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The 2021 Bitcoin Bubbles and Crashes—Detection and Classification

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Abstract: In this study, the Log-Periodic Power Law Singularity (LPPLS) model is adopted for real-time identification and monitoring of Bitcoin bubbles and crashes using different time scale data, and the modified Lagrange regularization method is proposed to alleviate the impact of potential LPPLS model over-fitting to better estimate bubble start time and market regime change. The goal here is to determine the nature of the bubbles and crashes (i.e., whether they are endogenous due to their own price evolution or exogenous due to external market and/or policy influences). A systematic market event analysis is performed and correlated to the Bitcoin bubbles detected. Based on the daily LPPLS confidence indictor from 1 December 2019 to 24 June 2021, this analysis has disclosed that the Bitcoin boom from November 2020 to mid-January 2021 is an endogenous bubble, stemming from the self-reinforcement of cooperative herding and imitative behaviors of market players, while the price spike from mid-January 2021 to mid-April 2021 is likely an exogenous bubble driven by extrinsic events including a series of large-scale acquisitions and adoptions by well-known institutions such as Visa and Tesla. Finally, the utilities of multi-resolution LPPLS analysis in revealing both short-term changes and long-term states have also been demonstrated in this study.

Keywords: bitcoin bubble; log-periodic power law singularity (LPPLS); LPPLS confidence indicator; cryptocurrency; financial bubble and crash; modified Lagrange regularization method

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Citation: Shu, M.; Song, R.; Zhu, W. The 2021 Bitcoin Bubbles and Crashes—Detection and Classification. *Stats* **2021**, *4*, 950–970. https://doi.org/10.3390/stats4040056

Academic Editor: Indranil SenGupta

Received: 25 October 2021
Accepted: 19 November 2021
Published: 21 November 2021

1. Introduction

In recent years, the cryptocurrency market has seen explosive growth and roller coaster volatility, attracting widespread attention from not only the investment community but also the academic community. Unlike the existing central banking systems, cryptocurrencies are decentralized digital currencies that can be traded through a distributed ledger blockchain network without intermediaries. The first cryptocurrency is Bitcoin, invented by pseudonymous Satoshi Nakamoto in 2008. It is based on the proof-of-work decentralized consensus mechanism using the SHA-256 hashing algorithm and was released on 3 January 2009 [1]. Since the Bitcoin code written in C++ is open source, other cryptocurrencies, often referred to as altcoins, have been swiftly produced by modifying certain features of the Bitcoin code on issuance, security, and governance functions, leading to the boom of the global cryptocurrency market with the number of cryptocurrencies (https://coinmarketcap.com (accessed on 22 July 2021)) growing from merely 7 in April 2013, to an incredulous 10,668 in June 2021. The total market capitalization of cryptocurrencies reached a new historical record high of \$2.481 trillion on 12 May 2021, accounting for about 2.3% of the global equity markets valuated at about \$107.15 trillion [2]. From its baseline of \$826.365 million on 6 July 2013, the total market value of the cryptocurrencies has increased by about 3000 times in less than 8 years (Figure 1).

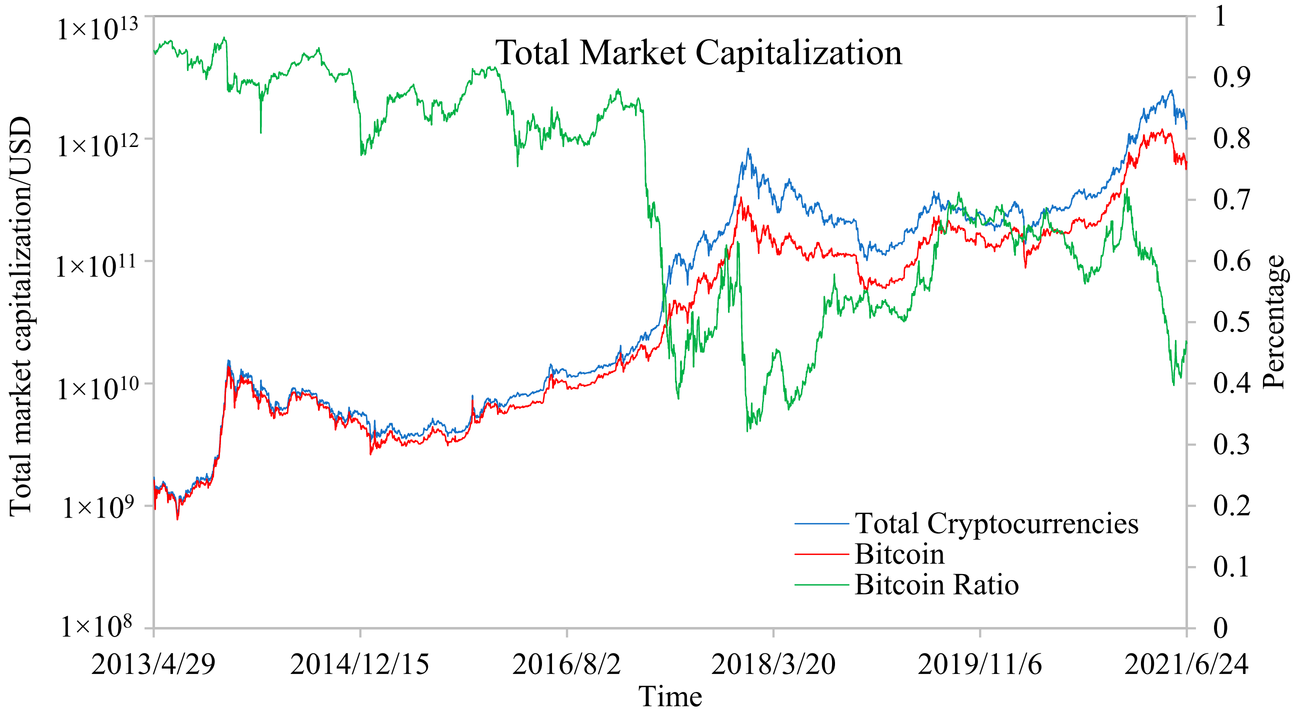
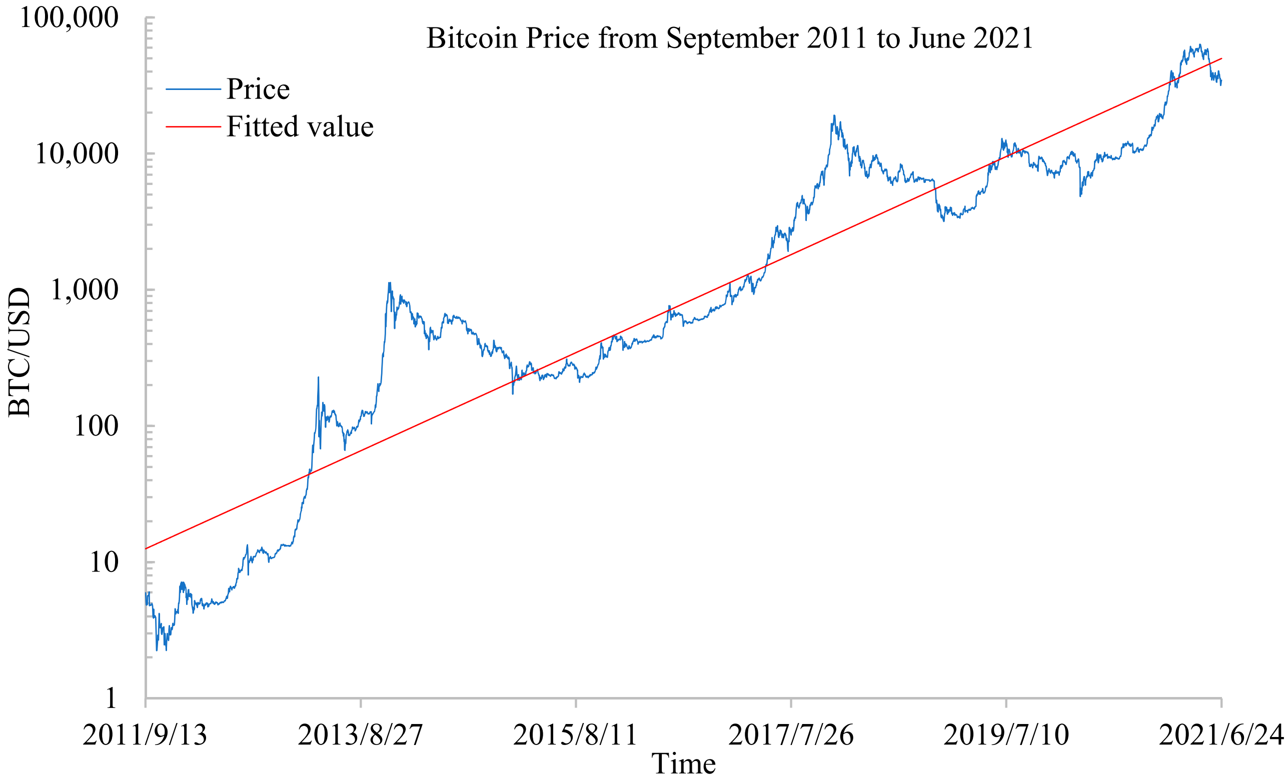


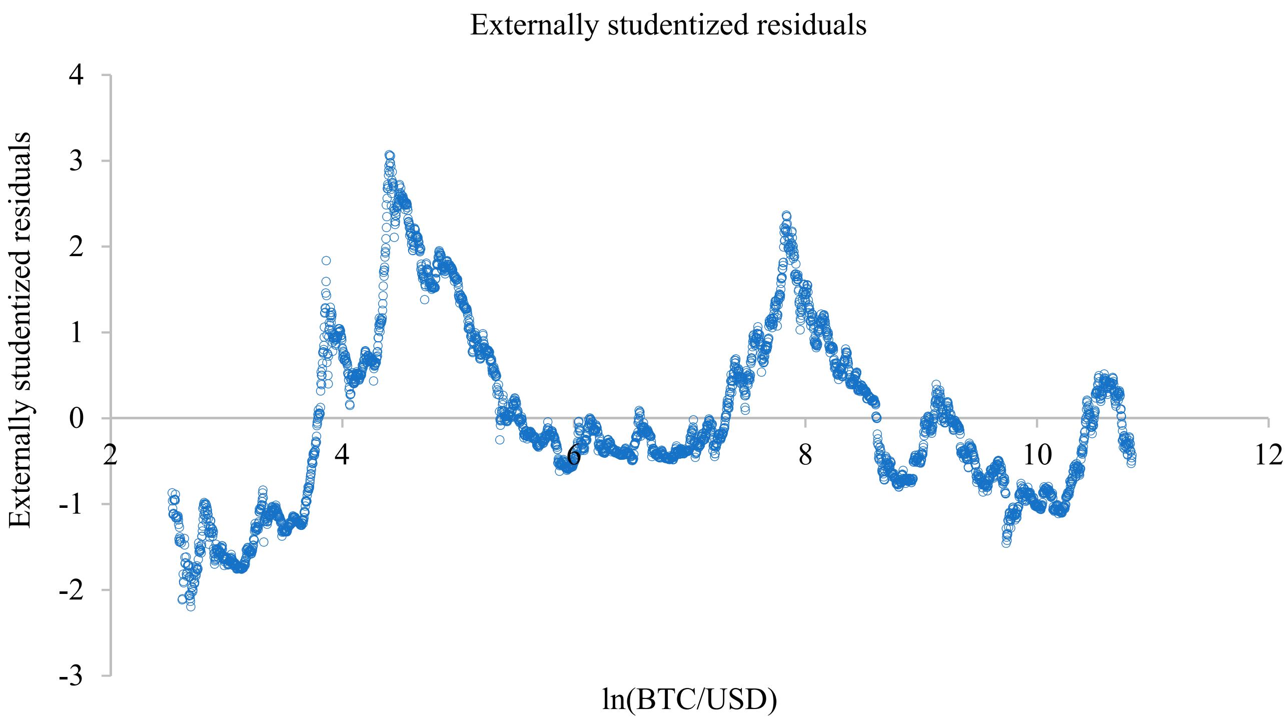
Figure 1. Evolution of the total market capitalization of cryptocurrencies and Bitcoin as well as Bitcoin ratio from 29 April 2013 to 24 June 2021.

Bitcoin, as the first decentralized cryptocurrency, has always maintained its dominant position in the cryptocurrency market (as shown in Figure 1). The total market capitalization of Bitcoin climbed to the historical peak of $1.198 trillion on 14 April 2021, accounting for 53.95% of the total cryptocurrency market value. The Bitcoin (total market capitalization) ratio has been fluctuating over time, with remarkable changes during bubble and crash phases. For example, the Bitcoin Ratio reached a peak of 96.55% on 18 November 2013, and fell to 32.14% on 1 January 2018. Bitcoin has become increasingly accepted in society which in turn, boosted its enduring rally. On 8 June 2021, EI Salvador became the first country to adopt Bitcoin as a legal tender [3].

The distinct characteristics of the Bitcoin price are extraordinary returns during the bubble phases and unpredictable large-scale crashes. Figure 2a presents the evolution of the daily price trajectories of Bitcoin from Bitstamp (https://bitcoincharts.com/ (accessed on 22 July 2021)). The price of Bitcoin rose from $2.24 on 20 October 2011 to $63,564.48 on 13 April 2021, resulting in a compound annual growth rate of 194.67%. The red line in Figure 2a represents the best exponential fit of Bitcoin price from 13 September 2011 to 24 June 2021, where the coefficient of determination R^2 is 0.8856 indicating that 88.56% of the total variation of Bitcoin price can be explained by the exponential regression model. It should be noted that while the regression line itself is well estimated, parametric statistical inference would be tenuous due to stringent underlying assumptions. Figure 2b shows the externally studentized residuals versus the corresponding fitted logarithm value of Bitcoin price, indicating that the variance of the errors is not constant, and errors are not independent and identically distributed. The normal Q-Q plot of the externally studentized residuals is shown in Figure 2c. The cumulative probability of the externally studentized residuals flattens at the extremes, indicating that the samples come from a distribution with heavier tails than normal. Nevertheless, the exponential model is still rather impressive. Figure 2a also shows that even though the long-term price of Bitcoin apparently follows the exponential growth, the Bitcoin price has suffered sustained super-exponential bubble growths and crashes on the short-time scale. In the recent super-exponential growth phase, the price of Bitcoin soared from $5033.42 on 16 March 2020 to $63,564.48 on 13 April 2021, an increase of 1062.85% in only 13 months. After the Bitcoin price peaked in mid-April, it fell sharply to $31,634.16 on 21 June 2021, plummeting by nearly half. Again, as we have witnessed time and again in its short history, we saw Bitcoin price bouncing back yet again, in October 2021, to narrowly below its record high in April 2021.

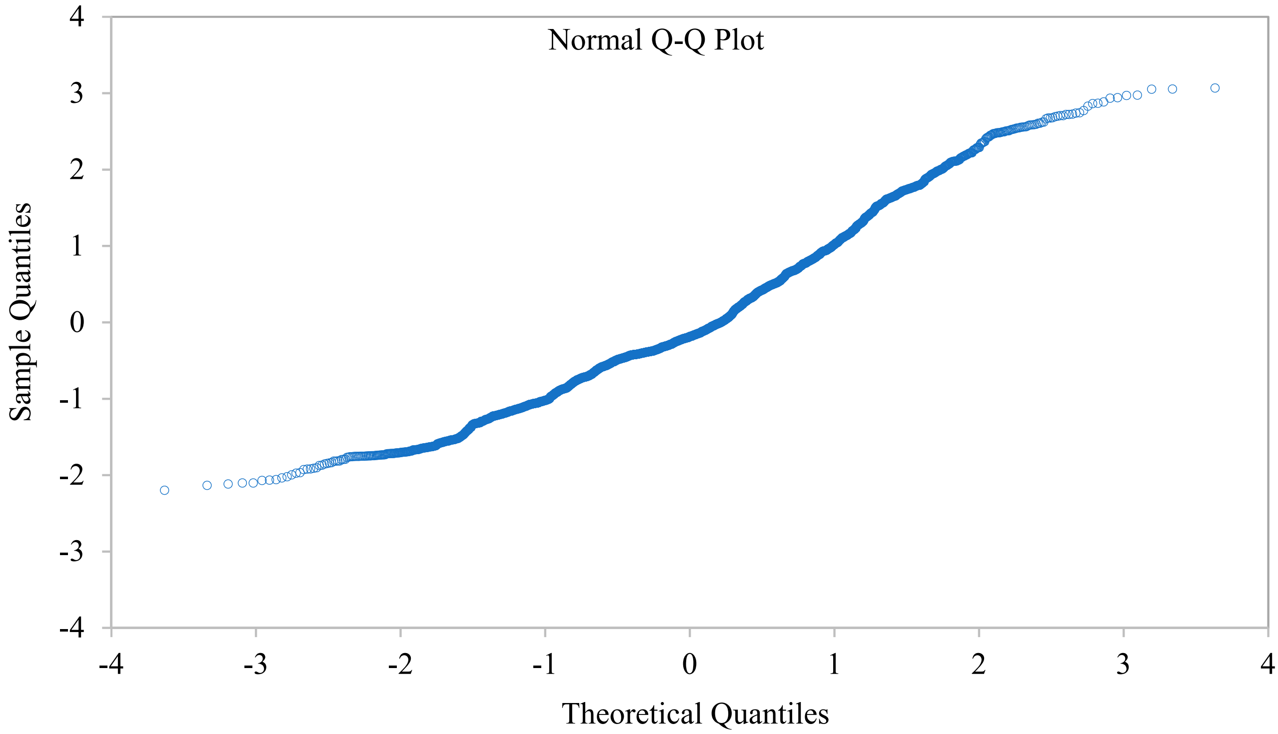


(**a**)



(**b**)

Figure 2. *Cont*.



(**c**)

Figure 2. Evolution of the daily price trajectories of Bitcoin from 13 September 2011 to 24 June 2021, along with the regression line: (**a**) Bitcoin daily price trajectories and regression line; (**b**) externally studentized residuals versus the corresponding fitted logarithm value of Bitcoin price; (**c**) the normal Q-Q plot of the externally studentized residuals.

The academic literature on Bitcoin price dynamics and extreme changes has been emerging in recent years. Kristoufek [4] quantified the bidirectional relationship between Bitcoin price and search queries on Google Trends and Wikipedia. Garcia et al. [5] identified two positive feedback loops that lead to Bitcoin bubbles in the absence of external stimuli: word of mouth and new Bitcoin adopters. Donier and Bouchaud [6] anticipated the amplitude of a potential crash of Bitcoin based on market liquidity. Blau [7] observed that speculative trading was not positively related to the Bitcoin's volatility. Balcilar et al. [8] conduced a causality-in-quantiles test on Bitcoin's returns, volatility, and trading volume, and found that the volume can predict returns except in bear and bull market regimes. Begušić et al. [9] estimated the scaling exponent of Bitcoin price fluctuations and found that the Bitcoin returns exhibit heavier tails than stocks and a finite second moment exists in Bitcoin return distribution. Takaishi [10] used Bitcoin1-min return data to investigate the statistical properties and multifractality. Ji et al. [11] examined dynamic connectedness through returns and volatility spillovers in six main cryptocurrency markets and found that connectedness does not depend on market size. Wheatley et al. [12] defined the market-to-Metcalfe value ratio to predict the Bitcoin bubbles by combing a generalized Metcalfe's law and the Log Periodic Power Law Singularity (LPPLS) model. Gerlach et al. [13] analyzed the Bitcoin bubbles from 2012 to 2018 using Epsilon drawdown detection method and multiple LPPLS confidence indicators. Shu & Zhu [14] proposed an adaptive multilevel times series detection method to identify the bubbles and crashes in real time. Enoksen et al. [15] studied price bubbles in eight major cryptocurrencies, and found that bubbles are positively correlated with higher volatility, trading volume, and transactions. Cheah et al. [16] examined impacts of multiple variables on Bitcoin return and found that Bitcoin return is influenced by Bitcoin-specific and external uncertainty factors rather than the common stock and bond market factors. García-Monleón et al. [17] developed a theoretical framework for evaluating the intrinsic value in cryptocurrencies. Bouri et al. [18] studied cumulative intraday return curves of Bitcoin to explore the predictability and profitable trading opportunities.

In this research, we systematically investigated the recent Bitcoin bubbles and crashes by using the LPPLS model [19–23]. The LPPLS model integrates the statistical physics of bifurcations and phase transitions, the economic theory of rational expectations, and the behavioral finance notion of trader-herding to define a positive (or negative) financial bubble as an unsustainable super-exponential increase (or decrease) in a finite time to reach an infinite return process, resulting in a short-term correction modelled by the symmetry of discrete scale invariance [24]. The LPPLS model captures the two significant characteristics of price trajectories in bubble regimes: the transient super-exponential growth caused by the positive feedback mechanism in the asset valuation by imitation and herding behavior of noise traders and actions from boundedly rational agents, and the accelerating log-periodic volatility oscillations stemming from the tension and competition between different traders and opposite market move predictions. The LPPLS model provides a flexible scheme to diagnose financial bubbles by analyzing the temporal price trajectory of an asset, without the need for complicated evaluation of the fundamental asset value. The limitation of the LPPLS model is that it can only detect endogenous bubbles, and not exogenous bubbles, in which an asset's fundamental value changes due to exogenous shocks. A brief review of its recent developments ensues. Kurz-Kim [25] applied the LPPLS model to capture the crash in German stock index. Geraskin and Fantazzini [26] reviewed the LPPLS model and employed three different calibration methods to fit the model. Filimonov and Sornette [27] transformed the traditional LPPLS formula, converting one nonlinear parameter into two linear parameters. Sornette et al. [28] developed the LPPLS confidence indicator and the trust indicator to measure the sensitivity of bubble calibration. Demos and Sornette [29] proposed the Lagrange regularization method to estimate the beginning time of bubbles. Ghosh et al. [30] detected the LPPLS signatures in the S&P BSE Sensex from 2000 to 2019. In addition, the LPPLS model has been widely adopted to detect endogenous bubbles in various financial markets, such as stock markets [28,31–37], cryptocurrency markets [12,14], and real estate markets [38,39].

In this study, the LPPLS model was employed to explore the underlying mechanism of the 2021 Bitcoin bubbles based on three different time scales: daily, weekly and hourly data. We also presented the realtime monitoring of Bitcoin bubbles and crashes from multiple time scales. This paper is organized as follows: Section 2 provides a brief review of the methodology, Section 3 presents the analyses and results, and Section 4 presents our conclusions.

2. Methodology

2.1. The Log-Periodic Power Law Singularity (LPPLS) Model and Calibration

The statistical formulation of the LPPLS model, originally called the Johansen–Ledoit–Sornette (JLS) model, can be written as [27]:

$$\mathrm{LPPLS}(t) \equiv E[\ln p(t)] = A + B(t_c - t)^m + C_1(t_c - t)^m \cos[\omega \ln(t_c - t)] + C_2(t_c - t)^m \sin[\omega \ln(t_c - t)] \quad (1)$$

Here $p(t)$ is the price trajectory of an asset, the critical time t_c is the most likely moment for the asset price trajectory to encounter a regime change in the form of a major crash or a significant change in growth rate terminating the accelerating oscillations. Additionally, $A > 0$ is the expected log-price at t_c. The power parameter $m \in (0, 1)$ ensures that the price stays finite and singular at t_c. Lastly, ω is the angular log-frequency of the oscillation.

In the network structure of the LPPLS model, the market participants are divided into two categories: rational traders who follow the well-established paradigm of rationality maintaining rational expectations for maximum utility and, in contrast, noise traders who do not trade based on the market realizations and reasonable expectations and are prone to herding and imitation behaviors, thus making irrational buying, selling, or holding decisions. It is assumed that asset prices can be destabilized by the collective behavior of noise trades (as we have witnessed repeatedly in history). The log periodic terms $\cos[\omega \ln(t_c - t)]$ and $\sin[\omega \ln(t_c - t)]$ originate from the pre-existing hierarchical structure in the scale of noise trader and/or from the interaction between market price impact inertia and nonlinear intrinsic value. The power law singularity $(t_c - t)^m$ leading to the formation of bubbles is derived from the positive feedback mechanism of the herding and imitation behaviors exhibited by noise traders. When time t reaches the critical time t_c—the power law singularity approaches the singularity—resulting in an infinite crash hazard rate.

The LPPLS formula in Equation (1) consists of three nonlinear parameters $(t_c\ , m,\ \omega)$ and four linear parameters $(A, B,\ C_1, C_2)$. Using the L^2 norm, the sum of squares of residuals of the LPPLS formula in Equation (1) can be determined as:

$$F(t_c, m, \omega, A, B, C_1, C_2) = \sum_{i=1}^{N}[\ln p(\tau_i) - A - B(t_c - \tau_i)^m - C_1(t_c - \tau_i)^m \cos(\omega \ln(t_c - \tau_i)) - C_2(t_c - \tau_i)^m \sin(\omega \ln(t_c - \tau_i))]^2 \tag{2}$$

Here $\tau_1 = t_1$ and $\tau_N = t_2$. The cost function $\chi^2(\ t_c\ , m,\ \omega)$ can be derived by slaving the four linear parameters $(A, B,\ C_1, C_2)$ to the three nonlinear parameters $(t_c\ , m,\ \omega)$:

$$\chi^2(t_c, m, \omega) = F_1(t_c, m, \omega) = \underset{\{A,B,C_1,C_2\}}{min} F(t_c, m, \omega, A, B, C_1, C_2) = F\left(t_c, m, \omega, \hat{A}, \hat{B}, \hat{C}_1, \hat{C}_2\right) \tag{3}$$

Here $\left(\hat{A}, \hat{B}, \hat{C}_1, \hat{C}_2\right)$ represent the estimates. We can estimate the three nonlinear parameters $(t_c\ , m,\ \omega)$ by solving the nonlinear optimization equation:

$$\left(\hat{t}_c\ , \hat{m},\ \hat{\omega}\right) = arg \min_{\{t_c\ , m,\ \omega\}} F_1(t_c\ , m,\ \omega) \tag{4}$$

After determining the best estimates of three nonlinear parameters, estimates of the four linear parameters $(A, B,\ C_1, C_2)$ can be obtained by solving:

$$\left(\hat{A}, \hat{B},\ \hat{C}_1, \hat{C}_2\right) = arg \min_{(A,B,\ C_1,C_2)} F(t_c\ , m,\ \omega, A, B,\ C_1, C_2) \tag{5}$$

The Equation (5) can be rewritten in the form of a matrix equation:

$$\begin{pmatrix} N & \sum f_i & \sum g_i & \sum h_i \\ \sum f_i & \sum {f^2}_i & \sum f_i g_i & \sum f_i h_i \\ \sum g_i & \sum f_i g_i & \sum {g^2}_i & \sum h_i g_i \\ \sum h_i & \sum f_i h_i & \sum g_i h_i & \sum {h^2}_i \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{B} \\ \hat{C}_1 \\ \hat{C}_2 \end{pmatrix} = \begin{pmatrix} \sum \ln p_i \\ \sum f_i \ln p_i \\ \sum g_i \ln p_i \\ \sum h_i \ln p_i \end{pmatrix} \tag{6}$$

where $f_i = (t_c - t_i)^m, g_i = (t_c - t_i)^m cos(\omega \ln(t_c - t_i))$, and $h_i = (t_c - t_i)^m sin(\omega \ln(t_c - t_i))$.

In this study, the covariance matrix adaptation evolution strategy (CMA-ES) [40] for nonlinear optimization has been integrated into the analysis pipeline. We obtained the best estimates of the three nonlinear parameters $(t_c\ , m,\ \omega)$ through minimizing the sum of residuals between the observed price and the simulated price trajectories from the calibrated model. The search space is bounded [14] as follows:

$$m \in [0, 1],\ \omega \in [1,\ 50],\ t_c \in \left[t_2,\ t_2 + \frac{t_2 - t_1}{3}\right],\ \frac{m|B|}{\omega\sqrt{C_1^2 + C_2^2}} \geq 1 \tag{7}$$

Based on the empirical evidence collected in previous investigations of financial bubbles, we have filtered the calibrated LPPLS model parameters to ensure the rigor of the model and the validity of the calibration results [14,28] as follows:

$$\begin{gathered} m \in [0.01, 0.99],\ \omega \in [2,\ 25],\ t_c \in \left[t_2,\ t_2 + \tfrac{t_2 - t_1}{5}\right], \tfrac{\omega}{2} \ln\left(\tfrac{t_c - t_1}{t_c - t_2}\right) \geq 2.5, \\ \max\left(\tfrac{|\hat{p}_t - p_t|}{p_t}\right) \leq 0.20, p_{lomb} \leq \alpha_{sign},\ \ln(\hat{p}_t) - \ln(p_t)\ \sim \text{AR}(1) \end{gathered} \tag{8}$$

2.2. LPPLS Confidence Indicator

In this study, the LPPLS confidence indicator, the fraction of the fitting windows of the calibrated LPPLS model that satisfies certain filter conditions [28], has also been utilized to quantify the sensitivity of selecting the start time t_1 in the fitting windows to the LPPLS bubble pattern. The higher the LPPLS confidence index, the more certain we are of concerning the bubble condition in the given fitting window, and thus the more reliable the price trajectory indicator is under the bubble state (and vice versa). It should be noted that a low value of LPPLS confidence indicator represents that the LPPLS bubble pattern presented in the price trajectories is highly sensitive to the selection of the starting time points of the fitting windows—hence the bubble signals is fragile.

For a given end point t_2, we shrank the length of fitting window (t_1, t_2) from 650 data points to 30 data points in steps of 5 data points, thereby generating a set of 125 fitting windows for each t_2. Then we calibrated the LPPLS model in Equation (1) within the search space of Equation (7) and used the filter constrains in Equation (8) to filter the calibrated LPPLS model parameters. Finally, the LPPLS confidence indicator was calculated as the fraction of qualified fitting windows out of the 125 total fitting windows. It should be noted that the step size in shrinking windows can be set to other values. The smaller the step size, the larger the number of fitting windows, the greater the computational cost, and the longer the computational time, and the higher the accuracy of the results. Since the calculation of the LPPLS confidence indicator is only based on the previous data of the specified endpoint t_2, independent to price trajectory after t_2, the LPPLS confidence indicator provides a real-time diagnosis of the bubble status when the t_2 is interpreted as the fictitious "present".

2.3. Modified Lagrange Regularization Method

In order to estimate the best start time of a new market regime, we combined the LPPLS model with the modified Lagrange regularization approach by removing outliers. For a specified end time point t_2, the bubble start time $t_1{}^*$ can be determined as the time corresponding to the minimization of a cost function of a set of LPPLS fits generated by varying fit window start time t_1. To compare the goodness-of-fit, the mean squared errors (MSE), defined as the sum of squared errors (SSE) $\sum_{i=t_1}^{t_2} r_i(\Phi)^2$ divided by the number of points $N = t_2 - t_1$ in each fitting window corrected by the number of degrees of freedom $P = 7$ of the model, is commonly used as the cost function $\chi^2(t_1, \Phi)$. Based on the MSE, the bubble start time $t_1{}^*$ can be estimated by:

$$t_1{}^* = arg \min_{t_1} \chi^2(t_1, \Phi) = arg \min_{t_1} \frac{1}{(t_2 - t_1) - P} \sum_{i=t_1}^{t_2} r_i(\Phi)^2 \tag{9}$$

where Φ denotes the set of model parameters $(t_c, m, \omega, A, B, C_1, C_2)$ to be fitted in the LPPLS model. The term $r_i(\Phi) = p_i{}^{data} - p_i{}^{model}(\Phi)$, where $p_i{}^{data}$ is the observed value of price time series, and $p_i{}^{model}(\Phi)$ is the fitted value of price based on the LPPLS model at time i. It should be noted that Equation (8) is a linear optimization process since the $r_i(\Phi)$ has been determined in the previous LPPLS model calibration.

For unbalanced lengths of fitting windows, when the number of data points is reduced relative to the number of degrees of freedom, the over-fitting problem will be an issue (i.e., the smaller fitting window would lead to a smaller value of the cost function $\chi^2(t_1, \Phi)$). To solve this problem, Demos and Sornette [29] proposed a simple Lagrange regularization term $\lambda(t_2 - t_1)$ to penalize the cost function $\chi^2(t_1, \Phi)$ with the length of fitting window, thus the bubble start time $t_1{}^*$ can be determined by:

$$t_1{}^* = arg \min_{t_1} \chi_\lambda{}^2(t_1, \Phi) = arg \min_{t_1} \frac{1}{(t_2 - t_1) - P} \sum_{i=t_1}^{t_2} r_i(\Phi)^2 - \lambda(t_2 - t_1) \tag{10}$$

where λ is the regularization parameter, which can be estimated empirically via a regression through the origin on $(t_2 - t_1)$ with λ representing the slope of the regression model quantifying the tendency of the model to overfit the data. The detrended cost function ${\chi_\lambda}^2(t_1,\, \Phi)$ can alleviate the unbalanced size bias to a certain extent enabling comparison of the fitting performance under different window sizes.

In practice, due to potential overfitting issue in the highly nonlinear LPPLS model, a set of MSEs may contain very few extreme values, which are considerably different from the majority of the data. These potential outliers can have dramatic effects on the fitted least squares regression function and distort the performance of detrended cost function ${\chi_\lambda}^2(t_1,\, \Phi)$. To improve the fitting performance, we proposed to detect and eliminate these outliers by applying the Bonferroni simultaneous test procedure.

We scaled the residuals of the cost function ${\chi_\lambda}^2(t_1,\, \Phi)$ using Equation (11) to generate the externally student residuals or studentized deleted residuals $t_{\lambda,i}$ as follows:

$$t_{\lambda,i} = r_{\lambda,i}\sqrt{\frac{N_\lambda - P_\lambda - 1}{SSE_\lambda(1 - h_{ii}) - {r_{\lambda,i}}^2}}\,,\ i = 1,\, 2,\, \ldots,\, N_\lambda \tag{11}$$

where N_λ is the number of total fitting windows, P_λ=1 is the degree of freedom of the no-intercept linear regression model, $r_{\lambda,i}$ and SSE_λ are residuals and sum of squared errors of the cost function ${\chi_\lambda}^2(t_1,\, \Phi)$, respectively, and h_{ii} is the diagonal elements of the hat matrix of the linear regression model.

Under the usual regression assumptions, the $t_{\lambda,i}$ will follow $t_{N_\lambda - P_\lambda - 1}$ distribution [41]. Based on the Bonferroni simultaneous test procedure, we tested all N_λ absolute values of the $t_{\lambda,i}$ to identify the potential outliers which satisfy the conditions in Equation (12). In this study, we used the significant level α = 0.10 in the following:

$$|t_{\lambda,i}| \geq t_{(\frac{\alpha}{2N_\lambda}),N_\lambda - P_\lambda - 1}\,,\ i = 1,\, 2,\, \ldots,\, N_\lambda \tag{12}$$

The detected outliers will be discarded from the group of MSEs, and the regularization parameter λ will be re-estimated, denoted by λ', based on the MSEs after removing outliers. The bubble start time ${t_1}^*$ can thence be estimated by:

$${t_1}^* = arg\min_{t_1} {\chi_{\lambda'}}^2(t_1,\, \Phi) = arg\min_{t_1} \frac{1}{(t_2 - t_1) - P}\sum_{i=t_1}^{t_2} r_i(\Phi)^2 - \lambda'(t_2 - t_1) \tag{13}$$

The modified Lagrange regularization approach in Equation (13) will reduce the impact of the potential overfitting problem of the LPPLS model to better evaluate a bubble start time and to identify potential change to a new market regime.

3. Empirical Analysis

3.1. Bubble Detection Using Daily Data

In this section, we used the daily data of Bitcoin price trajectory by moving the endpoint t_2 from 1 December 2019 to 24 June 2021 to detect positive bubbles associated with upwardly accelerating growth and vulnerable to regime change in the form of a crash or a significant reduction in growth rate, and negative bubbles associated with the downwardly accelerating decrease and susceptible to regime change in the form of a valley or a distinct reduction in decreasing rate. The length of fitting windows with an interval of 5 days is shrank from 650 days to 30 days, which is roughly from two years to one month. Figure 3 presents three typical fitting examples with a common endpoint t_2 = 3 January 2021, and three different sets of starting time: t_1 = 8 April 2020, t_1 = 5 September 2020, and t_1 = 29 November 2020—corresponding to the long, middle, and short time scale windows, respectively. The estimated parameters of the LPPLS model on the three typical fitting windows are shown in Table 1.

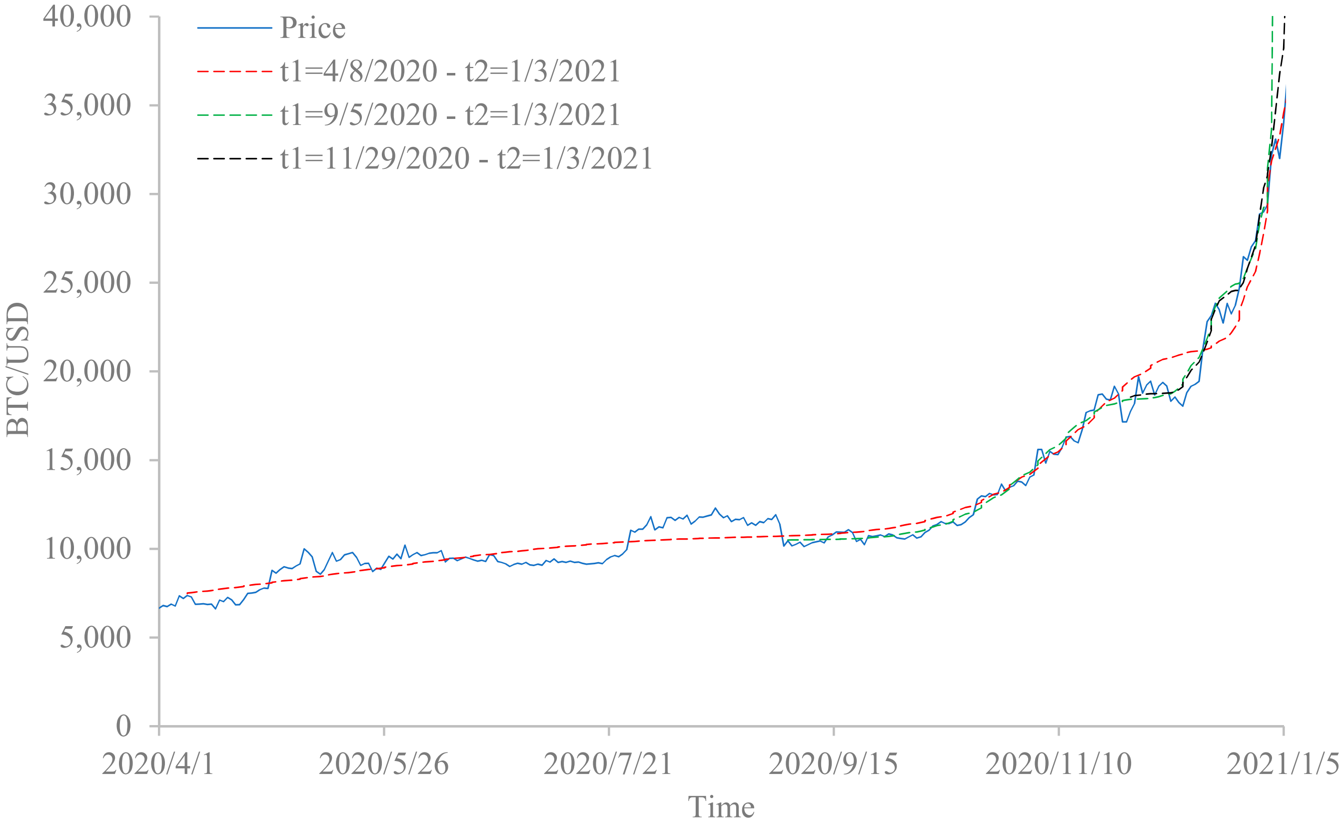


Figure 3. Three typical fitting examples with the same endpoint t_2 = 3 January 2021, and three different sets of starting time: t_1 = 8 April 2020, t_1 = 5 September 2020, and t_1 = 29 November 2020—corresponding to the long, middle, and short time scale windows, respectively.

Table 1. Samples of estimated parameters of the LPPLS model on the daily Bitcoin price.

t_1	t_2	A	B	C_1	C_2	t_c	m	ω
8 April 2020	3 January 2021	11.922	−3.024	−0.061	−0.028	7 January 2021	0.155	7.018
5 September 2020	3 January 2021	10.474	−2.168	−0.083	0.184	3 January 2021	0.494	4.977
29 November 2020	3 January 2021	13.175	−4.419	−0.093	−0.067	16 January 2021	0.132	4.311

Figure 4 shows the daily LPPLS confidence indicator for positive bubbles in red and negative bubbles in green as well as the Bitcoin price in blue from 1 December 2019 to 24 June 2021. If the value of LPPLS confidence indictor is close to zero (for example, less than 2%), it means the indicator is not robust to the choice of the starting time and few fitting windows are characterized by the two typical features of an endogenous bubble: the transient super-exponential growth and the accelerating log-periodic volatility oscillations. Thus we should handle the results carefully since there is a risk of overfitting.

In Figure 4, one cluster of negative bubble and two clusters of positive bubbles can be observed clearly. The cluster of negative bubbles took place between 6 December 2019 and 5 January 2020, with multiple peaks of 3.2%. During this period, the Bitcoin price reached a local trough of $6612.30 on 17 December 2019. The first cluster of positive bubbles occurred between 7 February 2020 to 17 February 2020 and the LPPLS confidence indicator reached the peak value of 5.6% on 12 February 2020. In the same period, the Bitcoin price rose to a local peak of $10,364.04 on 14 February 2020, increased by 56.7% within two months. After that, the price dropped to $5033.42 on 16 March 2020, losing nearly half of its value.

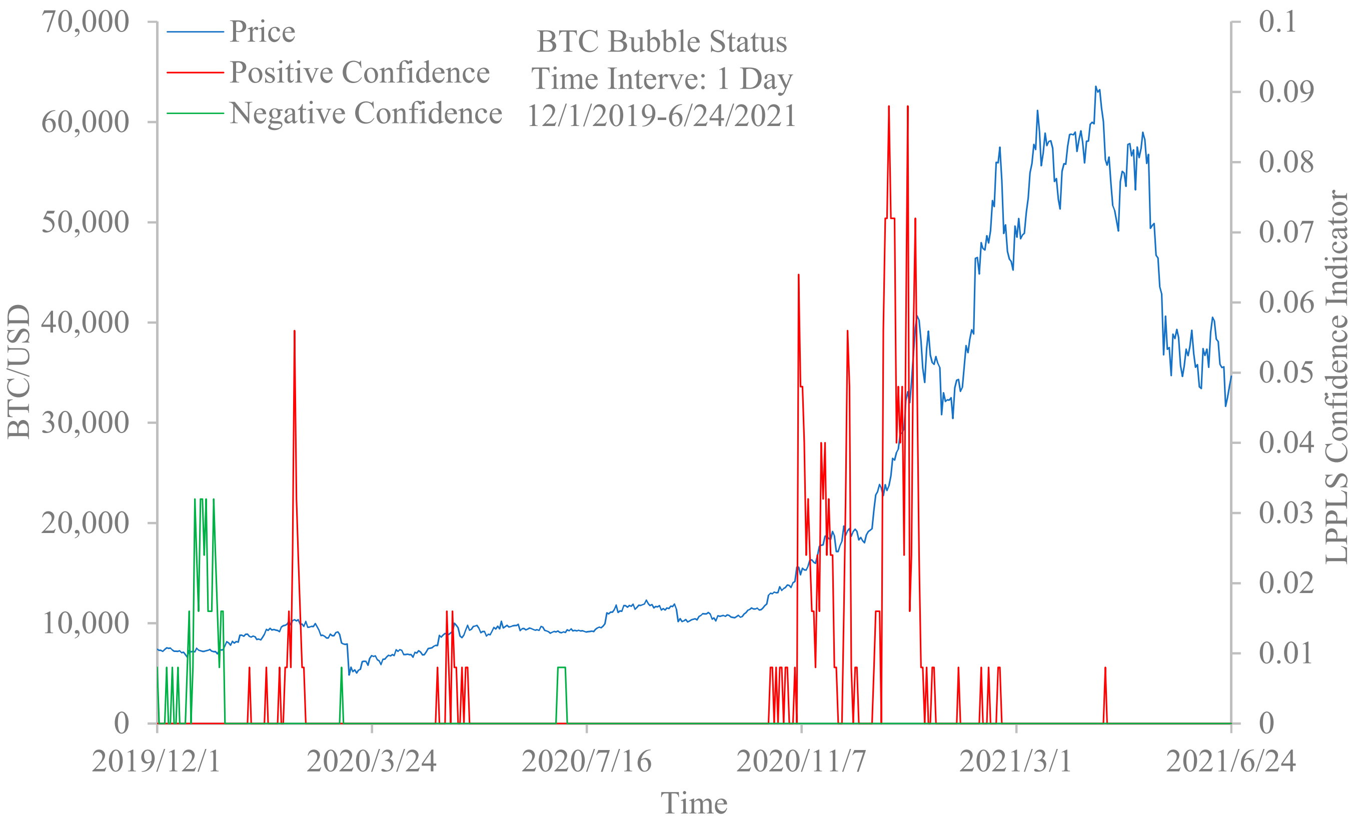


Figure 4. Daily LPPLS confidence indicator for positive bubbles is shown in red and negative bubbles in green (right scale), along with the Bitcoin price in blue (left scale), from 1 December 2019 to 24 June 2021.

The second cluster of positive bubbles in Figure 4 happened between 6 November 2020 and 17 January 2021. The two peaks occurred on 24 December 2020 and 3 January 2021, respectively, at 8.8%, indicating that the price trajectories of Bitcoin in 11 out of 125 fitting windows contained the obvious endogenous bubble pattern, and were indeed in positive bubble regime. The systemic instability of Bitcoin price is increasing, and the price in the positive bubble state is likely to change regime in the form of a crash or volatile sideway plateaus. The prediction is confirmed by the fact that the price of Bitcoin soared from $5033.42 on 16 March 2020 to $40,667.07 on 8 January 2021, a 7-fold increase in less than 10 months, and then plummeted to $30,818.18 on 21 January 2021, losing 24.2% of its value in just two weeks.

One of the interesting findings in Figure 4 is that from mid-January 2021 to June 2021, the LPPLS confidence indicator has few separate values of 0.8%, indicating the Bitcoin's price trajectory did not show the obvious endogenous bubble patterns during the boom period from 27 January 2021 with a price of $30,424.62 to the historical peak of $63,564.48 on 13 April 2021, an increase of 108.9% in three months. Thus, from the perspective of two-year daily data, the bubble formed during the boom period from mid-January to mid-April is less likely to be endogenous and may not stem from the self-reinforcement of cooperative herding and imitative behaviors of market participants. In contrast, the price spike during this period may be driven by the exogenous events, such as a significant acquisition—when Tesla disclosed that it has purchased $1.5 billion worth of Bitcoin on 8 February 2021 [42], and a notable currency adoption—when the Bank of New York Mellon on 11 February 2021 announced that it would start financing Bitcoin and other digital currencies, which was considered to be a validation of cryptocurrencies from a key bank [43]. In addition, between 8 May 2021 and 23 May 2021, the Bitcoin price plummeted from $58,984.75 to $34,706.79, a 41.2% drop in about two weeks. However, there was no negative LPPLS confidence indicator during the crash, indicating that this crash was caused by external shocks. For example, Elon Musk took back Tesla's commitment to accept Bitcoin as payment on 12 May 2021 and the Chinese financial institutions and businesses were banned to accept cryptocurrencies or use them to provide services [44]. Figure 5 presents the timeline of the main extrinsic events that might have affected the price of Bitcoin with the details of these events further tabulated in Table 2.

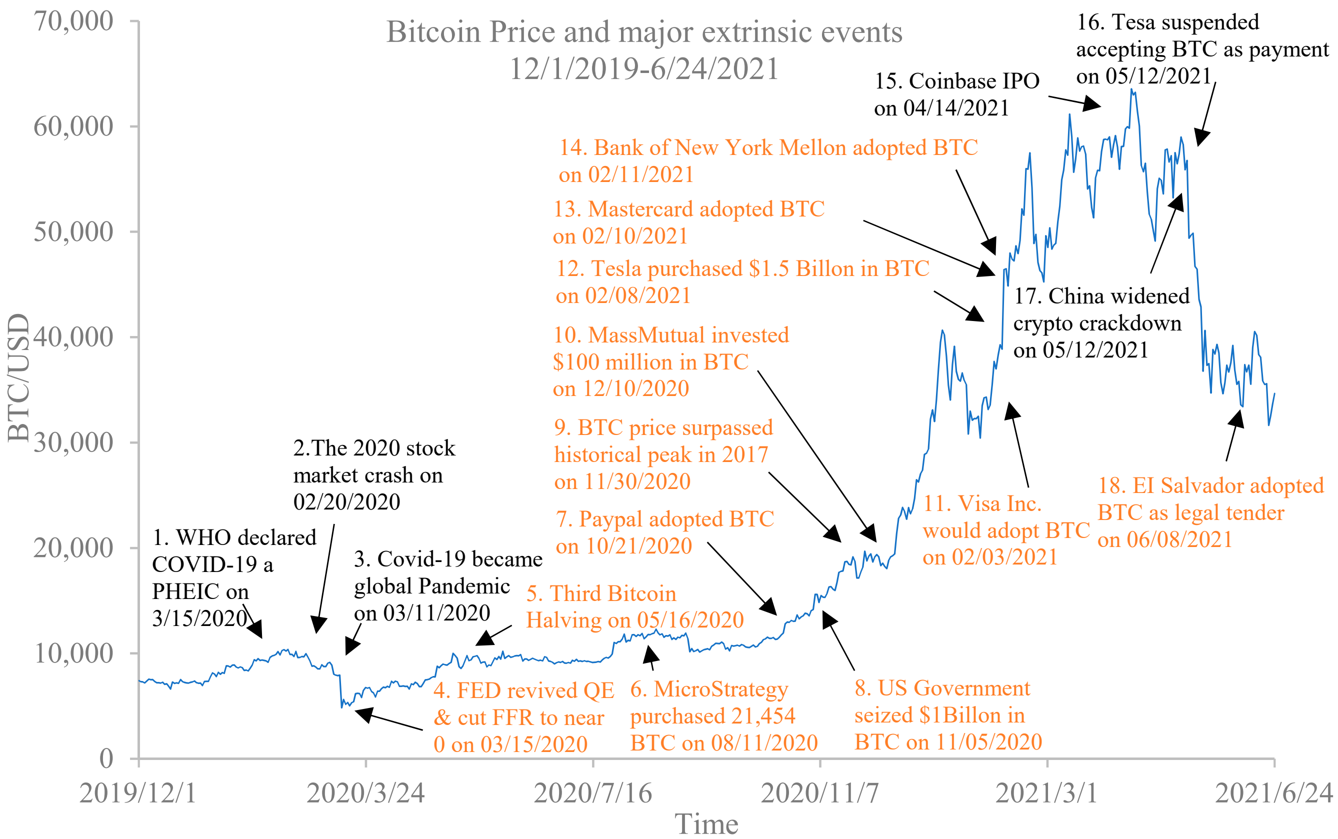


Figure 5. Timeline of main extrinsic events potentially affecting the price of Bitcoin with positive events marked in orange while negative events in black.

Table 2. Main extrinsic events that may have affected Bitcoin price from December 2019 to June 2021. Positive and negative events are marked in orange and black, respectively.

ID	Time	Events
1	30 January 2020	WHO issued the highest level of alarm: declaring the novel coronavirus outbreak a public health emergency of international concern (PHEIC) [45].
2	20 February 2020	Global stock markets started to crash and the S&P500 index fell by 33.9% in the next five weeks of 2020 [33]
3	11 March 2020	WHO declared the novel coronavirus COVID-19 a global pandemic [45].
4	15 March 2020	FED revived quantitative easing (QE) program and reduced the target range for the federal funds rate (FFR) to near zero [46].
5	16 May 2020	The third Bitcoin halving occurred, resulting in the mining rewards per every 10 min dropped from 12.5 to 6.25 BTC per block [47].
6	11 August 2020	The largest US independent publicly-traded business intelligence company MicroStrategy announced that it had spent $250 million to purchase 21,454 Bitcoins as primary treasury reserve asset [48].
7	21 October 2020	PayPal Holdings Inc. announced a new service allowing customers to buy, hold, or sell Bitcoin and other crypto coins using PayPal [49].
8	5 November 2020	US Government seized over $1 Billion in Bitcoin from Dark Web Marketplace Silk Road [50].
9	30 November 2020	Bitcoin reached a new all-time high of $19,860, surpassing the previous historical peak set in December 2017 [51].

Table 2. *Cont.*

ID	Time	Events
10	10 December 2020	The 169-year-old Massachusetts Mutual Life Insurance Co. invested $100 million in Bitcoin for its general investment account [52].
11	3 February 2021	Visa Inc. announced that a new crypto API program would launch later in the year to provide Bitcoin buying and trading services [53].
12	8 February 2021	Tesla disclosed that it had purchased $1.5 billion worth of Bitcoin and planned to accept Bitcoin as payment [42].
13	10 February 2021	Payments giant Mastercard announced that that it would support cryptocurrencies on its network from 2021 [54].
14	11 February 2021	The nation's oldest bank: Bank of New York Mellon, was the first global bank to start financing Bitcoin and other digital currencies, which was believed to be a validation of cryptocurrencies from a key bank [43].
15	14 April 2021	The largest cryptocurrency exchanges in the US launched the initial public offering (IPO) [55].
16	12 May 2021	Elon Musk took back Tesla's commitment and suspended accepting BTC as payment [44].
17	12 May 2021	Chinese financial institutions and businesses were banned to accept cryptocurrencies or use them to provide services [44].
18	8 June 2021	The first time Bitcoin was adopted as a legal tender by a country (EI Salvador) [3].

It should be noted that the Bitcoin price is characterized by a remarkably high volatility and the Bitcoin price can fluctuate greatly in a short period of time. As a result, the daily data of Bitcoin price cannot provide sufficient information about characteristics of severe price fluctuations, so the LPPLS model may fail to determine the bubble pattern effectively.

3.2. Estimation of Bubble Start Time

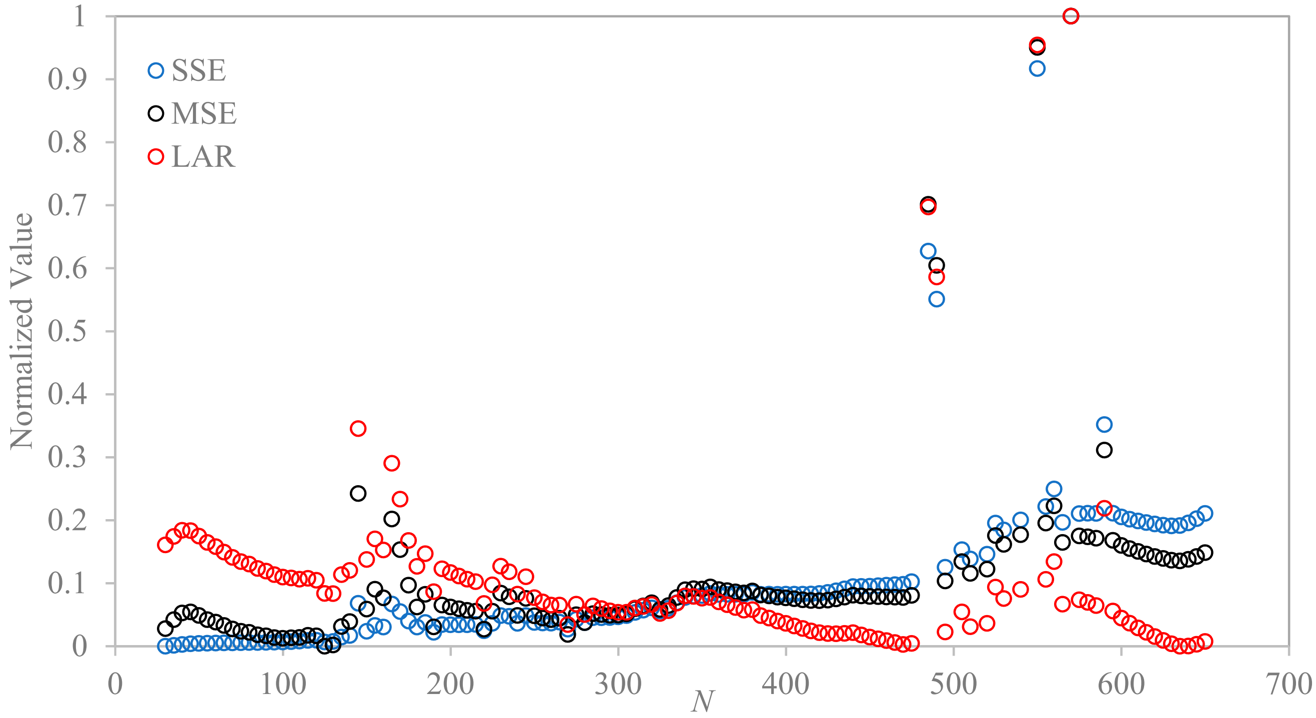
To demonstrate the modified Lagrange regularization method, we estimated the start time of the financial bubble peaking in early January 2021 by combining with the LPPLS model. The endpoint t_2 is set to 8 January 2021, and the length of fitting windows shrank from 650 data points to 30 data points in a step of 5 days.

Figure 6 shows three different goodness-of-fit measures for the shrinking fitting window in two different situations: (a) outliers are not removed, which is obtained based on the original Lagrange regularization method; and (b) outliers are removed, based on the modified Lagrange regularization method. The three measures include the sum of squared errors (SSE) $\sum_{i=t_1}^{t_2} r_i(\Phi)^2$, the mean squared errors (MSE) $\frac{1}{(t_2-t_1)-P}\sum_{i=t_1}^{t_2} r_i(\Phi)^2$, and the Lagrange regularization term (LAR) ${\chi_\lambda}^2(t_1,\ \Phi)$. For comparison, all three measures in Figure 6 are normalized. The X-axis label N represents the number of data points $t_2 - t_1$ in each fitting window or the length of each fitting window.

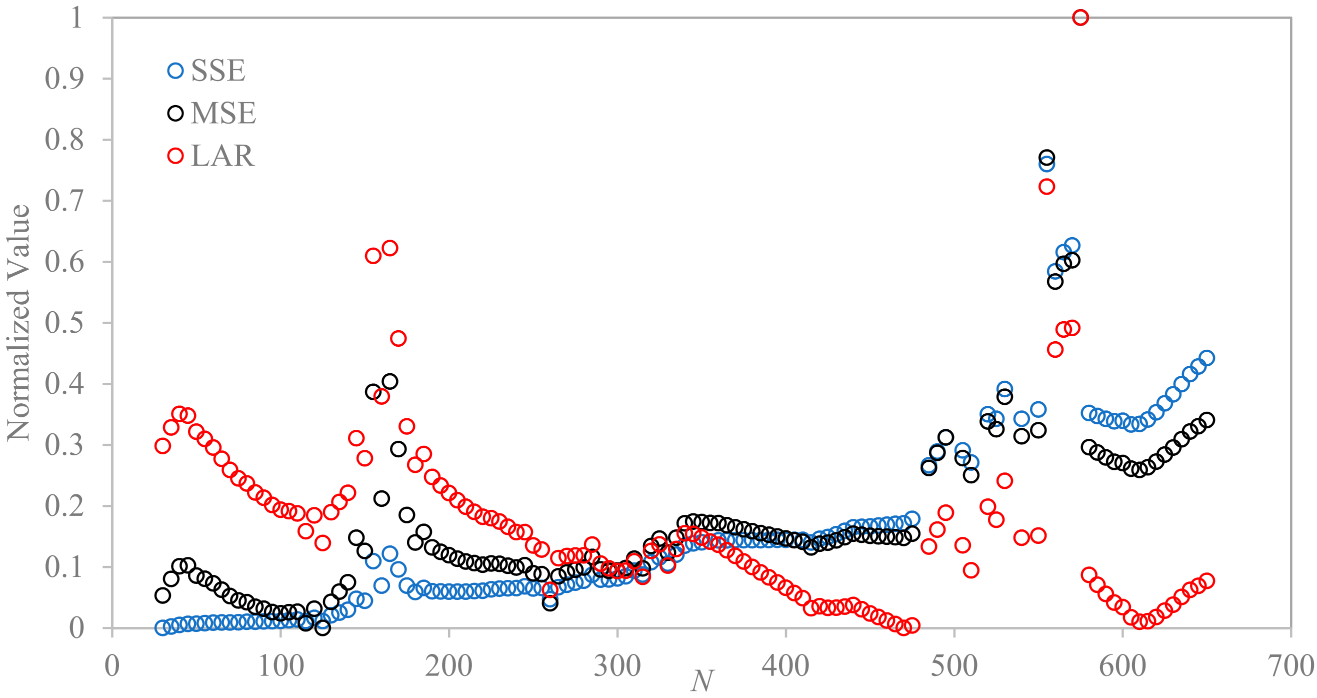
From Figure 6a,b, we can see clearly that the unbalanced size bias for SSE and MSE with the smaller length of fitting windows leads to a smaller value of the SSE and MSE when the number of data points N is reduced relative to the number of degrees of freedom. However, no significant unbalanced size bias is observed in LAR, since the length of the fitting window has been penalized in the cost function.

As shown in Figure 6a, the LAR reached the minimum value at N_λ= 610, which is corresponding to the bubble start time ${t_1}^* = 9$ May 2019. Few observations are separated in some fashion from the rest of the data, which can have a strong influence on the least squares line and distort the performance of the Lagrange regularization approach. The outliers may come from the over-fitting in the highly nonlinear LPPLS model. Based on the modified Lagrange regularization approach, we detected five outliers in total. After removing the outliers, the LAR in Figure 6b have the minimum value at $N_\lambda = 470$, corresponding to ${t_1}^* = 26$ September 2019, indicating the financial bubble peaking in early January 2021 had begun to form as early as 26 September 2019.

According to Figure 6b, we can also find that the start time of a financial bubble depends on the length of the fitting window analyzed. The location of the minimum LAR may be significantly different when the length of fitting windows analyzed is changed, e.g., the start time of a financial bubble is a relative time point, depending on the time frame analyzed.



(**a**)



(**b**)

Figure 6. Different goodness-of-fit measures for the shrinking fitting window with the endpoint t_2 = 8 January 2021: (**a**) outliers are not removed; (**b**) outliers are removed.

3.3. *Bubble Detection Using Weekly Data*

We used the weekly data in this section to investigate the 2021 Bitcoin bubble from a long-term time scale. We extracted data in the interval of the 5 trading days from the daily time series to generate a weekly time series of price trajectory. Based on the weekly time series, we calibrated the LPPLS model to obtain the weekly confidence indicator by shrinking the length of time windows $t_2 - t_1$ from 650 data points to 30 data points in steps of 5 data points, which corresponds to about nine years to half a year. Since the earliest date of the available historical daily data of Bitcoin we could retrieve is 13 September 2011, we calculated the weekly confidence indicator for the period from 29 August 2020 to 24 June 2021, which is shown in Figure 7.

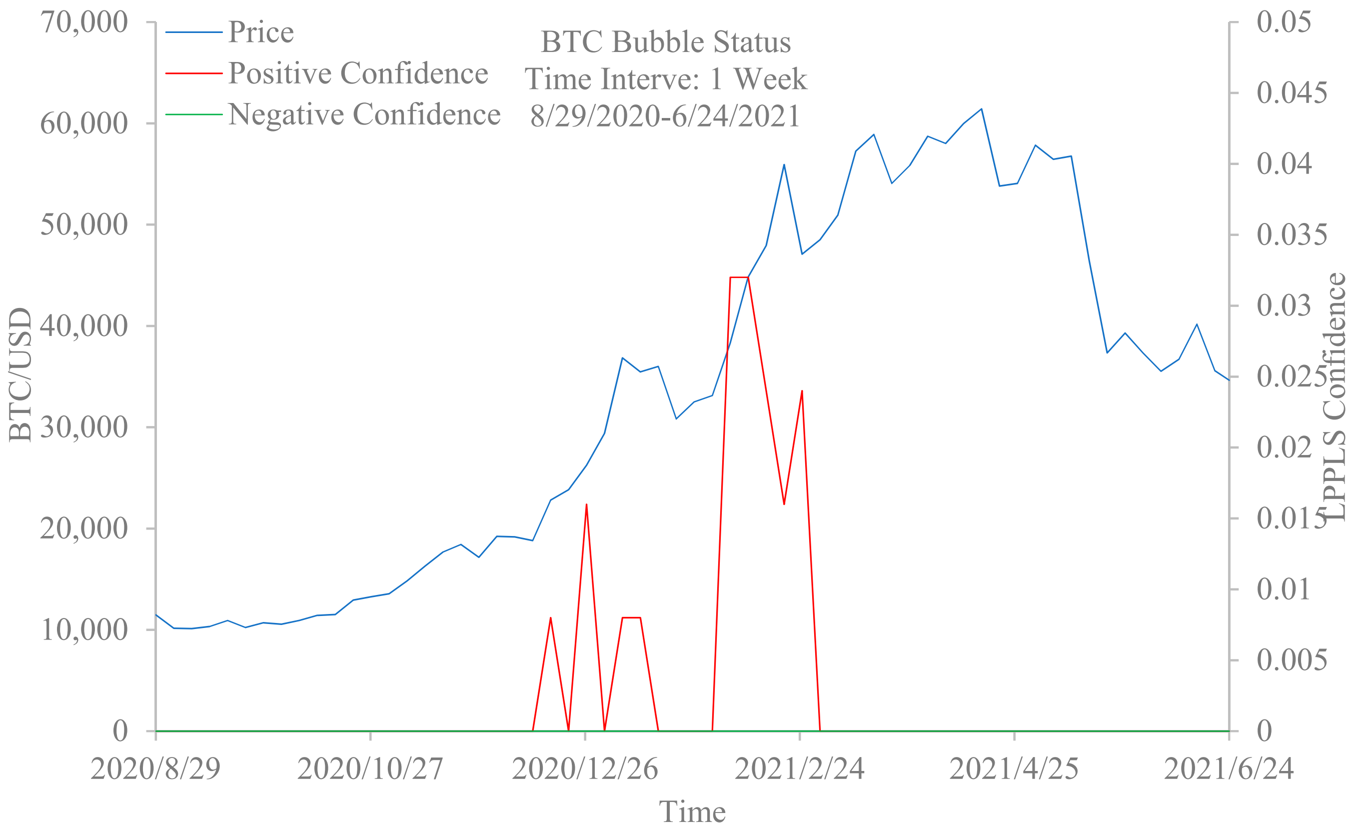


Figure 7. Weekly LPPLS confidence indicator for positive bubbles is shown in red and negative bubbles in green (right scale), along with the Bitcoin price in blue (left scale) from 29 August 2020 to 24 June 2021.

Two clusters of positive bubbles in Figure 7 formed between 17 December 2020 and 11 January 2021, and between 5 February 2021 and 25 February 2021. The weekly confidence indicator reached the peak value of 3.2% on 5 February 2021 and 10 February 2021, respectively, indicating that the significant signatures of the LPPLS model presented in the Bitcoin price trajectory in February 2021 and the Bitcoin price boom in February is an endogenous bubble. In this bubble regime, the Bitcoin price soared from USD 33,141.38 on 31 January 2021 to USD 55,936.04 on 20 February 2021—an increase of 68.78%. It should be noted that the results of weekly confidence indicator may not be consistent with that of the daily confidence indicator (e.g., the Bitcoin price boom in February based on daily confidence indicator in Figure 7 did not present the obvious endogenous bubble patterns), while it was characterized by clear endogenous bubble patterns based on weekly confidence indicator in Figure 7. This phenomenon is reasonable since an asset price trajectory exhibiting a super-exponential growth in a long-term scale may experience an exponential growth in a short-term scale, and vice versa.

3.4. Bubble Detection Based on Hourly Data

In order to overcome the shortcoming of daily data that lacks the ability to capture essential characteristics of Bitcoin's price trajectory when experiencing severe price fluctuations, we refined the time interval of price trajectory from one day to one hour. Based on the hourly price trajectory of Bitcoin downloaded from https://www.coinbase.com/ (accessed on 22 July 2021), we have detected the existence of positive bubbles and negative bubbles and monitored the changes in the bubble state from 1 December 2020 to 25 June 2021. Since the cryptocurrency market is usually open 24/7, we calibrated the LPPLS model for 24 endpoints t_2 for each day. For a specified endpoint t_2, a set of fitting windows is generated by shrinking the window length from 650 h to 30 h in step of 5 h, that is, approximately from one month to one day.

Figure 8 shows the hourly LPPLS confidence indicator for positive bubbles in red and negative bubbles in green along with the Bitcoin price in blue from 0:00 on 1 December 2020 to 0:00 on 25 June 2021. We can see that the hourly LPPLS confidence indicator can provide useful bubble diagnosis during the stage of dramatic price fluctuations and rapid state changes, which cannot be provided by the daily LPPLS confidence indicator.

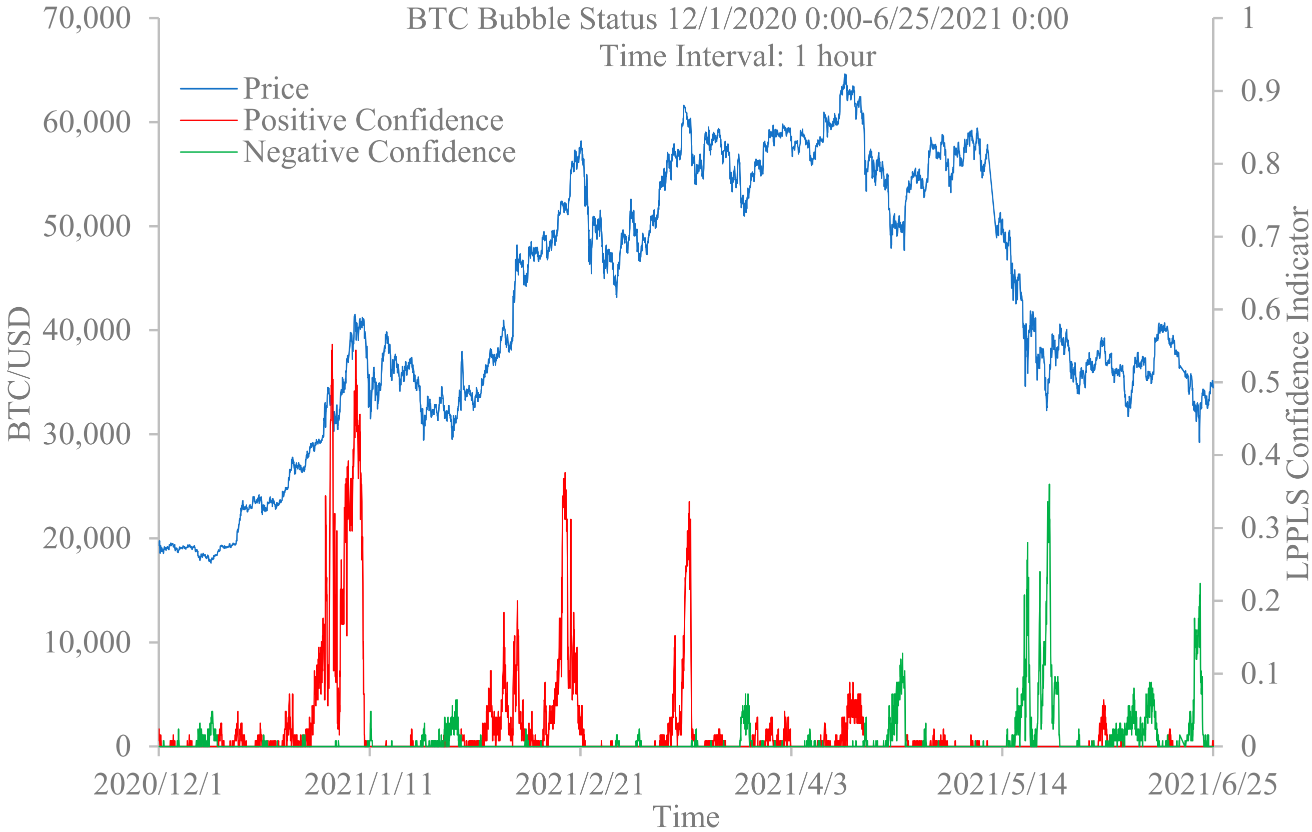


Figure 8. Hourly LPPLS confidence indicator for positive bubbles is shown in red and negative bubbles in green (right scale), along with the Bitcoin price in blue (left scale) from 1 December 2020 to 25 June 2021.

In Figure 8, five positive bubble clusters and six negative bubble clusters with peaks greater than 5% can be clearly observed. On 3 January 2021 at 22:00, the LPPLS confidence indicator for the positive bubble reached a peak of 55.2%, indicating that the endogenous bubble pattern has appeared in 69 out 125 fitting windows and the price trajectory has reached the system instability, and the bubble regime may change.

The first cluster of positive bubbles was formed from 26 December 2020 2:00 to 10 January 2021 5:00, with one peak of 55.2% on 3 January 2021 22:00, and another peak of 54.4% on 8 January 2021 12:00, followed by a negative bubble cluster from 21 January 2021 4:00 to 29 January 2021 6:00, with a peak of 6.4% on 28 January 2021 2:00. The detection of bubble regime changes is confirmed by the price movement of Bitcoin that surged to a local peak of $41,235.48 on 9 January 2021 19:00, and then dropped by 28.4% to $29,525.00 on 27 January 2021 8:00.

The second cluster of positive bubbles occurred from 2 February 2021 15:00 to 22 February 2021 7:00, with a peak of 37.6% on 18 February 2021 10:00. During this period, the price rocketed to $58,200.08 on 21 February 2021 12:00, increased by 97.1% from $29,525.00 on 27 January 2021 8:00. This bubble was terminated in the form of crash, which fell by 25.8% to $43,170.01 on 28 February 2021 11:00.

The third cluster of positive bubbles happened between 11 March 2021 4:00 and 16 March 2021 1:00 and reached a peak of 33.6% on 14 March 2021 16:00, followed by a negative bubble cluster from 24 March 2021 14:00 to 27 March 2021 0:00, with a peak of 7.2% on 25 March 2021 15:00. The Bitcoin price surged to $61,607.76 on 13 March 2021 14:00 with a 42.7% increase in two weeks, and then dropped by 17.2% to 50983.82 on 25 March 2021 10:00.

The fourth cluster of positive bubbles was observed from 13 April 2021 8:00 to 18 April 2021 6:00 and climbed to a peak of 8.8% on 15 April 2021 0:00, followed by a negative bubble cluster from 22 April 2021 13:00 and 26 April 2021 8:00 with a peak of 12.8% on 25 April 2021 8:00. The price of Bitcoin increased to the historical peak of $64,583.11 on 14 April 2021 6:00, and then fell by 25.8% to $47,895.01 on 23 April 2021 2:00.

Between 16 May 2021 17:00 and 26 May 2021 0:00, the largest cluster of negative bubbles was formed with a peak of 36.0% on 24 May 2021 0:00. During this period, the Bitcoin price suffered a notable crash in May 2021, dropped from $59,427.44 on 9 May 2021 22:00 to $32,270.87 on 23 May 2021 11:00, losing 45.7% of its value in two weeks and wiping out about $0.491 trillion in market value. During this period, the negative bubble regime changed in the form of volatile sideway plateaus, rather than a sharp rebound.

The last cluster of positive bubbles occurred from 2 June 2021 14:00 to 4 June 2021 8:00 and reached a peak of 6.4% on 3 June 2021 15:00, followed by a negative bubble cluster from 5 June 2021 14:00 to 14 June 2021 4:00. The price increased to a peak of $39,281.22 on 3 June 2021 5:00, and then decreased by 19.2% to $31,726.12 on 8 June 2021 10:00. Between 19 June 2021 22:00 and 23 June 2021 3:00, a negative bubble cluster was formed and reached a peak of 22.4% on 22 June 2021 11:00, corresponding to the price movement that the price of Bitcoin lost by 27.6% in one week from $40,390.26 on 16 June 2021 1:00 to $29,224.86 on 22 June 2021 8:00.

From Figure 8, we can see that most of the peaks (valleys) of Bitcoin price trajectory fall within the cluster of positive (negative) bubbles, demonstrating that the LPPLS model based on the hourly data has an excellent performance in detecting the existence of bubbles, monitoring the development of bubble state and accurately predicting the change of bubble regime, even when the Bitcoin price experience a roller coaster ride in a short period of time. In addition, the LPPLS confidence indicator is a reliable tool to quantify the probability of changes in bubble regime.

3.5. Short-Term and Long-Term Bubble Detection

In order to study the performance of LPPLS confidence indicator under different fitting window sizes, we divided the 125 fitting windows into two subgroups: short-term and long-term. In the short-term fitting window subgroup for a specified endpoint t_2, we shrank the length of the fitting window by moving the start time t_1 towards the end time t_2 from 200 data points to 30 data points in a step of 5 data points, generating 35 fitting windows. Similarly, we shrank the length of fitting window by moving the t_1 towards the t_2 from 650 data points to 205 data points in a step of 5 data points to generate 90 fitting windows for the long-term fitting window subgroup.

In this section, we investigated the performance of short-term and long-term LPPLS confidence indicators based on the hourly Bitcoin price trajectory. Thus, the short-term hourly LPPLS confidence indicator diagnosed bubble state based on the price data from 200 h previously to 30 h now, that is, approximately from one week to one day, and the long-term hourly LPPLS confidence indicator used the price data from 650 h to 205 h, that is, roughly from one month to one week.

Figures 9 and 10 show the short-term and long-term hourly LPPLS confidence indicator for positive bubbles in red and negative bubbles in green along with the Bitcoin price in blue from 0:00 on 1 December 2020 to 0:00 on 25 June 2021, respectively. It can be noted that the number of clusters of positive bubbles and negative bubbles in short-term subgroup in Figure 9 are much larger than those in the long-term subgroup in Figure 10, indicating that the bubble regime changes detected in the short-term time scale have a higher frequency than the long-term time scale, since the short-term LPPLS confidence indicator is more sensitive to the extreme fluctuations of price trajectory than long-term LPPLS confidence indicator. The short-term LPPLS confidence indicator provides a very useful diagnosis for the rapid changes of bubble regime on a short time scale, while the long-term LPPLS confidence indicator has a relative stable detection for the bubble state and can provide dynamics changes of the bubble regime on a long time scale. Based on the short-term and long-term hourly LPPLS confidence indicators, we can effectively detect the existence of bubbles and monitor the development of bubble regime on multiple time scales.

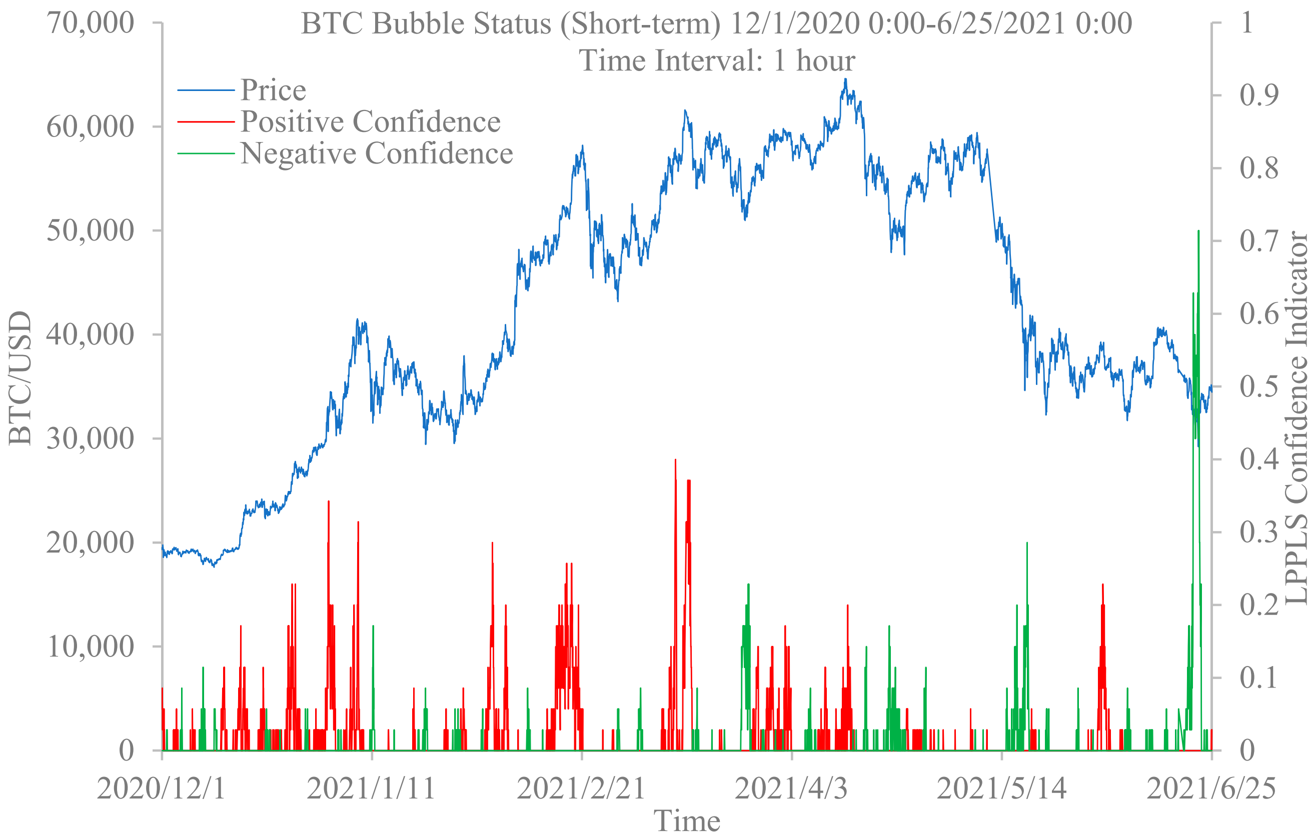


Figure 9. Short-term hourly LPPLS confidence indicator for positive bubbles is shown in red and negative bubbles in green (right scale), along with the Bitcoin price in blue (left scale) from 1 December 2020–25 June 2021.

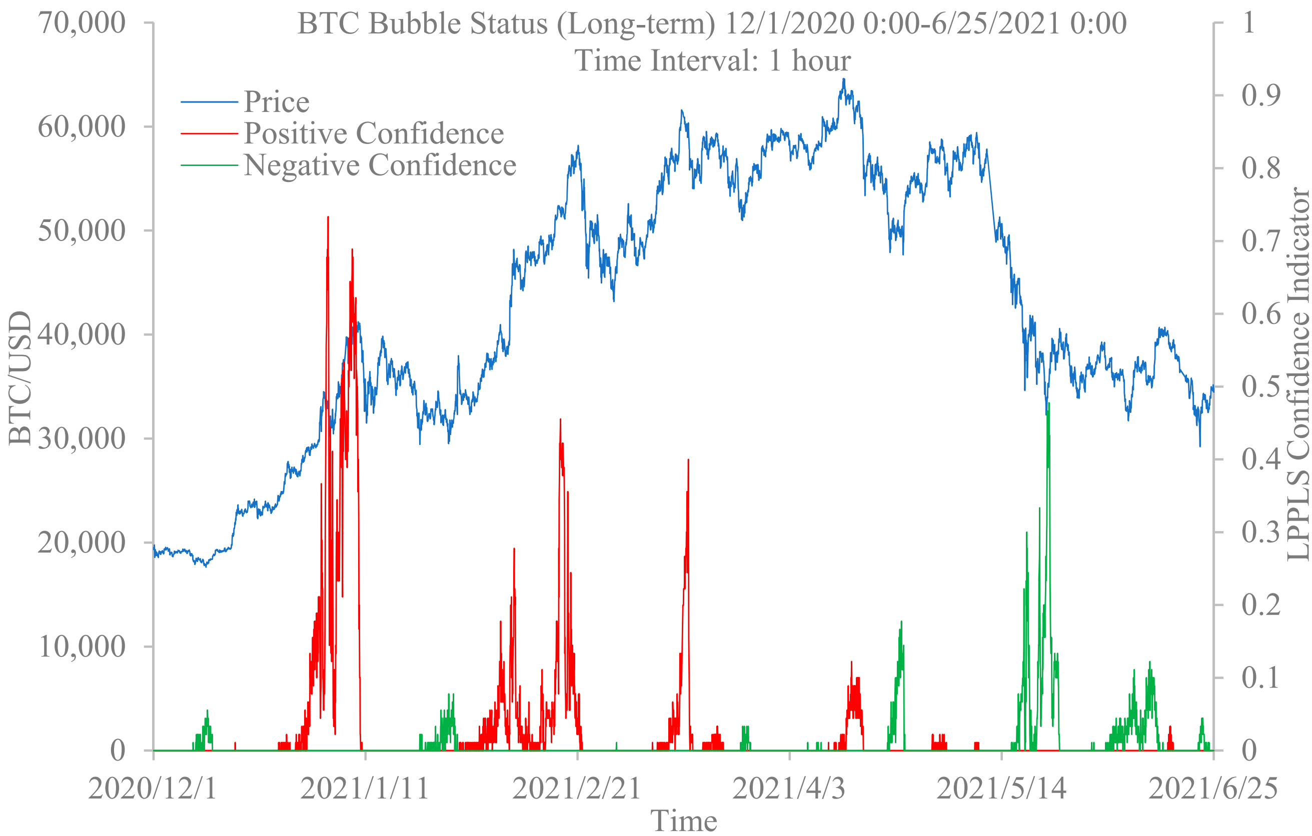


Figure 10. Long-term hourly LPPLS confidence indicator for positive bubbles is shown in red and negative bubbles in green (right scale), along with the Bitcoin price in blue (left scale) from 1 December 2020–25 June 2021.

4. Conclusions

Bitcoin, released in 2009 as the first decentralized cryptocurrency, has always dominated the cryptocurrency market and has experienced explosive growth as well as high volatility, attracting widespread attention with increasing impact on financial markets and society. The extreme volatility and rapid transition between Bitcoin price skyrocketing and plummeting have brought great challenges to predict the Bitcoin bubbles and crashes. In this study, we have adopted the LPPLS model to explore the underlying mechanism of the recent Bitcoin bubbles and crashes using a multi-resolution time-scale approach.

In summary, we have identified bubbles between 1 December 2019 and 24 June 2021 based on the daily data of Bitcoin price trajectory and detected the existence of one cluster of negative bubble from 6 December 2019 to 5 January 2020, and two clusters of positive bubbles between 7 February 2020 to 17 February 2020, and between 6 November 2020 and 17 January 2021, respectively. We have also found that the boom of Bitcoin from November 2020 to mid-January 2021 is an endogenous bubble, stemming from the self-reinforcement of cooperative herding and imitative behaviors of market participants, while the price spike from mid-January 2021 to mid-April 2021 is likely an exogenous bubble driven by exogenous events such as the large-scale acquisitions and adoptions by well-known institutes. Subsequently we have performed a systematic market event analysis and correlated them to Bitcoin bubbles and crashes, successfully.

To reduce the impact of the potential overfitting problem of LPPLS model on the estimation of financial start time, we have developed the modified Lagrange regularization method. We have also adopted a Bonferroni simultaneous test procedure to identify potential outliers. Based on the modified Lagrange regularization method, we estimated the financial bubble peaking in early January 2021 had sprouted from as early as September 2019. We have also used the weekly data to investigate the 2021 Bitcoin bubble from a long-term time scale and found that the daily and weekly data may feature different endogenous/exogenous causal profiles. In addition, using the hourly LPPLS confidence indictor from 1 December 2020 to 25 June 2021, we have discovered that most of the peaks and valleys of Bitcoin price trajectory fall within the clusters of positive and negative bubbles respectively, indicating that the LPPLS model based on the hourly data has an outstanding performance in detecting the change of bubble regime, especially when the Bitcoin price was on a roller coaster ride in a short period of time. Thus, the hourly LPPLS confidence indicator can provide useful bubble diagnostics during the stage of dramatic price fluctuations and rapid state changes, which is not available in the daily LPPLS confidence indicator. Lastly, it is shown that the short-term LPPLS confidence indicator is sensitive to the extreme fluctuations of price trajectory and can provide very useful diagnosis for the rapid changes of bubble regime on a short time scale. In contrast, the long-term LPPLS confidence indicator is not sensitive to the extreme price fluctuations and thus more attuned to detect the relative stable bubble state.

With the continuous increase in the acquisition and adoption of Bitcoin and other cryptocurrencies [56], the impact of bubble formation and its violent bursts continues to expand, rendering the establishment of a real-time bubble warning system, to foretell the impending bubbles and predict crash, an urgent issue. This study has created exactly such a paradigm for real-time bubble detection and monitoring, on a multi-resolution time scale. At present, the proposed analysis pipeline can only detect bubbles and crashes based on the LPPLS signature in price trajectory—further research is needed to incorporate other factors in the bubble/crash detection pipeline. Furthermore, robustness tests will be added in the follow up study as well.

Author Contributions: Conceptualization, M.S., R.S. and W.Z.; methodology, M.S. and R.S.; software, M.S. and R.S.; formal analysis, M.S. and R.S.; writing—original draft preparation, M.S. and R.S.; writing—review and editing, M.S., R.S. and W.Z.; project administration, M.S.; funding acquisition, M.S. All authors have read and agreed to the published version of the manuscript.

Funding: The New Faculty Start-Up Funds at the University of Wisconsin-Stout was approved on 6 November 2020. The Faculty Research Initiative Grant at the University of Wisconsin-Stout was approved on 11 November 2020. Both of two fundings are the internal fundings at UW-Stout. They do not have funding numbers.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The application datasets are publicly available.

Acknowledgments: The authors would like to thank the Blugold Supercomputing Cluster (BGSC) at the University of Wisconsin-Eau Claire.

Conflicts of Interest: The authors declare no conflict of interest.

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