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Review

Assessment of Reliability Allocation Methods for Electronic Systems: A Systematic and Bibliometric Analysis

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Abstract: Reliability allocation is the process of assigning reliability targets to sub-systems within a system to meet the overall reliability requirements. However, many traditional reliability allocation methods rely on assumptions that are often unrealistic, leading to misleading, unachievable, and costly outcomes. This paper provides a historical review of reliability allocation methods, focusing on the Weighing Factor Method (WFM), with a detailed analysis of its main findings, assumptions, and limitations. Additionally, the review covers methods for reliability optimization, redundancy optimization, and multi-state system optimization, highlighting their strengths and shortcomings. A case study is presented to demonstrate how the assumption of an exponential distribution impacts the reliability allocation process, showing the limitations it imposes on practical implementations. Furthermore, a bibliometric analysis is conducted to assess publication trends in the field of reliability allocation. Through examples, particularly in the context of electronic systems using commercial off-the-shelf (COTS) components, the challenges are discussed, and recommendations for alternative approaches to improve the reliability allocation process are provided.

Keywords: electronic systems; reliability allocation; redundancy allocation; weighing factor methods; bibliometric analysis

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1. Introduction

Reliability allocation is a critical process in the design and development of complex systems and sets the reliability requirements for the sub-systems within a system to meet the system reliability target. There are other definitions as well, but these tend to be more restrictive. For example, Mettas and Savva, 2001 [1] state that "reliability allocation refers to the optimization process on the reliabilities of all or some of the sub-systems of a given system in order to meet the target of overall system reliability with minimum cost". Rajeevan et al., 2018 [2] define "reliability allocation as a process by which the failure allowance for a system is apportioned in some lucid manner among its sub-system and elements". The system reliability requirement is often computed in terms of failure rate, reliability, or unreliability [3–5]. The importance of reliability allocation stems from the need to balance the reliability of individual sub-systems with the overall performance and cost of the system. As systems become increasingly complex, especially with the integration of advanced technologies and commercial off-the-shelf (COTS) components, traditional reliability allocation methods face significant challenges. In this paper, the system could be a product, and thus, the sub-systems might be considered modules or components.

The process of reliability allocation typically begins during the conceptual design and development stages of a new system [6–8]. It involves determining the reliability targets for each sub-system based on the overall system reliability requirements and ensuring these targets are achievable. The allocation process typically involves establishing system reliability requirements, constructing a reliability block diagram, and allocating reliability to the sub-systems [9–11]. In its simplest form, for a series system in which the 'i'th sub-system has reliability R_i, the system reliability R_s can be described as $R_\mathrm{s} = R_1 \times R_2 \times \ldots \times R_\mathrm{n}$, where n is the number of sub-systems.

Several methods and approaches have been proposed to compute the reliability requirements of each sub-system in a system [11–14]. However, the reliability allocation methods are developed based on certain assumptions, such as sub-systems having constant failure rates and the sub-systems failing independently [15–17]. The effectiveness of reliability allocation methods depends on the type of data required, the inclusion of failure dependencies, the accuracy of predicted life cycle conditions, interactions between the sub-systems, and system configuration [18–20]. Limitations come from assumptions that do not fulfill actual practices. Applying methods based on inappropriate assumptions may cause incorrect reliability design, so those methods are applicable only to a few practical systems. Newly developed methods are focused on establishing rational assumptions, but such problems are not fully cleared.

Recent studies have highlighted the need for more accurate reliability allocation approaches that consider these factors. For instance, the limitations of conventional methods in handling complex system configurations and failure dependencies are emphasized in [21]. The challenges of applying traditional methods to COTS components and suggesting alternative approaches that better accommodate real-world conditions are discussed in [22]. Recent advances in system design, particularly in electronics and embedded systems, have exposed significant limitations in traditional reliability allocation methods. For instance, the increasing use of COTS components and the growing complexity of systems have highlighted discrepancies between theoretical models and practical applications. Studies have shown that conventional methods often fail to account for real-world factors such as failure dependencies, varying failure rates, and complex system configurations [18,19]. The motivation for this paper arises from the need to address these shortcomings and provide a more accurate and practical approach to reliability allocation. By examining the limitations of existing methods and proposing alternative strategies, this paper aims to contribute to improving reliability design practices and enhancing modern systems' overall reliability.

Traditional reliability allocation methods often rely on assumptions that do not accurately reflect the complexities of modern systems. These assumptions, such as constant hazard rates and the independence of sub-system failures, can lead to unreliable and costly design outcomes. For example, many methods assume that failure rates are constant, which is rarely true for electronic systems where failure rates can vary significantly over time [23,24]. Indeed, for electronic components or electronic modules (especially for integrated circuits), it is usually assumed that the time to failure has a negative exponential distribution. This means that the failure rate is constant for a given operating regime (and not a function of time). However, the authors believe this assumption is acceptable in many cases; rather, for mechanical elements, this assumption should be accepted with great caution due to the physical wear that may occur during system operation, in which case a Weibull distribution might be more appropriate. This simplifying assumption is often introduced so that sub-system-level reliability can be calculated analytically based on Markov models [25]. Additionally, traditional methods often fail to account for the interactions between sub-systems and the impact of failure dependencies, leading to potential inaccuracies in reliability predictions [26–28]. Indeed, in most research works, it is assumed that the failure events that can affect the system components are stochastically independent. But a common cause of failure for multiple components is usually captured in the reliability model by introducing an additional element in series, precisely to reflect this aspect [28,29]. In order to address the issues with reliability allocation methods, the following objectives are set:

- To comprehensively review traditional reliability allocation methods, including equal allocation, weighing factors, and ORA methods. This review will outline the assumptions, applicability, and limitations of these methods.
- To critically assess these methods' limitations, particularly in modern systems with complex configurations and COTS components.
- To propose recommendations for improving reliability allocation practices. This includes suggesting alternative approaches that better align with real-world conditions and addressing the identified limitations.
- To discuss the practical implications of the findings for system designers and engineers, providing insights into how they can enhance their reliability allocation strategies.

In this paper, the reliability allocation methods, namely (1) equal allocation, (2) weighing factors, and (3) reliability optimization, are described along with their assumptions, applicability, and limitations. Section 2 provides a bibliometric analysis based on research articles published in the Scopus database. Section 3 describes the traditional and non-traditional reliability allocation methods. Section 4 assesses the reliability allocation methods and addresses the associated problems. Conclusions and recommendations are presented in Section 5.

2. Bibliometric Analysis

Reliability and redundancy allocation [30] are critical aspects of system design, especially in complex engineering systems where failure can lead to significant consequences. Over the decades, numerous methods and techniques have emerged to optimize system reliability while balancing resource constraints [31]. A bibliometric analysis of research in this domain offers insights into publication trends, key contributors, and the evolution of methodologies [32–34]. This study aims to systematically review and quantify the research output on reliability and redundancy allocation, highlighting significant advances, research gaps, and emerging directions that can guide future research efforts.

The bibliometric analysis of research articles (retrieved from the Scopus database) in the reliability and redundancy allocation domain covers an extensive period from 1940 to 2025, with citation data spanning 84 years (1940–2024). A total of 16,242 papers have been published, generating 312,780 citations. The field sees 3723.57 citations annually, with each paper receiving 19.29 citations. The analysis reveals significant author engagement, with 102,027 citations per author and an average of 3.56 authors per paper. The field exhibits strong scholarly influence, as shown by an h-index of 185 and a g-index of 356, underscoring the high impact of key contributions.

The extended period of publication activity reflects the growing complexity of systems and the increasing importance of reliability and redundancy in various sectors, especially as technological advancements demand more reliable systems. The high h-index (185) and g-index (356) underscore the foundational role that a core set of highly cited papers has played in shaping the field. These indices highlight the long-term value of pioneering research in reliability and redundancy allocation [13]. The collaborative nature of authorship suggests that the challenges posed by reliability and redundancy optimization require multidisciplinary approaches, with researchers from various fields contributing to the body of knowledge. The growing number of citations and papers over the years suggests that research in reliability and redundancy allocation will continue to evolve. Future research may focus on emerging areas such as fault-tolerant systems, artificial intelligence-driven optimization [35], and hybrid systems, where reliability is a critical factor. The high citation rates indicate a strong foundation upon which further innovations in reliability engineering can be built.

2.1. Country-Wise Co-Authorship Network Analysis

Figure 1 presents a co-authorship network visualization based on the number of publications related to "reliability allocations" across various countries, indicating global collaboration patterns in this research domain. China stands out as the most prominent contributor, having a significant number of publications, as indicated by its large node size [36]. United States, India, and Germany also show considerable research activity in this field, with strong connections to other nations. There is extensive collaboration between countries, as shown by the lines (edges) connecting the nodes. The thickness of these lines indicates the strength of cooperation. The United States and China have many connections, showing active partnerships with countries like India, Germany, and the United Kingdom. Germany, France, and Italy form a cluster, indicating close collaboration within Europe. Iran, Malaysia, and Taiwan also show significant regional collaboration, especially with China and South Korea. South Korea, Australia, and Canada are also actively engaged in this research domain, collaborating with larger countries and contributing to the global research output. This visualization highlights the global nature of research in reliability allocations. China and the United States play a central role, collaborating with various nations across continents. The interconnectedness of the countries suggests that reliability allocation research benefits from international collaboration, with a few central nations leading the research and others contributing regionally or through collaborations with larger research hubs.

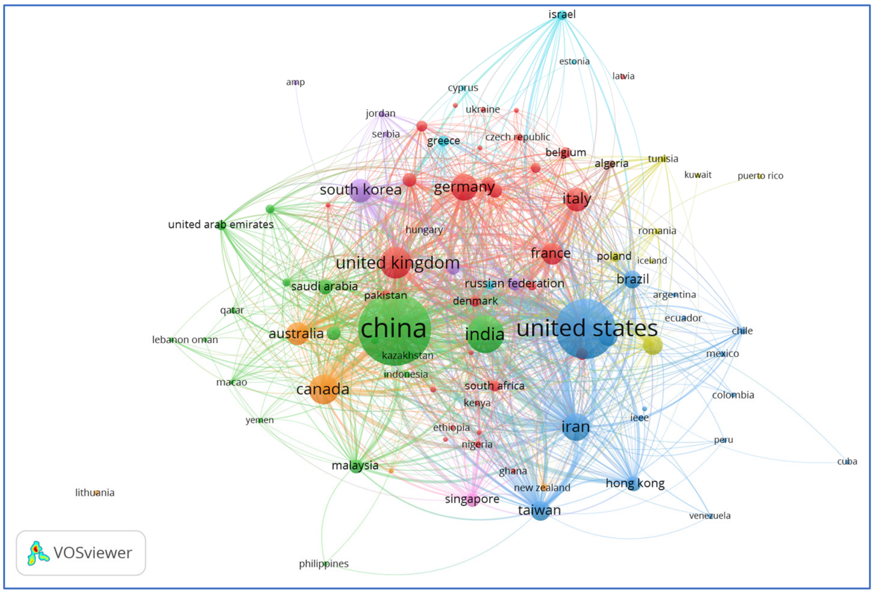

Figure 1. Co-authorship network visualization based on the number of publications related to "reliability allocations" across various countries.

2.2. Number of Citations per Country

Figure 2 shows the citation network of articles in the domain of "reliability allocations" across different countries. China and the United States dominate in terms of citations, with strong global collaborations, as depicted by large nodes and numerous connecting lines. Germany, India, and South Korea also have significant contributions. The network illustrates that research in this domain is highly interconnected, with international collaborations enhancing citation impact [37]. It also suggests that central countries serve as research hubs, influencing global reliability research trends.

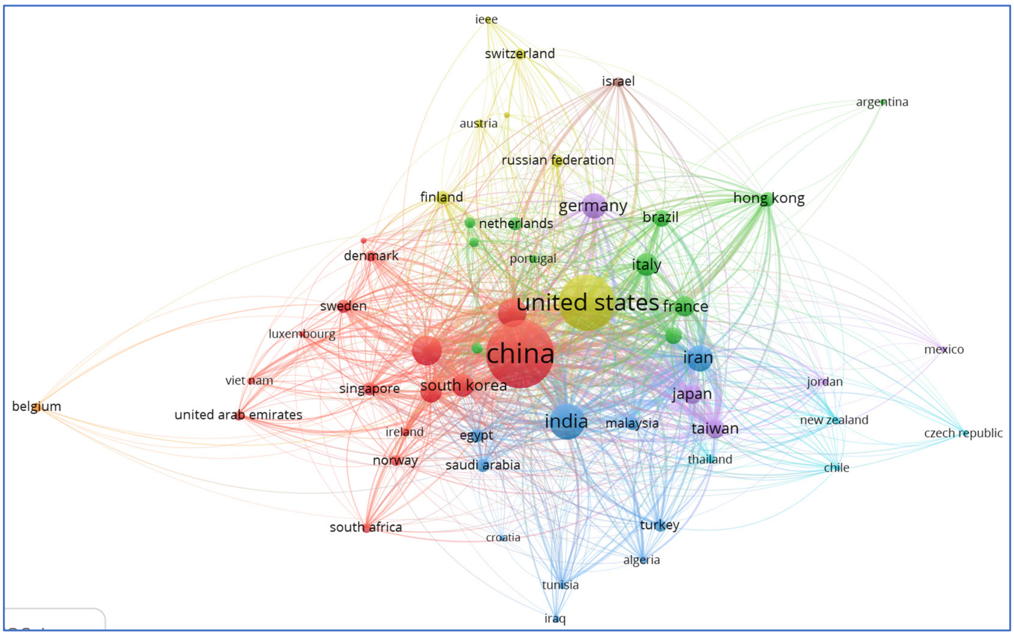

Figure 2. Citation network of articles in the domain of 'reliability allocations' across different countries and regions.

2.3. Co-Occurrence of Author's Keywords

Figure 3 illustrates the co-occurrence of keywords in articles related to "reliability allocations", revealing clusters of frequently associated terms. The size of the bubbles represents the frequency of the keywords, and the distance between bubbles indicates the strength of their association. Reliability and resource allocation [38] appear as central themes, with strong links to topics like optimization [39], genetic algorithms, and redundancy allocation [40]. Cloud computing, 5G, and machine learning [41] are also prominent emerging domains, indicating their growing relevance in reliability studies [42]. Observations suggest a convergence of classical reliability methods with modern technological challenges, while interpretations imply that future research may increasingly integrate machine learning and advanced algorithms into reliability and resource allocation frameworks. Optimization, machine learning, and deep reinforcement learning suggest a focus on improving reliability through intelligent algorithms. Terms such as "resource allocation", "power allocation", and "fault tolerance" highlight the importance of effective resource management for reliability. "Energy efficiency" and "distributed generation" indicate a growing concern for sustainable and efficient reliability solutions [38]. The visualization highlights the interconnections between various aspects of reliability allocation, emphasizing the need for optimization, efficient resource management, and consideration of energy efficiency and emerging technologies.

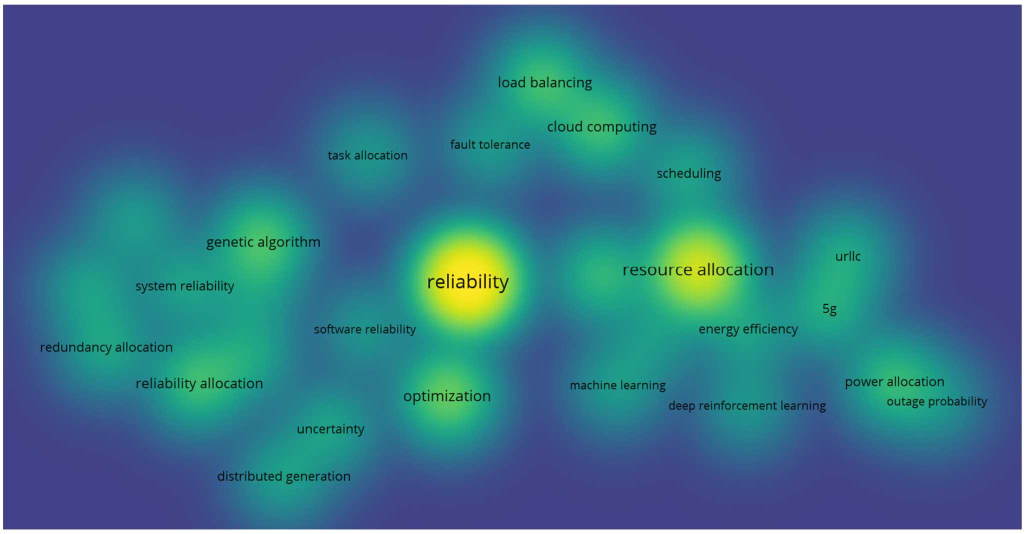

Figure 3. The co-occurrence of keywords in articles related to "reliability allocations", revealing clusters of frequently associated terms.

2.4. Number of Articles Published in Journals

Figure 4 shows the co-occurrences of journals in the field of reliability and redundancy allocation. The size of the bubbles represents the number of publications, and the thickness of the connecting lines indicates the strength of the association between journals. The journals "IEEE Transactions on Power Systems" and "IEEE Transactions on Vehicular Technology" occupy central positions, suggesting their significant influence in the field. "IEEE Transactions on Power Systems" and "Reliability Engineering and System Safety" form a cluster, indicating a strong focus on power system reliability. "IEEE Communications Magazine" and "IEEE Transactions on Computers" form another cluster, suggesting a connection between reliability and information technology. "Journal of Clinical Epidemiology" appears somewhat isolated, indicating a potential connection between reliability engineering and healthcare. The timeline included in the visualization shows a gradual increase in publications over the years, suggesting a growing interest and research activity in the field. The visualization highlights the interconnections between various journals in the field of reliability engineering and system safety, emphasizing the importance of power systems, communications, and computers in this domain. The growing number of publications suggests a continued evolution and expansion of research in the domain of reliability and redundancy allocation.

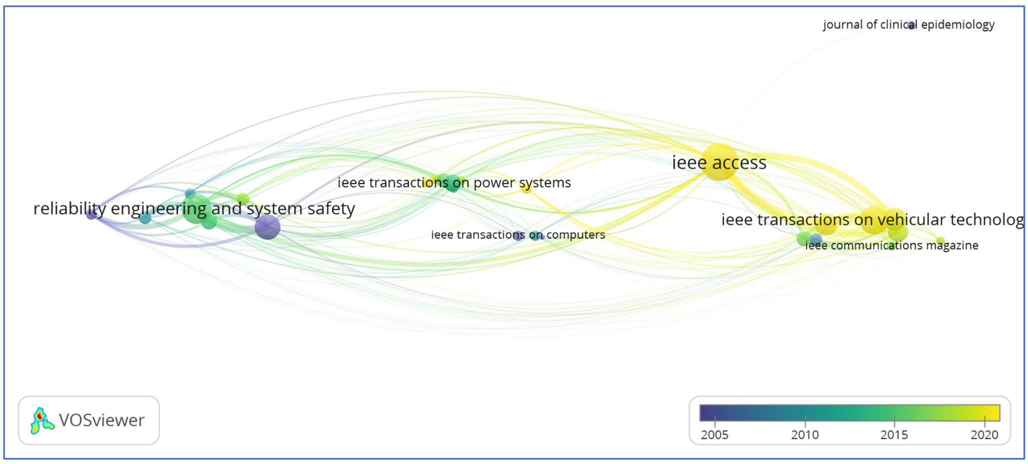

Figure 4. Co-occurrences of journals in which articles related to reliability and redundancy allocation were published.

3. Reliability Allocation Methods

Reliability allocation methods are broadly classified into three groups: equal allocation method [43,44], weighing factors methods [9,45], and optimal reliability allocation (ORA) methods [3,46], as shown in Figure 5. The methods are classified based on their approach and applicability. The equal allocation method distributes the reliabilities uniformly among all sub-systems. The weighing factor method (WFM) often entails using subjective and objective assessments to estimate the weight or allocation factors, which are thereafter used to calculate the failure rates of the sub-system. The objective assessments require past failure data and model definition, whereas the subjective assessments depend on expert judgments. The ORA uses programming techniques to determine the best number of sub-systems and their reliability ranks that, given certain limitations/constraints, either optimize system reliability or reduce system cost. Constraints, including cost, size, and weight, are also considered to meet system reliability requirements.

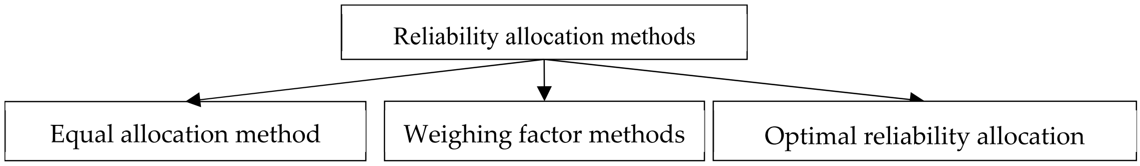

Figure 5. Types of reliability allocations.

3.1. Equal Allocation Method

The equal allocation method, also called the equal apportionment method, equally weighs all the serially connected sub-systems within a system [47,48]. A mathematical expression of the equal allocation method for a series system is given by Equation (1).

$$R_s = \prod_{i=1}^{n} R_i' \quad or \quad R_i' = (R_s)^{1/n} \tag{1}$$

where R_s—the system reliability goal, and R_i'—the sub-system reliability and n is the number of serially connected sub-systems.

Consider a system having three serially connected sub-systems that have a reliability goal of 0.9 for a warranty period of one year under the stated working conditions. The equal reliability allocation requires each sub-system to have a reliability of 0.9655 for the given warranty period.

Allocating equal reliability to all sub-systems is simple, but difficult to meet assigned estimates as different sub-systems have different materials, manufacturing processes, operating and loading conditions, failure mechanisms, and degradation rates [49,50]. As a result, sub-system goals are not given based on the degree of difficulty associated with achieving these goals [50]. The contribution of each sub-system toward the system's functional requirements may not also be the same. The equal allocation method does not consider the degree of difficulty associated with the achievement of these goals [51,52]. For example, smartphone sub-systems such as display, battery, and main logic board will have different functional requirements, failure mechanisms, and hazard rates; therefore, these sub-systems cannot be designed and manufactured with the same reliability. The equal allocation technique also presupposes that the sub-systems have a constant failure rate. The equal allocation approach presupposes that the system's sub-systems are connected in series; hence, it is not relevant to redundant, series-parallel, or complicated systems.

3.2. Weighing Factor Methods (WFMs)

The WFMs consider the significance of each sub-system's performance criteria to assign the system reliability requirements to the sub-systems. The system design team, based on the defined criteria and their judgments, needs to estimate the weighing factor for each sub-system numerically. The sub-systems are assumed to be connected in series and have constant failure rates [53]. The weighing factors of the sub-systems are calculated based on the failure data (subjective) and expert judgment (objective) assessments, and are normalized so that their sum is one [14,16]. The reliability requirement of a sub-system is then estimated as the system of the sub-system's weighing factor and the system reliability requirement. This section presents the widely used WFMs.

3.2.1. ARINC Method

ARINC method, developed by Aeronautical Radio, Incorporated (ARINC), expresses the reliability requirements in terms of failure rate [11,54]. It is assumed that the sub-systems have a constant failure rate (exponential distribution) [55]. This method can be used to serially connect sub-systems having independent failure rates [56]. The sub-system's failure rate to be allocated λ_i^* is estimated by multiplying the sub-system's weighing factor w_i to the system's maximum allowable failure rate λ^* given by Equation (2). The weighing factor w_i of a sub-system, i using the sub-system's known failure rate λ_i is estimated using Equation (3). As the weighing factors are estimated based on the sub-system's known failure rate, hence the applicability of the ARINC depends on the availability of field failure data of similar sub-systems [57].

$$\lambda_i^* = w_i \lambda^* \qquad \textit{such that} \ \sum_{i=1}^{n} \lambda_i^* \leq \lambda^* \tag{2}$$

$$w_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i} \tag{3}$$

3.2.2. AGREE Method

The AGREE method was formulated by the Advisory Group on Reliability of Electronic Equipment (AGREE). This methodology operates under the assumption that sub-systems possess constant failure rates and are interconnected in series [26,58]. System complexity is gauged by the number of sub-systems involved. The weighing factor, denoted as E_i, for each sub-system i, signifies the probability of system failure in the event of that particular sub-system's failure [59]. A weight of one indicates the essentiality of the sub-system for the smooth operation of the system, while a weight of zero implies a negligible impact on system functionality. The AGREE allocation method is articulated as follows:

$$\lambda_i = \frac{n_i \left[-log_e R_p(T) \right]}{N \times E_i \times t_i} \tag{4}$$

where λ_i—the allocated failure rate of sub-system i, n_i—the number of units in sub-system i, $R_p(T)$ signifies the system reliability at time t, t_i indicates the duration (in hours) for which sub-system i will be operational within T system hours, N represents the total number of sub-systems within the system.

The AGREE approach can be employed when information is available on the system complexity and the value of each sub-system to system success [60]. The accuracy of this method is low for the redundant systems, and the sub-system's importance factor is close to one [50,61].

3.2.3. Bracha Method

This method can be used only for serially connected sub-systems and involves determining four weighing factors: cutting-edge technology (A), intricacy (C), environmental circumstances (E), and functioning time (T) given by Equation (7) [3,62]. State-of-the-art technology (A) assesses the level of engineering advancement for each sub-system. Complexity (C) evaluates the number of units in a sub-system and their difficulty, as determined by Equation (5). Environmental circumstances (E) account for the sub-system's operational severity, determined by Equation (6). With these factors determined, the I_i index and the weighing factor w_i for the sub-system level can be calculated using Equation (8). Equation (9) provides the mathematical expression used to allocate system reliability $R_S(t)$ to its sub-systems $R_i(t)$.

$$C = 1 - e^{[-K_b + 0.6 \times K_p]} \tag{5}$$

where K_b—the ratio of the number of units in the specified sub-system to the total number of units in the entire system; K_p—the ratio of the number of redundant units in a certain sub-system to the total number of redundant units in the system.

$$E = 1 - \frac{1}{f} \tag{6}$$

where f is the external stress (0: minimum stress; 100: maximum stress).

$$T = \frac{T_m}{T_u} \tag{7}$$

where T_m is the system's mission total time and T_u sub-system operative time.

$$I_i = A_i \cdot (C_i + E_i + T_i); \quad w_i = \frac{I_i}{\sum\limits_{j=1,2,\ldots,n} I_j} \tag{8}$$

$$R_i(t) = R_S(t) w_i \tag{9}$$

3.2.4. Feasibility of Objectives (FoS) Method

The FoS method was developed to validate design feasibility and establish a reliability program to meet the system's reliability requirements [45,63]. In this approach, each sub-system's weighing factor is determined based on its relative difficulty in achieving the desired system reliability. The weighing factor for sub-system i is assessed through four factors: complexity (I_i), cutting-edge technology (S_i), task time (P_i), and environmental circumstances (E_i). Design engineering and expert judgments rate each factor on a numerical scale from one to ten, with detailed rating guidelines provided in referenced sources [45,50,64–66].

Intricacy is evaluated considering the likely number of units comprising the sub-system and the assembled complexity of these sub-systems. A sub-system with minimal complexity is rated as one, while a highly complex one is rated as ten. State-of-the-art reflects the current engineering progress across all fields, with a rating of ten assigned to the least developed design and one to the most advanced. Mission time rates the duration for which a sub-system operates during the mission, with a rating of ten for continuous operation and one for the shortest operating time. Environmental conditions are also rated from ten (harsh environments) to one (least severe environments) based on expected operational conditions.

The overall rating for a sub-system is determined by multiplying the ratings of these four factors (as given in Equation (10)), and then the sub-system's weighing factor is calculated by normalizing the ratings to ensure their sum equals one. Finally, the sub-system's failure rate (λ_i) to be allocated is determined using Equation (11), assuming all sub-systems have a constant failure rate.

$$w_i = \frac{I_i \cdot S_i \cdot P_i \cdot E_i}{\sum\limits_{i=1}^{k} I_i \cdot S_i \cdot P_i \cdot E_i}, \qquad i = 1, 2, \ldots, k \tag{10}$$

$$\lambda_i = \lambda \cdot w_i, \qquad i = 1, 2, \ldots, k \tag{11}$$

The FoS method is a powerful approach for evaluating the impact of redundancy on system reliability, particularly in series-parallel configurations. This method focuses on optimizing system reliability while adhering to predefined constraints, such as cost, weight, and volume, which are often critical in engineering design. In a series-parallel configuration, system reliability depends on the reliability of individual sub-systems connected in parallel to enhance redundancy. The FoS method enables designers to determine the optimal number and configuration of redundant components that maximize overall system reliability without exceeding resource limitations. By incorporating redundancy into reliability modeling, the method evaluates trade-offs between improved reliability and associated costs or complexities.

3.2.5. Hybrid Approaches

Hybrid approaches attempt to use the results of criticality or risk priority assessments derived from methods such as the Analytic Hierarchy Process (AHP) [67] and Failure Modes and Effects Analysis (FMEA) to estimate weighing factors that can be allocated to the sub-systems [9,65,68,69]. Yadav et al. [65] divide the system into three dimensions: functional requirements, physical structure, and time in service, and recognize the precarious sub-systems and failure modes that need to be considered during the allocation process at the design stage. Based on the supposition of linearity in rating scales, Itabashi-Campbell and Yadav [70] assign reliability to the sub-system using the Risk Priority Number (RPN) rank of FMEA. In practical terms, there is a nonlinear relationship between any prospective improvement in reliability and any of these scores. It can result in the sub-systems being assigned dependability requirements that are either greater or lower. The nonlinearity relationship problem was also tackled by Kim et al. [16], who proposed a reliability allocation strategy based on the relative frequency and severity of sub-system failures.

Several other WFMs have been published by many researchers. Including the critical flow method [62], the analytic critical flow method based on the analytic hierarchy process [71], the analytic integrated factor method [66,72], and the maximum entropy method ordered weighing averaging [45,73,74]. The WFMs try to overcome a few of the limitations of the equal allocation method. However, the WFMs were developed based on the assumption that the sub-systems are serially connected with a constant failure rate, which does not represent electronic systems. They also assume that the failures occur independently. The problems of the WFMs are addressed in Section 4.

Table 1 summarizes the various reliability allocation studies conducted using the Weighing Factor Method (WFM). The WFM literature review reveals diverse applications and adaptations of this approach across various fields, including automotive, aerospace, telecommunications, and electronic systems. Many studies highlight the integration of the WFM with other methodologies like AHP and FMEA to enhance reliability allocation and consider design constraints, such as cost, size, and operational factors, improving the WFM's practical applicability in real-world scenarios. Most studies do not address system redundancy, which is critical for improving reliability in more complex configurations. Few papers consider failure dependencies between sub-systems. This is a significant limitation as real-world systems often exhibit failure dependencies. The application of the WFM is predominantly limited to series configurations. Studies highlight the need for methods applicable to parallel or mixed configurations. Many studies assume constant failure rates, which might not be realistic for all types of systems, especially those with varying operational conditions. Research shows that WFM's applicability is limited when it comes to systems with redundancy or complex configurations. The assumption of independent failures overlooks the reality that sub-systems often exhibit dependencies that affect overall system reliability.

Table 1. Review of the WFM for their main findings, assumptions, and limitations.

Ref.	The Methods Used, Main Findings, and Limitations	Exponential Distribution	Failure Dependency	Series Configuration	Essential Factors Defined	Design Constraints	Sub-System Significance	Functional Requirement Parameters	Redundancy Information
[21,67]	Proposed approach for the WFM reliability allocation using hybrid AHP and GRA	Yes	No	Yes	Yes	No	Yes	Yes	No
[75]	FTA and ANN are used for the analysis of NC gear hobbing machines; need to collect more actual reliability data to enrich the model.	Yes	No	Yes	Yes	No	No	Yes	No
[76,77]	FMEA and T-spherical fuzzy sets techniques are used;	Yes	No	Yes	Yes	No	Yes	No	No
[78]	Expert judgment-based FMEA; the method is effective in the case of the iterative process.	Yes	No	Yes	Yes	No	Yes	Yes	No
[45]	Proposed Maximal Entropy Ordered Weighted Averaging was applied to fighter aircraft airborne radar systems.	Yes	No	No	No	No	No	No	No

Table 1. *Cont.*

Ref.	The Methods Used, Main Findings, and Limitations	Exponential Distribution	Failure Dependency	Series Configuration	Essential Factors Defined	Design Constraints	Sub-System Significance	Functional Requirement Parameters	Redundancy Information
[73]	The ME-OWA method is applied to the fighter aircraft's digital flight control computer.	Yes	No	No	No	No	No	No	No
[79]	Analytic algorithms and Monte Carlo simulation are applied to calculate the reliability of such a complex network structure.	Yes	No	No	No	No	No	No	No
[60]	The AGREE method is used to allocate reliability to the CNC cylindrical grinder.	Yes	No	No	No	No	No	No	No
[80]	The ordered weighted averaging tree and soft set approach with regard to flexible allocation product reliability.	Yes	No	No	No	No	No	No	No
[20]	Reliability allocation (RA) procedures are top-down techniques that allow the system reliability goal to be apportioned between its components.	Yes	No	No	No	No	No	No	No

Table 1. *Cont.*

Ref.	The Methods Used, Main Findings, and Limitations	Exponential Distribution	Failure Dependency	Series Configuration	Essential Factors Defined	Design Constraints	Sub-System Significance	Functional Requirement Parameters	Redundancy Information
[26]	Verification of the model under various boundary conditions has been carried out using the available data in the aerospace domain.	Yes	No	No	No	No	No	No	No
[81]	The AGREE method is used to allocate reliability to the high-speed train gear transmission system.	Yes	No	No	No	No	No	No	No
[82]	Reliability allocation method for traction power supply system of electrified railways in mountainous areas.	Yes	No	No	No	No	No	No	No
[83]	In a clearing system, both with and without machine failures, we show the existence of an optimal monotone policy.	Yes	No	No	No	No	No	No	No
[84–86]	Fuzzy linguistics and triangular fuzzy numbers are used to evaluate the uncertainty and subjective factors in the allocation process.	Yes	No	No	No	No	No	No	No

It is essential to develop methodologies that integrate redundancy and can be applied to parallel or mixed configurations. This would enhance the flexibility and applicability of the WFM. Several enhancements should be considered to improve the WFM for reliability allocation. First, incorporating failure dependency models into the WFM can better represent real-world systems by accounting for interdependencies between sub-systems. This may involve developing new algorithms or modifying existing ones to capture these dependencies accurately. Additionally, dynamic models that accommodate varying failure rates based on operational conditions and environmental factors should be explored. Moreover, there is a need to create detailed frameworks that systematically integrate all design constraints and functional requirements, ensuring that these factors are consistently accounted for in the WFM. Establishing guidelines or standards for this process could provide a structured approach.

3.3. *Optimal Reliability Allocation Method*

The allocation of individual sub-system dependability to achieve a desired degree of system reliability or cost, according to a set of restrictions like cost, weight, and volume, is generally the focus of Optimal Reliability Allocation (ORA). The methods used for reliability optimization, redundancy optimization, and multi-state optimization are summarized in Table 2. Three essential components are needed for an effective system dependability design: restrictions, an objective function or functions, and decision variables [87]. The variables that can be changed or choices that may be made to improve performance concerning the goal function are known as the decision variables. The sub-system type, reliability, redundancy configuration, and failure response are among the deciding variables. The choice of decision variables may be constrained by factors like financial constraints or a system's acceptable level of reliability. The system cost to be decreased, or the system dependability to be maximized might frequently be the objective function. Finding the best combination of variable values is made easier by the objective function, which assesses the system's performance based on the given values of the decision variables. Generally, the objective function can be thought of as either the system cost to be minimized or the system reliability to be increased.

3.3.1. Reliability-Based Optimization in Practical Engineering Applications

Reliability optimization plays a critical role in enhancing system performance while managing constraints such as cost and resource availability [3,88]. The objective function in reliability allocation typically involves minimizing system cost or maximizing reliability. Recent advancements have integrated sophisticated algorithms and adaptive techniques to improve optimization strategies [89,90]. For instance, hybrid adaptive Kriging combined with a water cycle algorithm has been applied to optimize the design of offshore wind turbine monopiles, addressing complex reliability-based design challenges. Similarly, Kriging-assisted hybrid reliability optimization, using a portfolio allocation strategy, has enhanced the structural reliability of offshore wind turbine support systems [91,92]. Additionally, multi-strategy enhanced circulatory system-based optimization (MECSBO) has demonstrated effectiveness in solving global optimization and reliability-based design problems [93]. These advanced methods provide dynamic, efficient, and precise approaches to reliability allocation, leveraging predictive modeling and simulation capabilities [94]. Incorporating these strategies into reliability allocation frameworks enhances decision-making accuracy, offering significant potential for future engineering applications and broader adaptability across diverse industries.

Table 2. Methods used for reliability optimization, redundancy optimization, and multi-state optimization.

Optimization Parameter	Details/Focus	Methods/Techniques
Reliability Optimization	focuses on improving the reliability of a system by optimizing design parameters, resource allocation, and operational strategies [95]	Genetic Algorithms (GAs); Simulated Annealing (SA); Particle Swarm Optimization (PSO); Linear Programming (LP); Dynamic Programming (DP);
Redundancy Optimization	involves designing systems with redundant components to increase reliability while optimizing the cost and effectiveness of redundancy	Mixed Integer Linear Programming (MILP); Fault Tree Analysis (FTA); Reliability Block Diagram (RBD); Genetic Algorithms (GAs).
Multi-State Optimization	deals with systems that can exist in multiple states beyond simple operational or failure states	Multi-State Reliability Modeling; Multi-State System Optimization; Stochastic Programming; Markov Decision Processes (MDPs) [96,97]; Monte Carlo Simulation;

ORA-based design methods assign redundancy based on sub-system-level reliability and aim to determine the trade-off between the system performance and the availability of resources. Several papers published since 1960 have addressed the redundancy allocation issues using different system configurations, structures, performance measures, optimization techniques, and other methods used for reliability optimization [98,99]. Tillman et al. [18] classified the heuristics methods developed to solve complex reliability optimization problems along with their benefits and shortcomings. The applications of different methods used for ORAs until 2000 are summarized in [100,101] and from 2000 to 2007 in [46]. Most recent works for ORA methods are more dedicated to active and standby redundancy, multi-objective optimization, multi-state system (MSS) optimization, fault-tolerance mechanisms, and hybrid optimization methods. Some key contributions made after 2007 and the most recent works available in the literature are highlighted in this section.

The ORA method is used to estimate the optimum number of sub-systems and their reliability levels by applying mathematical programming techniques [98]. In this method, reliability allocation to individual sub-systems is carried out in order to meet the desired system reliability level, subject to a set of constraints such as cost, weight, and volume [102,103]. This method aims to maximize system reliability or minimize system cost within given constraints. The reliability optimization process commences with constructing a model representing the entire system. This involves creating a system reliability block diagram that illustrates the reliability relationships among the sub-systems within the system. The potential impact of various sub-system modifications on system reliability can be estimated and weighed against the associated costs. Subsequently, an optimization analysis is conducted to identify the optimal combination of sub-system reliability enhancements that either meet or surpass the performance objectives at the lowest cost.

3.3.2. Redundancy Allocation and Optimization

The redundancy allocation and optimization methods can be used for simple systems that have series or parallel configurations; however, it is hard to apply these methods to complex configured systems. Dynamic programming, integer programming, mixed-integer, and nonlinear programming, as well as evolutionary algorithms, are the approaches used to solve complex equations [34,104]. Finding efficient optimization algorithms is always a challenge. Classification of the literature by reliability–redundancy optimization techniques is summarized in [105]. Metaheuristic methods, especially genetic algorithms, are widely used due to their robustness, although they are time-consuming. Several improvised algorithms are also developed to achieve an effective combination of genetic algorithms with heuristic algorithms, simulation annealing methods, neural network techniques, and other local search methods [106–109]. Such algorithms apply to active redundant systems, but it is unclear how they are applied to passive redundant systems. It is mentioned that "the dynamic programming methods used for reliability allocation are impractical if the number of variables is more than three" [54]. Therefore, this raises the question of how this method could be applied to complex systems with several variables.

Redundancy allocation problems focus on assigning redundant sub-systems in parallel or combining structural redundancy to achieve the desired system reliability. The research specifically addresses the case of reliability redundancy allocation problems and does not delve into alternative approaches involving combined sub-system enhancements. The emphasis is on general reliability redundancy allocation problems rather than other forms of redundancy allocation. In practice, two main redundancy schemes are available: active and cold standby. While cold standby redundancy offers higher reliability, it is challenging to implement due to difficulties in detection failure. Reliability design problems typically consider only active redundancy, but the optimal design may incorporate either active redundancy, cold standby redundancy, or a combination of both. However, any improvement effort usually requires resources. In many cases, a single objective may not adequately capture the complexities of a real-world problem requiring optimal design.

As used by [110], the Sharma and Venkateswaran method [111] is a simple heuristic method that may be applied to multistage system problems with an arbitrary number of constraints. Essentially, the method involves augmenting redundancy to the stage with the lowest reliability. The first step in the process involves allocating a redundant sub-system to the stage with the lowest reliability. If any constraint is breached, the most recently added redundant sub-system is removed. The subsequent count represents the optimal allocation for that stage, which is then excluded from further consideration. If all stages are eliminated from consideration, the current count of allocated redundant sub-systems represents the optimal system configuration. Otherwise, the process iterates.

Problems involving multistage series systems with nonlinear constraints have been successfully solved using this technique [110,112]. It was used to formulate a minimization issue subject to two exponential function constraints for a five-stage system configuration with three nonlinear constraints, and it produced an optimal solution. Additionally, it has been applied to bridge configurations and multistage series configurations, producing suboptimal solutions in both scenarios. Nevertheless, the approach was used by Nakagawa and Miyazaki [107] to solve a parallel-series reliability issue with a single linear constraint, and it was observed that as the number of stages increases, this method may generate inaccurate solutions.

3.3.3. Zero-One Integer Programming Methods

Zero-one integer programming methods are particularly effective in addressing Reliability Allocation Problems (RAP) and Total Reliability Allocation Problems (TRAP) [113], especially in systems requiring binary decision-making, such as whether to incorporate redundancy into specific components [114]. Unlike general linear programming approaches, which are suitable for continuous variables, zero-one integer programming provides a more precise modeling framework by explicitly handling discrete variables [115]. This enables better alignment with practical reliability optimization scenarios where design choices involve selecting or rejecting individual reliability-enhancing strategies [25].

3.3.4. Multi-State System Optimization

The classical reliability analysis and allocation consider only two states of the system, i.e., operating or failed [116]. The binary-state system reliability analysis methods play a significant role in practical reliability engineering [117]. However, complex systems may have more than two states. In the literature, complex systems are called MSSs, and the system performance level can be expressed as degraded states (e.g., excellent, very good, good, average, and poor) [18,118]. The MSS analysis methods have been developed since the 1970s. Since 1988, several methods and algorithms have been developed, and their applicability for reliability optimization and optimal design has advanced [105,110–112]. Determining the relationship between the states of the sub-systems and the system is the main goal of the MSS reliability optimization; in particular, several efforts have been made to solve reliability allocation problems for mixed-configured systems and MSSs [119]. The meta-heuristic approach is often used to solve such problems; however, it is a time-consuming job, and approximated algorithms or guidance are not proposed [120–122]. Furthermore, reliability is studied as the main performance measure. However, the system's availability needs to be considered as one of the performance measures of the MSS. Furthermore, maintainability parameters such as mean-time-to-failure (MTTF) and percentile life need to be taken into account when the system task period is unspecified, as in maximum real-world cases. "However, the evaluation of the MSS reliability is more difficult and potentially mathematically cumbersome" [102]. Various studies conducted on methods for reliability optimization, redundancy optimization, and multi-state optimization are summarized in Table 3.

Markov processes are widely used for modeling and analyzing reliability in the MSS due to their ability to capture system behavior where components or sub-systems can exist in multiple performance states rather than a binary functioning or failed condition [96,97]. Unlike traditional binary-state reliability models, the MSS models account for partial failures and performance degradation, providing a more comprehensive view of system reliability [123]. Markov processes represent state transitions as stochastic events governed by transition probabilities or rates, enabling dynamic modeling of time-dependent reliability [124]. In reliability modeling, each state represents a specific performance level of a system or component, with transitions between states driven by failure and repair processes. The steady-state probabilities derived from solving the Markov model allow the calculation of key reliability metrics such as system availability, mean time to failure, and failure frequency. Additionally, Markov processes can incorporate dependencies between components by defining transition rates that reflect shared failure mechanisms or common causes, offering more accurate reliability assessments for complex engineering systems.

Table 3. Review based on methods for reliability optimization, redundancy optimization, and multi-state optimization.

Ref.	Methods Used	Name of the Systems/Product	Main Findings	Limitations/ Assumptions/ Considerations	Scope for Future Work
[125]	Maximum likelihood estimators and uniformly minimum variance unbiased estimators	Gas pipeline	Optimize system reliability and cost	Simulation-based study; Pumps exceed the period of infant mortality but fail before the period of wear-out.	More complex real-life reliability problems; availability optimization
[126]	Analytical target cascading-based exterior penalty function (ATC-EPF)	Parallel system	Highly efficient	Real-life applications missing	Real-life applications
[127]	Weibull probability density function (PDF)	Wind Turbine (WT) subassemblies'	The model can combine permanent and intermittent faults	Human factors need to be considered; Sub-systems are connected in series	Maintenance strategies need to be developed.
[128]	A reliability redundancy allocation problem (RRAP) methodology is proposed.	Two experiments (details are not mentioned)	Highly efficient	Series-parallel system	There is a need to validate this by applying it to a real-time system.
[129]	Edge-computing data flows	Active data-caching systems	Total network throughput increased by up to 25%.	A limited number of data frame	Exploit multi-modal data in the MEC networks to further improve the proposed model.
[130]	A novel study of the RRAP as a many-objective optimization problem formulating the many-objective RRAP (MaORRAP).	Over-speed gas turbine systems, pharmaceutical plants, and large-scale systems.	--	Series-parallel systems, complex bridge systems	The model can be applied to fault tolerance, thermal stability, and energy efficiency.

Table 3. *Cont.*

Ref.	Methods Used	Name of the Systems/Product	Main Findings	Limitations/ Assumptions/ Considerations	Scope for Future Work
[131]	General network reliability model	Power grid system	System reliability has increased by more than 60%	Protection strength increases linearly with the number of resources spent on protection.	Cost–benefit analysis
[132]	RRAP	Multi-state flow network (MFN)	A prior estimate and a posterior check are conducted to ensure a feasible solution with a high probability.	Simulation-base study; the graph is connected and has no self-loops;	The methodology can be applied to more redundancy strategies, such as mixed, K-mixed, and G-mixed strategies.
[95,133]	Particle swarm optimization algorithm	Reliability–redundancy allocation problem (RRAP)	Particle swarm optimization with more uniform initial particle distribution converges faster and better	Individual link sub-systems and assume that all nodes are perfect; the probability of failure is zero until the standby component is required to operate as a replacement for a failed component	Incorporate heterogeneous components into the RRAP and apply more redundancy strategies in system design.
[106]	Integer Programming, Monte Carlo Simulation	Communication Networks	Identified optimal redundancy levels for minimizing downtime	Exponential failure rates assumed; Computationally expensive for large networks	Develop faster algorithms for large-scale networks

4. Problems of Reliability Allocation Methods

Reliability allocation methods are developed based on several assumptions and limitations such as the assumption [11] of constant failure rates, failures are independent, sub-systems within the system have series configuration, unavailability of reliability data, and approach used to estimate failure rates as discussed in Section 2. The reliability allocation methods are also typically developed for non-repairable systems, whereas many real-world systems are repairable [134]. This section presents the problems of the reliability allocation methods. The effect of assumptions on the allocated reliability and its propagation from the system level to the sub-system level is also assessed.

4.1. Reliability Data and Distributions Under Considerations

The WFMs do not specify anything about the type of data required; for example, how the data are to be collected and combined, and the life cycle conditions under which the data should be collected [135,136]. Such information plays a vital role, as most of the reliability allocation needs to be performed at the design stage. This may raise concerns about the validity of the failure data (assumed life-cycle conditions and actual life-cycle conditions). The ARINC method assumes that the mission time for each sub-system equals the system mission time [8,119]. Furthermore, the ARINC method largely depends on the availability of bills of materials or field data from similar sub-systems.

The reliability requirements of all types of sub-systems within the system are estimated assuming a constant failure rate (exponential distribution) [137,138]. This assumption is misleading as most of the electronic sub-systems in actual practice do not always have constant failure rates and are, therefore, not representative of most of the systems. As reliability allocation is performed for a given time, this does not mean the allocation is correct for a shorter or longer mission, as it depends on the hazard rates of individual sub-systems, which are generally not constant.

Here is a mathematical example. The time-to-failure (TTF) data of an electronic sub-system used in a machine tool are collected from the field and given in operating hours as 190, 510, 1359, 1399, 3000, 4561, 8731, and 15,601 (h). Table 4 presents the effects of selected reliability distributions on the reliability metrics. There is a significant variation in the reliability values for the given time, in the design life for the given reliability, and in the MTTF for the selected reliability distributions. Reliability allocation using only exponential distribution cannot be a true representative of this sub-system if the most suitable distribution is other than exponential, especially Weibull 2-parameter, Gamma, or lognormal. If the target reliability of this sub-system is 0.8 at the end of 1000 h of operation, this target cannot be achieved as the hazard rate of this sub-system is not constant.

Table 4. Reliability metrics for failure data.

Reliability Distribution	Reliability				Design Life (h)			MTTF (h)
	100 h	500 h	1000 h	3000 h	R = 0.95	R = 0.9	R = 0.8	
Exponential	0.98	0.91	0.83	0.58	281	578	1226	5493
Weibull 2-parameter	0.94	0.816	0.71	0.45	80	209	564	4844
Weibull 3-parameter	0.98	0.80	0.68	0.44	136	223	503	5373
Lognormal	0.97	0.81	0.67	0.41	148	266	538	7485
Normal	0.79	0.77	0.74	0.60	--	--	10	4418
Gamma	0.92	0.79	0.69	0.45	44	141	461	4733

4.2. Estimation of Weighing Factor and Failure Rate of a Sub-System

Each reliability allocation method uses its own ways to estimate the weighing factor, which is then multiplied by the system reliability requirement to obtain a sub-system reliability requirement, especially a failure rate [80]. Tables 5 and 6 summarize the reliability allocation or weighing factors considered by different methods and assumptions made, respectively. Another critical issue with these methods is that the basic failure rate of the sub-system is normalized by multiplying it by the failure rate allocation factor. It is also assumed that redundant sub-systems are repairable. The methods use system complexity, but the complexity type used is not described. The WFMs are applicable only to serially configured systems, and it can be observed that the word "complexity" refers to a system having a large number of serially connected sub-systems and not a system having mixed (series/parallel) configuration or complex configuration. The Wang et al. [54] method uses various factors and data design, manufacturing, and maintenance activities to estimate the reliability allocation factor. However, because the technology, materials, and manufacturing processes are changing rapidly with time, the feasibility and usefulness of the method must be assessed.

Table 5. Reliability allocation factors.

Consideration of Weighing Factor \ Method	AGREE [11,26,54,71,79,139,140]	ARINC [11,69,137,141–143]	Bracha Method [3,62]	Feasibility of Objectives (FoS) Method [45,50,64–66]	Jeong and Koh [11,144]	Wang et al. [11,14]	Aggarwal's Method	Integrated Factor Method [64,87,145]	Karmiol Method [98]
System reliability	*								
Mission time	*	*	*	*			*	*	
System complexity			*	*	*	*	*	*	*
Number of sub-systems	*								
Sub-system significance	*				*				
Impact of failure					*	*			
Frequency of failure		*				*			
Criticality of failure						*			
Manufacturing technology						*	*		
Working conditions				*		*			*
Cost						*	*		
Redundancy information							*		
Maintenance							*		
Criticality								*	*
Functionality								*	
Cutting-edge technology			*	*				*	*

* indicates the factors considered in the different methods.

Table 6. Assumptions and applicability of WFMs.

Assumptions and Applicability	AGREE [11,26,38,54,71,119,139,140]	ARINC [11,69,137,141–143]	Bracha Method [3,62]	Feasibility of Objectives (FoS) Method [45,50,64–66]	Jeong and Koh [46]	Wang et al. [11]	Aggarwal's Method	Integrated Factor Method [64,87,145]	Karmiol Method [98]
Does it consider reliability distribution other than exponential distribution?	No	No	No	No	No	No	No	No	No
Does it consider failure dependency?	No	No	No	No	No	No	No	No	No

Table 6. *Cont.*

Assumptions and Applicability	AGREE [11,26,38,54,71,119,139,140]	ARINC [11,69,137,141–143]	Bracha Method [3,62]	Feasibility of Objectives (FoS) Method [45,50,64–66]	Jeong and Koh [46]	Wang et al. [11]	Aggarwal's Method	Integrated Factor Method [64,87,145]	Karmiol Method [98]
Is it applicable to systems other than the series configuration?	No	No	No	No	No	No	No	No	No
Does it define the essential factors to be considered for reliability allocation?	No	No	No	No	No	Yes	Yes	No	No
Does it consider the design constraints such as cost, size, weight, and volume?	No	No	No	No	No	No	No	No	No
Does it need the information on sub-system significance?	Yes	Yes	No	No	No	No	No	No	No
Does it consider functional requirement parameters for allocation?	Yes	Yes	Yes	No	No	No	No	Yes	Yes
Does it consider the redundancy information?	No	No	No	No	No	No	Yes	No	No

4.3. Uncertainty in Reliability Allocation

Uncertainty in reliability allocation arises from factors such as technology maturity, cost variations, and environmental adaptability. To manage these uncertainties, reliability predictions should incorporate technology readiness levels for better allocation target adjustments. Probabilistic models, sensitivity analyses, and cost–benefit trade-offs help balance reliability and cost constraints. Environmental adaptability can be addressed using techniques like fuzzy logic and Bayesian inference to model varying reliability conditions. Employing these strategies enhances the robustness of reliability allocation, promoting well-informed and adaptive decision-making. Future research should explore integrating artificial intelligence and digital twins for real-time reliability assessment and dynamic uncertainty management.

4.4. Failure Dependency

In most reliability analyses, it is assumed that the failures of the sub-systems within a system are statistically independent [135,146,147]. This means that the failure of one sub-system does not affect the functions of the other sub-systems. However, in actual reliability analyses, this assumption is often not true as different electronic sub-systems function using internal interactions [148,149]. After the reliability state is decreased by one of the sub-systems, the internal interactions among the remaining sub-systems may cause further reliability states to decrease. The WFMs do not consider the failure dependency during the reliability allocation process that flows from the system to sub-systems. Therefore, reliability allocation without considering failure dependency does not make any sense [137]. For example, the failure or degraded performance of the cooling fan used in a laptop can increase the temperature of the nearby electronic sub-systems and result in a decrease in the expected reliability and lifetime. In another example, the life of rotating and reciprocating systems is significantly affected by the quality of the lubrication and the condition of the lubricant over the operation. An effective reliability allocation method should consider the sub-systems' failure dependency and their impact during the allocation process.

4.5. Redundant and Complex System Reliability Allocation

The WFMs apply to series systems but not to redundant and complex configured systems. They also rely largely on the importance of each sub-system to the system's success [150]. The AGREE method can be applied to redundant systems if the weighing factor is close to 1, or else the reliability estimates are inaccurate [11,54]. The complexity is assessed based on the number of sub-systems within the system. The Jeong and Koh method [12] allocates reliability requirements in two stages: first, the requirements are allocated to the sub-systems (first level), assuming that they are in series, and then to the units of each sub-system (second level), assuming they have series/parallel or mixed configuration. The method is flexible to a certain extent and can be applied at multiple levels. The problems of the reliability allocation methods discussed throughout this section are summarized in Table 3.

4.6. Digital Twins and Artificial Intelligence in Reliability Allocations

In recent years, digital twins [35] and artificial intelligence (AI) have emerged as transformative technologies in reliability engineering and reliability allocation processes. Digital twins are virtual representations of physical systems that mirror their real-time behavior using sensor data and advanced simulation models. These digital counterparts allow engineers to predict system performance, diagnose potential failures, and optimize reliability allocation strategies throughout a system's life cycle [151]. The continuous feedback loop between physical systems and their digital twins improves the precision of failure predictions and enables proactive maintenance strategies, reducing downtime and enhancing system reliability.

AI techniques, particularly machine learning (ML) [41] and deep learning (DL), have revolutionized reliability assessment by enabling data-driven approaches to identify complex patterns in large datasets [3,152]. AI-driven models are capable of handling nonlinear dependencies, dynamic failure rates, and interrelated sub-systems, which traditional reliability allocation methods often struggle to address. Methods such as neural networks, support vector machines, and reinforcement learning are increasingly being integrated into reliability prediction frameworks. These approaches facilitate adaptive reliability allocations based on evolving system conditions and usage profiles.

Combining digital twins with AI creates a robust framework for predictive reliability, where real-time data analytics inform dynamic reliability allocation decisions. This integration allows for the continuous refinement of reliability estimates as system conditions change, leading to more accurate and cost-effective reliability designs. Future research could explore hybrid models that incorporate both traditional allocation methods and AI-enhanced digital twins to address challenges in complex and redundant systems. The synergy between these technologies holds immense potential for advancing reliability engineering practices.

4.7. Key Issues with Optimal Reliability Allocation Methods

The ORA method uses mathematical programming approaches to obtain the optimal solution [153]. They require defining the cost models in terms of reliability, system reliability model, and confidence bounds, especially upper and lower bounds, on the sub-systems. Optimal reliability methods apply to series, redundant, and complex configured systems. However, as the number of sub-systems in the system increases, computational efficiency becomes a concern, and mathematical programming may not provide an optimal solution for given constraints [154]. Heuristic [107], meta-heuristic [155], and exact methods [156], as well as published advancements in those methods, such as ant colony optimization and hybrid genetic algorithms, are often used in optimal design and allocation problems and have shown significant improvement in computational efficiency [39]. Even though uncertainty, sub-system dependence, identification of failure mechanisms, and assumption of constant failure rate are still critical areas, they were often not considered in ORA methods [85,150].

The ORA problems are typically formulated and solved under certain common assumptions, although not always universally applicable. These assumptions often include:

- Assuming a constant hazard rate simplifies the analysis but may not always reflect real-world conditions accurately.
- The assumption of independent and identically distributed sub-systems facilitates analysis but may oversimplify the interdependencies among sub-systems.
- While active redundancy is common, it may not always be the case in practical systems.
- The assumption of a perfect switching of redundant sub-systems in standby redundancy may not hold true in all scenarios due to practical limitations [11].
- The constraint 'limitations on mixing functionally equivalent sub-systems within a parallel structure' may simplify analysis but may not accurately represent real-world configurations.
- Assuming binary states of sub-systems and the system simplifies the problem but may not capture the complexity of real-world system behavior.

While these assumptions help simplify the original problem and facilitate easier solutions, they may also limit the usefulness of the methods. In some cases, artificially constraining the problem may yield solutions that deviate from real-world conditions. Therefore, it is essential to carefully analyze systems and formulate the problem to depict realistic behaviors accurately.

5. Conclusions and Recommendations

One of the key issues in a new system design process is to allocate reliability specifications to the sub-systems within the system. Reliability allocation methods were developed for this purpose and are classified into three groups: equal allocation method, weighing factors methods, and ORA methods. The literature and numerous examples presented in this paper show that most of the reliability allocation methods are fraught with assumptions that are generally unacceptable and often result in misleading, costly, and unachievable outcomes.

The allocation methods assume that the hazard rate is constant, but that is generally not true for electronic sub-systems and systems. The failure of the sub-systems within a system is statistically assumed to be independent, which does not represent most of the real systems. These methods are developed especially for serially configured systems and do not apply to systems that have redundant and complex configurations. The reliability allocation methods do not consider the failure modes and mechanisms associated with the sub-systems during the allocation process.

This study reveals that the concept of reliability allocation is faulty as the allocation process can lead to wrong sub-system selection. If the reliability of a sub-system is known, we can determine whether or not the design will work per the requirements. This is especially the case for electronic systems, where the sub-systems are commercially off-the-shelf (COTS) items. A correct approach is to select the sub-systems by estimating their reliabilities based on the system's configuration and the desired reliability goal rather than on reliability allocation.

5.1. Recommendations

5.1.1. Integration of Advanced Optimization Techniques

Combining various optimization techniques, such as genetic algorithms, particle swarm optimization, and simulated annealing, can yield better results by leveraging the strengths of each method. Future research should focus on developing and testing hybrid algorithms adapting to complex and dynamic system requirements. Incorporating machine learning algorithms to predict failure rates and optimize reliability allocation dynamically can enhance the accuracy and efficiency of reliability models. AI-based methods can help adapt to changing system conditions and failure patterns in real time.

5.1.2. Reliability Allocation Based on Best-Fit Probability Distribution

In reliability allocation, using the best-fit probability distribution such as Weibull, lognormal, normal, or gamma, over the commonly assumed exponential distribution, enhances accuracy. The exponential distribution is appropriate for constant failure rates, but real-world data often exhibit varying failure behaviors, especially for electronic components and products. The Weibull distribution is versatile for modeling increasing or decreasing failure rates, while lognormal is suitable for life data that are positively skewed. The gamma distribution effectively handles variable failure rates and is beneficial for complex systems. Selecting the most appropriate distribution improves reliability estimates, leading to better-informed decision-making and optimized system performance.

5.1.3. Consideration of Complex System Configurations

Future studies should address the limitations of current methods by incorporating nonlinear dependencies and interactions between components. Research into methods that handle complex failure dependencies and multi-state systems will be crucial. Explore dynamic allocation methods that can adjust redundancy in real time based on changing operational conditions and failure data. This approach can improve system reliability by adapting to evolving scenarios and usage patterns.

5.1.4. Enhanced Modeling and Simulation Techniques

Develop more accurate failure models considering varying failure rates, environmental conditions, and operational stresses. Improved simulation models can lead to better predictions and reliability assessments. Research into multi-state reliability models that account for different performance levels and states of components can provide a more comprehensive view of system reliability. This includes integrating these models into optimization frameworks for better results.

5.1.5. Consideration of Uncertainty and Variability

Implement robust optimization techniques to handle uncertainties in failure data, system parameters, and environmental conditions [85]. Future research should focus on developing methods that can provide reliable solutions despite these uncertainties. Use stochastic optimization techniques incorporating randomness and variability in failure rates and system loads. This can improve the robustness of reliability allocations in systems with high uncertainty.

5.1.6. Application to Emerging Technologies

As electric and autonomous vehicles become more prevalent, developing reliability allocation methods specific to these technologies will be important. This includes addressing the unique challenges and failure modes associated with these systems. With the growth of IoT, there is a need for reliability optimization techniques tailored to distributed systems with numerous interconnected devices. Research should focus on optimizing reliability across complex networks of IoT devices.

5.1.7. Scalability and Computational Efficiency

Develop scalable algorithms that can efficiently handle large-scale systems with numerous components and complex configurations. This includes improving computational efficiency without compromising the accuracy of the reliability assessment. Utilize parallel computing techniques to speed up the optimization process, especially for large and complex systems. Leveraging high-performance computing resources can significantly reduce computation times.

5.1.8. Human Factors and Practical Implementation

It is required to create user-friendly tools and software for implementing reliability allocation techniques, allowing practitioners to easily apply advanced methods without requiring extensive expertise in optimization [157]. Incorporate field data and real-world performance metrics into reliability models to ensure that the allocation methods are practical and reflect actual system performance. This includes developing methods for integrating feedback and updating models based on real-world data.

5.1.9. Regulatory and Standards Compliance

Ensure that reliability allocation methods comply with industry standards and regulatory requirements. Research should focus on aligning methods with existing standards and contributing to developing new standards as technologies evolve. Establish benchmarking procedures and validation techniques to assess the effectiveness of reliability allocation methods. This includes comparing different methods and validating their performance against real-world systems.

5.1.10. Interdisciplinary Research

Encourage interdisciplinary research that combines reliability engineering with other fields, such as systems engineering, operations research, and economics. This can lead to more holistic and innovative approaches to reliability allocation. Explore applying reliability allocation methods across different industries to identify best practices and transfer knowledge between sectors. This can provide valuable insights and solutions applicable to diverse fields.

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