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Developing the Urban Fire Safety Co-Management System in China Based on Public Participation

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Abstract: The new situations, problems, and challenges facing urban fire safety work are gradually increasing in China, so innovating urban fire safety governance modes is an urgent task. In the fire management practice of the Chinese government, the establishment of an urban fire safety co-management system is an important measure for aggregating fire safety management resources and improving the level of urban fire safety prevention, as well as control. In order to reveal and clarify the interacting relationships and influencing mechanisms among multiple subjects in an urban fire safety co-management system, we constructed an urban fire safety co-management game model comprising fire supervision departments, production management units, and the public based on evolutionary game theory. The stability of the urban fire safety co-management game system is explored from the perspective of game subjects. The influencing factors of strategy selection between game subjects in the game system were investigated using numerical simulation analysis. The research results show that elevating the informatization level of co-management, the risk perception level of the public, and the disclosure level of fire safety information are conducive to stimulating the public's positivity to participate in co-management. Strengthening the accountability of the superior government is conducive to ensuring the supervision level of fire supervision departments. The above measures have positive value for optimizing China's urban fire safety co-management systems, establishing urban fire safety management synergy, and ensuring the stability of social fire safety situations.

Keywords: urban fire safety; social co-management; fire supervision; evolutionary game theory; game systems

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1. Introduction

Fire safety is a socialized systematic project involving all aspects and the whole process of social governance [1,2]. In recent years, with the rapid development of China's economy and society, the new situations, problems, and challenges facing fire safety work have gradually increased [3–5]. The situation of urban fire prevention and control is gradually becoming more complicated, and the level of urban fire safety management is not encouraging [6,7]. On the one hand, there are more types of urban fire risks that are more difficult to prevent and control, as well as a wider range of fires [8]. On the other hand, non-traditional and traditional fire safety problems interweave, influence, and penetrate each other [9,10], and the problem of fire risk remediation and rebound are very prominent. The uncertainties and uncontrollable factors of fire are constantly increasing [11,12]. It is therefore urgent to further consolidate the foundation of urban fire safety, strengthen the ability of fire safety assurance, and comprehensively improve the level of public fire safety in the whole of society.

In 2023, the Chinese government set up the National Fire and Rescue Administration as a component of the State Council, which is administered by the Ministry of Emergency Management, to supervise and manage fire safety across the country [13]. As an important component of the government's public management, fire management refers to law enforcement actions such as fire supervision departments conducting fire safety supervision and inspection on production management units and punishing them for violating fire management policies and regulations according to the law [14]. Performing fire safety management and improving the level of fire prevention is of great value to improve urban resilience and ensures the stable development of the economy and society, but there are still many challenges facing urban fire safety management in China [15]. Among them, the more prominent problem of the insufficient number of grassroots fire safety management personnel has led to insufficient coverage of fire safety management, and thus the phenomenon of management omission occurs from time to time [16,17].

Therefore, it is necessary to innovate the urban fire safety management mode and fully mobilize the proclivity of the public to participate in fire co-management. Promoting the establishment and improvement of an urban fire safety co-management system can help expand the coverage of fire risk rectification, establish fire safety management synergy, and ensure the stability of urban fire safety situations. Compared with the traditional fire safety management mode, having public participation in fire safety co-management causes the relationships and roles of fire supervision departments and production management units in the supervision process to change and present new characteristics [18]. Therefore, it is necessary to analyze the interacting mechanism among various subjects in urban fire safety co-management from a systematic perspective and clarify the key factors affecting the inter-subject relationships, which has important value for improving the level of urban fire safety co-management.

At present, the existing studies have mainly focused on the main subjects of fire management, the content of fire management, and the strategy of fire management. Some scholars have summarized the fire management problems associated with densely populated places, major fire risks, inflammable and explosive units, hazardous chemical enterprises, and other units. Through identifying the potential weak links [19–22] and assessing the fire risks of urban fire safety [23–26], countermeasures and suggestions to improve fire management are given accordingly. In identifying the potential weak links of urban fire safety, Ardianto and Chhetri applied the Markov chain method to model the spatio-temporal dynamics of urban residential fire risk and predict the possibility of fire occurrence [19]. Noori et al. constructed the urban vulnerability index map by using the Fuzzy-VIKOR approach in a Geographic Information System (GIS) [20]. Then, the regions with urban vulnerability to potential fire hazards are identified based on the multi-criteria decision-making (MCDM) method. Based on the geographically weighted regression (GWR) model and the novel machine learning framework, Kumar et al. analyzed the relationships between urban patterns and fire causes, which can improve the fire management level of decision-makers over time and space [21]. Singh et al. conducted a hotspot analysis of structure fires based on GIS-based spatial analysis and statistical techniques [22]. Further, hotspots were delineated by the kernel density estimation (KDE) and Getis-Ord Gi* with inverse distance weighting (IDW). In assessing the fire risks of urban fire safety, Bai and Liu proposed the theoretical framework for urban safety resilience based on the triangle model, which includes the indicators of fire hazard, regional characteristics, and fire resilience. Furthermore, the entropy weight method and cloud model are used to analyze the urban safety resilience and risk level ratings. Wang et al. developed the urban fire risk assessment method based on the points of interest (POIs) data. Further, the distribution of the fire risks can be obtained through the POIs and the kernel density analysis (KDA). Rani et al. integrated hazard mapping of physical (PVS) and social (SVS) parameters based on a Geographic Information System (GIS) and the analytic hierarchy process (AHP) in order to achieve the hierarchical integrated spatial fire risk assessment. Masoumi et al. drew fire risk maps and vulnerability maps based on the spatial information produced using unmanned aerial vehicles (UAVs) in order to realize the fire risk assessment of urban buildings. Meanwhile, some studies have analyzed the key links that need to be paid attention to in urban fire safety management and discussed how to realize the connection between fire safety management practice and fire management policies [19,21,23,24]. On the whole, the existing studies show the tendency to attach importance to the method design. There are few research results on the interaction relationships and strategy changes among various subjects in urban fire safety management.

In an urban fire safety co-management system, coordination and cooperation among various subjects are inevitably involved. Meanwhile, fire safety management behaviors among game subjects are mutually influenced. Some scholars have introduced game theory to study the strategy behavior between fire management departments and social organizations. Bai et al. constructed the game model of fire safety management between fire management departments and social organizations and analyzed the relationships between punishment intensity and sampling ratio on strategy selection [27]. In further research, the supervision game model based on penalty and responsibility was established [28].

However, compared with game theory, evolutionary game theory is more suitable for the game problem of subjects in fire safety management. This is because evolutionary game theory mainly includes the following characteristics: Firstly, when the evolutionary game model is constructed, the game subjects are not required to be completely rational. Secondly, evolutionary game theory is a research method that combines game theory with a dynamic evolution process, which emphasizes a dynamic equilibrium. As a long-term and dynamic process rather than a static one, fire safety management is more compatible with evolutionary game theory [29]. Thirdly, in the process of evolutionary game analysis, there are both selection and mutation processes in the evolution process of game subjects, which are more in line with the actual situation of fire safety management [30]. To explore this further, this study will introduce evolutionary game theory to describe the interactive relationships of subjects in an urban fire safety co-management system. Evolutionary game theory is an important method for exploring and describing strategy selection and the interacting relationships among game subjects. It has experienced rapid development and application in environmental management [31], project management [32], emergency management [33], information management [34], public health management [35], marine management [36], and other government management problems. By summarizing and analyzing the game and interactive relationships between the governments and the managed objects, the existing studies have accordingly built the evolutionary game models to explore the decision-making modes of each subject in the process of government management. Meanwhile, some studies add the public as the main subject of the game or split the types of governments, which increases the complexity of the game models. In addition, some studies have enriched the influencing factors of game subjects' strategy selection by introducing mechanisms of rewards, punishments, and superior government supervision.

In the research direction of safety management, scholars have analyzed management strategies in key areas such as coal mine safety [37], agricultural safety [38], environmental safety [39], food safety [40], traffic safety [41], and building safety [42] management based on evolutionary game theory. Liu et al. built the evolutionary game model of coal mine safety regulation, which is composed of a coal mine regulator and multiple coal mine enterprises. The evolution strategy of game subjects discussed under the penalty strategy is static payment or dynamic payment based on the system dynamics method [37]. Wang and Xu constructed the agriculture product supply chain game model based on agricultural product suppliers, agricultural product processors, and quality inspection departments. Meanwhile, consumer feedback and governments' management constraints are introduced into the model as parameter variables to explore the decision-making selections of each subject in the supply chain that affect the quality of agricultural products [38]. Jiang et al. constructed the evolutionary game model of environmental regulation to analyze the dynamic game relationships between polluting enterprises, local government regulators, and central government planners. The study clarified and revealed the action plans to jointly improve the efficiency of environmental regulation by central and local governments [39]. Gao et al. proposed an evolutionary game model of food safety risk behavior based on food suppliers, manufacturers, and sellers by introducing the concept of co-management.

The study analyzed the types of risk links in the food supply chain and explored the relationship between different types of risk links and the level of social co-management [40]. Feng et al. introduced a public supervision mechanism in constructing the railway transportation safety regulation game model. The dynamic decision-making process between the National Railway Administration, China Railway, and the public is discussed. By analyzing the effectiveness of the game system, the measures that are effective in improving railway safety productivity are revealed [41]. Jiang et al. proposed the evolutionary game model of construction safety supervision by analyzing the supervision strategies among government supervision agencies, general contractors, and supervision engineers. Then, suggestions are put forward that are helpful for policymaking in terms of construction safety supervision [42].

In conclusion, evolutionary game theory can effectively describe the interaction and coordination among subjects in fire safety management, and it is a feasible method to identify the game mechanism in an urban fire safety co-management system. This study will build the urban fire safety co-management game model based on evolutionary game theory. Through stability and numerical simulation analyses, the stable state and evolution trajectory of the urban fire safety co-management game system can be clarified and illustrated. Based on the strategy selection and influencing factors of fire supervision departments, production management units, and the public, the study proposes suggestions in line with urban fire safety situations. Furthermore, the multiplier effect of an urban fire safety co-management system can be realized in order to reduce urban fire risks and the negative impact of fire on social development.

The structure of this paper is as follows: Section 2 puts forward the main research questions and sets the hypotheses and parameters of an urban fire safety co-management game model. The replication dynamic equations of fire supervision departments, production management units, and the public are constructed and obtained. In Section 3, the strategy stability of game subjects in urban fire safety co-management game systems is analyzed. The strategy evolution phase diagrams of fire supervision departments, production management units, and the public are drawn. Combined with numerical simulation analysis, Section 4 explores the influencing factors of interaction and game relationships among fire supervision departments, production management units, and the public in urban fire safety co-management. The dynamic evolution law of the urban fire safety co-management game system is summarized. In Section 5, there is further discussion based on the research results, summarizing the limitations and gaps for further exploration. Section 6 summarizes the research conclusions and proposes the implications for improving urban fire safety co-management systems by strengthening fire safety publicity and education, optimizing the information interacting mechanism, expanding co-management participation channels, and improving the accountability system.

2. Research Problems and Model Construction

2.1. Research Problems

Fire supervision departments are the main subjects responsible for urban fire safety, and they are mainly responsible for fire prevention, fire risk reduction, and emergency rescue. Production management units mainly refer to public gathering places such as shopping malls, schools, hospitals, transportation hubs, and business units such as construction projects, warehouses, and workshops. At present, the public is playing an increasingly obvious role in the government practice and exploration of promoting an urban co-management system. Therefore, the establishment of an urban fire safety co-management system with the introduction of the public can effectively integrate urban fire management resources, cover fire safety management, improve the efficiency of fire safety management, and maintain the stability of social fire safety situations.

In urban fire safety co-management systems, fire supervision departments, production management units, and the public each have different functions and strategies. There are both fire supervision departments to supervise production management units and the public's whistleblowing for production management units. Moreover, there are interaction and game relationships between fire supervision departments, production management units, and the public; the strategic selections among the above subjects are dynamic and mutually restricted.

Specifically, the supervision intensity of fire supervision departments, whether production management units operate safely, and whether the public participates in co-management will all affect the fire safety co-management strategy of game subjects. The strategy selections of the game subjects are closely related to the urban fire situation and the effect of fire safety management. Therefore, exploring the influencing factors of risk perception and strategy selections of fire supervision departments, production management units, and the public, as well as clarifying how to support the effective implementation of urban fire safety co-management systems, are the focuses of this study.

2.2. Assumptions of Urban Fire Safety Co-Management Game Model

The urban fire safety co-management game model is constructed in this study, and the interaction and game relationships among fire supervision departments, production management units, and the public are shown in Figure 1. In order to clarify the behavioral strategy selections and dynamic game mechanism of fire supervision departments, production management units, and the public, the following specifications are proposed for the urban fire safety co-management game model.

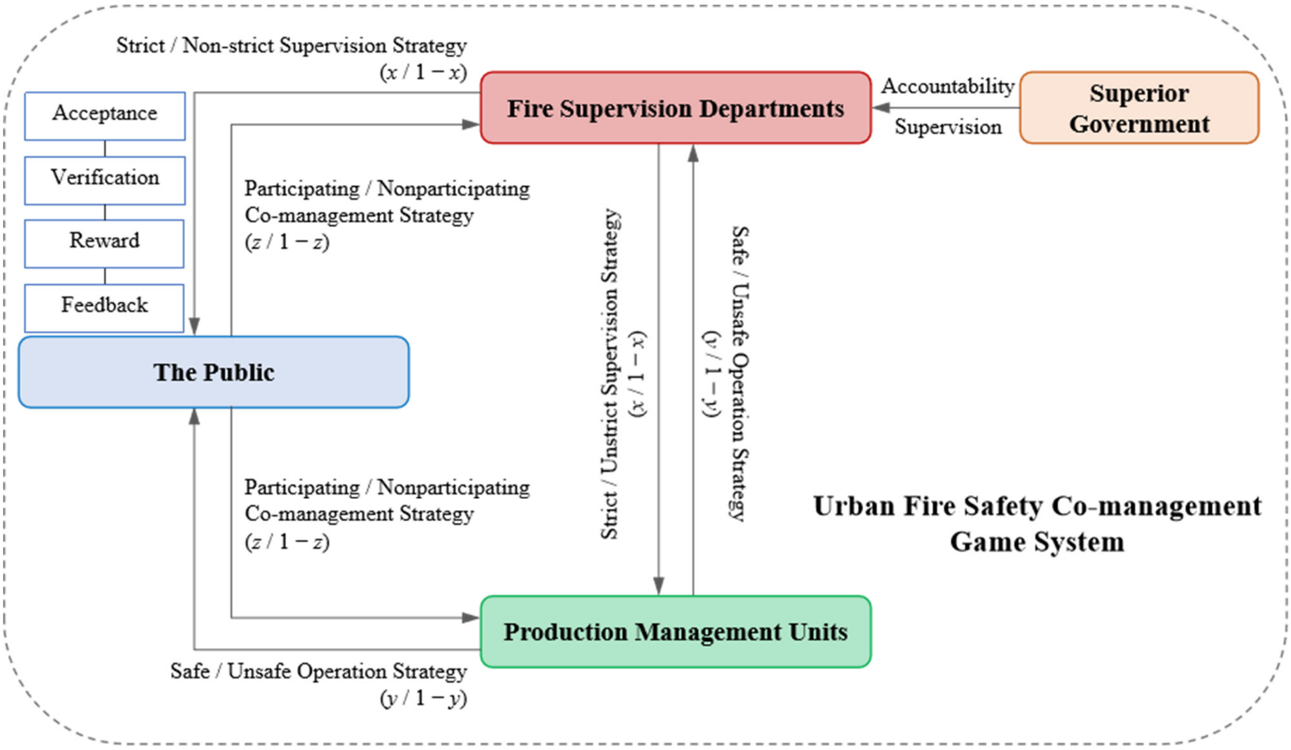

Figure 1. The interacting and game relationships between subjects in the urban fire safety co-management game model.

Specification 1. Fire supervision departments, production management units, and the public in the urban fire safety co-management game model are all bounded by rationality. There is a certain information asymmetry among the tripartite subjects of the game system, and the subjects' behavior selections are all aimed at maximizing their own interests.

Specification 2. Fire supervision departments can select a strict supervision strategy or a non-strict supervision strategy in the urban fire safety supervision process. The probabilities of strict supervision strategy and non-strict supervision strategy are x $(0 \leq x \leq 1)$ and $1 - x$, respectively. Production management units will comprehensively consider operating costs and benefits in business activities and select the safe or unsafe operation strategy. The corresponding probabilities are y $(0 \leq y \leq 1)$ and $1 - y$, respectively. In the face of potential fire risks in production management units and fire supervision departments' assistance needs, the public can select participating or nonparticipating. The probabilities of a participating co-management strategy and a nonparticipating co-management strategy are z $(0 \leq z \leq 1)$ and $1 - z$, respectively. In addition, fire supervision departments' responses to the public's whistleblowing of fire risks include acceptance, verification, reward, and feedback.

Specification 3. When fire supervision departments carry out fire safety supervision on production management units, they will pay certain supervision costs C_f, such as organization, manpower, and resources. Meanwhile, when the public selects the participating co-management strategy, fire supervision departments will carry out on-site verification of the whistleblowing information and pay the corresponding verification costs C_r. Under the strict supervision strategy, fire supervision departments will obtain certain reputation interests and social safety interests R due to their timely response to the public's whistleblowing information, as well as the disclosure of fire risks and fire safety violations of production management units. When production management units select the unsafe operation strategy, fire supervision departments will receive additional social safety and economic development interests G from the on-site verification of the whistleblowing information. The fire supervision intensity of fire supervision departments is α. When α is larger, fire supervision departments will pay higher fire safety supervision costs and on-site verification costs and will gain more reputation benefits and social safety benefits. μ is the degree of fire safety information disclosure in a fire safety co-management system. A high-level μ means that fire supervision departments are more willing and informative in disclosing information to society, and the information transparency of urban fire safety is higher. Correspondingly, the reputation interests R obtained by fire supervision departments are proportional to the degree of fire safety information disclosure μ.

When fire supervision departments select the non-strict supervision strategy, the supervision intensity of production management units and the response degree to the public's whistleblowing information will be reduced. This will cause fire supervision departments to suffer a certain loss of reputation and credibility L_f due to the failure of their response to the public's whistleblowing and information disclosure.

In order to consolidate the responsibility of urban fire safety management, fire supervision departments will be punished by the superior government in the event of fire accidents under the non-strict supervision strategy. The corresponding accountability and penalty losses are P. η is the accountability intensity of the superior government to fire supervision departments. A high-level η means that fire supervision departments will be punished with greater losses due to non-strict supervision and dereliction of duty.

Specification 4. Under the safe operation strategy, the operating incomes of production management units are M. Production management units' input costs are C_s in the aspects of fire safety daily management, purchase and maintenance of firefighting facilities, self-inspection, and rectification of fire risks. In order to measure the input intensity of the fire safety management of production management units, β is introduced. Under the unsafe operation strategy, the input costs of the fire safety management of production management units are βC_s. The penalty losses and additional operating losses of production management units punished by fire supervision departments are F_1. Meanwhile, due to the public's whistleblowing of fire risks or fire safety violations, the penalty losses and additional operating losses of production management units punished by fire supervision departments are F_2. Correspondingly, when fire supervision departments select the non-strict supervision strategy, the penalty losses and additional operating losses of production management units are αF_1 and αF_2, respectively.

In addition, the reputation losses and operating losses of production management units from the public's whistleblowing and information disclosure are K. It can be seen that when the degree of fire safety information disclosure μ is larger and the input intensity of fire safety management β is smaller, the reputation losses and operating losses K of production management units will increase, which are $(1-\beta)\,\mu K$.

Specification 5. When the public selects a participating co-management strategy for fire safety, it needs to pay certain management and time costs C_p. For the whistleblowing of fire risks or fire safety violations, the public will receive certain rewards and recognition incomes W. In practice, the public can participate in fire safety co-management through a special telephone line for whistleblowing, internet platforms, and other ways, so the informatization level of co-management λ is introduced in this study. Specifically, a larger λ means lower costs for the public to participate in fire safety co-management.

Meanwhile, since the public's ability to detect fire risks or fire safety violations are different and random, the study introduces the risk perception level of the public θ to describe the probability of the public's identification of potential fire risks. When the risk perception level of the public is greater, the public also has a higher probability of identifying fire risks and fire safety violations.

In addition, the public's perceived potential losses due to the possibility of fires in production management units are L_p. When the input costs of the fire safety management of production management units are lower, the public's perception of losses will be higher.

Further, Table 1 provides the definition and scope of parameters in the urban fire safety co-management game model.

Table 1. Parameters of urban fire safety co-management game model.

Parameters	Concept	Value
M	The operating incomes of production management units under safe operation [43].	$M > 0$
C_s	The input costs of fire safety management of production management units under safe operation [43].	$C_s > 0$
F_1	The penalty losses and additional operating losses of production management units under unsafe operation [37].	$F_1 > 0$
F_2	The penalty losses and additional operating losses of production management units from the public's whistleblowing [30].	$F_2 > 0$
K	The reputation losses and operating losses of production management units from the public's whistleblowing and information disclosure [18].	$K > 0$
C_f	The fire safety supervision costs of fire supervision departments [43].	$C_f > 0$
C_r	The on-site verification costs of fire supervision departments about whistleblowing information [44].	$C_r > 0$
G	The social safety and economic development interests of fire supervision departments from on-site verification [44].	$G > 0$
R	The reputation interests of fire supervision departments due to the timely response to the public's whistleblowing and information disclosure [35].	$R > 0$
L_f	The reputation and credibility losses of fire supervision departments due to the failure response to the public's whistleblowing and information disclosure [35].	$L_f > 0$
P	The accountability and penalty losses of fire supervision departments from the superior government due to non-strict supervision [18].	$P > 0$
C_p	The costs of the public due to the participation in fire safety co-management [41].	$C_p > 0$
W	The incentive incomes of the public due to the participation in fire safety co-management [33].	$W > 0$
L_p	The public's perception of potential damage from the fire risks of production management units [35].	$L_p > 0$
α	Fire supervision intensity of fire supervision departments [40,45].	$0 \le \alpha \le 1$
β	The input intensity of fire safety management of production management units [41]	$0 \le \beta \le 1$
θ	The risk perception level of the public [35].	$0 \le \theta \le 1$
λ	The informatization level of co-management [46].	$0 \le \lambda \le 1$
μ	The degree of fire safety information disclosure [47].	$0 \le \mu \le 1$
η	The accountability intensity of the superior government [39,48].	$0 \le \eta \le 1$

2.3. Construction of Urban Fire Safety Co-Management Game Model

According to the game situation of fire supervision departments, production management units, and the public in urban fire safety co-management, the payoff matrix of urban fire safety co-management game system is obtained, as shown in Table 2.

Table 2. The payoff matrix of urban fire safety co-management game system.

Fire Supervision Departments	Production Management Units	The Public	
		Participating Co-Management Strategy	Nonparticipating Co-Management Strategy
Strict supervision strategy	Safe operation strategy	$-C_f - C_r + \mu R$, $M - C_s$, $-(1-\lambda)C_p + \theta W$	$-C_f$, $M - C_s$, 0
	Unsafe operation strategy	$-C_f - C_r + \mu R + G$, $M - \beta C_s - F_1 - F_2 - (1-\beta)\mu K$, $-(1-\lambda)C_p + \theta W - (1-\beta)L_p$	$-C_f$, $M - \beta C_s - F_1$, $-(1-\beta)L_p$
Non-strict supervision strategy	Safe operation strategy	$\alpha(-C_f - C_r + \mu R) - L_f$, $M - C_s$, $-(1-\lambda)C_p + \alpha\theta W$	$-\alpha C_f$, $M - C_s$, 0
	Unsafe operation strategy	$\alpha(-C_f - C_r + \mu R + G) - L_f - P$, $M - \beta C_s - \alpha F_1 - \alpha F_2 - (1-\beta)\mu K$, $-(1-\lambda)C_p + \alpha\theta W - (1-\beta)L_p$	$-\alpha C_f - P$, $M - \beta C_s - \alpha F_1$, $-(1-\beta)L_p$

2.3.1. Replication Dynamic Equation of Fire Supervision Departments

The expected benefit of the strict supervision strategy of fire supervision departments U_{11} is as follows:

$$U_{11} = yz\left(-C_f - C_r + R\right) + y(1-z)\left(-C_f\right) + z(1-y)\left(-C_f - C_r + \mu R + G\right) + (1-y)(1-z)\left(-C_f\right) \quad (1)$$

Further, the expected benefit of the non-strict supervision strategy of fire supervision departments U_{12} is as follows:

$$U_{12} = yz\left[\alpha\left(-C_f - C_r + R\right) - L_f\right] + y(1-z)\left(-\alpha C_f\right) + z(1-y)\left[\alpha\left(-C_f - C_r + \mu R + G\right) - L_f - P\right] + (1-y)(1-z)\left(-\alpha C_f - P\right) \quad (2)$$

The average expected benefit of fire supervision departments' fire safety supervision strategy is:

$$\overline{U_1} = xU_{11} + (1-x)U_{12} \quad (3)$$

Therefore, the replication dynamic equation of fire supervision departments' fire safety supervision strategy is as follows:

$$F(x) = \frac{dx}{dt} = x\left(U_{11} - \overline{U_1}\right) = x(1-x)(U_{11} - U_{12}) = x(1-x)\left[-(1-\alpha)C_f + z(1-\alpha)(-C_r + \mu R) + zL_f + zG(1-\alpha)(1-y) + P - yP\right] \quad (4)$$

2.3.2. Replication Dynamic Equation of Production Management Units

The expected benefit of the safe operation strategy of production management units U_{21} is as follows:

$$U_{21} = xz(M - C_s) + z(1-x)(M - C_s) + x(1-z)(M - C_s) + (1-x)(1-z)(M - C_s) \quad (5)$$

Further, the expected benefit of the unsafe operation strategy of production management units U_{22} is as follows:

$$U_{22} = xz[M - \beta C_s - F_1 - F_2 - (1-\beta)\mu K] + z(1-x)[M - \beta C_s - \alpha(F_1 + F_2) - (1-\beta)\mu K] + x(1-z)(M - \beta C_s - F_1) + (1-x)(1-z)(M - \beta C_s - \alpha F_1) \quad (6)$$

The average expected benefit of production management units' operation strategy is:

$$\overline{U_2} = yU_{21} + (1-y)U_{22} \quad (7)$$

Therefore, the replication dynamic equation of production management units' operation strategy is as follows:

$$G(y) = \frac{dy}{dt} = y(U_{21} - \overline{U_2}) = y(1-y)(U_{21} - U_{22}) = y(1-y)[-(1-\beta)C_s + z(1-\beta)\mu K + (zF_2 + F_1)(x + \alpha - x\alpha)] \quad (8)$$

2.3.3. Replication Dynamic Equation of the Public

The expected benefit of the participating co-management strategy of the public U_{31} is as follows:

$$U_{21} = xz(M - C_s) + z(1-x)(M - C_s) + x(1-z)(M - C_s) + (1-x)(1-z)(M - C_s) \quad (9)$$

Further, the expected benefit of the nonparticipating co-management strategy of the public U_{32} is as follows:

$$U_{22} = xz[M - \beta C_s - F_1 - F_2 - (1-\beta)\mu K] + z(1-x)[M - \beta C_s - \alpha(F_1 + F_2) - (1-\beta)\mu K] + x(1-z)(M - \beta C_s - F_1) + (1-x)(1-z)(M - \beta C_s - \alpha F_1) \quad (10)$$

The average expected benefit of the public's fire safety co-management strategy is:

$$\overline{U_3} = zU_{31} + (1-z)U_{32} \quad (11)$$

Therefore, the replication dynamic equation of the public's fire safety co-management strategy is as follows:

$$H(z) = \frac{dz}{dt} = z(U_{31} - \overline{U_3}) = z(1-z)(U_{31} - U_{32}) = z(1-z)[x(1-\alpha)\theta W + \alpha\theta W - (1-\lambda)C_p] \quad (12)$$

To sum up, the urban fire safety co-management game system can be obtained:

$$\begin{cases} F(x) = \frac{dx}{dt} = x(U_{11} - \overline{U_1}) = x(1-x)(U_{11} - U_{12}) = x(1-x)[-(1-\alpha)C_f + z(1-\alpha)(-C_r + \mu R) + zL_f + zG(1-\alpha)(1-y) + P - yP] \\ G(y) = \frac{dy}{dt} = y(U_{21} - \overline{U_2}) = y(1-y)(U_{21} - U_{22}) = y(1-y)[-(1-\beta)C_s + z(1-\beta)\mu K + (zF_2 + F_1)(x + \alpha - x\alpha)] \\ H(z) = \frac{dz}{dt} = z(U_{31} - \overline{U_3}) = z(1-z)(U_{31} - U_{32}) = z(1-z)[x(1-\alpha)\theta W + \alpha\theta W - (1-\lambda)C_p] \end{cases} \quad (13)$$

3. Strategy Stability Analysis of Urban Fire Safety Co-Management Game Model

3.1. Strategy Stability Analysis of Fire Supervision Departments

According to the stability theorem of the replication dynamic equation, the probability of fire supervision departments' selection of the fire safety supervision strategy is stable when $F(x) = 0$ and $F'(x) < 0$.

Therefore, it follows that when $y = y_0 = \frac{-(1-\alpha)C_f + z(1-\alpha)(-C_r+\mu R) + zL_f + z(1-\alpha)G + P}{P + z(1-\alpha)G}$ or $z = z_0 = -\frac{-(1-\alpha)C_f + P - yP}{(1-\alpha)(-C_r+\mu R) + L_f + G(1-\alpha)(1-y)}$, then $F(x) \equiv 0$. At this time, fire supervision departments adopt which fire safety supervision strategy will not change with the evolution of the urban fire safety co-management game system.

Further, the derivation of the replication dynamic equation of the fire supervision departments $F(x)$ can be obtained, which is $F'(x) = (1-2x)[-(1-\alpha)C_f + z(1-\alpha)(-C_r + \mu R) + zL_f + zG(1-\alpha)(1-y) + P - yP]$. When $y > y_0$ or $z < z_0$, both $F'(x)|_{x=0} < 0$ and $F'(x)|_{x=1} > 0$ are true. Accordingly, $x = 0$ (non-strict supervision strategy) is the stable strategy of fire supervision departments. On the contrary, when $y < y_0$ or $z > z_0$, both $F'(x)|_{x=0} > 0$ and $F'(x)|_{x=1} < 0$ are true. Accordingly, $x = 1$ (strict supervision strategy) is the stable strategy of fire supervision departments.

To sum up, the evolutionary phase diagram of the fire supervision strategy of fire supervision departments can be obtained, as shown in Figure 2.

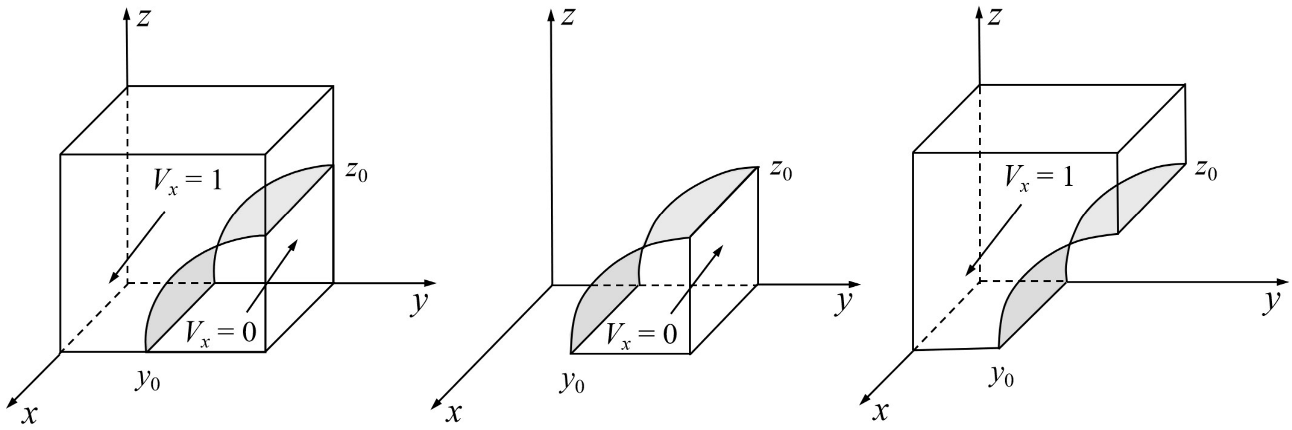

Figure 2. Evolutionary phase diagram of fire supervision departments' strategies.

As can be seen from Figure 2, when the y_0 is larger or the z_0 is smaller, there is a greater probability that fire supervision departments select the strict supervision strategy ($x = 1$). On the contrary, when y_0 is smaller or z_0 is larger, fire supervision departments are more inclined to select the non-strict supervision strategy.

Therefore, the study takes the partial derivative of y_0 with respect to C_f, Cr, R, L_f, P, and G. It can be concluded that y_0 is the decreasing function of fire safety supervision costs C_f and on-site verification costs C_r, and the increasing function of reputation interests R, reputation and credibility losses L_f, accountability and penalty losses P, as well as social safety and economic development interests G. That is, when C_f and C_r decrease, or when R, L_f, P, and G increase, fire supervision departments have a greater probability of selecting the strict supervision strategy.

3.2. Strategy Stability Analysis of Production Management Units

Similarly, when $G(y) = 0$ and $G'(y) < 0$ are satisfied, the probability of production management units' selection of the operation strategy is in a stable state.

Therefore, it can be obtained that when $x = x_0 = -\frac{-(1-\beta)C_s+z(1-\beta)\mu K+\alpha(zF_2+F_1)}{(zF_2+F_1)(1-\alpha)}$ or $z = z_0 = \frac{-[-(1-\beta)C_s+F_1(x+\alpha-x\alpha)]}{(1-\beta)\mu K+F_2(x+\alpha-x\alpha)}$, there is $G(y) \equiv 0$. At this time, production management units adopt an operation strategy that will not change with the evolution of the urban fire safety co-management game system.

Further, the derivation of the replication dynamic equation of production management units $G(y)$ can be obtained, which is $G'(y) = (1-2y)[-(1-\beta)C_s + z(1-\beta)\mu K + (zF_2 + F_1)(x + \alpha - x\alpha)]$. When $z > z_0$ or $x > x_0$, both $G'(y)|_{y=1} < 0$ and $G'(y)|_{y=0} > 0$ are true. Accordingly, $y = 1$ (safe operation strategy) is the stable strategy of production management units. On the contrary, when $z < z_0$ or $x < x_0$, both $F'(x)|_{x=0} > 0$ and $F'(x)|_{x=1} < 0$ are true. Accordingly, $y = 0$ (unsafe operation strategy) is the stable strategy of production management units.

To sum up, the evolutionary phase diagram of the operation strategy of production management units can be obtained, as shown in Figure 3.

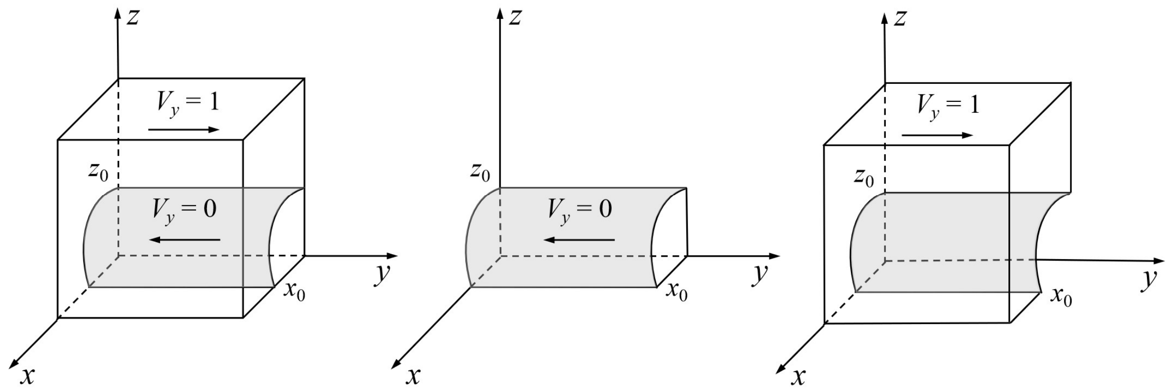

Figure 3. Evolutionary phase diagram of production management units' strategies.

As can be seen from Figure 3, when the x_0 and the z_0 are smaller, there is a greater probability that production management units select the safe operation strategy ($y = 1$). On the contrary, when x_0 and z_0 are larger, production management units are more inclined to select the unsafe operation strategy.

Therefore, this study takes the partial derivative of z_0 with respect to C_s, K, F_1, and F_2. It can be concluded that z_0 is the decreasing function of the input costs of fire safety management C_s, and the increasing function of the reputation losses and operating losses K, as well as the penalty losses and additional operating losses F_1 and F_2. That is, when C_s decreases, or when K, F_1, and F_2 increase, production management units have a greater probability of selecting the safe operation strategy.

3.3. Strategy Stability Analysis of the Public

Similarly, the probability of the public's selection of the fire safety co-management strategy is stable when $H(z) = 0$ and $H'(z) < 0$.

Therefore, it can be obtained that when $x = x_0 = -\frac{\alpha\theta W-(1-\lambda)C_p}{(1-\alpha)\theta W}$, then $H(z) \equiv 0$. At this time, the public adopt a fire safety co-management strategy that will not change with the evolution of the urban fire safety co-management game system.

Further, the derivation of the replication dynamic equation of the public $H(z)$ can be obtained, which is $H'(z) = z(1-2z)\left[x(1-\alpha)\theta W + \alpha\theta W - (1-\lambda)C_p\right]$. When $x > x_0$, both $H'(z)|_{z=1} < 0$ and $H'(z)|_{z=0} > 0$ are true. Accordingly, $z = 1$ (participating co-management strategy) is the stable strategy of the public. On the contrary, when $x < x_0$, both $H'(z)|_{z=1} > 0$ and $H'(z)|_{z=0} < 0$ are true. Accordingly, $z = 0$ (nonparticipating co-management strategy) is the stable strategy of the public.

To sum up, the evolutionary phase diagram of the fire safety co-management strategy of the public can be obtained, as shown in Figure 4.

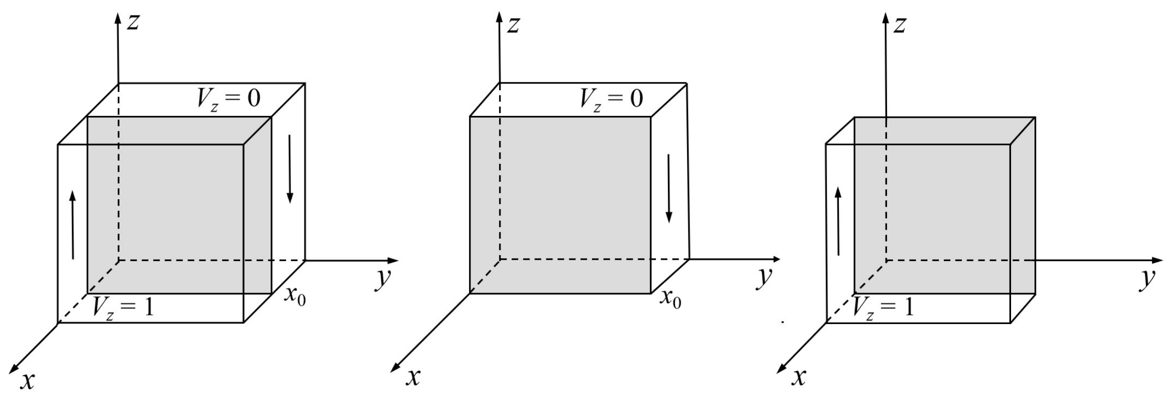

Figure 4. Evolutionary phase diagram of the public's strategies.

As can be seen from Figure 4, when the x_0 is larger, the greater the probability that the public will select the nonparticipating co-management strategy ($z = 0$). On the contrary, when x_0 is smaller, the public are more inclined to select the participating co-management strategy.

Then, taking the partial derivative of x_0 with respect to C_p and W, it can be concluded that x_0 is the decreasing function of the incentive incomes W and the increasing function of the costs of the participation C_p. This indicates that the incentive incomes W has a positive impact on the public participating in fire safety co-management and that the costs of whistleblowing C_p has a negative impact on the public participating in fire safety co-management. When W increases or C_p decreases, the public is more likely to participate in fire safety co-management.

4. Numerical Simulation Analysis of the Urban Fire Safety Co-Management Game System

Numerical simulation analysis can further clearly describe the interaction and game relationships between fire supervision departments, production management units, and the public in urban fire safety co-management, so as to realize the analysis of the dynamic evolution of the urban fire safety co-management game system.

Therefore, this study will discuss the influence of the informatization level of co-management λ, the risk perception level of the public θ, the degree of fire safety information disclosure μ, and the accountability intensity of the superior government η on the game subjects' strategy selection. Specifically, the Delphi method is used to score the questionnaires of experts in the field of fire safety management, and the initial values of the strategy parameters in the game model are obtained. Assume that $\alpha = 0.7$, $\beta = 0.3$, $\mu = 0.5$, $\theta = 0.5$, $\lambda = 0.5$, $\eta = 0.5$, $C_f = 18$, $C_r = 10$, $C_s = 16$, $C_p = 12$, $F_1 = 8$, $F_2 = 12$, $K = 15$, $G = 20$, $W = 10$, $R = 15$, $L_f = 12$, and $P = 28$. In addition, the initial state of the urban fire safety co-management game system is set as (0.5, 0.5, 0.5).

4.1. The Influence of the Informatization Level of Co-Management λ on the Game System

Figure 5 shows the evolution trajectories of the urban fire safety co-management game system at different informatization levels of co-management λ. It can be seen that the stable state of the urban fire safety co-management game system evolves from (1, 0, 0) to (1, 1, 1). For fire supervision departments, no matter what state λ is in, the departments are encouraged to converge on the strict supervision strategy.

When the informatization level of co-management is at a low level ($\lambda = 0.1$, 0.3, or 0.5), the costs of the public participating in fire safety co-management are higher. Therefore, the probability of the public selecting a participating co-management strategy is low; it will evolve in the direction of $z = 0$. Meanwhile, the expected penalty losses and operating losses caused by the unsafe operation strategy of production management units are lower and will converge to $y = 0$. When the informatization level of co-management is at a high level ($\lambda = 0.7$ or 0.9), it will encourage the public to participate in urban fire safety co-management. This is because the improvement of the informatization level can simplify the management procedures of the public, improve the flexibility of the public to participate in co-management, and reduce the management costs of the public. Correspondingly, at this time, production management units realize that the public's participation will increase the penalty losses, reputation losses, and business losses caused by whistleblowing, so production management units will be inclined to select the safe operation strategy.

Meanwhile, with the improvement of the informatization level of co-management λ, the urban fire safety co-management game system will evolve faster to (1, 1, 1). In addition, when $\lambda = 0.5$, as the probability of production management units selecting the safe operation strategy and the public selecting the participating co-management strategy increases, fire supervision departments' strategy trajectories present the phenomena of fluctuation and repetition. This indicates that the introduction of the public participating in urban fire safety co-management will help fire supervision departments to reduce supervision costs to a certain extent. This will enhance the consciousness of production management units to operate safely, but it will also cause the phenomenon of the lax supervision of fire supervision departments.

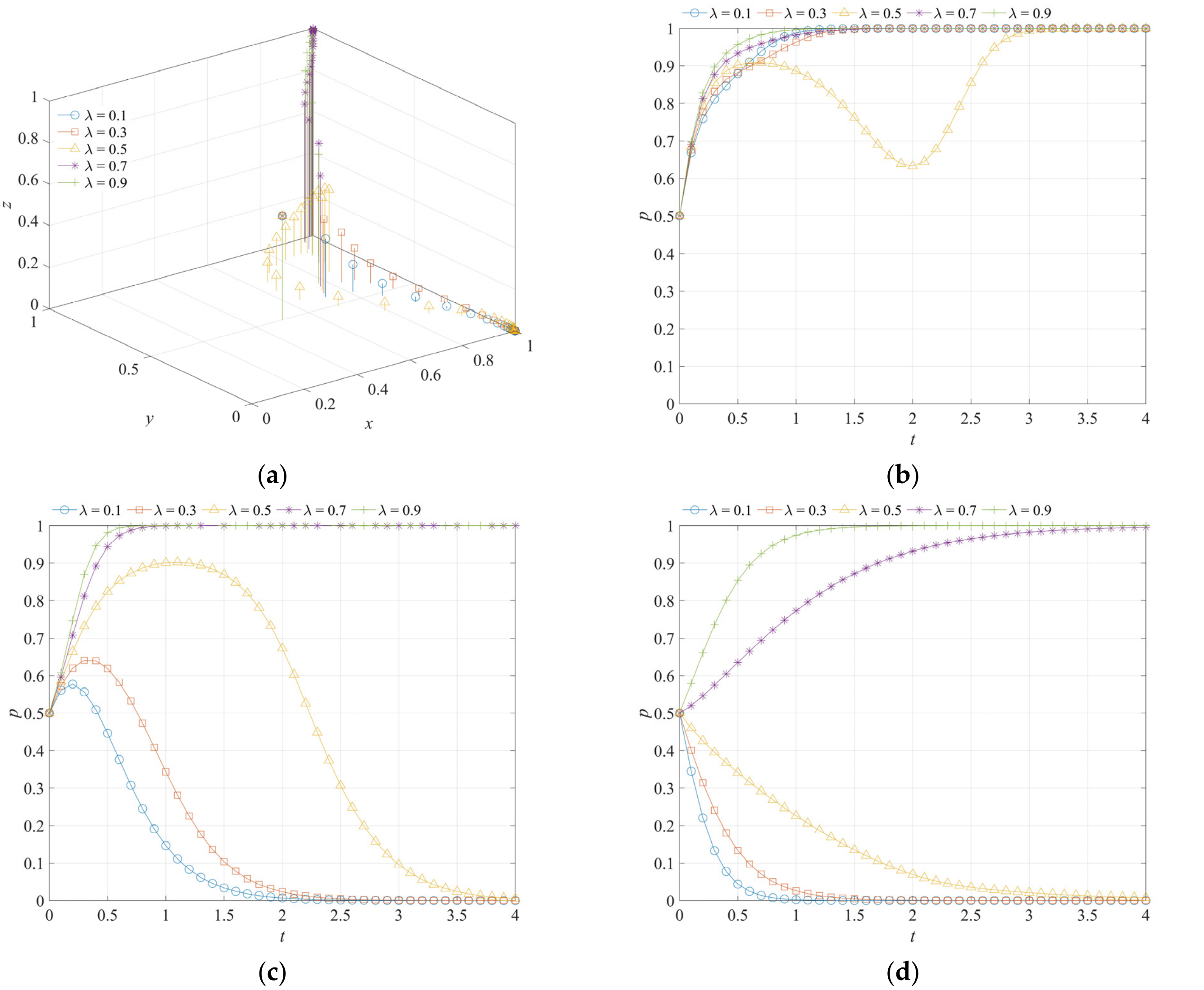

Figure 5. Numerical simulation results under different informatization levels of co-management λ. Evolutionary trajectories, under different λ, of (**a**) game systems, (**b**) fire supervision departments' strategies, (**c**) production management units' strategies, and (**d**) the public's strategies.

4.2. The Influence of the Risk Perception Level of the Public θ on the Game System

The evolution trajectories of the urban fire safety co-management game system for when the risk perception level of the public θ is 0.3, 0.4, 0.5, 0.6, and 0.7 are shown in Figure 6. The risk perception level θ reflects the public's understanding and cognition of fire risks and fire safety and is the parameter to measure the probability of the public finding fire risks or fire safety violations of fire supervision departments. That is, an increase in the risk perception level θ indicates that the public is more sensitive to potential fire risks, the public will receive greater reward incomes from participating in co-management, and the fire safety violations of production management units are more likely to be discovered by the public.

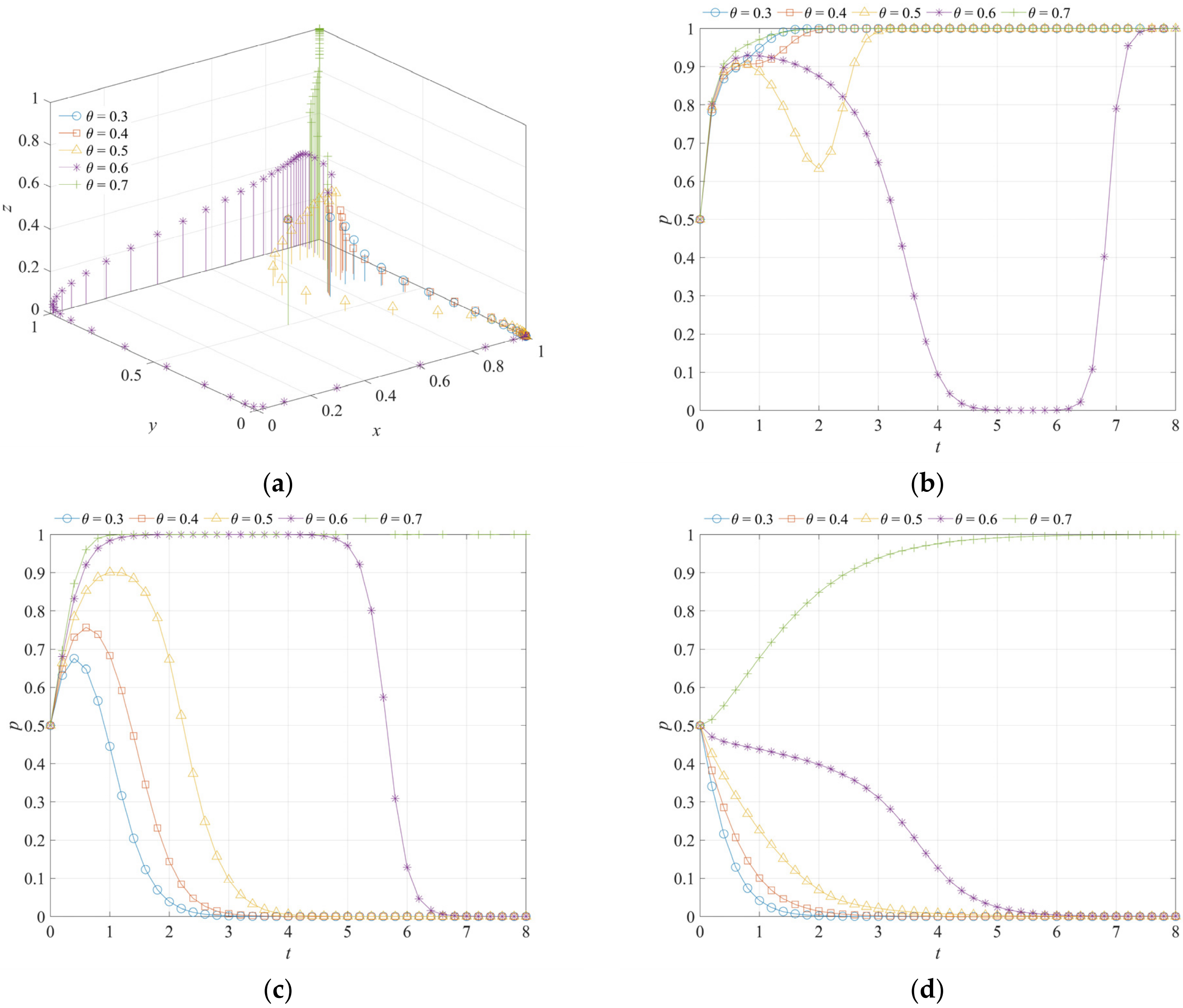

Figure 6. Numerical simulation results under different risk perception levels of the public θ. Evolutionary trajectories, under different θ, of (**a**) game systems, (**b**) fire supervision departments' strategies, (**c**) production management units' strategies, and (**d**) the public's strategies.

When θ is set as 0.3, 0.4, 0.5, and 0.6, the strategy combination of strict supervision, unsafe operation, and a nonparticipating co-management strategy is the stable state of the urban fire safety co-management game system. Specifically, the increase in the risk perception level θ effectively enhances production management units' willingness to select the safe operation strategy. On the contrary, the risk perception level θ represents a certain block to fire supervision departments selecting the strict supervision strategy, which is reflected in the increasing convergence time of fire supervision departments to the stable strategy. This phenomenon is particularly pronounced when θ = 0.6. This is because fire supervision departments will reduce the regulatory initiative when production management units adopt the safe operation strategy. Meanwhile, production management units also tend to be insecure when the supervision intensity decreases. Therefore, the strategy evolution trajectories of fire supervision departments and production management units show obvious characteristics of fluctuation. When θ = 0.7, the stable state of the urban fire safety co-management game system evolves from (1, 0, 0) to (1, 1, 1). From the perspective of the game system, the risk perception level of the public is conducive to the formation of the stable strategy combination of strict supervision, safe operation, and participating co-management. From the perspective of the fire safety management practice, the risk perception level of the public is conducive to improving the level of urban fire safety management, effectively reducing the possibility of fire accidents and thus realizing the stability of urban fire safety situations.

4.3. The Influence of the Degree of Fire Safety Information Disclosure μ on the Game System

The evolutionary trajectories of fire supervision departments, production management units, and the public obtained after simulation when the degree of fire safety information disclosure μ is set as 0.1, 0.3, 0.5, 0.7, and 0.9 are shown in Figure 7. As shown in Figure 7a, the urban fire safety co-management game system forms the stable state of (1, 0, 0). That is, fire supervision departments, production management units, and the public form a stable strategy combination of strict supervision, unsafe operation, and a nonparticipating co-management strategy.

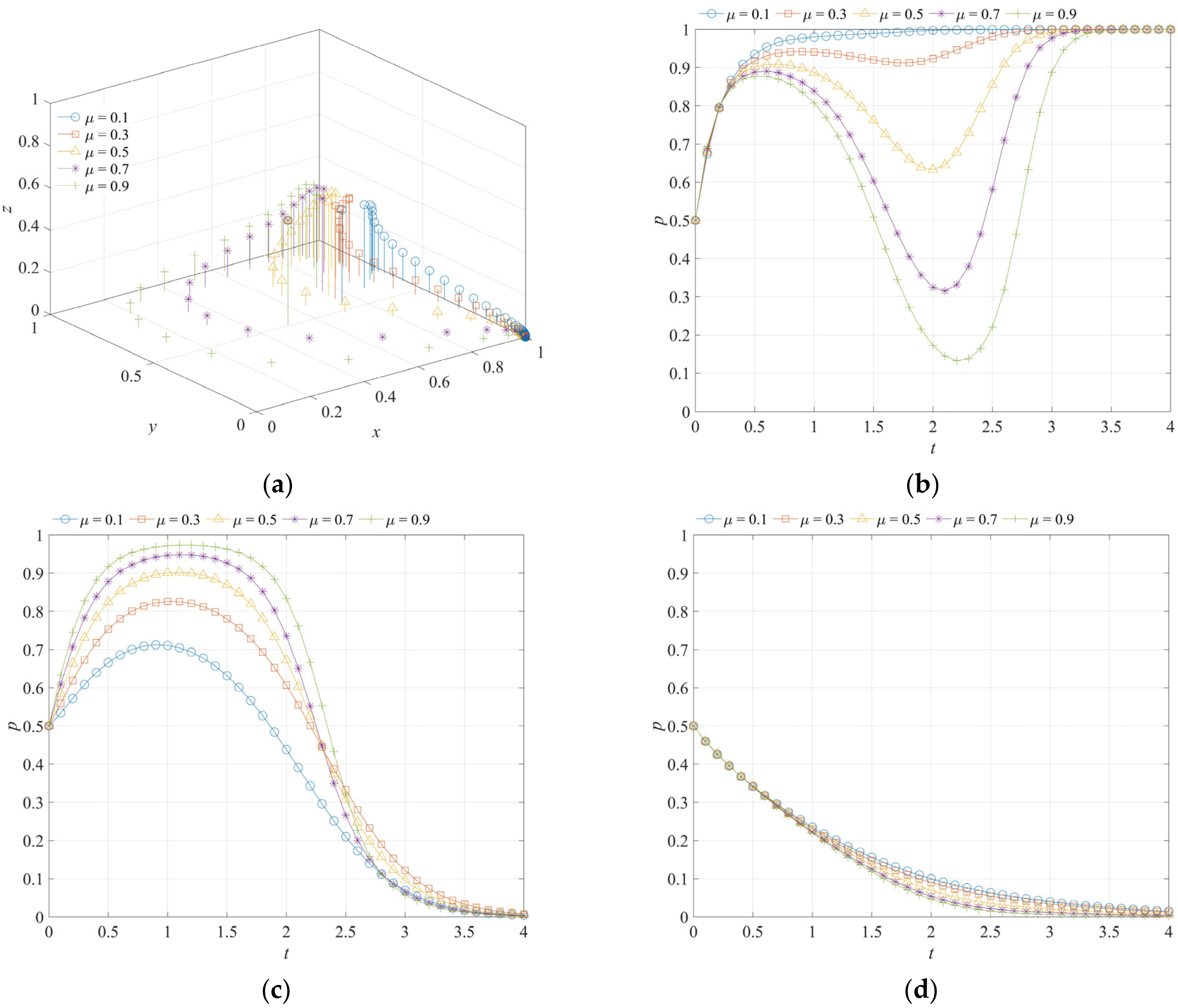

Figure 7. Numerical simulation results under different degrees of fire safety information disclosure μ. Evolutionary trajectories, under different μ, of (**a**) game systems, (**b**) fire supervision departments' strategies, (**c**) production management units' strategies, and (**d**) the public's strategies.

In Figure 7b, as the degree of fire safety information disclosure μ increases, fire supervision departments' strategy evolution trajectories fluctuate to a greater extent. This is because the openness of fire safety information can promote fire supervision departments to obtain more reputation interests and social safety interests, so fire supervision departments are less willing to select the strict supervision strategy. However, since production management units still have unsafe operation conditions, fire supervision departments still select the strict supervision strategy to fulfill the supervision duties and finally stabilize at $x = 0$. As can be seen from Figure 7c, with the increase in the degree of fire safety information disclosure μ, the fire safety violations and fire risks existing in production management units become clearer. In this case, the reputation losses and corresponding operation losses of production management units will also increase, so the speed of the strategy evolution trajectories converging to the unsafe operation strategy will gradually slow down. Meanwhile, in the early stage of evolution, production management units tend to converge in the direction of the safe operation strategy. This indicates that the improvement of the degree of fire safety information disclosure can inhibit the unsafe operation behavior of production management units. In Figure 7d, the public's willingness to participate in fire safety co-management is gradually decreasing. This is because the improvement of the degree of fire safety information disclosure μ strengthens the public's knowledge of fire risks in production management units and the trust in fire supervision departments. Therefore, the higher the transparency of fire safety information and the greater the amount of information, the more inclined the public is to converge to $z = 0$.

4.4. The Influence of the Accountability Intensity of the Superior Government η on the Game System

The strategy selection situations of the game subjects in the urban fire safety co-management game system when the accountability intensity of the superior government η is set as 0.00, 0.25, 0.50, 0.75, and 1.00 are shown in Figure 8. η is a variable that is a measure of the degree to which fire supervision departments are supervised by the superior government. A larger η means that fire supervision departments will suffer higher penalty losses for not performing the fire safety supervision function. As can be seen from Figure 8, the accountability intensity of the superior government η has a direct impact on fire supervision departments' game strategies. When η is at a lower level ($\eta = 0.00$ or 0.25), fire supervision departments will select the non-strict supervision strategy because of the lower perceived losses of accountability penalty. However, with the increase in η, fire supervision departments are more active in adopting strict supervision strategies. Although the accountability intensity of the superior government η has no direct effect on production management units or the public, with the change of fire supervision departments' strategy selection, production management units and the public, belonging to the urban fire safety co-management game system, also respond by increasing the probability of safe operation and participating in a co-management strategy. Specifically, the convergence rate of production management units to the unsafe operation strategy and the public to the nonparticipating co-management strategy slows down with the increase in the accountability intensity of the superior government.

In conclusion, the increase in the accountability intensity of the superior government η promotes the transformation of the stable strategy combination of the urban fire safety co-management game system from (0, 0, 0) to (1, 0, 0). The accountability intensity of the superior government can effectively restrain fire supervision departments from forming supervision inertia and compacting the responsibility of urban fire safety management. Meanwhile, η can guide production management units and the public in slowing down the convergence rate to $y = 0$ and $z = 0$ to some extent. This is the variable that guarantees and supports the stable operation of urban fire safety co-management game systems.

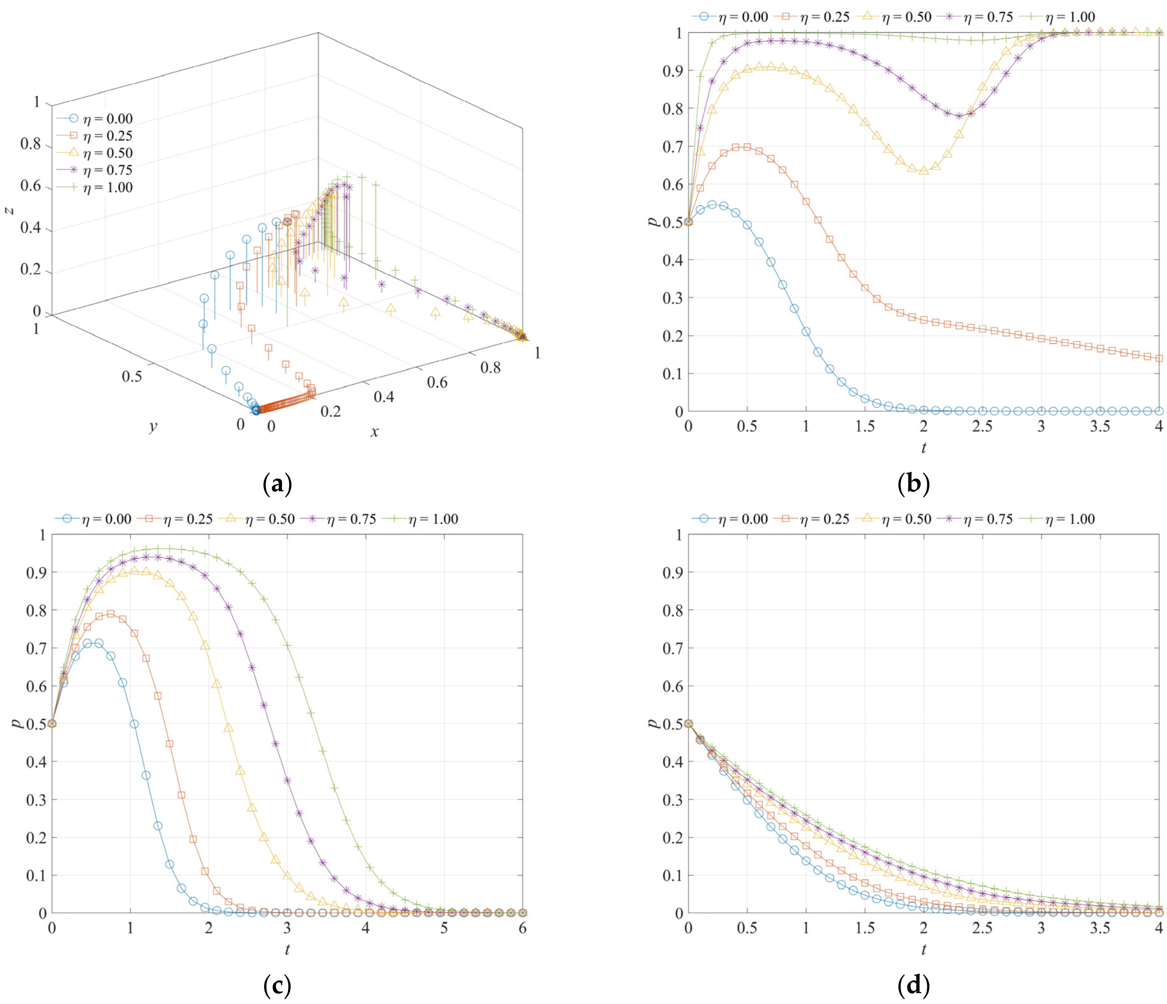

Figure 8. Numerical simulation results under different accountability intensities of the superior government η. Evolutionary trajectories, under different η, of (**a**) game systems, (**b**) fire supervision departments' strategies, (**c**) production management units' strategies, and (**d**) the public's strategies.

5. Discussion

Urban fire safety management is an interactive process of multi-subject coordination and game relationships. The establishment of an urban fire safety co-management system with the introduction of the public is an innovative measure aiming to crack the current problem of fire safety management and improve its level in China. Therefore, understanding the operation mode of urban fire safety co-management systems and clarifying the interacting relationships of multiple subjects in the system is critical. The urban fire safety co-management game model established in this study, composed of fire supervision departments, production management units, and the public, constructed from China's fire safety management practice, can effectively depict this problem.

5.1. Discussion on Strategy Stability Analysis

Stability analysis is an important part of analyzing the game system stability, which can help to clarify the evolution trend of the game system when the game subject's strategy changes. In the urban fire safety co-management game system, we discussed the influence of the initial strategy of the game system and parameters on the strategy selection of the game subjects combined with stability analysis.

On the one hand, it can be found that the initial state of the game system has different effects on the strategy selection of fire supervision departments, production management units, and the public. For fire supervision departments, when $y < y_0$ or $z > z_0$, the strict supervision strategy is the stable strategy of fire supervision departments. Therefore, when y_0 is smaller or z_0 is larger, fire supervision departments are more likely to select the strict supervision strategy. For production management units, when $z > z_0$ or $x > x_0$, the safe operation strategy is the stable strategy of production management units. When x_0 is smaller or z_0 is smaller, production management units are more likely to select the safe operation strategy. For the public, when $x > x_0$, the participating co-management strategy is the stable strategy of the public. That is, the probability that the public will select the participating co-management strategy is inversely proportional to x_0.

On the other hand, the parameters of the urban fire safety co-management game model have a direct impact on the strategic stability of game subjects. For fire supervision departments, the probability of selecting the strict supervision strategy is proportional to R, L_f, P, and G, and inversely proportional to C_f and C_r. For production management units, the probability of selecting the safe operation strategy is directly proportional to K, F_1, and F_2, and inversely proportional to C_s. For the public, the probability of selecting the participating co-management strategy is proportional to W and inversely proportional to C_p. In addition, the influence of the informatization level of co-management λ, the risk perception level of the public θ, the degree of fire safety information disclosure μ, and the accountability intensity of the superior government η on the strategy selection of game subjects are mainly indirect, which are discussed in the section on numerical simulation.

5.2. Discussion of Numerical Simulation Analysis

According to the simulation analysis results of an urban fire safety co-management game system, improving the informatization level of co-management λ, the risk perception level of the public θ, and the degree of fire safety information disclosure μ are conducive to stimulating the public's enthusiasm to participate in fire safety co-management. Meanwhile, according to the information from Figures 5a, 6a and 7a, the public's participation has triggered the regulatory inertia of fire supervision departments to a certain extent. This is mainly because the public's participation in co-management will expand the coverage of fire safety management and reduce the supervision cost of fire supervision departments. Moreover, there will be feedback on certain reputation incentives from society to fire supervision departments. Therefore, it is crucial to further explore how to enhance the public's participation in co-management and strict supervision by fire supervision departments at the same time, so as to realize the mutual promotion between the public and fire supervision departments in the urban fire safety co-management system.

Meanwhile, the introduction of vertical administrative constraints, which strengthen the supervision and accountability of the superior government to fire supervision departments, is conducive to maintaining the probability of fire supervision departments choosing the strict supervision strategy. Therefore, in the operation of urban fire safety co-management systems, comprehensive measures should be taken. It is necessary to continuously improve the whistleblowing mechanism, broaden whistleblowing channels, strengthen information disclosure, and strengthen fire safety publicity for the public while, at the same time, supervising the performance of responsibilities of fire supervision departments. Furthermore, fire supervision departments should further strengthen the investigation and analysis of the public's whistleblowing. They should not only be able to quickly and timely verify the accuracy of whistleblowing information but also investigate and summarize the weak environment and prominent problems of fire safety in the region. Further, it is necessary to improve the pertinence of fire safety management measures and avoid the blind area of fire safety supervision in the region.

This research represents an attempt to design and explore the interaction between organizations in urban fire safety management. Compared with existing studies, mathematical models were selected to describe the problem of urban fire safety co-management, and the dynamic evolution of strategy selection among game subjects is explored from a systematic perspective. The factors affecting the effectiveness of urban fire safety co-management systems are also clarified. It can be seen that the evolutionary game model is an effective tool for simulating the interaction between organizations in the process of fire safety management and provides an effective solution to explore the multiple game relationships in fire safety management. In addition, this study proposes that the urban fire safety co-management system based on public participation has shown outstanding advantages. Compared with the traditional urban fire safety management system, the characteristics of the urban fire safety co-management system are shown in Table 3.

Table 3. The advantages and characteristics of urban fire safety co-management system.

System	Traditional Fire Safety Management System	Urban Fire Safety Co-Management System
Subjects	(1) Fire supervision departments (2) Production management units	(1) Fire supervision departments (2) Production management units (3) The public
Management mode	(1) Fire supervision departments to production management units	(1) Fire supervision departments to production management units (2) Fire supervision departments to the public (3) The public to production management units
Advantages and Disadvantages	(1) The insufficient number of grassroots fire safety management personnel (2) The insufficient coverage of fire safety management (3) The high management costs of fire supervision department	(1) Wider coverage of fire safety management (2) Lower management costs of fire supervision department (3) Increased awareness of production management units being punished (4) Increased public awareness of fire safety (5) More transparency of fire safety information
Fire safety interests	(1) The interests of fire safety management on production management units	(1) The interests of fire safety management on production management units (2) The reputation interests of information disclosure (3) The safety interests of the fire safety on-site verification (4) The safety interests of the enhanced fire safety atmosphere
Influencing factors of system	(1) The management intensity of fire supervision departments to production management units (2) Whether production management units comply with the fire safety management regulations	(1) The management intensity of fire supervision departments to production management units (2) Whether production management units comply with the fire safety management regulations (3) Fire supervision departments' feedback to the public's whistleblowing (4) The fire risk perception level of the public (5) The transmission efficiency of the public's whistleblowing (6) The accountability intensity of the superior government

First of all, the urban fire safety co-management system has more diverse participants and richer management modes. Meanwhile, the urban fire safety co-management system has more prominent advantages, including a wider coverage of fire safety management, lower management costs of the fire supervision department, increased awareness of production management units being punished, increased public awareness of fire safety, and more transparency of fire safety information. In terms of social safety interests and economic development interests expected by the government, the urban fire safety co-management system can also provide more support and response, such as the reputation interests of information disclosure, the safety interests of the fire safety on-site verification, and the safety interests of the enhanced fire safety atmosphere. Furthermore, since the urban fire safety co-management system involves more aspects, there are more factors to maintain the level of urban fire safety management. Specifically, we should focus on fire supervision departments' feedback on the public's whistleblowing, the fire risk perception level of the public, the transmission efficiency of the public's whistleblowing, and the accountability intensity of the superior government to achieve the significant improvement of urban fire safety management efficiency. In conclusion, the establishment of the urban fire safety co-management system has positive significance for improving the level of urban fire safety co-management.

6. Conclusions and Implications

6.1. Research Conclusions

Building a fire safety co-management system is an important measure to aggregate fire safety management resources and improve the level of urban fire risk prevention. In order to clearly analyze the interaction relationships in an urban fire safety co-management system, an urban fire safety co-management game model was constructed in this study that includes fire supervision departments, production management units, and the public. The stability characteristics of the urban fire safety co-management game system were explored, and the influencing factors of game system construction and game subjects' strategy selection were discussed based on numerical simulation analysis. The research results show that:

(1) For the public, improving the informatization level of fire safety co-management can effectively reduce the costs of the public's participation in co-management. Meanwhile, when the public's perception level of fire risks is higher, the public will receive greater reward incomes, and there is a greater probability that production management units' fire safety violations will be discovered. All the above have promoted the enthusiasm and initiative of the public to participate in co-management, which is conducive to the formation of the stable state (1, 1, 1) of the urban fire safety co-management game system.

(2) Improving the informatization level of fire safety co-management can not only increase the reputation incomes of fire supervision departments but can also strengthen the perception of production management units of various losses. In other words, improving the openness of fire safety information can inhibit the non-strict supervision of fire supervision departments and the unsafe operation behavior of production management units.

(3) Through its construction, the urban fire safety co-management system can actively mobilize the enthusiasm of fire supervision departments, production management units, and the public. It is conducive to establishing urban fire safety management synergy. However, due to the allocation of management costs, fire supervision departments may lack the motivation for supervision and management. Therefore, it is necessary to ensure the effective implementation of an urban fire safety co-management system by strengthening the inspection and accountability of the superior government.

In general, the novelty and research contributions of this study mainly include: This study extends evolutionary game theory to the inter-organizational relationships of fire safety management and provides a new perspective for the study of fire safety management. Meanwhile, the research constructs the urban fire safety co-management game model and clarifies the factors that affect the strategy selection of game subjects in urban fire safety co-management systems through strategy stability analysis and numerical simulation analysis. Further, the key factors and feasible strategies for improving the performance of urban fire safety co-management are discussed and clarified. This study contributes to the improvement of urban fire safety co-management systems and the level of urban fire safety management.

In addition, there are still some limitations and spaces for further exploration in this line of research. On the one hand, this study includes model derivation and numerical simulation analysis of urban fire safety co-management systems based on evolutionary game theory, which has a wide breadth of adaptability. However, it can still be combined with actual case data of urban fire safety management to improve the accuracy of adaptation to the research problem. On the other hand, the urban fire safety co-management game model can be further adjusted and improved for the practice of urban fire safety management, such as by further developing and increasing the types of game subjects for different types of fires, classifying production management units in combination with specific fire risks, or introducing more influencing factors and mechanisms of urban fire safety management for analysis.

6.2. Implications

In order to further improve the urban fire safety co-management system and promote its performance, along its practice and the conclusions, the following recommendations consistent with fire risk characteristics and urban fire safety management scenarios are presented.

(1) Strengthening the social fire safety training publicity and education. The establishment of an urban fire safety co-management system depends on the coordination and cooperation between the governments and various social organizations. Therefore, fire supervision departments should take instructional measures to strengthen the fire safety awareness and prevention level of production management units and the public. On the one hand, fire supervision departments should strengthen the perception and popularization of fire management policies, so as to improve the awareness and understanding of production management units on fire management policies. Meanwhile, the help, guidance, and technical services for production management units can be increased according to the actual conditions of cities and regions. In addition, it is also necessary to publicize typical urban fire cases through various types of media, increase media exposure of fire risks, and strengthen fire safety warning education in order to enhance the initiative of production management units choosing safe operations. On the other hand, fire supervision departments should enhance the publicity of fire safety and broaden the publicity mode of fire safety, so as to improve the concept and cognition of fire safety of society and the public's understanding of fire risks. This can not only enhance the public's enthusiasm to participate in urban fire safety co-management but also improve the public's accuracy in identifying fire risks. The overall efficiency of urban fire safety co-management can be improved by creating a positive social fire safety atmosphere.

(2) Optimizing the interaction mechanism of fire risks and fire safety information. The effective interaction mechanism of fire safety information is an important guarantee to improve the performance of urban fire safety co-management [49]. On the one hand, it is necessary for fire supervision departments to improve the publicity scope and intensity of production management units' fire risks and fire safety violations. By comprehensively using traditional and emerging media, fire safety information should be released to society through multiple directions and channels to increase the perception of production management units of reputation loss, operation loss, as well as penalty loss. The strategy selection of production management units should be promoted to evolve in the direction of safe operation. Accordingly, the strategic selection of production management units can be promoted to evolve in the direction of safe operation. Meanwhile, strengthening fire safety information disclosure has a positive effect on improving the social reputation interests of fire supervision departments and can also improve the efficiency of the urban fire safety co-management system. On the other hand, urban fire management resources can be integrated by considering fire safety information in the internal audit of various industries and fields. Therefore, fire supervision departments should establish information docking with the competent departments of production management units' corresponding industries, such as commerce, education, and health, so as to strengthen the joint management of production management units.

(3) Enriching and expanding the participation channels of urban fire safety co-management. The connecting platform and channel between fire supervision departments and the public are the foundation for supporting the construction of an urban fire safety co-management system and establishing urban co-management synergy. On the one hand, it is necessary to use informatization measures to optimize the process of the public's whistleblowing. By building and exploring a diversified whistleblowing plat-

form, combining online and offline elements, the channels for the public to participate in urban fire safety co-management are expanded, and the costs of participation will be reduced, such as through a special telephone line, a government affairs platform, or other models. Meanwhile, the public's whistleblowing of fire risks and fire safety violations should be enriched, such as through fire safety information registration and uploading photos of illegal behavior, in order to improve the accuracy and comprehensiveness of whistleblowing information. On the other hand, it is necessary to improve the reward mechanism for the public's whistleblowing of fire risks and fire safety violations. The public should be commended and rewarded through multiple means for participating in urban fire safety co-management, and typical cases of the public participating in urban fire safety co-management should be publicized. Forming positive public opinion guidance can further encourage and guide the public to participate in urban fire safety co-management.

(4) Establishing and improving the accountability mechanism for fire safety supervision. Strengthening the administrative constraints of fire safety management is an important means to urge fire supervision departments to carry out management functions. Meanwhile, it is helpful to avoid lax and lagging supervision of fire supervision departments caused by the construction of urban fire safety co-management systems. Therefore, on the one hand, the superior government and relevant departments should strengthen supervision and spot-check the performance of fire supervision departments by consulting the account, conducting field visits, and other modes to promote the evolution of fire supervision departments' stability strategy toward the direction of a strict supervision strategy. On the other hand, it can be combined with the fire backward investigation and accountability mechanism to review the fire safety supervision process that has an impact on the occurrence of fire and punish fire supervision departments that fail to perform supervision functions. In addition, the superior governments should implement dynamic adjustments to the construction of urban fire safety co-management systems in view of the fire safety management problems. By improving the matching degree of fire safety management measures, fire safety co-management systems, and urban fire safety practices, the effectiveness of urban fire safety co-management will be enhanced.

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