Supplemental Material for Closed Form Bayesian Inferences for Binary Logistic Regression with Applications to American Voter Turnout

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Abstract

This supplemental material section contains simulation results and more detailed analysis regarding the results presented in "Closed Form Bayesian Inferences for Binary Logistic Regression with Applications to American Voter Turnout."

1 Numerical Simulations to Test Parameter Recovery

In this section, we illustrate the efficacy of our method by performing a series of numerical simulations. In particular, we conducted a series of simulations for $p = 2, 3$, and 4 attributes, allowing $I = 1000$, $JN_i = 1$, and 300 terms in the series expansion (i.e. 1000 households, 1 category, and 1 occasion). These simulations

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are a subset of a larger number of simulations conducted that is available upon request. For each vector of parameters (μ_1,...,μ_P,...,σ_1^2,...,σ_P^2), we performed 25 simulations. For each such simulate, we simulated $I = 1000$ values of $(x_{i,1}, ..., x_{i,P})$, rescaled them by dividing by a constant to allow for sufficient 0/1 variation of y_i, numerically approximated $P(Y|\Omega)$ via Equation 21, and maximized the resulting marginal likelihood as a function of Ω.[1]

These simulations were run using MATLAB on an AMD 2.2 GHz Triple Core Processor with 8 GB of RAM. Our results are summarized in Tables S1-S9, consisting of the true values of (μ_1,...,μ_P,...,σ_1^2,...,σ_P^2), the mean and standard deviation of each of these values, and the corresponding t-statistics. The simulations corresponding to $p = 4$ attributes is split up into several tables (Tables S5-S9) due to space constraints. In order to determine whether the values were in numerical correspondence with the true values, we conducted t-tests for each of the parameters. This resulted in comparing the computed t-statistics to a t-distribution with 24 degrees of freedom, as a result of performing 25 simulations for each set of parameter estimates. After instituting conservative but commonly accepted Bonferroni corrections, we used critical values of 3.791 for $p = 2$ attributes, 4.051 for $p = 3$ attributes, and 4.244 for $p = 4$ attributes to compare the calculated t-statistics against. All of our values are well below these critical values. These significance tests therefore do not suggest any meaningful disparity between the estimated parameter values and the true values. As these runs were based on simulated data from a plethora of normal distributions, these simulations demonstrate the strong accuracy of our polynomial approximations as well as the efficacy of our new technique. Additionally, the final few terms of the tails of the truncated series approximations were essentially zero (according to MATLAB output) for a variety of choices of truncation levels. This fact suggested that, at least for these simulations, the truncated series expansions based on 300 terms were reasonable approximations to the marginalized likelihood.

[1]The judgment about the magnitude of this constant can easily be made by looking at the consequences of choices on the intermediary output in common statistical computing packages.

Table S1: Numerical Simulations for $P = 2$ attributes

(μ_1, μ_2)	(σ_1^2, σ_2^2)	$(\overline{\mu_1}, \overline{\mu_2})$	$(\overline{\sigma_1^2}, \overline{\sigma_2^2})$	$(\sigma_{\mu_1}, \sigma_{\mu_2})$	$(\sigma_{\sigma_1^2}, \sigma_{\sigma_2^2})$
(-8, -3)	(3, 8)	(-7.970, -2.972)	(2.556, 7.658)	(1.581, 1.417)	(1.195, 1.387)
(-4, -14)	(5, 6)	(-3.728, -14.175)	(4.772, 6.511)	(1.188, 1.236)	(1.273, 1.037)
(-7, -5)	(5, 3)	(-7.300, -5.078)	(4.691, 2.710)	(1.434, 1.426)	(1.582, 1.366)
(5, -4)	(4, 5)	(4.865, -3.842)	(4.265, 5.239)	(1.001, 1.210)	(1.432, 1.150)
(-5, 4)	(4, 5)	(-4.801, 4.123)	(4.385, 5.072)	(1.408, 1.318)	(1.252, 1.252)
(5, 4)	(4, 5)	(5.062, 3.893)	(4.429, 5.312)	(1.377, 1.279)	(1.441, 1.086)
(8, -3)	(3, 8)	(8.069, -3.232)	(2.771, 8.290)	(1.742, 1.816)	(0.969, 0.636)
(-8, 3)	(3, 8)	(-7.601, 2.904)	(2.942, 8.389)	(1.587, 1.414)	(0.745, 0.885)
(8, 3)	(3, 8)	(8.182, 2.970)	(2.173, 8.484)	(1.329, 1.362)	(1.387, 1.873)
(4, -14)	(5, 6)	(3.779, -14.461)	(5.259, 5.584)	(1.077, 1.353)	(1.242, 1.303)
(-4, 14)	(5, 6)	(-3.945, 13.877)	(5.167, 5.557)	(0.945, 1.135)	(1.088, 1.012)
(7, -5)	(5, 3)	(7.061, -4.359)	(5.005, 3.416)	(1.253, 1.111)	(1.686, 1.374)
(-7, 5)	(5, 3)	(-7.666, 5.115)	(4.806, 2.929)	(1.036, 1.319)	(1.344, 1.542)
(7, 5)	(5, 3)	(7.327, 4.565)	(4.786, 2.678)	(1.410, 1.240)	(0.858, 1.323)

Table S2: t-statistics for $P = 2$ attributes

(μ_1, μ_2)	(σ_1^2, σ_2^2)	t-stat (μ_1)	t-stat (μ_2)	t-stat (σ_1^2)	t-stat (σ_2^2)
(-8, -3)	(3, 8)	0.095	0.098	1.859	1.232
(-4, -14)	(5, 6)	1.147	-0.708	0.897	-2.461
(-7, -5)	(5, 3)	-1.047	-0.272	0.976	1.063
(5, -4)	(4, 5)	-0.672	0.652	-0.924	-1.040
(-5, 4)	(4, 5)	0.706	0.466	-1.538	-0.289
(5, 4)	(4, 5)	0.224	-0.418	-1.490	-1.437
(8, -3)	(3, 8)	0.197	-0.640	1.180	-2.275
(-8, 3)	(3, 8)	1.257	-0.339	0.392	-2.201
(8, 3)	(3, 8)	-0.683	0.111	2.981	-1.291
(4, -14)	(5, 6)	-1.025	-1.704	-1.042	1.597
(-4, 14)	(5, 6)	0.292	-0.542	-0.765	2.187
(7, -5)	(5, 3)	0.242	2.887	-0.013	-1.514
(-7, 5)	(5, 3)	-3.212	0.435	0.723	0.230
(7, 5)	(5, 3)	1.159	-1.756	1.246	1.216

Table S3: Numerical Simulations for $P = 3$ attributes

(μ_1, μ_2, μ_3)	$(\sigma_1^2, \sigma_2^2, \sigma_3^2)$	$(\overline{\mu_1}, \overline{\mu_2}, \overline{\mu_3})$	$(\overline{\sigma_1^2}, \overline{\sigma_2^2}, \overline{\sigma_3^2})$	$(\sigma_{\mu_1}, \sigma_{\mu_2}, \sigma_{\mu_3})$	$(\sigma_{\sigma_1}, \sigma_{\sigma_2}, \sigma_{\sigma_3})$
(-5, -6, -7)	(3, 4, 3)	(-5.010, -6.298, -6.883)	(3.268, 3.973, 3.232)	(1.591, 1.453, 1.704)	(1.037, 1.091, 0.941)
(5, -6, -7)	(3, 4, 3)	(5.030, -5.690, -7.658)	(3.028, 4.098, 2.679)	(1.788, 1.897, 1.382)	(1.397, 1.128, 1.357)
(-5, 6, -7)	(3, 4, 3)	(-5.283, 6.043, -6.691)	(2.685, 3.947, 3.189)	(1.159, 1.369, 1.617)	(1.288, 0.748, 0.923)
(-5, -6, 7)	(3, 4, 3)	(-4.803, -5.962, 7.247)	(2.930, 4.096, 2.885)	(1.242, 1.541, 1.396)	(1.250, 0.882, 1.139)
(5, -6, 7)	(3, 4, 3)	(4.745, -5.889, 7.093)	(3.130, 3.925, 3.263)	(1.295, 1.600, 1.501)	(1.054, 1.038, 0.766)
(5, 6, 7)	(3, 4, 4)	(4.964, 6.290, 6.987)	(2.972,4.339, 4.123)	(1.542, 1.423, 1.593)	(0.970, 1.617, 1.397)
(-15, -4, -6)	(2, 7, 4)	(-15.347, -3.751, -6.367)	(2.116, 7.619, 3.472)	(2.045, 1.894, 2.162)	(1.009, 2.977, 2.961)
(15, -4, -6)	(2, 7, 4)	(14.352, -3.913, -5.930)	(2.005, 7.058, 4.202)	(1.708, 1.568, 1.579)	(0.825, 2.255, 1.830)
(-9, -8, 4)	(3, 3, 5)	(-9.203, -7.712, 3.560)	(3.024, 3.637, 4.689)	(1.567, 1.737, 1.919)	(1.011, 1.888, 1.167)
(-15, -4, 6)	(2, 7, 4)	(-14.975, -3.882, 5.610)	(1.888, 6.995, 4.364)	(1.890, 1.528, 1.790)	(0.981, 2.478, 2.290)
(15, -4, 6)	(2, 7, 4)	(14.601, -3.780, 6.152)	(1.761, 6.503, 4.535)	(2.174, 1.919, 1.853)	(0.940, 2.714, 2.688)
(15, 4, 6)	(2, 7, 4)	(14.818, 4.131, 5.782)	(2.169, 6.344, 4.371)	(1.891, 2.127, 1.729)	(0.986, 2.706, 2.787)
(-9, -8, -4)	(3, 3, 5)	(-9.039, -8.245, -3.967)	(2.721, 2.975, 5.219)	(1.641, 1.504, 1.533)	(1.989, 2.321, 2.394)
(-9, 8, -4)	(3, 3, 5)	(-9.112, 8.124, -4.107)	(3.191, 3.364, 4.949)	(1.205, 1.686, 1.434)	(1.022, 0.762, 0.598)
(-9, -8, 4)	(3, 3, 5)	(-9.203, -7.712, 3.560)	(3.024, 3.637, 4.689)	(1.567, 1.737, 1.919)	(1.011, 1.888, 1.167)
(9, -8, 4)	(3, 3, 5)	(9.699, -7.759, 3.298)	(3.104, 2.810, 4.954)	(1.591, 1.361, 1.421)	(0.557, 0.782, 0.557)
(-9, 8, 4)	(3, 3, 5)	(-9.176, 7.820, 4.318)	(2.949, 3.260, 5.068)	(1.440, 1.403, 1.431)	(1.157, 1.054, 0.993)
(9, 8, 4)	(3, 3, 5)	(9.394, 8.074, 3.907)	(2.709, 2.310, 4.487)	(1.693, 1.531, 1.629)	(2.403, 2.344, 2.304)

Table S4: t-statistics for $P = 3$ attributes

(μ_1, μ_2, μ_3)	$(\sigma_1^2, \sigma_2^2, \sigma_3^2)$	t-stat (μ_1)	t-stat (μ_2)	t-stat (μ_3)	t-stat (σ_1^2)	t-stat (σ_2^2)	t-stat (σ_3^2)
(-5, -6, -7)	(3, 4, 3)	-0.030	-1.024	0.342	-1.293	0.125	-1.231
(5, -6, -7)	(3, 4, 3)	0.083	0.818	-2.382	-0.101	-0.433	1.185
(-5, 6, -7)	(3, 4, 3)	-1.221	0.156	0.955	1.222	0.352	-1.023
(-5, -6, 7)	(3, 4, 3)	0.791	0.123	0.883	0.279	-0.544	0.507
(5, -6, 7)	(3, 4, 3)	-0.984	0.347	0.310	-0.614	0.364	-1.717
(5, 6, 7)	(3, 4, 4)	0.118	-1.016	0.041	0.147	-1.048	-0.442
(-15, -4, -6)	(2, 7, 4)	-0.848	0.657	-0.849	-0.572	-1.040	0.892
(15, -4, -6)	(2, 7, 4)	-1.897	0.279	0.223	-0.031	-0.129	-0.552
(-9, -8, 4)	(3, 3, 5)	0.648	-0.829	1.146	0.121	1.686	-1.333
(-15, -4, 6)	(2, 7, 4)	0.066	0.385	-1.089	0.570	0.010	-0.794
(15, -4, 6)	(2, 7, 4)	-0.917	0.573	0.409	1.273	0.916	-0.996
(15, 4, 6)	(2, 7, 4)	-0.481	0.307	-0.630	-0.856	1.213	-0.666
(-9, -8, -4)	(3, 3, 5)	0.118	0.814	-0.107	-0.702	-0.054	0.456
(-9, 8, -4)	(3, 3, 5)	0.465	-0.369	0.372	0.932	2.392	-0.423
(-9, -8, 4)	(3, 3, 5)	0.648	-0.829	1.146	0.121	1.686	-1.333
(9, -8, 4)	(3, 3, 5)	-2.197	-0.887	2.472	0.936	-1.215	-0.411
(-9, 8, 4)	(3, 3, 5)	0.611	0.640	-1.111	-0.222	1.235	0.342
(9, 8, 4)	(3, 3, 5)	-1.163	-0.242	0.285	-0.605	-1.472	-1.113

Table S5: Numerical Simulations for $P = 4$ attributes

$(\mu_1,\, \mu_2,\, \mu_3,\, \mu_4)$	$(\sigma_1^2,\, \sigma_2^2,\, \sigma_3^2,\, \sigma_4^2)$
(-5, -14, -3, -2)	(3,6,4,6)
(-10,-12,-5,-5)	(3,3,3,4)
(-7,-5,-5,-3)	(4,6,5,4)
(5,-14,-3,-5)	(3,6,4,6)
(-5,14,-3,-5)	(3,6,4,6)
(-5,-14,-3,5)	(3,6,4,6)
(5,14,-3,5)	(3,6,4,6)
(5,-14,3,-5)	(3,6,4,6)
(5,14,3,5)	(3,6,4,6)
(10,-12,-5,-5)	(3,4,3,4)
(-10,12,-5,-5)	(3,4,3,4)
(-10,-12,5,-5)	(3,4,3,4)
(-10,-12,-5,5)	(3,4,3,4)
(10,-12,5,-5)	(3,4,3,4)
(-10,12,-5,5)	(3,4,3,4)
(10,12,5,5)	(3,4,3,4)
(7,-5,-5,-3)	(4,6,5,4)
(-7,5,-5,-3)	(4,6,5,4)
(-7,-5,5,-3)	(4,6,5,4)
(7,-5,5,-3)	(4,6,5,4)
(-7,5,-5,3)	(4,6,5,4)
(7,5,5,3)	(4,6,5,4)

Table S6: Numerical Simulations for $P = 4$ attributes (continued)

$(\overline{\mu_1}, \overline{\mu_2}, \overline{\mu_3}, \overline{\mu_4})$	$(\overline{\sigma_1^2}, \overline{\sigma_2^2}, \overline{\sigma_3^2}, \overline{\sigma_4^2})$
(-4.957, -13.727, -3.254, -2.323)	(2.368, 6.014, 3.736, 5.074)
(-9.651, -11.951, -5.249, -4.794)	(2.753, 3.035, 2.806, 3.984)
(-7.269, -4.924, -5.028, -3.163)	(3.523, 5.720, 4.367, 3.839)
(5.088, -14.096, -3.556, -4.840)	(2.437, 5.938, 4.067, 6.112)
(-5.618, 14.514, -2.732, -4.960)	(2.832, 6.048, 3.760, 5.845)
(-4.707, -14.606, -2.770, 5.084)	(2.694, 5.589, 4.116, 6.014)
(-5.120, 14.078, -3.168, 5.304)	(3.410, 5.987, 4.348, 5.793)
(4.987, -14.345, 3.140, -4.963)	(2.412, 5.604, 3.876, 5.818)
(5.062, 14.542, 2.360, 5.343)	(2.597, 5.537, 3.947, 5.476)
(9.624, -11.746, -5.127, -5.109)	(2.659, 4.061, 3.088, 3.973)
(-10.161, 12.526, -4.934, -5.120)	(3.124, 4.026, 3.069, 4.073)
(-10.167, -12.214, 4.796, -4.899)	(2.685, 3.774, 3.066, 3.636)
(-9.792, -11.717, -5.133, 4.834)	(3.243, 4.206, 3.034, 3.243)
(10.076, -11.867, 5.134, -4.972)	(2.969, 4.024, 2.934, 4.088)
(-9.833, 12.462, -5.085, 4.123)	(2.964, 3.948, 3.008, 3.934)
(10.432, 11.902, 5.244, 4.756)	(3.054, 4.280, 2.398, 3.640)
(7.045, -5.497, -4.710, -3.204)	(4.347, 6.055, 4.975, 3.883)
(-7.097, 5.174, -5.122, -2.976)	(3.875, 5.943, 4.816, 3.799)
(-6.715, -5.053, 5.164, -3.247)	(4.037, 6.064, 4.898, 4.066)
(7.361, -4.920, 4.834, -2.846)	(3.907, 5.966, 5.073, 3.997)
(-7.234, 4.965, -5.234, 3.456)	(4.026, 5.923, 5.091, 4.172)
(6.766, 5.419, 5.249, 3.109)	(3.757, 5.210, 4.864, 4.174)

Table S7: Numerical Simulations for $P = 4$ attributes (continued)

$(\sigma_{\mu_1}, \sigma_{\mu_2}, \sigma_{\mu_3}, \sigma_{\mu_4})$	$(\sigma_{\sigma_1^2}, \sigma_{\sigma_2^2}, \sigma_{\sigma_3^2}, \sigma_{\sigma_4^2})$
(1.956, 1.762, 1.672, 1.712)	(1.231, 1.878, 1.525, 1.599)
(1.938, 1.898, 1.712, 1.827)	(2.301, 2.506, 2.298, 2.501)
(1.652, 1.479, 1.743, 1.574)	(1.845, 2.094, 2.099, 2.007)
(1.595, 1.482, 1.471, 1.601)	(1.593, 0.967, 0.737, 1.153)
(1.277, 1.576, 1.337, 1.517)	(1.365, 0.934, 0.749, 0.565)
(1.382, 1.612, 1.570, 1.670)	(0.914, 1.146, 0.939, 0.492)
(1.707, 1.542, 1.451, 1.287)	(1.502, 0.906, 1.061, 0.875)
(1.440, 1.811, 1.365, 1.349)	(1.132, 0.789, 0.786, 0.724)
(1.380, 1.726, 1.839, 1.850)	(1.822, 2.091, 2.155, 2.075)
(1.372, 1.680, 1.645, 1.379)	(1.313, 0.848, 1.171, 0.761)
(1.588, 1.542, 1.434, 1.666)	(0.740, 0.549, 0.887, 0.660)
(1.805, 1.776, 1.723, 1.866)	(1.928, 2.108, 2.050, 2.163)
(1.798, 1.769, 1.545, 1.692)	(1.936, 1.835, 1.826, 1.678)
(1.656, 1.866, 1.496, 1.623)	(0.502, 0.479, 0.235, 0.294)
(1.564, 1.680, 1.755, 1.518)	(0.780, 0.575, 0.763, 0.775)
(1.753, 1.815, 1.588, 1.806)	(2.352, 2.435, 2.215, 2.432)
(1.138, 1.449, 1.528, 1.654)	(1.340, 0.880, 1.124, 1.040)
(1.519, 1.793, 1.945, 1.562)	(1.009, 0.880, 0.857, 0.854)
(1.681, 1.850, 1.458, 1.728)	(0.469, 0.898, 0.277, 0.689)
(1.294, 1.450, 1.624, 1.433)	(0.657, 0.267, 0.215, 0.528)
(1.376, 1.826, 1.736, 1.493)	(1.185, 0.630, 0.966, 0.527)
(1.590, 1.671, 1.557, 1.679)	(1.903, 2.122, 2.165, 2.243)

Table S8: Numerical Simulations for $P = 4$ attributes (continued)

t-stat (μ_1)	t-stat (μ_2)	t-stat (μ_3)	t-stat (μ_4)
0.111	0.774	-0.758	-0.942
0.899	0.129	-0.727	0.563
-0.813	0.257	-0.079	-0.519
0.276	-0.325	-1.891	0.500
-2.419	1.631	1.003	0.132
1.060	-1.881	0.731	0.250
-0.351	0.254	-0.580	1.180
-0.045	-0.951	0.513	0.138
0.225	1.570	-1.740	0.928
-1.370	0.756	-0.387	-0.396
-0.508	1.705	0.231	-0.360
-0.463	-0.603	-0.591	0.272
0.580	0.799	-0.431	-0.491
0.230	0.356	0.449	0.085
0.534	1.375	-0.241	-2.889
1.233	-0.269	0.768	-0.675
0.199	-1.713	0.949	-0.616
-0.320	0.486	-0.313	0.078
0.848	-0.144	0.562	-0.715
1.394	0.274	-0.512	0.538
-0.849	-0.097	-0.674	1.526
-0.737	1.255	0.801	0.324

Table S9: Numerical Simulations for $P = 4$ attributes (continued)

t-stat (σ_1^2)	t-stat (σ_2^2)	t-stat (σ_3^2)	t-stat (σ_4^2)
2.568	-0.038	0.866	2.897
0.537	-0.070	0.421	0.032
1.294	0.668	1.508	0.400
1.768	0.324	-0.452	-0.485
0.615	-0.255	1.606	1.372
1.675	1.792	-0.616	-0.147
-1.366	0.070	-1.639	1.183
2.597	2.513	0.790	1.255
1.107	1.106	0.124	1.262
1.300	-0.359	-0.376	0.175
-0.841	-0.236	-0.388	-0.555
0.817	0.537	-0.160	0.841
-0.627	-0.560	-0.094	2.255
0.306	-0.248	1.405	-1.490
0.229	0.455	-0.054	0.425
-0.115	-0.576	1.360	0.741
-1.294	-0.310	0.112	0.563
0.622	0.326	1.072	1.177
-0.392	-0.354	1.841	-0.481
0.708	0.628	-1.689	0.028
-0.108	0.608	-0.471	-1.629
0.638	1.860	0.314	-0.388

2 Statistically Significant Regression Results

In this section, we present our statistically significant regression results. Point estimates largest in magnitude are presented in Figures 2-4 of the manuscript.

Table S10: μ_p Estimates from Equations 22-23 Estimated via MML

	2.5th Percentile	50th Percentile	97.5th Percentile
Intercept	0.023	2.001	4.795
Election Year 2008	-2.041	-1.082	-0.120
Arkansas	0.287	2.336	6.201
Illinois	0.021	1.604	3.666
Maine	0.410	2.601	6.606
Michigan	0.040	1.448	3.263
Minnesota	0.124	2.821	5.047
Wisconsin	0.694	2.506	5.561
Black	3.065	5.263	8.495
Latinx	0.622	1.684	3.514
Other	0.022	1.218	2.433
20-40K	-5.975	-3.053	-0.646
Age 30-44	-8.496	-6.586	-2.798
Age 45-64	-4.119	-2.447	-0.360
Age 65+	0.007	0.665	2.088
HS	-8.829	-6.053	-3.490
Some College	-3.908	-2.644	-0.826
Post-Grad	-3.326	-1.630	-0.237
Married	0.262	1.935	4.575
Kids	0.421	1.964	4.584
Whites, $20-40k	-4.186	-1.834	-0.155
Whites, $40-75k	0.113	1.932	3.999
Latinx, $0-20k	-6.587	-3.090	-0.026
Other, $0-20K	-8.232	-2.867	-0.391
District of Columbia, $40-75k	0.335	2.168	4.002
Maine, $40-75k	0.292	1.683	4.414
Mississippi, >$150k	0.163	1.388	4.427
Arizona, Black	0.248	3.044	8.414
Michigan, Black	0.486	3.476	7.688
Pennsylvania, Black	0.596	3.304	7.065
South Dakota, Black	0.257	2.348	5.676
Virginia, Black	0.040	3.287	8.649
Arizona, Latinx	0.026	1.588	3.679
New Mexico, Latinx	0.070	1.485	6.230
Virginia, Latinx	0.354	2.093	4.889
Alabama, Other	0.721	2.731	6.808
Texas, Other	0.286	1.733	3.689
Utah, Other	0.160	2.590	15.482

Table S11: σ_p^2 Estimates from Equations 22-23 Estimated via MML

	2.5th Percentile	50th Percentile	97.5th Percentile
Intercept	0.355	2.145	6.988
Election Year 2008	0.135	0.670	3.561
Arkansas	3.518	9.421	17.676
Illinois	4.157	4.348	10.199
Maine	1.510	3.549	4.543
Michigan	0.390	1.829	6.889
Minnesota	0.024	1.526	11.136
Wisconsin	0.271	0.360	0.651
Black	0.049	0.149	0.218
Latinx	0.317	0.393	0.598
Other	0.005	0.770	1.347
20-40K	0.168	1.809	2.824
Age 30-44	4.081	13.496	33.255
Age 45-64	7.197	14.330	24.708
Age 65+	0.249	0.531	4.020
HS	3.580	26.479	83.324
Some College	1.472	9.257	19.374
Post-Grad	0.437	3.682	6.068
Married	0.140	0.190	3.947
Kids	0.022	0.476	0.969
Whites, $20-40k	0.085	0.227	0.347
Whites, $40-75k	0.166	0.256	0.412
Latinx, $0-20k	0.011	0.895	63.421
Other, $0-20k	0.082	4.006	6.680
District of Columbia, $40-75k	0.027	0.435	0.457
Maine, $40-75k	1.141	2.974	7.380
Mississippi, >$150k	0.226	2.304	4.628
Arizona, Black	0.101	1.589	2.470
Michigan, Black	0.740	1.566	28.776
Pennsylvania, Black	0.001	0.482	0.907
South Dakota, Black	0.921	1.074	1.576
Virginia, Black	0.276	0.361	1.759
Arizona, Latinx	0.079	2.616	3.014
New Mexico, Latinx	0.962	8.287	63.628
Virginia, Latinx	0.097	0.318	0.733
Alabama, Other	0.757	4.064	17.419
Texas, Other	5.156	6.403	12.933
Utah, Other	0.014	2.326	3.500

References

[1] Kevin Dayaratna. *Contributions to Bayesian Statistical Modeling in Public Policy Research.* PhD thesis, University of Maryland, 2014.