

Article

Measuring Cross-Correlations, Contagion and Long-Range Behavior between Fires in Brazil and Some Time Series Related to Its Economic Growth

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Abstract: Fires bring up the debate about their impact on Brazil's economic growth. Some processing tools such as cointegration and, especially, the correlation have been applied for identifying possible transmission or contagion mechanisms between distinct time series. This paper adopts the detrended cross-correlation analysis (DCCA) and rolling window approach to investigate the dynamic coupling between fires and the evolution of some key variables related to Brazil's economic growth (e.g., agricultural planted area, ethanol production, rainfall in the midwest region and gross domestic product) covering two periods, namely from January 2012 to August 2016 (before the Brazilian presidential impeachment occurred in 2016) and from September 2016 to April 2021, covering the post-impeachment scenario, with the new government policies in the environmental sector. The results show a positive cross-correlation between the level of fires versus planted area of all cereals, leguminous and oleaginous in Brazil (mostly Soybean and Corn) and versus ethanol production (a renewable energy generation). It is also possible to verify some impact level on the Brazilian gross domestic product. Furthermore, we observed quantitatively, by means of the adopted methods that fires in Brazil have the potential to damage economic growth and some activities addressed in this study can also harm the environment in both mid and long-term.

Keywords: time series; DCCA method; commodities; rolling window



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1. Introduction

Brazil is an emerging country and historically played a major role in fulfilling its climate commitments with the global summit. The country started to monitor deforestation in the Amazon in 1985. Since this year, more than half a million square kilometers of forest have been destroyed for land occupation, logging, pastures, agricultural crops, mining, and among others [1]. Between 2004 and 2012, Brazil reduced the deforestation rate in the Amazon by 83% [2]. The effective application of national environmental laws, the creation of large protected areas, the introduction of commitments in the soy and beef production chains, the restrictions on access to credit for rural producers that do not comply with environmental regulations, and the use of real-time satellite imagery to monitor and locate illegal logging were very important initiatives that contributed to that success [3]. Nonetheless, after 2012, several misguided measures and budget reductions for environmental enforcement agencies were implemented. Consequently, the rate of deforestation increased again [4,5].

Indeed, a dramatic increase occurred in 2019, when deforestation increased 85%, according to the National Institute for Space Research (INPE) [6]. From January to December 2019, a total of 9174 were deforested, compared to 4951 in the same period in 2018 [6]. Nonetheless, deforestation is not the only driver that can lead to fires. Fire can either be

used in a farm-fallow context to prepare the area for agriculture using the cyclic slash-and-burn system and also as means of pasture management in cattle ranches. Moreover, climate changes and climatic extremes can lead to uncontrolled fire on open lands or in the forest [7–9].

Accordingly to the Paris Agreement on Climate Change, Brazil has committed to end illegal deforestation by 2030 [10]. Considering its own National Policy on Climate Change, Brazil had also committed to reduce deforestation in the Amazon to less than 3925 square kilometers per year by 2020 [11]. However, from January to July 2020, deforestation was recorded at 4739.92 square kilometers. This area not only considerably exceeded the limit that Brazil included in its climate commitments for the entire year 2020 but also is larger than the area deforested in the same period in 2019 [11]. For the same period of 2021, the Legal Amazon suffered a deforestation of 5026.52 square kilometers, thus showing a growth of 6% in one year.

The destruction of Brazilian vegetation, especially in the Amazon, has consequences that go far beyond Brazil. Forests act as natural carbon storage areas, absorbing and storing it over time. When a forest burns, it can release hundreds of years of stored carbon in the form of carbon dioxide, one of the main greenhouse gases driving climate change, into the atmosphere in a matter of hours [12]. The Amazon plays an exceptional role against climate change, storing approximately 100 billion tons of carbon—an amount equivalent to ten years of global greenhouse gas emissions, having 2018 as the reference year—and removes approximately 600 million of tons per year of the atmosphere [13,14].

Fires do not occur naturally in the humid ecosystem of the Amazon basin. In fact, they are started by people who complete the deforestation process when the most valuable trees have already been removed, often illegally.

Fire can also spread from newly deforested areas and old grasslands to forested areas. Fires, caused by natural ignition, like lightning, are extremely rare in the rainforest and are estimated to occur only every 500 years or more [15].

Accordingly, the development of new technologies, methods and models able to contribute to mitigate the occurrence of fires is very important. Acknowledging the factors that contribute to igniting fires is becoming extremely necessary for not only planning control and suppression but also to avoid social and economic losses in the upcoming years [5]. In this context, a fire risk model was constructed by Zhao et al. [16] with Geographic Information System (GIS) and a multi-layer hierarchical analysis allowing for evaluating the impact of various factors on fire occurrence in a more precise manner.

In turn, Zhang et al. [17] proposed a new firefighting distance criterion (FFDC) to evaluate the actual firefighting coverage of the road network improving the ability and shortening the response time of firefighting activities.

Methods, analysis and modelling using statistics applicable to the fires phenomena have also been developed. As an example, in order to improve statistical approaches for near real-time land cover change detection in non Gaussian time series (TS) data, Anees et al. [18] proposed a supervised land cover change detection framework in which a TS is modeled as a triply modulated cosine function using the extended Kalman filter, and the trend parameter of the triply modulated cosine function is used to derive repeated sequential probability ratio test (RSPRT) statistics.

Scientists warn that the government's failure to contain the accelerating pace of forest loss could push the Amazon to a 'tipping point', when vegetation can be replaced by a type closer to a savanna. In this case, huge amounts of greenhouse gases would be released into the atmosphere and could have catastrophic consequences for the Brazilian economy and for the global efforts to mitigate climate change [19].

It is well known that the advances related to Brazil's environmental agenda projected until the year 2012 fell short of expectations. Nonetheless, the national public policy agenda remained aligned with the declared commitment to international policies, and the country's leading role can be measured by the international reference that the Rio+20 meeting [20] in 2012 assumed before the international community. After 2014, an imbalance in relation

to environmental policy in Brazil allows for pointing to an initial crisis in this sector that began with the impeachment in August 2016 and deepened after 2019 [21]. According to Araújo [22], there is evidence of the gravity of the destruction of environmental protection policy that has taken and continues to take place in Brazil by changing non-statutory rules and cutting budgets.

The aforementioned moving towards the relaxation of environmental policy may have contributed to increase fires justifying this investigation and the introduction of a structural break in the fires TS.

The most recent adoption of environmental policies that generate doubtful or unsatisfactory results affects the Brazil's credibility and may contribute to increasing the risk aversion of external and even internal investments, especially long-term ones. Moreover, this theme also concerns the international community due to the related environmental aspects and, therefore, has a global reach.

The main goal of this work is to investigate possible cross-relations and contagion between the level of fires in Brazil versus the evolution of some time series (TS) that represent some variables of interest that can be impacted by the fires, such as the agricultural planted area (AGR), ethanol (ETH) production, rainfall in the midwest region (RMW) and gross domestic product (GDP). For this purpose, we adopt the detrended cross-correlation analysis (DCCA) method [23–25].

Stemming this ideas, the remaining sections are organized as follows; in Section 2, the selected TS are described and the necessary mathematical and computational methods are introduced. In Section 3, the results obtained are discussed. Finally, in Section 4, the main conclusions are outlined.

2. Data and Methodology

2.1. Data Characteristics

We consider the TS describing the fires and the other four key variables aforementioned, related to the Brazil's economic growth. We understand that the specific variables selected are key regarding the possible cross-correlation between fires in Brazil and some time series related to its economic growth due the main following reasons that are supported by their respective cited references: firstly, a large part of Brazil's GDP comes from the agricultural sector [26,27], which demands an increase in planted area year by year [26,27]. In addition, there is some evidence that the harvesting of ethanol in some Brazilian regions is related to fires [28]. In addition, Brazil has increased the ethanol production from corn crops [5], which can lead to a higher demand for planted areas if productivity does not increase at the same rate [5]. Finally, the Brazilian Midwest is a key region regarding agriculture production, and the rainfall level is a variable that can be close to both agricultural production and fires [29]. We analyze the evolution of all TS during approximately nine years (from January 2012 to April 2021) with monthly frequency (112 observations). The data characteristics and source are shown in Table 1. The datasets for fires, AGR, ETH, RMW, and GDP are obtained from the National Institute for Space Research (INPE) [30], the Brazilian Institute of Geography and Statistics (IBGE), the National Agency of Petroleum, Natural Gas and Biofuels (ANP), the Brazilian National Institute of Meteorology (INMET), and the Central Bank of Brazil (BCB), respectively.

Table 1. Description of key variables and their TS.

Variable	Unit	Frequency	Measure	Scope	Source
Fires	Square Kilometers (km ²)	Monthly	Proxy	Whole Country	INPE
AGR	Hectares (ha)	Monthly	Proxy	Whole Country	IBGE
ETH	Cubic meters (m ³)	Monthly	Proxy	Whole Country	ANP
RMW	Parts per Million (ppm)	Monthly	Direct	Midwest Region	INMET
GDP	Brazilian Reals (R\$)	Monthly	Proxy	Whole Country	BCB

Figure 1 depicts the dynamics of fires in the last decade in Brazil. The behavior of the TS related to the other four variables considered in this study, namely AGR, ETH, RMW and GDP, during the same period, is presented by Figure 2. The AGR includes the total planted area of all cereals, leguminous and oleaginous in Brazil (mostly Soybean and Corn).

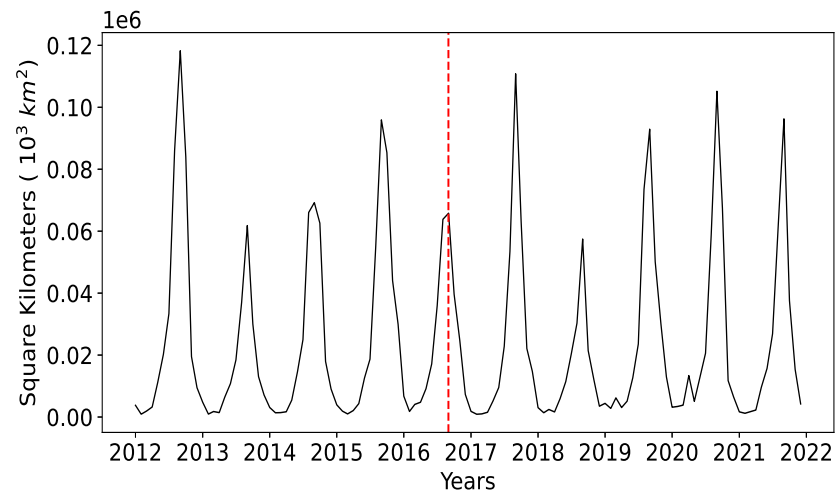
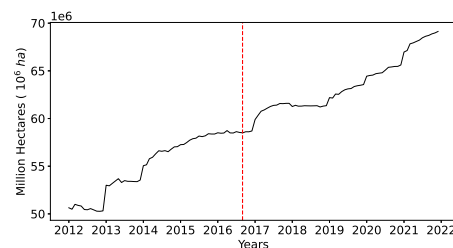
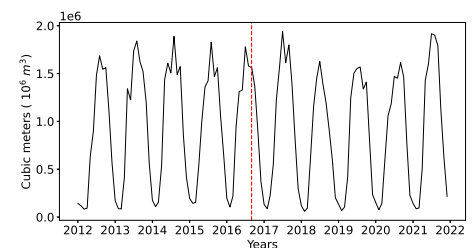


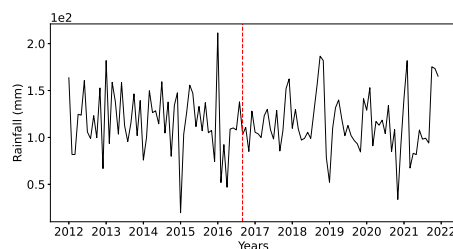
Figure 1. The fires TS before seasonality.



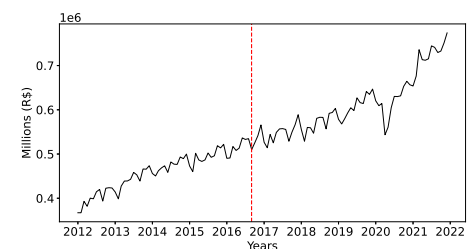
(a) The AGR TS



(b) The ETH TS



(c) The RMW TS



(d) The GDP TS

Figure 2. The key variables TS before seasonality decomposition.

In order to analyze the dynamics of cross-correlations, we consider two distinguished periods, P_1 and P_2 , where the first denotes the period prior to the impeachment occurred in Brazil (from January 2012 to August 2016), and the second denotes the period after the impeachment (from September 2016 to April 2021). The dashed lines between the years 2016 and 2017 in both Figures 1 and 2 point out the exact date of the impeachment.

We have chosen to deseasonalize the data for a few reasons, namely: first, we analyze the relation of time series with different seasonality components. The AGR, ETH, and RMW are strongly affected by seasonal components since they are agricultural and climate variables, but the seasonal component diverges from each other. On the other hand, the GDP does not comport an important seasonal component. Second, we desire to evaluate the cross-correlation using specifically the trend component, in order to address the level of influence of forest fires in those variables and vice versa. This approach adopted in this paper is based on Granger [31]. We also performed the unit root test (see Table 2) and compare the series before and after the deseasonalization process. One can note the same conclusions about the behavior of time series.

Table 2. The ADF unit root test for the variables. Significance level of 95% (t -value $< \tau - 5$ pct) rejects the null hypothesis of non-stationarity.

Variable	Model	t -Value—lag 1	t -Value— <i>diff</i>	τ —5 pct	Stationary at Level?	Stationary at First Difference?
<i>Fires</i>	Trend	−3.157	−5.038	−3.430	No	Yes
	Drift	−3.175	−5.037	−2.880	Yes	Yes
	None	−0.949	−5.052	−1.950	No	Yes
<i>Fires-Adj</i>	Trend	−4.953	−9.003	−3.430	Yes	Yes
	Drift	−4.952	−9.045	−2.880	Yes	Yes
	None	−1.665	−9.088	−1.950	No	Yes
<i>AGR</i>	Trend	−1.770	−6.754	−3.430	No	Yes
	Drift	0.006	−6.765	−2.880	No	Yes
	None	5.137	−5.358	−1.950	No	Yes
<i>AGR-Adj</i>	Trend	−1.770	−6.754	−3.430	No	Yes
	Drift	0.362	−6.765	−2.880	No	Yes
	None	5.137	−5.358	−1.950	No	Yes
<i>ETH</i>	Trend	−2.780	−5.443	−3.430	No	Yes
	Drift	−2.189	−5.461	−2.880	No	Yes
	None	0.207	−5.474	−1.950	No	Yes
<i>ETH-Adj</i>	Trend	−4.806	−9.536	−3.430	Yes	Yes
	Drift	−4.511	−9.571	−2.880	Yes	Yes
	None	−0.811	−9.613	−1.950	No	Yes
<i>RMW</i>	Trend	−7.383	−9.273	−3.430	Yes	Yes
	Drift	−7.378	−9.287	−2.880	Yes	Yes
	None	−0.318	−9.329	−1.950	No	Yes
<i>RMW-Adj</i>	Trend	−7.383	−9.273	−3.430	Yes	Yes
	Drift	−7.378	−9.287	−2.880	Yes	Yes
	None	−0.318	−9.329	−1.950	No	Yes
<i>GDP</i>	Trend	−1.919	−10.673	−3.430	No	Yes
	Drift	0.447	−10.662	−2.880	No	Yes
	None	2.668	−9.995	−1.950	No	Yes
<i>GDP-Adj</i>	Trend	−2.266	−8.938	−3.430	No	Yes
	Drift	0.266	−8.904	−2.880	No	Yes
	None	2.492	−8.119	−1.950	No	Yes

We use the DCCA method [24] and the correlation coefficient [32], which can be used with variables that are not stationary, avoiding the necessity of performing stationarity tests. The ADF unit root test (see Table 2) assures that the series are non-stationary.

Furthermore, by using the correlation coefficient, it is possible not only to deal with contemporary correlation but also to analyse the correlation for different time scales [33]. In addition, it allows the observation of possible nonlinearities [34].

The methodology adopted is employed as follows. Firstly, an additive decomposition of the TS is considered to remove any periodic seasonal behavior. Afterwards, the cross-correlation method is applied in order to calculate the value of the coefficient ρ_{DCCA} between the considered series, making it possible to identify the TS behavior over time. A rolling window approach [33,35–38] is applied to obtain the coefficient ρ for the time scales (in months) $4 \leq n \leq 12$. Considering that the series has 112 observations, we divided it into two subperiods of $N = 56$ observations in order to estimate the correlation coefficient for P_1 and P_2 . The R software was used for all analyses presented.

2.2. Detrended Cross-Correlation Analysis Method

The DCCA method was proposed by Podobnik and Stanley [24], in order to evaluate the power law cross correlations between different simultaneous TS, even in the presence of non-stationarity [24]. The procedure consists of four steps that, at the end, allows for obtaining the cross-correlation coefficient, as follows:

(i) Let us consider two different series y_K and x'_K , $K = 1, 2, 3, \dots, N$. The series y_K and x'_K are integrated, and two new series are obtained according to Equation (1):

$$\begin{aligned} Y_K &= y_1 + y_2 + y_3 + \dots + y_K \\ X'_K &= x'_1 + x'_2 + x'_3 + \dots + x'_K \end{aligned} \quad (1)$$

(ii) The integrated series are divided into $(N - n)$ overlapping boxes of equal size n . We also employ a rolling window approach to slide the boxes as a function of time.

(iii) A local trend for each box ($Y_{K,i}$, $\tilde{X}_{K,i}$) is calculated and then the covariance function of the residuals in each box is obtained by means of Equation (2):

$$f_{DCCA}^2(n, i) = \frac{1}{(n+1)} \sum_{K=i}^{i+n} (Y_K - \tilde{Y}_{K,i}) (X'_K - \tilde{X}'_{K,i}) \quad (2)$$

(iv) Afterwards, the average for all boxes must be calculated to obtain the covariance function from Equation (3):

$$F_{DCCA}^2(n) \equiv \frac{1}{N-n} \sum_{i=1}^{N-n} f_{DCCA}^2(n, i) \quad (3)$$

Finally, the cross-correlation coefficient ρ_{DCCA} is calculated, defined as the relationship between the detrended covariance F_{DCCA}^2 and the detrended variation F_{DFA} . This coefficient has a limited range between $-1 \leq \rho_{DCCA} \leq 1$, given by the following expression:

$$\rho_{DCCA}(n) \equiv \frac{F_{DCCA}^2(n)}{F_{DFA1}(n)F_{DFA2}(n)} \quad (4)$$

The extreme values of ρ_{DCCA} correspond to perfect cross correlation (1) and perfect anti-cross correlation (−1), while the null value means non-cross correlation [39–41].

2.3. Rolling Window Approach and the Statistical Test for $\Delta\rho_{DCCA}$

Different statistical tests have been adopted to evaluate the detrended cross-correlation coefficients [42–45]. In this work, we applied the statistical test proposed by Guedes et al. [25,43,46] to evaluate $\Delta\rho_{DCCA}$. The estimation of $\Delta\rho_{DCCA}$ from dynamic cross-correlation coefficients is called dynamic contagion, and this concept is based on the works of Forbes and Rigobon [47], Guedes et al. [43,46], and Tilfani et al. [35]. For it, we calculate the $\Delta\rho_{DCCA}$ between any correlation in moment t until $t - 56$ using four different window sizes (W_1 to W_4), which captures the difference between the correlation coefficient

in that time. The concept of contagion is understood as a shift increase or decrease in correlation after a certain phenomenon, normally applied to distinct periods that are separated (before and after) by this phenomenon [35,47]. In addition, Tilfani et al. [35] firstly applied this concept of contagion using dynamic cross-correlation coefficients in order to estimate the shift increase or decrease of cross-correlation continuously. This work applies the dynamic contagion concept to analyze the dynamic coupling between fires and the evolution of some key variables related to Brazil's economic growth covering two periods, pre and post Brazilian impeachment in 2016.

The coefficient is represented by:

$$\Delta\rho_{DCCA}(n) = \rho_{DCCA}^{P_2}(n) - \rho_{DCCA}^{P_1}(n) \quad (5)$$

where $\rho_{DCCA}^{P_2}(n)$ and $\rho_{DCCA}^{P_1}(n)$ are the DCCA coefficients for the periods P_1 and P_2 , respectively. The subsequent test consists of calculating the probability distribution function (PDF) of the $\Delta\rho_{DCCA}(n)$, supposing that it obeys a normal distribution. This comprises the below steps [25]:

- Generate two TS with long-range cross-correlation by the ARFIMA process [24,48–50];
- Divide the TS for periods P_1 and P_2 and shuffle these pairs;
- Estimate $\rho_{DCCA}(n)$ and the periods' difference $\Delta\rho_{DCCA}(n)$;
- Repeat step 2 several times;
- Obtain the distribution of $\Delta\rho_{DCCA}(n)$, and
- (Additional step) Evaluate the normality of the distribution [38].

In general, the PDF of $\Delta\rho_{DCCA}(n)$ converges to a normal distribution, as shown by [25]. However, we decided to conduct D'Agostino and Pearson's normality test [51,52] to verify the normality of the distribution. Hereafter, the following contagion hypothesis is tested with a t -test for the mean of the $\Delta\rho_{DCCA}(n)$ parametric group and the Wilcoxon signed-rank test for the non-parametric group:

$$H_0: \Delta\rho_{DCCA}(n) = \langle \Delta\rho_{DCCA} \rangle;$$

$$H_1: \Delta\rho_{DCCA}(n) \neq \langle \Delta\rho_{DCCA} \rangle;$$

where the null hypothesis (H_0) consists of the statement that the contagion does not exist. Alternatively, H_1 evaluates the existence of contagion between the two periods. In addition, the $\langle \Delta\rho_{DCCA} \rangle$ is the sample mean, which is approximately equal to zero [25]. Thus, for each PDF defined by window size W —in this study, $W_1 = 18$ months, $W_2 = 24$ months, $W_3 = 30$ months and $W_4 = 36$ months, where $W_{min} = W_1$ and $W_1 > \frac{N_{P_1}}{4}$ as suggested by [25]—and n time scales, we can obtain the positive critical point defined as $\Delta\rho_c(n)$ for 95% confidence levels as follows (see Table A1):

$$\langle \Delta\rho_{DCCA} \rangle \pm Z_{\alpha/2} \frac{SD}{\sqrt{N}} \quad (6)$$

where $Z_{\alpha/2}$ is the value for the chosen confidence level α , SD is the standard deviation, and N is the sample size.

3. Results and Discussion

As previously stated, this study adopts the DCCA cross-correlation coefficient and the rolling window approach to assess the relationships between the TS that represents fires and those TS related to the other four key variables of interest for the Brazil's economic growth. We highlight that the rolling window approach strengthens the results, i.e., it provides robustness to the achieved results since it involves an analysis with explicit time variation.

Figure 3a,b and point out the *Fires*–AGR ρ_{DCCA} coefficient for periods P_1 and P_2 , respectively. One can observe that the cross-correlation strengthened for P_2 in comparison to P_1 but could not overpass the 0.7 threshold.

Similarly, from Figure 3c,d, it is possible to note a non-correlated pattern between *Fires*–*ETH* pair later in the last decade (P_1). However, as the period approaches the impeachment date, we note a rapid increase of cross-correlation that is also sustained for P_2 . Most of Brazil's ethanol production is predominantly concentrated in the southeast region (mainly from sugarcane in Sao Paulo state), and, therefore, relatively distant from the Amazon forest zone. One of the reasons that can explain the increase of cross-correlation in later P_1 and succeeded by P_2 is due to an increase of ethanol corn-based production in the midwest region, which has been intensively supported by the Brazilian government in both periods. Secondly, as fires data include all the biomes in Brazil, another possible relation between fires and ETH is due to the application of fires in sugarcane harvest in some regions [28]. One may also note that this pattern weakened by the end of P_2 , but it can easily be explained by COVID-19 outbreak's impact on fuel demand, which forced a decrease in the fuel supply in Brazil. Therefore, on a regular basis, one can expect that the co-movement between *Fires*–*ETH* is likely positive, but it reversed in an extreme fuel market condition. It can indicate that fires dynamics are not dependent on ETH. In other words, ETH production does not cause most of the movement in fires.

On the other hand, the *Fires*–*RMW* showed a weak cross-correlation during P_1 , but this pattern shifted for P_2 , where a negative cross-correlation for most parts of the period can be observed. This pattern is only expected for the case where the lack of rainfall (drought) in the midwest, which is an Amazon forest zone, could cause an increase in fires. Figure 4c reinforces this conclusion since the periods showed a predominant difference, i.e., $\Delta\rho_{DCCA} > 0$ for every applied time scales (n). Moreover, it is also possible to note that the *Fires*–*GDP* pair only showed a significant cross-correlation during the periods close to the impeachment process, i.e., in the five months prior to the event and the subsequent five months. However, it is possible to note that, during this short time span, it reverted from a positive cross-correlation to a negative pattern. The lack of evidence of a clear pattern might indicate that this is not a causation relation, since it is expected that the *GDP* variable shows a pattern similar to the *AGR*, considering the vast influence of agricultural scope to the Brazilian GDP.

Table 3 summarizes the descriptive statistics for the $\Delta\rho_{DCCA}$ distributions as a function of n with the different sizes of W (18, 24, 30 and 36 months). Differently from expected and observed by some authors [25,38], the distributions' mean values are not close to zero and the standard deviation (SD) does not decrease for greater W sizes. In addition, mostly skewness and kurtosis diverged from values observed from normal distributions, i.e., $Kurtosis \approx 3$ and $Skewness \approx 0$ for different combinations of n and W . Multiple pieces of evidence tend to affect the normality of the distributions. For this reason, the D'Agostino and Pearson's normality test [51,52] is conducted, and the results are shown in Table 4. For the ones that reject the null hypothesis of normality (marked in Table 4), the contagion test must be conducted by different statistical approaches such as a non-parametrical test.

Hence, the contagion hypothesis can be tested for each $\Delta\rho_{DCCA}$ distribution. Table 5 depicts the significance test, where the t -test is applied to parametric (normal) distributions and the Wilcoxon signed-rank test for non-parametric (non-normal) distributions. In general, there is evidence of a contagion, i.e., differences for P_1 and P_2 , for every set of pairs. Combining the specific data in Table 5 illustrates that the *Fires*–*AGR* has more expressive contagion in the mid-term—as pointed by $W1$ and $W2$ —than in the long-term ($W3$ and $W4$). The fact that the *AGR* was shown to have impacted the fires in the long term of P_2 as much as in P_1 might indicate that the agricultural impact on fires has not been influenced by any particular policy. Differently, the *Fires*–*ETH* and *Fires*–*RMW* revealed a contagion for every window size. Therefore, these factors presented the most relevant shift in pattern for the post-impeachment period (P_2). Some factors such as the *RMW* are not controlled by any sort of government policy but have a clear causation aspect. On the other hand, as pointed earlier, the *ETH* variable might be influenced by the Brazilian energy downstream policies, which have been intensively incentivizing the production of the ethanol corn-based.

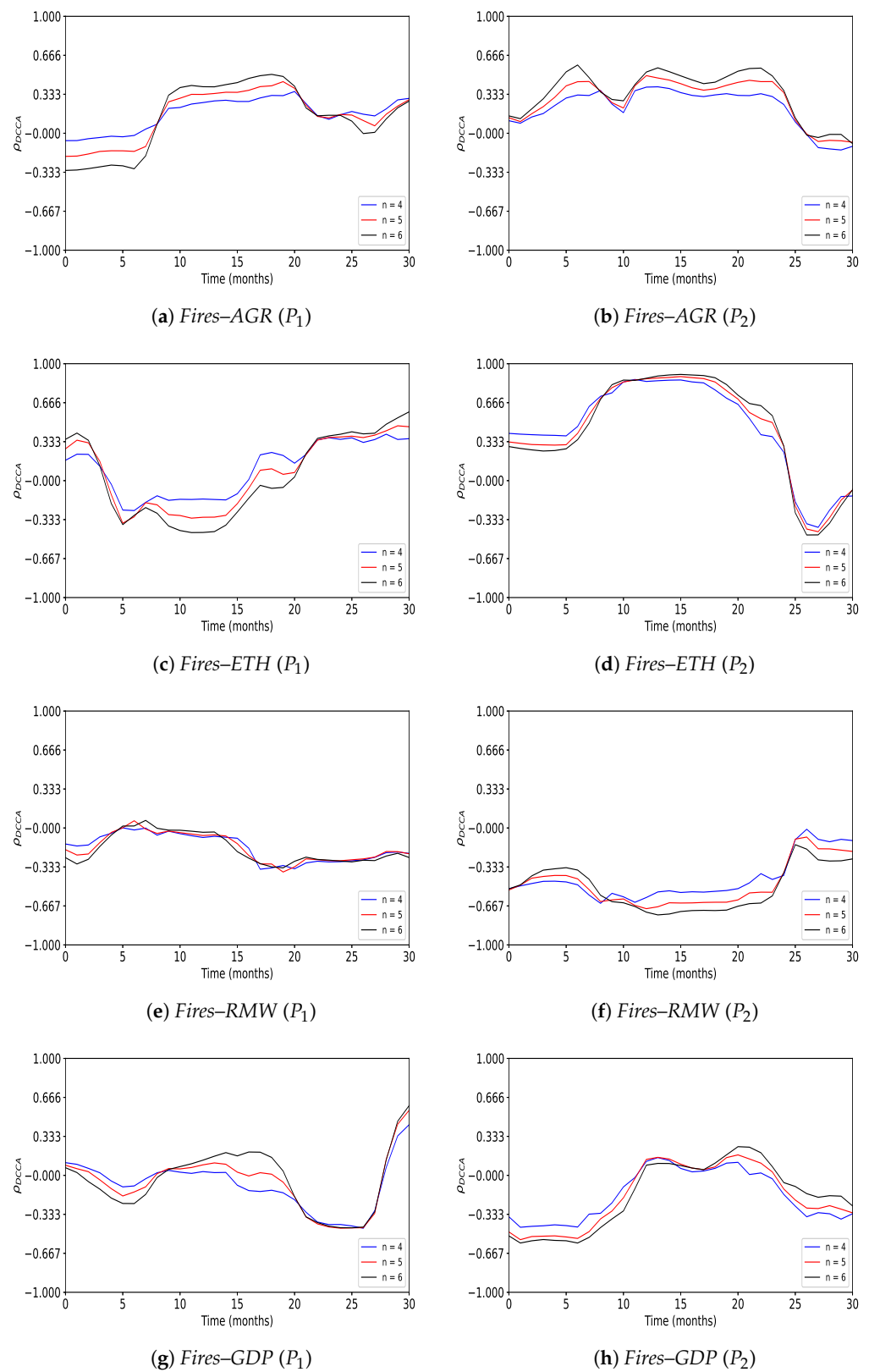


Figure 3. The ρ_{DCCA} comparing the TS of P_1 vs. P_2 for W1.

Table 3. The descriptive summary of $\Delta\rho_{DCCA}$ for W1 to W4.

Window Size	Time Scale	Fires–AGR				Fires–ETH				Fires–RMW				Fires–GDP			
		Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis
18 months	$n = 4$	−0.1311	0.1251	−0.3435	−1.1539	−0.6774	0.3030	0.3824	−1.2147	0.4112	0.1304	−0.5332	−0.8343	0.1209	0.2865	−0.0195	−1.5758
	$n = 6$	−0.2474	0.3296	−0.7230	−0.9882	−0.7962	0.4873	0.7273	−0.6848	0.4413	0.1720	−0.0726	−1.1151	0.2382	0.2504	−0.7832	0.5222
24 months	$n = 4$	−0.0775	0.1418	−0.2976	−0.6844	−0.6274	0.2243	−0.0599	−1.4473	0.3703	0.1232	−0.4540	−0.9713	0.0322	0.2150	0.5116	−1.0904
	$n = 6$	−0.1581	0.2144	−0.6384	−0.6550	−0.8347	0.3175	0.8201	−0.2804	0.4762	0.1241	−0.2440	−1.2724	0.1149	0.2113	0.3811	0.6770
	$n = 8$	−0.2092	0.2614	−0.4005	−1.5032	−0.9696	0.3938	0.9268	0.1081	0.5304	0.2062	−0.9370	0.2389	0.1601	0.1735	−1.5322	2.4042
30 months	$n = 4$	−0.0507	0.1440	−0.2511	−1.6438	−0.5390	0.1331	0.2464	−0.5591	0.3218	0.0924	−0.3881	−0.7618	0.0808	0.1418	0.1515	−1.1318
	$n = 6$	−0.1401	0.2318	−0.3811	−1.5357	−0.7429	0.2334	0.6411	−0.9884	0.4685	0.0670	−0.9759	−0.2386	0.1227	0.1501	0.6116	−0.7964
	$n = 8$	−0.1767	0.2965	−0.4974	−1.2443	−0.8781	0.3367	0.9367	−0.1471	0.5454	0.0940	−0.0718	−0.9515	0.1275	0.1419	0.4320	−1.4784
	$n = 10$	−0.2157	0.3162	−0.3705	−1.2296	−0.9652	0.3742	1.0330	0.3002	0.5937	0.1354	−0.2801	−1.1400	0.0965	0.1462	−0.0532	−1.5226
36 months	$n = 4$	−0.0175	0.0999	−0.4592	−1.4662	−0.4604	0.0720	−0.4023	−0.5844	0.2417	0.1252	−0.8624	−0.7513	0.1327	0.0665	0.2261	−0.9130
	$n = 6$	−0.1006	0.2029	−0.6398	−1.3508	−0.6087	0.1618	−0.0473	−1.3451	0.4200	0.0606	−0.6323	−0.7115	0.1484	0.0963	0.0468	−1.4076
	$n = 8$	−0.1362	0.2672	−0.8354	−1.0675	−0.7081	0.2634	0.2993	−1.1698	0.5077	0.0503	0.7061	−0.6327	0.1088	0.1588	0.4527	−1.0570
	$n = 10$	−0.1999	0.3215	−0.4842	−1.4403	−0.7755	0.3411	0.4382	−1.1548	0.5468	0.0758	0.8838	−0.8161	0.0378	0.2291	0.5088	−1.1043
	$n = 12$	−0.2251	0.3565	−0.4696	−1.3862	−0.8256	0.3787	0.5333	−1.1290	0.5883	0.1002	0.3235	−1.4106	−0.0263	0.2623	0.1747	−1.4177

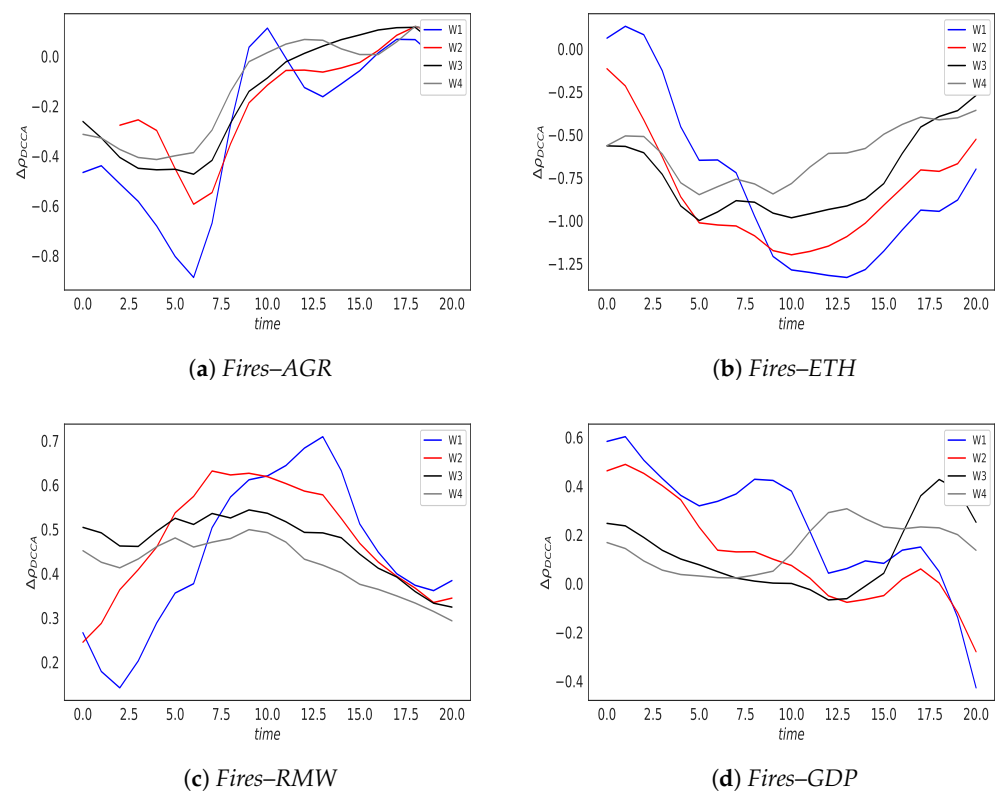


Figure 4. The $\Delta\rho_{DCCA}$ for the different TS pairs and time scale of $n = 6$.

Table 4. The normality test of $\Delta\rho_{DCCA}$ for W1 to W4. Significance level of 95% rejects the null hypothesis of normality, i.e., p -value < 0.05 (highlighted in bold).

D'Agostino and Pearson's Normality Test									
Pair	Time Scale	W1		W2		W3		W4	
		χ^2	p -Value	χ^2	p -Value	χ^2	p -Value	χ^2	p -Value
Fires-AGR	$n = 4$	2.7585	0.2518	0.5786	0.7488	12.3765	0.00205	7.6693	0.0216
	$n = 6$	3.4081	0.1819	1.9603	0.3753	9.1118	0.0105	6.3679	0.0414
	$n = 8$	-	-	8.3197	0.0156	4.2427	0.1199	4.5637	0.1021
	$n = 10$	-	-	-	-	4.1287	0.1269	8.3575	0.0153
	$n = 12$	-	-	-	-	-	-	7.0745	0.0291
Fires-ETH	$n = 4$	3.9899	0.1360	7.3777	0.0250	0.3638	0.8337	0.8922	0.6401
	$n = 6$	2.7157	0.2572	3.1629	0.2057	3.2838	0.1936	5.1915	0.0746
	$n = 8$	-	-	4.3827	0.1118	4.0996	0.1288	3.2026	0.2016
	$n = 10$	-	-	-	-	5.5181	0.0634	3.5562	0.1690
	$n = 12$	-	-	-	-	-	-	3.7495	0.1534
Fires-RMW	$n = 4$	1.9982	0.3682	2.2331	0.3274	1.1397	0.5656	3.7523	0.1532
	$n = 6$	2.2438	0.3257	4.3140	0.1157	4.3020	0.1164	2.1977	0.3333
	$n = 8$	-	-	4.6866	0.0960	1.1180	0.5718	2.5028	0.2861
	$n = 10$	-	-	-	-	2.8490	0.2406	4.0844	0.1297
	$n = 12$	-	-	-	-	-	-	7.0282	0.0298
Fires-GDP	$n = 4$	11.3901	0.0034	3.3079	0.1913	2.4871	0.2884	1.1680	0.5577
	$n = 6$	4.1842	0.1234	0.9333	0.6270	2.2916	0.3180	6.4369	0.0400
	$n = 8$	-	-	14.3877	0.0008	9.1201	0.0105	2.7773	0.2494
	$n = 10$	-	-	-	-	9.5206	0.0086	3.4102	0.1818
	$n = 12$	-	-	-	-	-	-	6.8116	0.0332

Table 5. The significance (contagion) test of $\Delta\rho_{DCCA}$ for W1 to W4. Significance level of 95% rejects the null hypothesis of $\Delta\rho_{DCCA} = 0$, i.e., p -value < 0.05 (highlighted in bold).

<i>t</i> -Test or Wilcoxon Signed-Rank Test for Significance at Differences									
Pair	Time Scale	W1		W2		W3		W4	
		Statistic	<i>p</i> -Value	Statistic	<i>p</i> -Value	Statistic	<i>p</i> -Value	Statistic	<i>p</i> -Value
<i>Fires</i> – <i>AGR</i>	$n = 4$	−4.5693	2.0000e-04	−2.3852	2.8300e-02	65.0000	2.2730e-01	77.0000	4.6880e-01
	$n = 6$	−3.2732	4.2000e-03	−3.2136	4.8000e-03	49.0000	6.4100e-02	73.0000	3.7600e-01
	$n = 8$	-	-	31.0000	1.0000e-02	−2.5970	1.8200e-02	−2.2218	3.9400e-02
	$n = 10$	-	-	-	-	−3.1254	5.3000e-03	62.0000	6.2900e-02
	$n = 12$	-	-	-	-	-	-	57.0000	4.2000e-02
<i>Fires</i> – <i>ETH</i>	$n = 4$	−10.2450	2.1026e-09	0.0000	5.9570e-05	−18.5511	4.5154e-14	−29.2884	6.7118e-18
	$n = 6$	−7.4877	3.1906e-07	−12.0490	1.2650e-10	−14.5878	4.0187e-12	−17.2400	1.8012e-13
	$n = 8$	-	-	−11.2836	4.0021e-10	−11.9520	1.4592e-10	−12.3208	8.5172e-11
	$n = 10$	-	-	-	-	−11.8199	1.7753e-10	−10.4193	1.5787e-09
	$n = 12$	-	-	-	-	-	-	−9.9918	3.2079e-09
<i>Fires</i> – <i>RMW</i>	$n = 4$	14.4500	4.7830e-12	13.7703	1.1526e-11	15.9543	7.6760e-13	8.8440	2.3933e-08
	$n = 6$	11.7578	1.9477e-10	17.5891	1.2352e-13	32.0251	1.1652e-18	31.7302	1.3974e-18
	$n = 8$	-	-	11.7896	1.8571e-10	26.5776	4.4693e-17	46.2588	8.2008e-22
	$n = 10$	-	-	-	-	20.0915	9.8982e-15	33.0695	6.2032e-19
	$n = 12$	-	-	-	-	-	-	0.0000	5.9570e-05
<i>Fires</i> – <i>GDP</i>	$n = 4$	63.0000	6.8000e-02	0.6871	0.4999	2.6111	1.6700e-02	9.1361	1.4133e-08
	$n = 6$	4.3592	3.0000e-04	2.4926	2.1600e-02	3.7461	1.3000e-03	0.0000	5.9570e-05
	$n = 8$	-	-	30.0000	3.0000e-03	25.0000	1.7000e-03	3.1400	5.2000e-03
	$n = 10$	-	-	-	-	48.0000	1.9000e-02	7.5540e-01	4.5880e-01
	$n = 12$	-	-	-	-	-	-	104.0000	6.8940e-01

4. Conclusions

This paper investigated the impact of fires on some variables of interest related to the economy and growth of Brazil. We adopted the DCCA and used the rolling window approach to explore the dynamic coupling between the TS studied.

The results showed a positive cross-correlation between the fires versus AGR and against ETH, where the first presented higher contagion evidence in the mid-term rather than in the long-term, while the second revealed evidence of contagion for every window size considered. This might indicate that the agricultural impact on fires has not been influenced by any long-term policy, but this is different for ethanol, in which energy downstream policies have been intensively incentivizing the production of the ethanol corn-based in the last few years. There was also some impact on the RMW level in the midwest, which is something that deserves further investigation for its causality aspect, since it may be evidence of the impact of fires on the climate of this region, which could jeopardize future agricultural production in the Brazilian midwest, one of the most important agricultural regions in the country. Even the impact specifically on Brazil's GDP is not clear since the cross-correlation patterns for P_1 and P_2 are relatively different and close to zero, the present study was able to quantify the dynamic cross-correlations contagion and verify the mid or long-range behaviors of fires time series vis-à-vis some key variables represented by TS related to Brazil's economic growth.

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Abbreviations

The following abbreviations are used in this manuscript:

TS	Time Series
AGR	Agricultural planted area
ETH	Ethanol
RMW	Rainfall in the midwest region
GDP	Gross domestic product
INPE	National Institute for Space Research
IBGE	The Brazilian Institute of Geography and Statistics
ANP	The National Agency of Petroleum, Natural Gas and Biofuels
INMET	The Brazilian Institute of Meteorology
BCB	The Central Bank of Brazil
DCCA	Detrended Cross-Correlation Analysis

Appendix A

Table A1. The critical values of $\Delta\rho_{DCCA}$ with 95% confidence level for W1 to W4.

Pair	Time Scale	W1	W2	W3	W4
<i>Fires-AGR</i>	$n = 4$	0.0746	0.1556	0.1861	0.1469
	$n = 6$	0.2946	0.1945	0.2411	0.2331
	$n = 8$	-	0.2207	0.3111	0.3033
	$n = 10$	-	-	0.3044	0.3289
	$n = 12$	-	-	-	0.3613
<i>Fires-ETH</i>	$n = 4$	-0.1790	-0.2584	-0.3200	-0.3419
	$n = 6$	0.0053	-0.3125	-0.3590	-0.3425
	$n = 8$	-	-0.3219	-0.3243	-0.2749
	$n = 10$	-	-	-0.3497	-0.2145
	$n = 12$	-	-	-	-0.2028
<i>Fires-RMW</i>	$n = 4$	0.6256	0.5730	0.4738	0.4476
	$n = 6$	0.7242	0.6803	0.5788	0.5197
	$n = 8$	-	0.8695	0.7000	0.5905
	$n = 10$	-	-	0.8164	0.6714
	$n = 12$	-	-	-	0.7530
<i>Fires-GDP</i>	$n = 4$	0.5921	0.3858	0.3141	0.2421
	$n = 6$	0.6500	0.4625	0.3697	0.3067
	$n = 8$	-	0.4454	0.3609	0.3700
	$n = 10$	-	-	0.3370	0.4145
	$n = 12$	-	-	-	0.4052

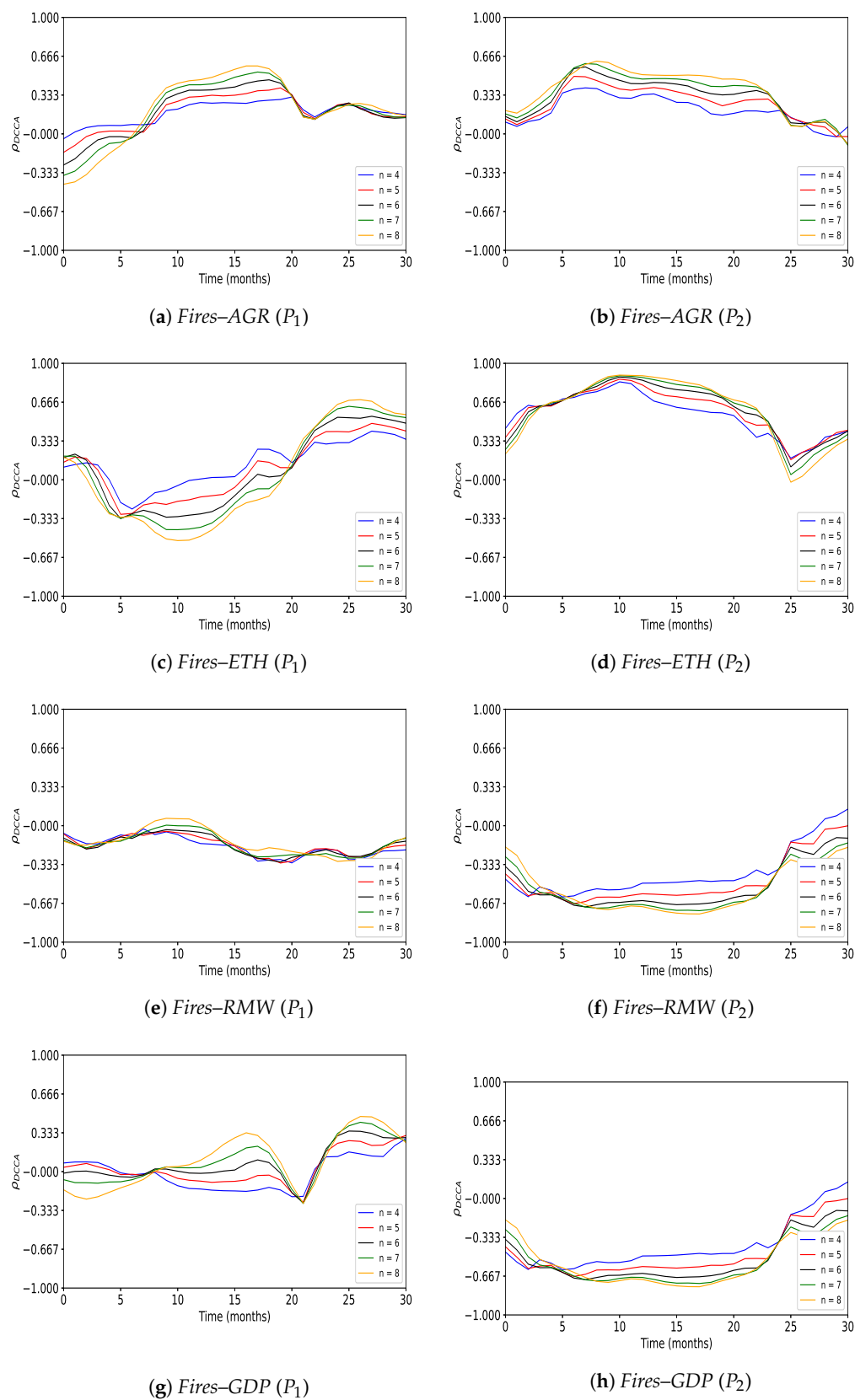


Figure A1. The ρ_{DCCA} comparing TS of P_1 vs. P_2 for W2.

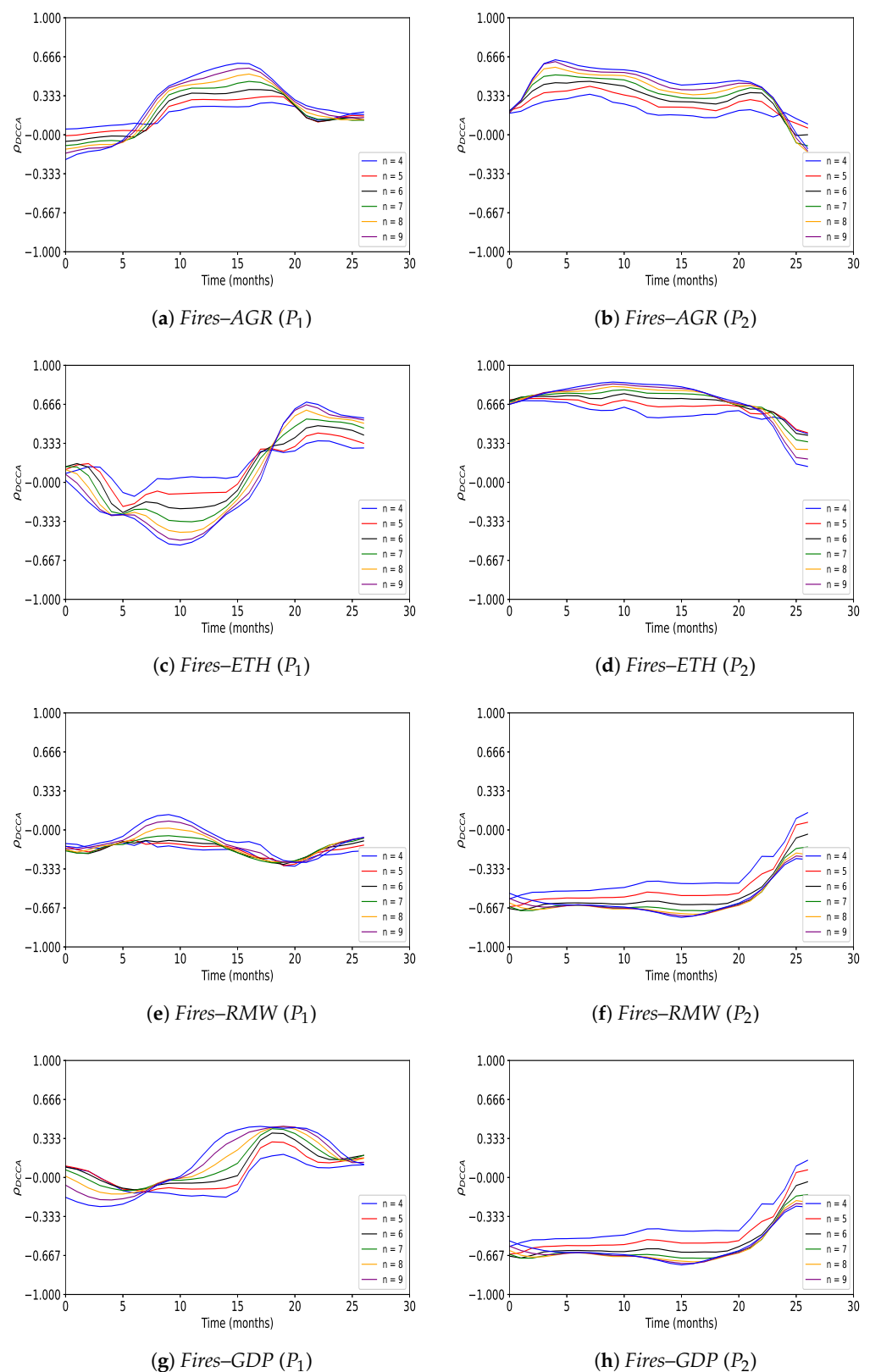


Figure A2. The ρ_{DCCA} comparing TS of P_1 vs. P_2 for W3.

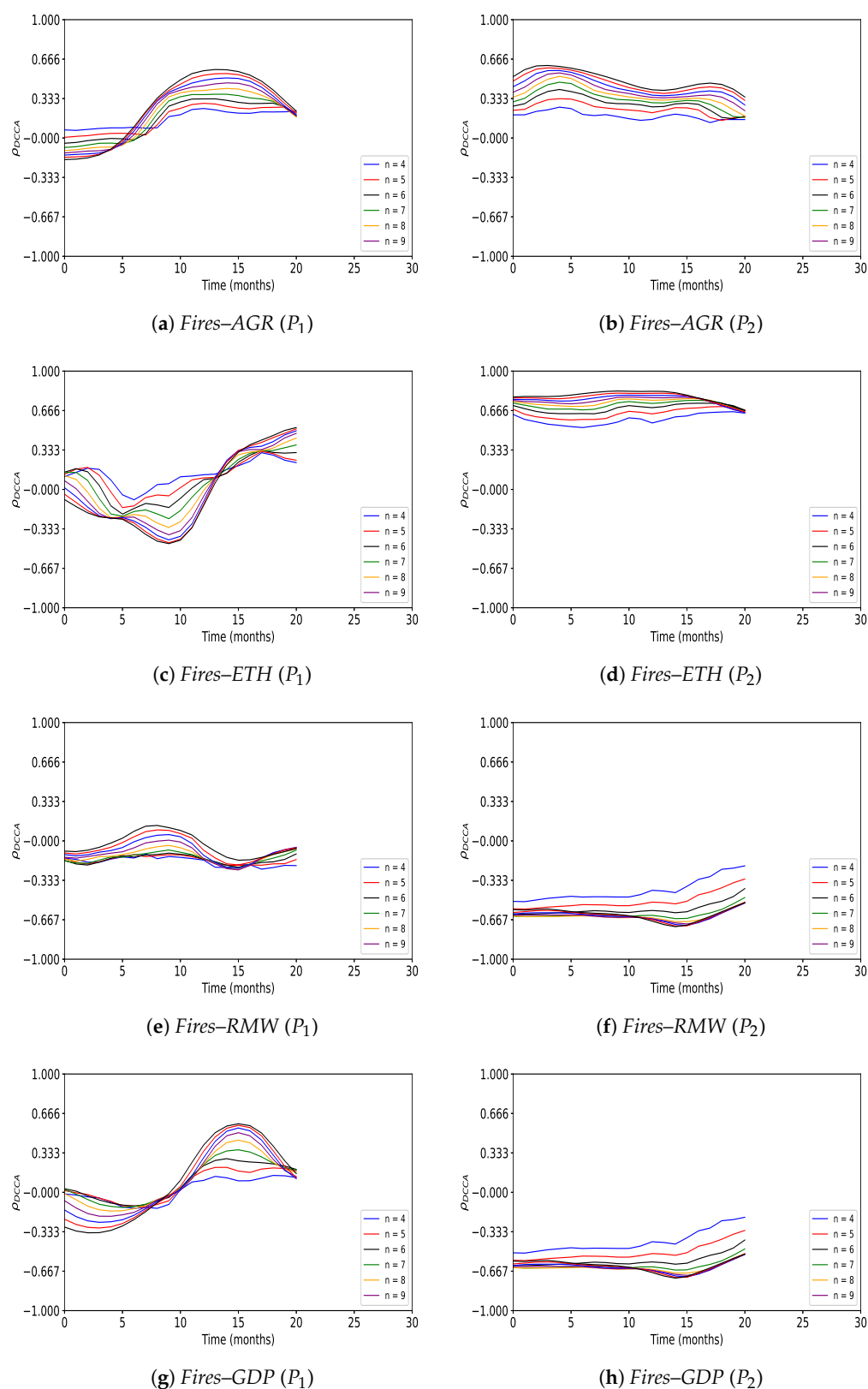


Figure A3. The ρ_{DCCA} comparing TS of P_1 vs. P_2 for W4.

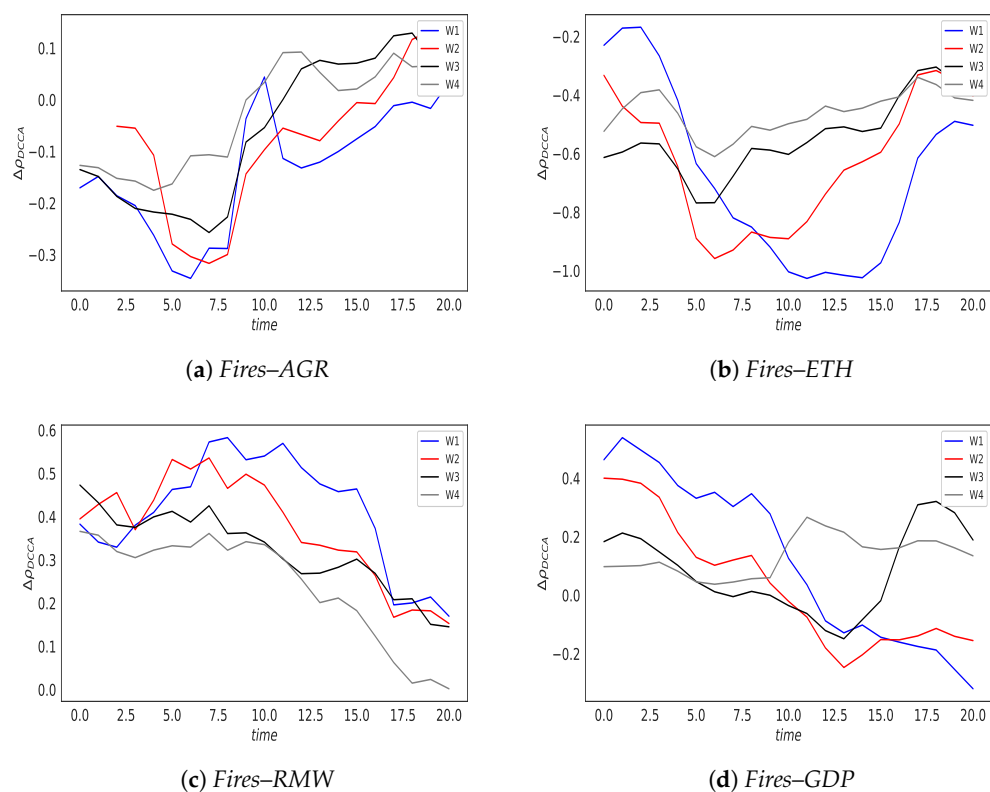


Figure A4. The $\Delta\rho_{dcca}$ for the different TS pairs and time scale $n = 4$.

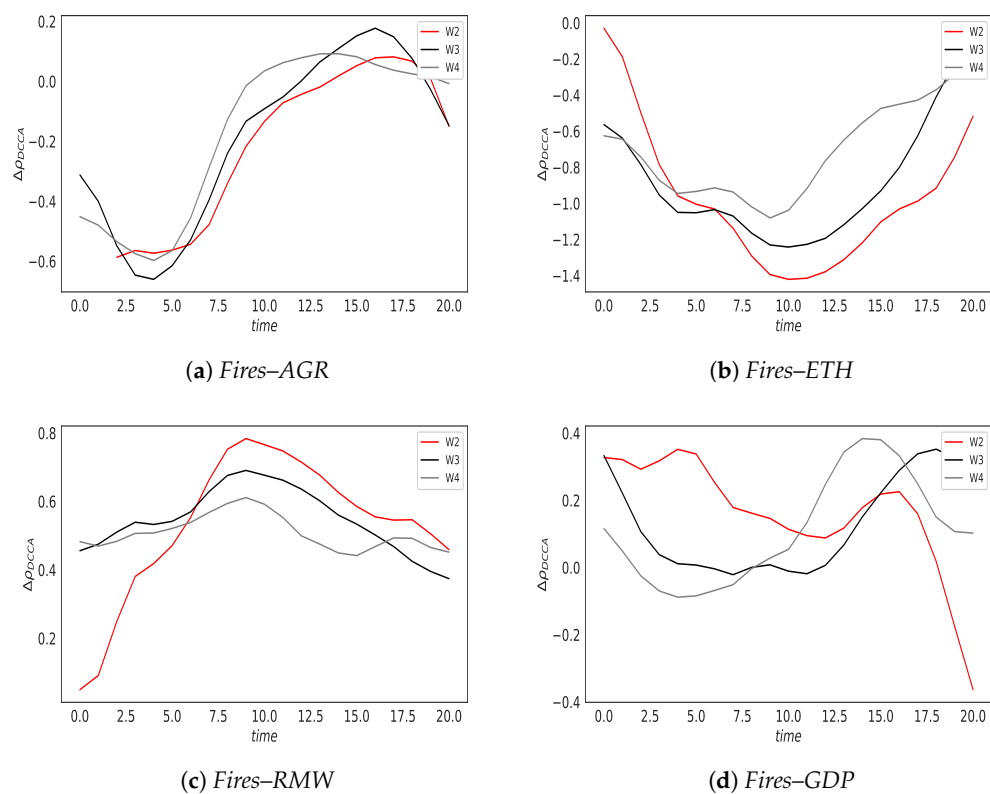


Figure A5. The $\Delta\rho_{dcca}$ for the different TS pairs and time scale $n = 8$.

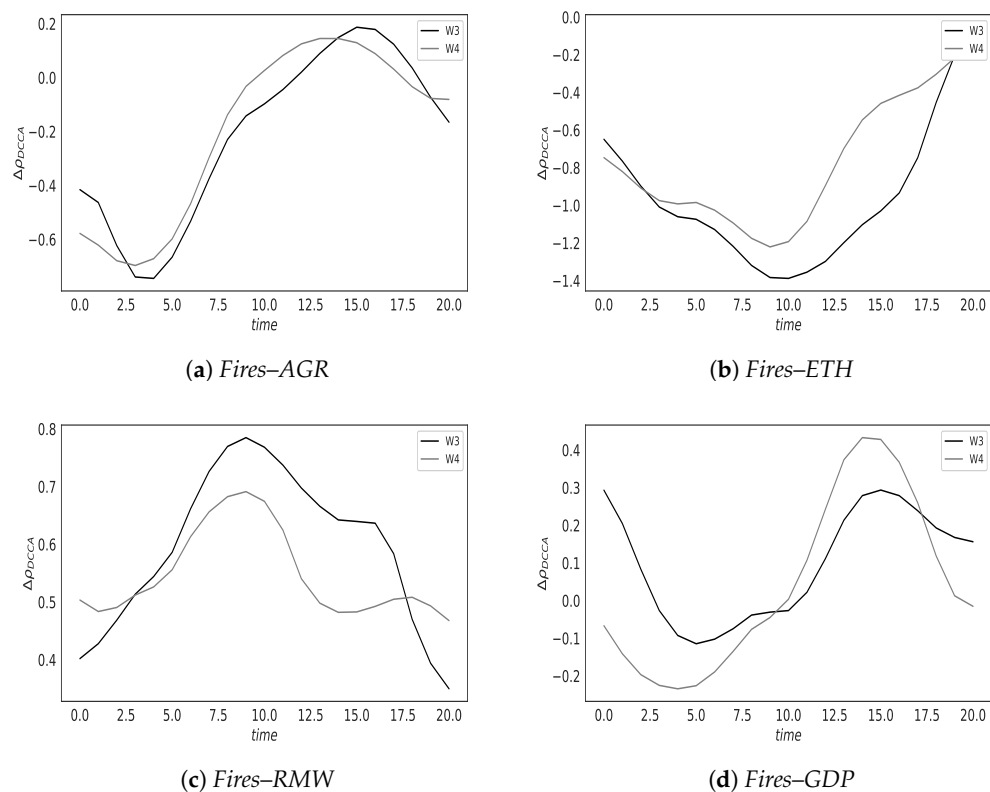


Figure A6. The $\Delta\rho_{dcca}$ for the different TS pairs and time scale $n = 10$.

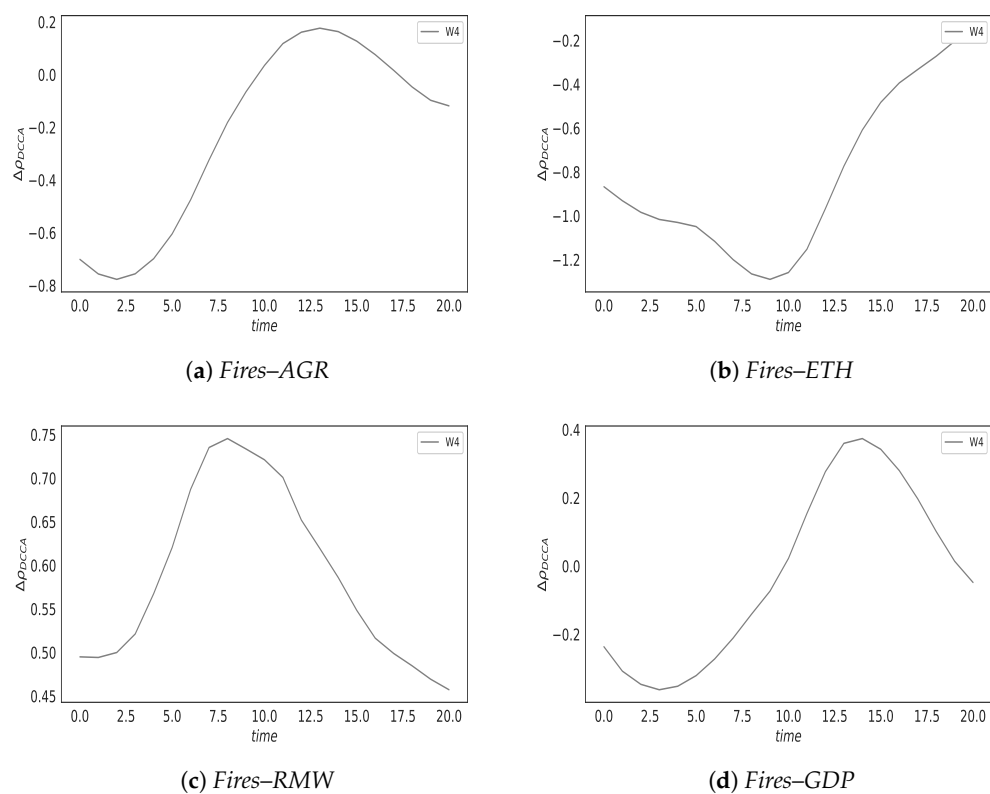


Figure A7. The $\Delta\rho_{dcca}$ for the different TS pairs and time scale $n = 12$.

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