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Brief Report

The Invariance of Inelastic Overlap Function

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Abstract: In this study, we consider the symmetry property of the inelastic overlap function and its relation to the reflective scattering mode appearance. This symmetry property disfavors an exclusion of one of the scattering modes—the reflective mode—when approaching the asymptotic limit. Predominance of the particular mode correlates with the energy and impact parameters ranges.

Keywords: unitarity; symmetry property of inelastic overlap function; reflective scattering mode

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1. Introduction: Unitarity of Partial Amplitude

Unitarity written for the partial wave amplitude $f_l(s)$ of elastic scattering (spin degrees of freedom are neglected) has a familiar form at high energies, i.e.,

$$\mathrm{Im} f_l(s) = |f_l(s)|^2 + \eta_l(s), \tag{1}$$

where the function $\eta_l(s)$ represents the contribution of the intermediate inelastic states into the product SS^+ ($SS^+ = 1$). The respective elastic S-matrix element is $S_l(s) = 1 + 2if_l(s)$.

Unitarity in the impact parameter representation ($b = 2l/\sqrt{s}$) connects the elastic and inelastic overlap functions introduced by Van Hove [1] with $h_{tot}(s,b) \equiv \mathrm{Im} f(s,b)$ by the following relation

$$h_{tot}(s,b) = h_{el}(s,b) + h_{inel}(s,b) \tag{2}$$

The respective cross-sections are determined by the integrals of the functions h_i over b:

$$\sigma_i(s) = 8\pi \int_0^\infty bdbh_i(s,b) \tag{3}$$

where $i = tot, el, inel$. The impact parameter representation has an appealing feature providing a geometric, semiclassical meaning to the scattering picture of hadrons.

This note is devoted to the inelastic overlap function, $h_{inel}(s,b)$, namely, its energy behavior at $b = 0$. Based on unitarity we consider this quantity as a function of elastic scattering amplitude. The essential role of elastic scattering is determined by the strong coherence of hadron constituents during this process. Despite hadrons being extended objects, they demonstrate a significant probability to survive under interaction, i.e., a rather significant contribution to pp-interactions is provided by the elastic scattering with the ratio of elastic to total cross-sections $\sigma_{el}(s)/\sigma_{tot}(s)$ increasing with energy. The increase of this ratio is a result of the effective increase with energy of the intensity of soft and strong interactions. Hence, the geometrical characteristics of hadron collisions and dynamics related to the elastic scattering are essential for the hadron interaction understanding, i.e., for the development of QCD in the soft region where the confinement and collective effects in constituents interactions play a crucial role. Thus, this process is an important tool for the confinement studies in QCD. In this nonperturbative region, an explicit relation of the intensity of interaction with the running coupling constant of QCD has not been found due to the unsolved problem of confinement and an existence of the spontaneous chiral symmetry breaking phenomenon.

It is important to note that unitarity is formulated for the asymptotic colorless hadron on-mass shell states and is not directly associated with the fundamental fields of QCD —quarks and gluons. The operator S is defined in the Hilbert space and spans over vectors corresponding to the observable physical particles. The unitarity property of this operator is formulated in terms of the physical particles. Even an extension to the off-mass shell colorless states leads to significant changes in the predicted behavior of observables.

Unitarity allows for variation of the scattering amplitude in the interval $0 \leq |f| \leq 1$, which covers both the absorptive and reflective scattering modes when $|f| \leq 1/2$ and $|f| > 1/2$, respectively. Consideration of the inelastic overlap function provides valuable insight on the nature of the scattering. Transition to the reflective scattering mode results in the peripheral behavior of the inelastic overlap function as well as changes to the structure of the hadron interaction region [2].

The main point of this note is that the symmetry property of the inelastic overlap funcion does disfavor an exclusion of one of the scattering modes—the reflective mode —when approaching the asymptotic limit. A predominance of thisparticular mode correlates with the energy and impact parameters variation ranges.

The size and form of the reflective scattering mode interaction region are consistent with the results of the impact parameter analysis performed at the LHC energies and can be interpreted as a manifestation of the color conducting phase formation. The observation of the reflective scattering mode underlines the importance of events classification regarding the impact parameter value of the collision. This mode most significantly affects collisions with small impact parameters.

Reflective scattering mode does not imply any kind of hadron transparency in the head-on collisions. Rather, it is about the geometrical elasticity. The term transparency is relevant for the energy and impact parameter range related to the shadow scattering regime only, i.e., where $f < 1/2$.

The emerging physical picture of the high energy hadron interaction region in a transverse plane can be visualized then in the form of a reflective disk (with its albedo approaching to complete reflection at the center at $s \to \infty$), which is surrounded by a black ring (with complete absorption, $h_{inel} = 1/4$) since the inelastic overlap function h_{inel} has a prominent peripheral form at $s \to \infty$ in this scattering mode. The reflection mode implies that despite the limiting behavior of $S(s, b)$ corresponds to $S \to -1$ at $s \to \infty$ and fixed b, the gap survival probability tends to zero at $s \to \infty$ and, in particular, contribution of the central production processes is consistent with unitarity. Indirect information on the reflective ability of the interaction region can, in principle, be extracted from the differential cross-section of the deep-elastic scattering. The scattering in the deep-elastic region, i.e., at large transferred momenta at sufficiently high energies, is sensitive to the reflective ability of the interaction region arising due to presence of the inner core in proton structure.

The interpretation of the negative values of S as corresponding to reflective scattering is based on analogy with optics [2]. The reflective scattering is a natural interpretation of unitarity saturation based on the optical concepts in high energy hadron scattering and can be interpreted as a result of the continuous increasing density of the interacting hadrons with energy. Thus, there is a close analogy with the light reflection off a dense medium when the phase of the reflected light is changed by 180°.

The reflective scatting can also be useful for the explanation of the non-monotonous energy dependence of inelasticity and other phenomena in the cosmic rays studies. The hadron interactions and mechanism of particle generation would be changing in the region of $\sqrt{s} = 3 - 6$ TeV. However, the interpretation of the cosmic ray data is complicated since the primary energies of cosmic particles are far beyond the energies of modern accelerators. Despite this, one may use interpretation of the knee existence in the cosmic ray spectrum as an indication to the effect of changing hadron interaction dynamics related to appearance of the reflective scattering mode. The logarithmic increase of the inelastic cross-section is of interest for interpretation of the results obtained for the inelastic processes measured in the extended air showers under the cosmic rays studies. The existing cosmic data suggest a decreasing dependence of the ratio $\sigma_{inel}^{p-air}(s)/\sigma_{tot}^{pp}(s)$ up to the energy $\sqrt{s} = 10^2$ TeV. Despite the cosmic data not being very conclusive due to low statistics, they reveal general trends with certainty. The data do not indicate flattening of this ratio with the energy, i.e., the data do not exclude the possibility that the ratio continuously decreases approaching zero at $s \to \infty$.

The reflective scattering mode leads to formation of a peripheral impact parameter dependence of the inelastic interactions probability because of unitarity. The peripherality increases with the increase of the colliding energy. This reflective scattering mode significantly affects collisions with small impact parameters, suppressing the production of secondary particles in such collisions. Emerging ring-like dependence of the inelastic overlap function can be associated with the effect of self-dumping inelastic channels contribution. This effect can be understood as a depletion of the radiative state mechanism manifestation and can be associated with a randomization of the phases of multiparticle production amplitudes. Such randomization can be considered as a result of the color-conducting collective formation in the intermediate state of hadron interactions. Its role increases with the energy increase.

The enhancement of the peripheral particle production would destroy the balance between orbital momentum in the initial and final states; most of the particles in the final state would carry out orbital momentum. To compensate for this difference in the orbital momentum, the spins of the particles should be lined up, i.e., the spins in the ring-like events should demonstrate significant correlations.

An essential feature of the reflective scattering mode is the decoupling of elastic scattering from multiparticle production The knowledge of decoupling mechanism is important for the development of the nonperturbative QCD, where confinement plays a leading role. Information on the interaction dynamics of soft hadron collisions have remained rather limited for a long time.

General principles play a guiding role in the soft hadron interaction studies. Unitarity regulates the relative strength of elastic and inelastic interactions. This regulatory function of unitarity is the main issue of this study, and the next section will quantify the degree of reflection of the interaction region and its necessity.

2. Symmetry of Inelastic Overlap Function

We consider a pure imaginary case of elastic scattering amplitude with replacement $f \to if$. Inelastic overlap function can then be expressed through the scattering amplitude $f(s,b)$ in the following form due to unitarity

$$h_{inel}(s,b) = f(s,b)[1 - f(s,b)]. \tag{4}$$

To clarify the symmetry property of the inelastic overlap function, we will consider the energy variation of the scattering amplitude f under a fixed value of the impact parameter. It is enough to consider the case of $b = 0$. The region of variation of the scattering amplitude covers the range $0 \le f \le 1$ and $h_{inel} \equiv h_{inel}(s,0)$, invariant under replacement

$$f \leftrightarrow 1 - f. \tag{5}$$

Thus, the Equation (4) is invariant for the two amplitude variation intervals (0, 1/2] and [1/2, 1), i.e.,

$$(0, 1/2] \leftrightarrow [1/2, 1) \tag{6}$$

and both ranges of amplitude variation correspond to a single range of inelastic overlap function variation $(0, 1/4]$. These two intervals are equivalent in the sense that h_{inel} repeats its values. Note, that the use of U-matrix unitarization [3] provides a continuous transition from absorptive to reflective scattering mode, covering the whole range of the amplitude variation allowed by unitarity.

The energy evolution of the elastic S-matrix scattering element S ($S \equiv S(s,0)$), the elastic scattering amplitude f ($f \equiv f(s,0)$), and the inelastic overlap function from some initial energy s_i to a final value s_f across the energy s_m where a h_{inel} has its maximal value, ($h^m_{inel} = 1/4$):

$$s_i \to s_m \to s_f \tag{7}$$

has been discussed in Ref. [3]. It is illustrated by the following relations:

$$S^i > 0 \to S^f < 0, \text{ i.e., } f^i < 1/2 \to f^f > 1/2, \tag{8}$$

and

$$h^i_{inel} < 1/4 \to h^f_{inel} < 1/4. \tag{9}$$

Thus, the inelastic overlap function h_{inel} can perform a loop variation with increasing energy in accordance with Equation (7). It varies as

$$h^i_{inel} \to 1/4 \to h^f_{inel} \tag{10}$$

and $h^f_{inel} = h^i_{inel}$ provided that $f^i + f^f = 1$.

Equation (9) means the appearance of the reflective scattering mode and it takes place regardless of the scattering amplitude form. Indeed, such behavior of h_{inel} implies an onset of decrease (i.e., $\partial h_{inel}(s,b)/\partial s$ at $b = 0$ becomes negative) of the inelastic overlap function and is associated with the appearance of reflection at the LHC energy range. The U-matrix form of unitarization and its relation to the symmetry property of h_{inel} was used for continuous extrapolation to higher energies in Ref. [3]. It incorporates both scattering modes, providing a ground for their simultaneous presence. The impact parameter profile of $h_{inel}(s,b)$ becomes peripheral when $s > s_m$.

Thus, the invariance of the inelastic overlap function h_{inel} is in favor of coexistence of the absorptive and reflective scattering modes in elastic scattering (see a discussion of the latter in Refs. [2,3]) and corresponds to the transition of the elastic scattering matrix element:

$$S \leftrightarrow -S. \tag{11}$$

It is not clear whether this property has a meaning of a separate physical concept. However, it is coherent with the saturation of unitarity limit for the amplitude f implied by the principle of maximal strength of strong interactions proposed long ago by Chew and Frautchi [4,5]. They noted that a "characteristic of strong interactions is a capacity to "saturate unitarity condition at high energies". Factor -1 in Equation (11) is interpreted as a result of a reflection by analogy with optics. It can also be considered as a result of an analogue of Berry phase appearance [2] or of color-conducting matter formation in the interaction region of reaction [6]. The color-conducting matter formation can be used in its turn for explanation of the correspondence of above symmetry property to a quark-hadron duality (confinement) [7]. For discussion of correlation of the S-matrix unitarity and confinement, see Ref. [8]. It is also suggested to associate this mode with effective resonance formation resulting from " an exceptional intermediate state that unites the correlated patrons" [9–11].

3. Real Part of the Scattering Amplitude

It should be recollected that the above symmetry property for the inelastic overlap function takes place for the pure imaginary scattering amplitude.

This section discusses the changes in the symmetry properties due to the real part of the scattering amplitude. By relaxing the requirement of a pure imaginary elastic scattering amplitude and taking $b = 0$, the unitarity condition in the impact parameter representation can be rewritten in the form:

$$\mathrm{Im} f[1 - \mathrm{Im} f] = h_{inel} + [\mathrm{Re} f]^2. \tag{12}$$

It is evident from Equation (12) that

$$[\mathrm{Re}f]^2 \leq 1/4 - h_{inel}. \tag{13}$$

Thus, $|\mathrm{Re}f| \to 0$ in both cases: when $h_{inel} \to 1/4$ (full absorption, $S = 0$) and/or $\mathrm{Im}f \to 1$ (full reflection, $S = -1$). Without neglect of the small real part of the scattering amplitude, one should consider invariance of the function $h_{inel} + [\mathrm{Re}f]^2$ under replacement

$$\mathrm{Im}f \leftrightarrow 1 - \mathrm{Im}f. \tag{14}$$

An account of a small real part of the scattering amplitude does not qualitatively change the results, Numerical calculations based on the model-independent analysis of available experimental data [12] give $h_{inel} = 1/4 - \alpha$ with small positive function α, $\alpha \ll 1/4$, at the LHC energies.

4. Conclusions

The symmetry property of the inelastic overlap function and simultaneous presence of the two scattering modes are a straightforward consequence of unitarity. It is in favor of the coexistence of the absorptive and reflective scattering modes at small impact parameter values. When addressing the asymptotics with the use of the respective relations, for example, the Gribov–Froissart projection formula [13,14], one should not expect the symmetry realization since the scattering is reduced to a purely absorptive mode at large impact parameter values. In contrast, the considered symmetry property disfavors an *ad hoc* exclusion of one of the scattering modes (i.e., the reflective mode) under approaching the asymptotic limit $s \to \infty$. Predominance of the particular mode is to be correlated with the energy and impact parameters ranges under consideration. An important problem is how to estimate the impact parameter value in a given pp-collision event from the data. In view of the prominent collective effects observed in small systems, such as pp-collisions, together with indication on the reflective scattering mode presence at the LHC, an introduction of a type of analog of centrality accounting these phenomena is necessary to classify the events of interest. It is evident that the event classification by multiplicity of the final state is not relevant for that purpose, since a contribution from the elastic channel is then almost neglected, and measurements of the transverse energy is a more adequate experimental tool. In other words, the most relevant observable seems to be a sum of the transverse energies of the final state particles. Such a sum obtains a significant elastic channel contribution.

In addition, an important issue is the continued search for the unambiguous experimental manifestations of the reflective mode in elastic and inelastic hadron interactions. It is evident that the experiments at the LHC are essential for understanding the proton structure as well as the structure of the proton interactions region. Usage of the impact parameter picture and addressing the reflective scattering mode allow us to combine the analyses of the results from those experiments for physics interpretation of the reflective scattering.

QCD is a theory of hadron interactions with colored objects confined inside those entities. One can imagine that the color conducting medium is being formed instead of the color insulating one when the energy of the colliding hadrons increases beyond some threshold value. The properties of such a medium are actively being studied in nuclear collisions, but the color conducting phase can be generated in hadron interactions too. The appearance of the reflective scattering mode can be associated with the formation of the color conducting medium in the intermediate state of hadron interaction. Keeping in mind the quark-gluon hadron structure, it would be tempting to relate the reflective scattering mode to the quark-gluon plasma formation in small systems.

An experimentally deconfined state of matter has been discovered at RHIC, where the highest values of energy and density have been reached. This deconfined state appears to be a strongly interacting collective state of matter with properties of the perfect liquid. The importance of the experimental discoveries at RHIC is that the matter reveals a high degree of coherence when it is well beyond the critical value of density and temperature.

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References

1. Van Hove, L. High-energy collisions of strongly interacting particles. *Rev. Mod. Phys.* **1964**, *36*, 655–665. [CrossRef]
2. Troshin, S.M.; Tyurin, N.E. Reflective scattering from unitarity saturation. *Int. J. Mod. Phys. A* **2007**, *22*, 4437–4449. [CrossRef]
3. Troshin, S.M.; Tyurin, N.E. Energy evolution of the overlap functions: increasing ratio of $\sigma_{el}(s)/\sigma_{tot}(s)$ and black ring emergence. *J. Phys. G Nucl. Part. Phys.* **2022**, *49*, 115001. [CrossRef]
4. Chew, G.F.; Frautchi, S.C. Unified approach to high- and low-energy strong interactions on the basis of the Mandelstam representation. *Phys. Rev. Lett.* **1960**, *5*, 580–583. [CrossRef]
5. Chew, G.F.; Frautchi, S.C. Dynamical theory for strong interactions at low momentum transfers but arbitrary energies. *Phys. Rev.* **1961**, *123*, 1478–1486. [CrossRef]
6. Troshin, S.M.; Tyurin, N.E. Reflective scattering and color conductivity in hadron collisions. *J. Phys. G Nucl. Part. Phys.* **2019**, *46*, 105009. [CrossRef]
7. Brodsky, S. J.; Roberts, C. D.; Shrock, R.; Tandy, P.C. Confinement contains condensates. *Phys. Rev. C* **2012**, *85*, 065202. [CrossRef]
8. Troshin, S.M.; Tyurin, N.E. Collective effects in multiparticle production at the LHC. *Int. J. Mod. Phys. A* **2011**, *26*, 4703–4730. [CrossRef]
9. Anisovich, V.V.; Nikonov, V.A.; Nyiri, J. Hadron collisions at ultrahigh energies: Black disk or resonant disk modes? *Phys. Rev. D* **2014**, *90*, 07405. [CrossRef]
10. Anisovich, V.V. Gribov universality of hadron cross sections at ultrahigh energies. *Int. J. Mod. Phys. A* **2016**, *31*, 1645010. [CrossRef]
11. Nekrasov, M.L. Resonant mode of elastic scattering at ultrahigh energies. *Phys. Rev. D* **2022**, *106*, 014028. [CrossRef]
12. Csörgo, T.; Pasechnik, R.; Ster, A. Proton structure and hollowness from Lévy imaging of pp elastic scattering. *Eur. Phys. J. C* **2020**, *80*, 126. [CrossRef]
13. Gribov, V.N. Possible asymptotic behavior of elastic scattering. *JETP Lett.* **1961**, *41*, 667-669.
14. Froissart, M. Asymptotic behavior and subtractions in the Mandelstam representation. *Phys. Rev.* **1961**, *123*, 1053–1057. [CrossRef]