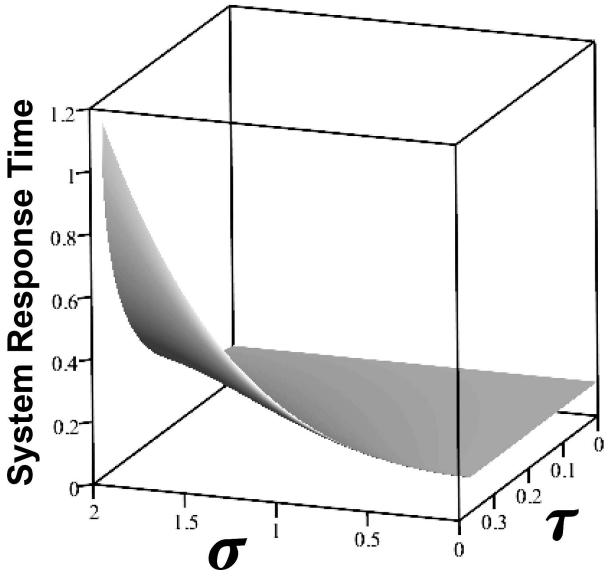



**Figure 9.** Response time $T = Z^{-2}$, $\alpha = 1$, $\beta = 3$. For $S(n, \alpha, \tau)$, $n = -1$, $\alpha = 1$. $T$ increases synergistically with $\sigma$ and $\tau$ until the critical point $\alpha\tau = \exp[-1] \approx 0.37$, after which instability becomes manifest.

## 5. A Multiplicative System

The Erlang/Gamma Distribution applies to a sequential series of independent, exponentially distributed, 'events'. It is, perhaps, conceivable that events might have multi-

plicative rather than additive influence, like an exploding infection or other reproducing population, or cancer metastasis, leading to a Lognormal Distribution model:

$$\rho(x,\,\mu,\,\sigma) = \frac{\exp[-(\ln(x)-\mu)^2/(2\sigma^2)]}{x\sigma\sqrt{2\pi}}$$
$$g_{\pm} = \frac{F}{e^{\mu\pm\sigma\sqrt{-\ln(2\pi\,F^2\sigma^2)}}}$$
$$L = \frac{\mathrm{erf}\left(\frac{\sqrt{2}\,(\ln(g)-\ln(R_0)+\mu)}{2\sigma}\right)}{2} + \frac{1}{2} \tag{22}$$

We impose an exponential model on $dZ/dt$, generating the expression for $F$ of Equation (6). Here, $\alpha = 1$, $C_1 = -1$, $C_2 = 3$, and $\beta$ is again the index of 'arousal'.

Figure 10 shows the corresponding real- and imaginary-valued cognition rates $L(\beta)$ for $g_-$, setting $R_0 = \mu = \sigma = 1$. This recovers an analog to the sequential additive model of Figure 5, with only two, very narrow, real-valued spikes. The rest follows.

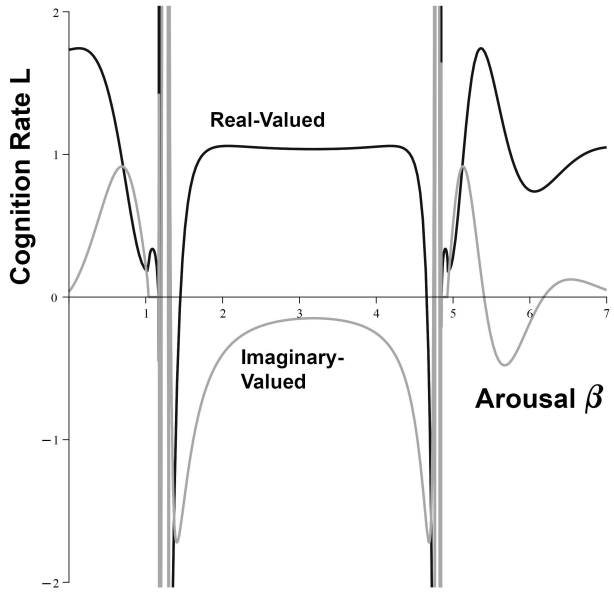



**Figure 10.** Analog to Figure 5 for the multiplicative model of Equation (22), using $g_-$. $R_0 = \mu = \sigma = 1$, $C_1 = -1$, $C_2 = 3$. There are only two narrow stable cognition rate spikes.

## 6. A Crack-Propagation Model

Crack-propagation system failure models are often based on the Weibull Distribution leading, here, to the relations

$$\rho(k,\,\lambda,\,x) = \frac{k\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}}{\lambda}$$
$$g = \frac{F\left(\frac{F}{k}\right)^{-\frac{1}{k}}\left(-\frac{F}{kW\left(n,-\frac{F}{k}\right)}\right)^{\frac{1}{k}}}{\lambda}$$
$$L = e^{-\left(\frac{R_0}{g\lambda}\right)^k} \tag{23}$$

Taking $F$ as in Equation (6), with $n = 0, -1$, $R_0 = 1$, $C_1 = -1$, $C_2 = 3$, leads to the starkly fragmented cognitive dynamics of Figure 11. Again, there are only two narrow stable cognition spikes with rising arousal $\beta$.

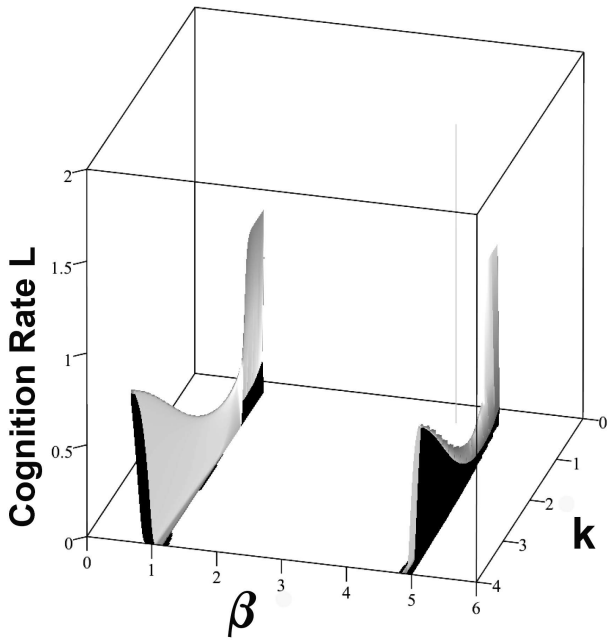



**Figure 11.** Stable cognitive dynamics structure for a Weibull Distribution crack-propagation model, $n = 0, -1$, $R_0 = 1$ under the conditions of Equation (6), $C_1 = -1$, $C_2 = 3$. Again, there are only two narrow stable cognition spikes.

## 7. Determining the Underlying Probability Distribution

Predicting the dynamics of system cognition depends, in this model, on identification of the foundational probability distribution—or the spectrum of distributions under different forms of stress—characterizing the system under study. The most direct method might be through empirical observation of the system's 'hazard rate', the form of the function $Q(x)$ representing the rate at which it detects a new signal, given a signal rate $x$, for different signal classes.

For a distribution $\rho(x)$ with associated hazard rate $Q(x)$, the basic relations are

$$Q(x) \equiv \frac{\rho(x)}{1 - \int_0^x \rho(u)du}$$
$$\rho(x) = \lim_{\epsilon \to 0} Q(\epsilon) e^{\int_\epsilon^x \left( -Q(Z) + \frac{\frac{d}{dZ} Q(Z)}{Q(Z)} \right) dZ} \tag{24}$$

For the exponential distribution $\rho(x) = \lambda \exp[-\lambda x]$, $Q(x) = \lambda$.

For the Gamma distribution in arbitrary $k$, $\lambda$,

$$Q(k, \lambda, x) = -\frac{x^{k-1} \lambda^k k(k+1)}{-e^{\lambda x} \Gamma(k+2) + \left( (\lambda x)^{-\frac{k}{2}} M_{\frac{k}{2}, \frac{k}{2}+\frac{1}{2}}(\lambda x) e^{\frac{\lambda x}{2}} + k + 1 \right) x^k \lambda^k} \tag{25}$$

where $M$ is the Whittaker M Function of appropriate order, recalling that the Erlang Distribution is defined for integer $k$.

For the multiplicative system Lognormal Distribution,

$$Q(\mu, \sigma, x) = -\frac{e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}} \sqrt{2}}{x\sigma\sqrt{\pi} \left( -1 + \mathrm{erf}\left( \frac{\sqrt{2}\,(\ln(x)-\mu)}{2\sigma} \right) \right)} \tag{26}$$

where erf is the Error Function.

For the 'crack propagation' Weibull Distribution,

$$Q(k, \lambda, x) = k\lambda^{-k}x^{k-1} \tag{27}$$

Figure 12 shows the form of these relations for the Gamma (including the Boltzmann) and the Lognormal Distributions. For the Gamma, $\lambda = 1$, and for the Lognormal, $\mu = \sigma = 1$.

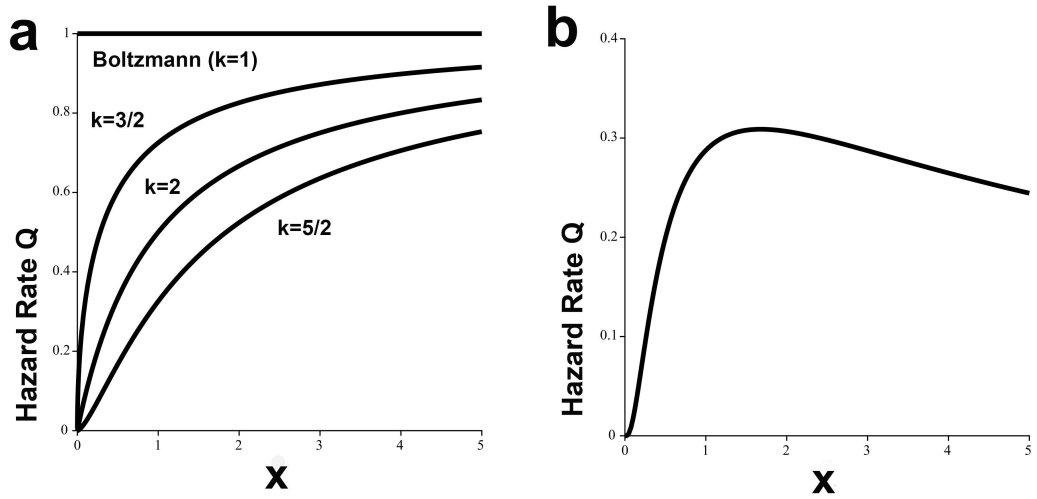



**Figure 12.** Hazard rate forms. (**a**) For the Gamma Distribution at various $k$, setting $\lambda = 1$, including the Boltzmann Distribution, k = 1. (**b**) For the Lognormal Distribution, $\mu = \sigma = 1$.

Assuming a simple monotonically declining hazard rate $Q(\lambda, x) = \lambda/(\lambda x + 1)$ implies $\rho(\lambda, x) = \lambda/(\lambda x + 1)^2$, a 'fat-tailed' distribution without finite first or second moments. Calculating 'temperature' $g$ and cognition rate $L$ as above, then imposing the exponential model of Equation (6) on $F$ as in Figure 3 leads—somewhat surprisingly—to a close analog to Figure 3 for cognition rate vs. arousal. Details are left as an exercise.

As shown in the Mathematical Appendix, imposing a U-shaped hazard function, using a generalized Gompertz Distribution, via numerical methods, leads to severely constrained cognition, replicating the two narrow real-valued peaks of Figure 5.

In sum, empirical observation of a cognitive system's hazard rate or rates under varying signal strengths and modes permits inference of the underlying probability distribution (or the spectrum of distributions under different excitations), and hence inference of cognitive dynamics and dysfunction under differing selection pressures.

Further, since the form of the underlying probability distribution has profound impact on the dynamics of cognition, it is likely an object of evolutionary process, leading to 'distribution tuning' under stress.

## 8. Discussion

Those of us accustomed to acting within 'normal' bureaucracies will, perhaps, recognize something of the degradation, distortion, and stunting imposed by increasing levels of 'decision structure' under the critical time constraints so fondly exploited by John Boyd and his disciples.

The striking difficulties encountered by rigidly-controlled Russian forces in the Ukraine are well known and summarized in the Appendix from this multi-stage perspective by the Perplexity AI system.

Indeed, the term 'workplace autonomy' [43] is often cited in civilian enterprise as a central tool for ensuring both high productivity and retention of employees.

What this work suggests, among other matters, is that the emergence of an Erlang/Gamma Distribution of order above one in an institutional pattern of times-to-decision is very much not a good thing.

In addition, the probability models explored here, based firmly on the asymptotic limit theorems of information and control theories, might, with some effort, be converted into robust statistical tools for the study and remediation of pathologies of cognition at and across a considerable spectrum of scales and levels of organization not confined to the study of organized conflict.

Unfortunately, as the works of the master statisticians of the early 20th Century demonstrate, the conversion of asymptotic limit theorems into robust, verifiable statistical tools is a fraught enterprise. First, the limit theorems associated with 'ordinary' statistics are, themselves recondite, as those who have forged through the proofs of the Central Limit, Renewal, Martingale, and related Theorems have found. The Theorems that characterize and constrain information and control theories are no less deep and subtle. In addition, the first-order 'statistical mechanics' and 'nonequilibrium thermodynamics' methods used to derive the probability models developed here are simplistic—indeed entry-level—approximations abducted from physical theory. Analysis of real-world, real-time data would be needed to map out the necessary extensions and modifications of both theory and the tools that follow from that theory. This was, indeed, much the experience in the emergence of 'ordinary' parametric and nonparametric statistical methodology.

In sum, the mathematical structures invoked here, complicated as they may seem, provide only the most limited initial starting point. After all, as Maturana and Varela [15] assert, cognition lies at the very heart of biological mystery. A full-scale address, it seems, requires even deeper formal approaches than explored here, such as phase transition driven by nonergodic groupoid symmetry-breaking and associated arguments [44,45].

As reviewers of this paper have noted, we do not provide empirical validation of model outputs such as cognitive transitions or collapse thresholds. One put the matter this way,

> …[A] supervised learning approach is proposed in which the target variable corresponds to one of the latent quantities that cannot be directly observed but are essential to the system's dynamics, such as decision delay ($\tau$) or the level of operational distortion ($\sigma$). These parameters, which are crucial for determining system stability and transitions toward dysfunctional cognitive states, can be estimated from historical or simulated data using regression models.
>
> The observable features used for learning might include the average duration of decisions, the number of hierarchical levels involved, the frequency of communication between operational units, the variability of decision outcomes, or the amount of resources used. These variables, derived from operational logs or synthetic datasets, would serve to train a predictive model capable of inferring the value of the theoretical parameter selected as the target.

At present, the author simply does not have access to the resources needed for such studies. A clever reader, however, can take this material and run with it.

## 9. Appendix

### *9.1. Two-State Models*

Here, we assume there are only two possible states, 'on' with $R_+ = R_0 + \delta$, $\delta \ll R_0$, and 'off', $R_- = R_0 - \delta$, again under the Boltzmann and two-stage Erlang probability distributions. We set $R_0 = 1$. Under these distributions, the essential relations for the partition functions and cognition rates become

$$\begin{aligned} \mathrm{e}^{-\frac{F}{g}} &= \mathrm{e}^{-\frac{1+\delta}{g}} + \mathrm{e}^{-\frac{1-\delta}{g}} \\ \frac{F\,\mathrm{e}^{-\frac{F}{g}}}{g} &= \frac{(1+\delta)\mathrm{e}^{-\frac{1+\delta}{g}}}{g} + \frac{(1-\delta)\mathrm{e}^{-\frac{1-\delta}{g}}}{g} \end{aligned} \tag{28}$$

$$\begin{aligned} L &= \frac{\mathrm{e}^{-\frac{1+\delta}{g}}}{\mathrm{e}^{-\frac{1+\delta}{g}} + \mathrm{e}^{-\frac{1-\delta}{g}}} \\ L &= \frac{(1+\delta)\mathrm{e}^{-\frac{1+\delta}{g}}}{g\left(\frac{(1+\delta)\mathrm{e}^{-\frac{1+\delta}{g}}}{g} + \frac{(1-\delta)\mathrm{e}^{-\frac{1-\delta}{g}}}{g}\right)} \end{aligned} \tag{29}$$

Equation (28) can be expanded as a series in $\delta$, giving first-order approximations for $g$ as

$$\begin{aligned} g &\approx \frac{1-F}{\ln(2)} + \mathcal{O}(\delta^2) \\ g &\approx \frac{1-F}{\ln(2/F)} + \mathcal{O}(\delta^2) \end{aligned} \tag{30}$$

We drop the terms of order $\delta^2$.

$F$ is taken as in Equation (6), setting $\alpha = 1$, $C_1 = -1$, $C_2 = 3$, with $\beta$ the index of arousal. This generates the cognition rate graphs of Figure 13a for the Boltzmann and Figure13b for the two-stage Erlang distribution. The two-stage model is bifurcated and, again, grossly distorted, in comparison with the Boltzmann example.

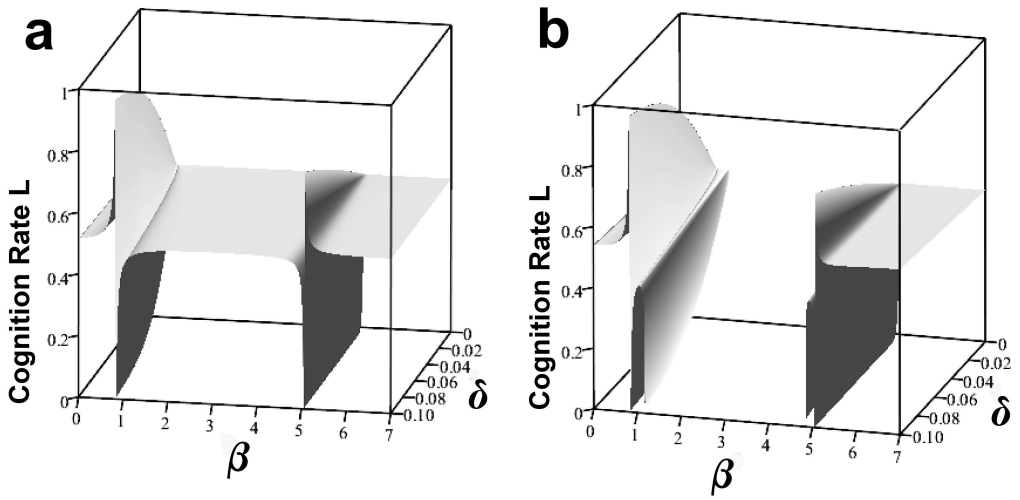



**Figure 13.** $F$ is as in Equation (6), $\alpha = 1$, $C_1 = -1$, $C_2 = 3$, with $\beta$ the index of arousal. (**a**) Cognition rate for the Boltzmann Distribution. (**b**) Cognition rate for the two-stage Erlang distribution. Again, the two-stage model is grossly distorted, in comparison with the Boltzmann example. Two-state examples based on the Lognormal Distribution are recognizably similar to the two-stage Erlang case.

For the two-stage Lognormal Distribution, again setting $R_0 = 1$,

$$g \approx \sqrt{F}\, 2^{\frac{\sigma^2}{\ln(F)}} e^{\sigma^2 - \mu} + \mathcal{O}(\delta^2)$$

$$L = \frac{g\sqrt{2}\, e^{-\frac{\left(\ln\left(\frac{1+\delta}{g}\right)-\mu\right)^2}{2\sigma^2}}}{2(1+\delta)\sigma\sqrt{\pi}\left(\frac{g\sqrt{2}\, e^{-\frac{\left(\ln\left(\frac{1+\delta}{g}\right)-\mu\right)^2}{2\sigma^2}}}{2(1+\delta)\sigma\sqrt{\pi}} + \frac{g\sqrt{2}\, e^{-\frac{\left(\ln\left(\frac{1-\delta}{g}\right)-\mu\right)^2}{2\sigma^2}}}{2(1-\delta)\sigma\sqrt{\pi}}\right)} \tag{31}$$

The algebraic expression for $g$ is different, but the graph of $L(\delta, \beta)$ is recognizably similar to the two-stage Erlang case.

For the Weibull Distribution crack propagation model, setting $\lambda = R_0 = 1$,

$$g \approx \frac{1-F}{\ln\left(2^{\frac{1}{k}} F^{-\frac{k-1}{k}}\right)} + \mathcal{O}(\delta^2)$$

$$L = \frac{k\left(\frac{\delta+1}{g}\right)^{k-1} e^{-\left(\frac{\delta+1}{g}\right)^k}}{k\left(\frac{\delta+1}{g}\right)^{k-1} e^{-\left(\frac{\delta+1}{g}\right)^k} + k\left(\frac{1-\delta}{g}\right)^{k-1} e^{-\left(\frac{1-\delta}{g}\right)^k}} \tag{32}$$

The expression for $g$ is analogous to those for the Boltzmann and Erlang Distributions. Fixing $\delta \ll 1$, the graph of $L(k, \beta)$, however, is singularly irregular and very highly bifurcated.

### *9.2. 'Best Case' Analysis for the Gamma Distribution*

Using Equation (10), we can 'easily' recapitulate the analysis leading to Figure 4, again, in the definition for $F$ setting $C_1 = 1$, $C_2 = 33/20$ and taking $R_0 = 1$. The result is shown in Figure 14. The result is bifurcated, with the upper section taking $n = 0$ in the Lambert W-function, and the lower corresponding to $n = -1$. Multi-layer decision structures, indexed by $k$, remain cognitively stunted in this model.

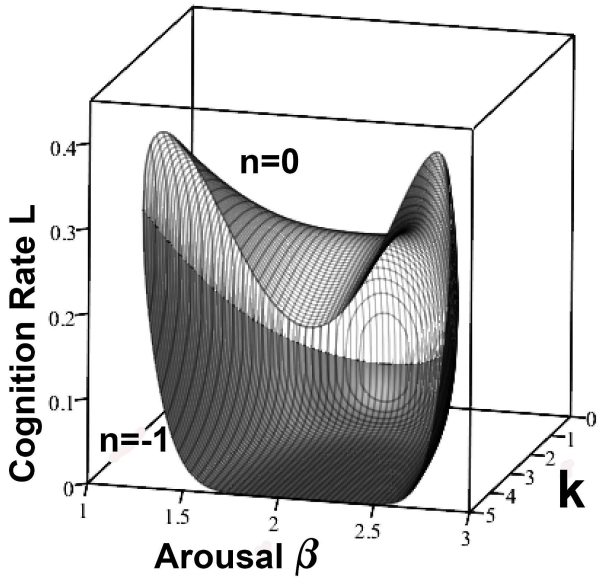



**Figure 14.** Extension of the 'best case' from Figure 4 to the Gamma Distribution model with variable $k$, the depth-of-command. The solution is bifurcated according to the order of the Lambert W-function defining $g$. Multi-layer decision structures remain cognitively stunted in this model.

*9.3. U-Shaped Hazard Function*

The generalized Gompertz Distribution is characterized by the relations

$$\rho(\lambda, c, \theta, x) = \theta\lambda\, e^{cx} e^{-\frac{\lambda\left(e^{cx}-1\right)}{c}} \left(1 - e^{-\frac{\lambda\left(e^{cx}-1\right)}{c}}\right)^{\theta-1}$$

$$Q = \frac{\theta\lambda\, e^{cx-\frac{\lambda\left(e^{cx}-1\right)}{c}} \left(1 - e^{-\frac{\lambda\left(e^{cx}-1\right)}{c}}\right)^{\theta-1}}{1 - \left(1 - e^{-\frac{\lambda\left(e^{cx}-1\right)}{c}}\right)^{\theta}}$$

$$\theta\lambda\, e^{\frac{cF}{g}} e^{-\frac{\lambda\left(e^{\frac{cF}{g}}-1\right)}{c}} \left(1 - e^{-\frac{\lambda\left(e^{\frac{cF}{g}}-1\right)}{c}}\right)^{\theta-1} = g$$

$$L = 1 - \left(1 - e^{-\frac{\lambda\left(e^{\frac{cR_0}{g}}-1\right)}{c}}\right)^{\theta} \tag{33}$$

Setting $\lambda = c = 1$, $\theta = 1/2$, $R_0 = 1$ and imposing Equation (6) with $\alpha = 1$, $C_1 = -1$, $C_2 = 3$ allows numerical solution to the third expression of Equation (33) for $g$ in terms of the arousal $\beta$, leading to the results of Figure 15. Here, a U-shaped hazard rate replicates the two narrow cognition rate peaks of Figure 5.

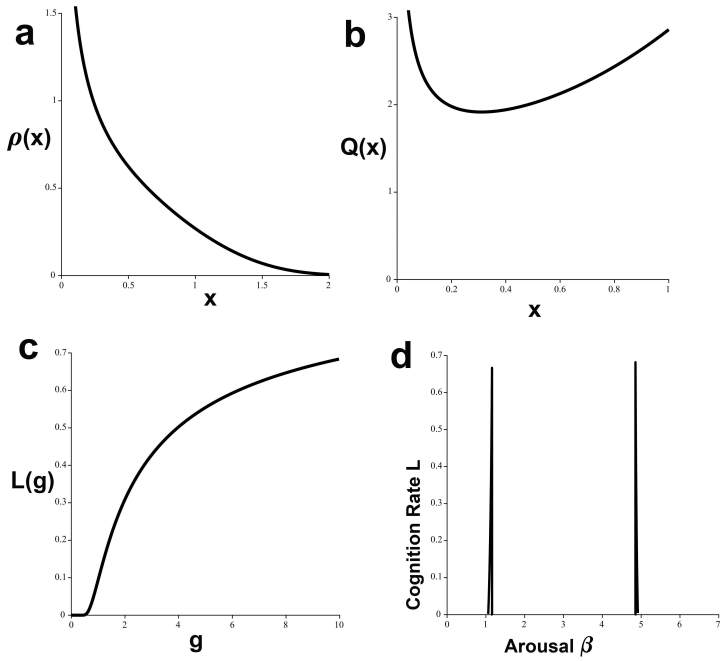



**Figure 15.** Cognitive dynamics for the generalized Gompertz Distribution, setting $\lambda = c = 1$, $\theta = 1/2$, $R_0 = 1$, under Equation (6). The relation between $F$ and $g$ has been solved numerically. (**a**) $\rho(x)$. (**b**) Hazard rate. (**c**) Cognition Rate as a function of $g$. (**d**) Cognition rate vs. 'arousal', replicating Figure 5.

*9.4. Russia and the Ukraine*

As compiled by Perplexity AI, 2/21/2025.

Recent research on institutional cognition... suggests that hierarchical, detailed command structures can be modeled using a multi-stage cancer model based on the Erlang

distribution [1]. This perspective provides an interesting lens through which to analyze the relative failure of the Russian military effort in Ukraine.

The Erlang distribution describes the probability of multiple independent random events occurring by a given time [3]. In the context of institutional cognition and military command, these 'events' can be interpreted as decision-making stages or information transmission steps within a hierarchical structure.

## 10. Russian Command Structure and Its Failures

The Russian military's command structure in Ukraine exhibited several characteristics that align with this multi-stage model:

1. **Hierarchical and Centralized Decision-Making:** The Russian forces relied heavily on a top-down approach, with detailed orders coming from higher echelons [6]. This aligns with a multi-stage model where each decision or order must pass through multiple layers before reaching the tactical level.

2. **Lack of Initiative at Lower Levels:** Russian units often stopped and waited for instructions from senior commanders when faced with unforeseen circumstances [10]. This behavior is consistent with a system where multiple 'events' (approvals or decisions) must occur before action can be taken.

3. **Delayed Response to Changing Situations:** The Russian military demonstrated poor adaptability to rapidly changing battlefield conditions [10]. In the Erlang distribution model, this could be interpreted as a system requiring too many stages (events) to complete before a response can be formulated.

4. **Vulnerability of Command Posts:** Ukrainian forces successfully targeted Russian command posts multiple times, significantly degrading their ability to plan and conduct coordinated operations [9]. This vulnerability is exacerbated in a system that relies heavily on centralized command, as modeled by the multi-stage approach.

## 11. Contrast with Mission Command

The Ukrainian military, on the other hand, adopted a more flexible approach aligned with modern mission command principles:

1. **Decentralized Decision-Making:** Ukrainian forces demonstrated a capacity for mission-oriented command, allowing for more autonomous operations at lower levels [6].

2. **Adaptability:** The Ukrainian military showed greater ability to adapt to changing situations, consistent with a system requiring fewer 'stages' or decision points.

3. **Resilience:** By distributing decision-making authority, the Ukrainian command structure was more resilient to disruptions, unlike the vulnerable multi-stage Russian system.

## 12. Implications

Viewing the Russian command failures through the lens of ... [a]... multi-stage model highlights how a hierarchical, detailed command structure can become a liability in modern warfare. Each stage in the decision-making process introduces potential delays and vulnerabilities, much like how multiple stages in the Erlang distribution model of cancer development increase the waiting time for an event to occur [4].

This perspective underscores the advantages of mission command, which reduces the number of 'stages' required for decision-making and action, thereby increasing responsiveness and resilience on the battlefield. The Russian experience in Ukraine serves as a stark illustration of how outdated command structures can significantly hamper military effectiveness in contemporary conflicts.

*Citations*

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