Supplementary Material

SA Affect of Node Removal on Network Structural and Positional Features

The goal of the following discussion is to evaluate the effectiveness of different structure-aware measures in capturing the impact of different event types. Figures. S1 to S6 illustrate different type of events pertaining to graph evolution.

Figure. S1 shows the original graph. The degree centrality, node betweenness centrality and edge betweenness centrality for all nodes and edges in the graphs are reported in table S3 and S5. We also report the entropy variation based on the underlying three centrality measures in table S1. Looking at table S3, we observe that nodes 3 and 5 rated slightly different from the point of view of node betweenness centrality, which is not the case in degree centrality that considers the number of immediate neighbours. The drawback of node betweenness centrality is that the side nodes 1, 2, 6 and 7 are scored as 0. In other words, the metric does not distinguish the importance of the corner nodes. On the other hand, edge betweenness centrality measures the rate of which an edge plays the role of a "bridge-like connector" between two sides of the network. As observed, there are 4 different values assigned to 6 edges compared to 3 values in degree and node betweenness centrality. We expect that relatively higher expressive power of edge betweenness centrality led to better performance in dynamic link prediction task. We compare the entropy variations after the occurrence of events.

Figure. S2 shows the graph after adding the edge (3, 6). As we see in table S9, the degree centrality and node betweenness centrality of nodes 3 and 6 increased, and the betweenness centrality of nodes 4 and 5 decreased as they are less likely to be reached after the new edge is added. Analogously, the betweenness centrality of edges (3, 4), (4, 5), (5, 6) and (5, 7) decreased, but the same metric almost tripled for edge (6,7), since it shares a node with newly added edge (3, 6). As for entropy variation, the degree centrality and node betweenness centrality-based values increased; and the one for edge betweenness centrality slightly decreased. Additionally, we observe 6 different values of betweenness centrality assigned to 7 edges whereas there are 4 values for the node-based measures.

Figure. S3 illustrates the graph after a new node 8 is added and a new edge (4, 8) is being formed as a result. Expectedly, the importance of node 4 has increased and thus reflected in degree centrality and node betweenness centrality values. At this stage nodes 3, 4 and 5 adopt same value of degree centrality. However, this is not the case for node betweenness centrality, as this measure is different for all three nodes. As for edge betweenness centrality the value for all edges, except the newly added (4,8) have dropped, as adding a node and its connection to the bridge node 4, slightly downgraded the importance of the existing side edges. In addition, the degree centrality and edge betweenness centrality-based entropy variation has dropped negligibly, whereas the one for node betweenness has increased. Remarkably. This may lead to the conclusion that node betweenness centrality-based entropy variation could better highlight the impact of the structural change (which is link formation in this case).

Figure. S4 demonstrates the graph once edge (6, 7) is removed from the original graph. As can be seen the degree centrality of nodes 6 and 7 and betweenness centrality of node 5 reduced as a result. In addition, the betweenness centrality of (5, 6) and (5, 7) slightly increased. The increase in entropy variation is more highlighted once measured based on node betweenness centrality.

The graph after removal of node 3 has shown in Figure. S5. The edges (1, 3), (2, 3) and (3, 4) are removed and nodes 1 and 2 turned into isolation. In this scenario the node betweenness centrality for all nodes except node 5 will score zero, and node 5 takes the role of the only bridge in the graph with an increase in betweenness centrality of its connected edges. The graph entropy based on the underlying measure changes into a zero which indicate an extreme change of entropy variation. Although the node betweenness centrality might not be an indicative node feature for start-like graph, the derived/extracted metric, entropy variation could significantly express the impact of such structural events.

Figure. S6 shows the graph after node 4, the main bridge connector of the graph with highest node betweenness centrality and edge betweenness of its edges, is removed. Like the previous case, the graph entropy variation based on node centrality will drop to zero. The graph breaks into 2 isolated, connected components. This is the scenario wherein a new feature, ratio of changes in the number of components come into effect. It's noted that we disregard the underlying feature in our experiments, since the graphs of autonomous system are connected graphs and therefore, there is

only connected component. Moreover, the degree centrality and edge level centrality provide the same level of information in this scenario, since the entire graph turned into very simple graphs.

If the graph breaks down into multiple components or the number of connected components changes after a node is removed, denoted by $\sigma_{\mathcal{G}_{\bar{v}}}$, we can consider another node feature integrated into our model $1 - \frac{\sigma_{\mathcal{G}_{\bar{v}}}}{\sigma_{\mathcal{G}}}$. The higher the underlying coefficient would be, the more structural change would be imposed to the graph after removal of node v. We will not add such feature to our model at this point, since AS-733 and AS-Oregon-2 graphs are perpetually connected.
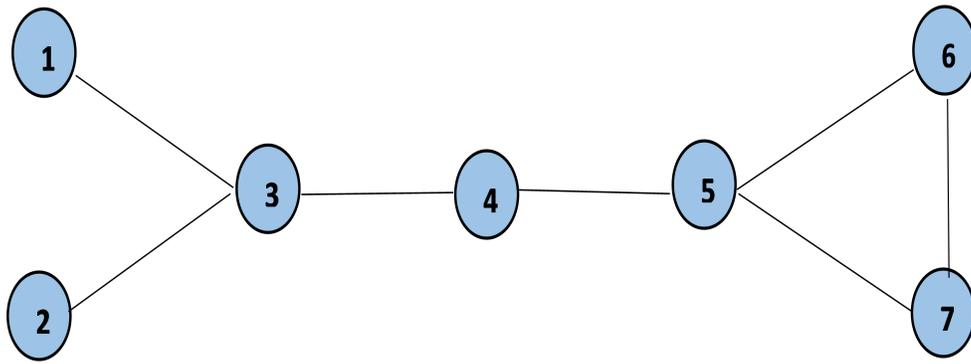
Figure S1: Original graph represenation

Table S2: Original graph information

	degree centrality	node betweenness centrality	edge betweenness centrality
entropy (e)	1.87	2.27	1.78

Table S4: Original graph information

nodes	degree centrality	node betweenness centrality
1	0.16	0
3	0.5	0.6
2	0.16	0
4	0.33	0.6
5	0.5	0.533
6	0.33	0
7	0.33	0

Table S6: Original graph information

| edges	edge betweenness centrality
(1,3)	0.285
(3,2)	0.285
(3,4)	0.571
(4,5)	0.571
(5,6)	0.238
(5,7)	0.238
(6,7)	0.47
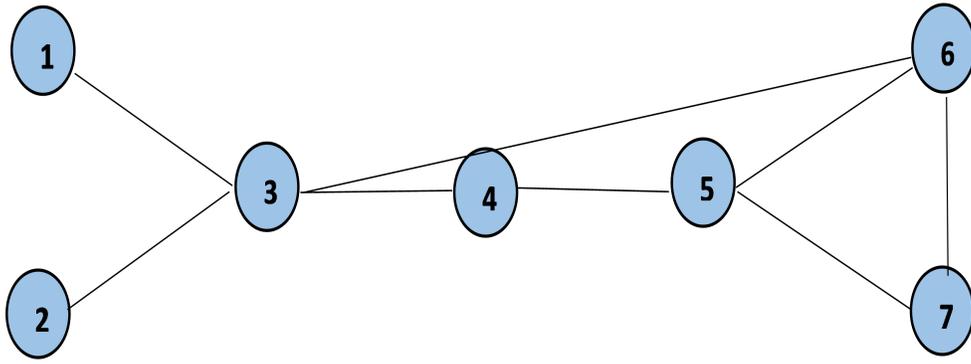
Figure S2: Adding edge (3, 6) to the original graph

Table S8: Adding edge (3,6) to the original graph

	degree centrality	node betweenness centrality	edge between centrality
entropy (e)	1.84	2.007	1.84
entropy variation (Δe)	0.016	0.115	-0.03

Table S10: Adding edge (3,6) to the original graph

nodes	degree centrality	node betweenness centrality
1	0.166	0
3	0.666	0.633
2	0.166	0
4	0.333	0.1
5	0.5	0.1
6	0.5	0.3
7	0.333	0

Table S12: Adding edge (3,6) to the original graph

edges	edge betweenness centrality
(1,3)	0.285
(3,2)	0.285
(3,4)	0.238
(3,6)	0.380
(4,5)	0.190
(5,6)	0.142
(5, 7)	0.95
(6, 7)	0.190
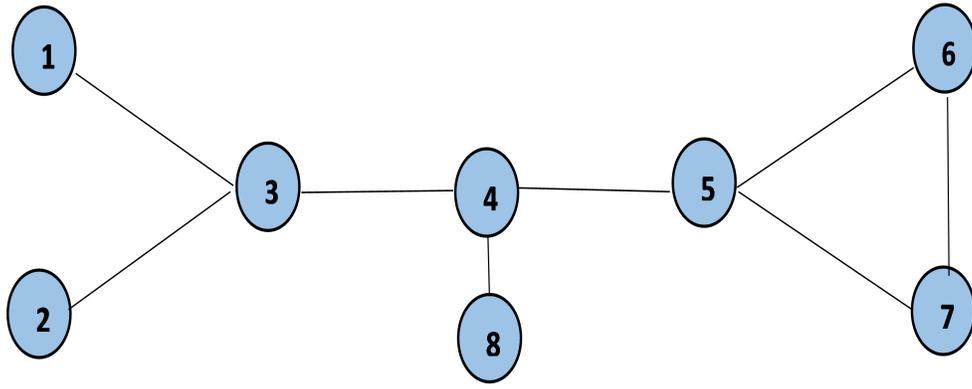
Figure S3: Adding node 8 to the original graph

Table S14: Adding node 8 to the original graph

	degree centrality	node betweenness centrality	edge betweenness centrality
entropy (e)	1.98	1.08	1.91
entropy variation (Δe)	-0.058	0.52	-0.073

Table S16: Adding node 8 to the original graph

nodes	degree centrality	node betweenness centrality
1	0.142	0
3	0.428	0.523
2	0.142	0
4	0.428	0.714
5	0.428	0.476
6	0.285	0
7	0.285	0
8	0.142	0

Table S18: Adding node 8

edges	edge betweenness centrality
(1,3)	0.25
(3,2)	0.25
(3,4)	0.535
(4,5)	0.535
(4,8)	0.25
(5, 6)	0.214
(5,7)	0.214
(6,7)	0.035
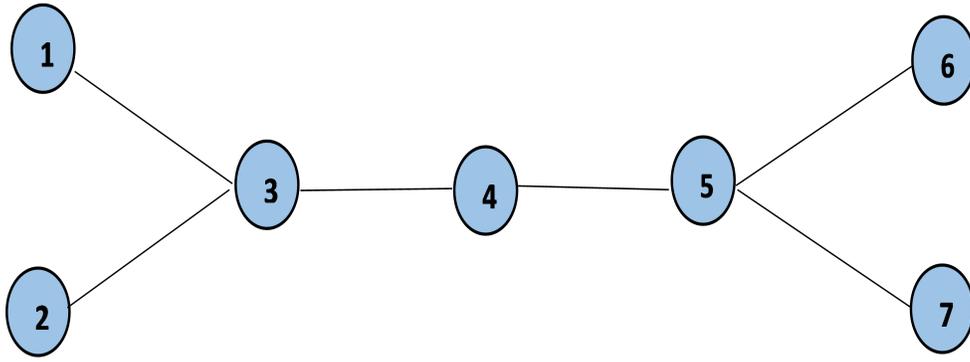
Figure S4: Removing edge (6, 7) from the original graph

Table S20: Removing edge (6,7) from the original graph

	degree centrality	node betweenness centrality	edge betweenness centrality
entropy (e)	1.82	1.09	1.73
entropy variation (Δe)	0.026	0.519	0.028

Table S22: Removing edge (6, 7) from the original graph

nodes	degree centrality	node betweenness centrality
1	0.166	0
3	0.5	0.6
2	0.166	0
4	0.33	0.6
5	0.5	0.6
6	0.166	0
7	0.166	0

Table S24: Removing edge (6,7) from the original graph

edges	edge betweenness centrality
(1,3)	0.285
(3,2)	0.285
(3,4)	0.571
(4,5)	0.571
(5,6)	0.285
(5,7)	0.285
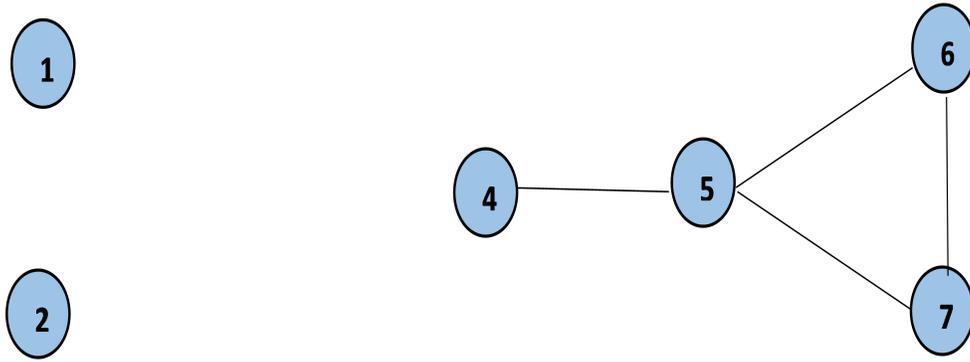
Figure S5: Removing node 3 from the original graph

Table S26: Removing node 3 from the original graph

	degree centrality	node betweenness centrality	edge betweenness centrality
entropy (e)	1.320	0	1.32
entropy variation (Δe)	0.294	1	0.258

Table S28: Removing node 3 from the original graph

nodes	degree centrality	node betweenness centrality
4	0.333	0
5	1	0.666
6	0.666	0
7	0.666	0

Table S30: Removing node 3

edges	edge betweenness centrality
(4,5)	0.5
(5, 6)	0.333
(5,7)	0.333
(6,7)	0.166

Figure S6: Removing node 4 from the original graph

Table S32: Removing node 4 from the original graph

	degree centrality	node betweenness centrality	edge betweenness centrality
entropy (e)	1.74	0	1.549
entropy variation (Δe)	0.069	1	0.13

Table S34: Removing node 4 from the original graph

nodes	degree centrality	node betweenness centrality
1	0.2	0
3	0.4	0.1
2	0.2	0
5	0.4	0
6	0.4	0
7	0.4	0

Table S36: Removing node 4

edges	edge betweenness centrality
(1,3)	0.133
(3,2)	0.133
(5,6)	0.666
(5,7)	0.666
(6,7)	0.666
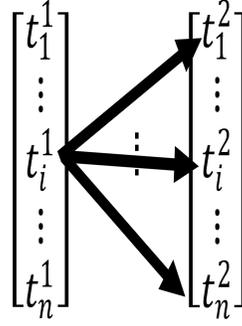
Figure S7: Correlation between two one-dimensional temporal point process

SB   Correlation Analysis between two Temporal point processes using Ripley K function

The subsequent section solely considered the temporal dimension of the two-point process, ignoring the spatial dimension for the time being. The goal here is to investigate the potential correlation among the two point processes corresponding to addition and deletion types of events using K1D, a R package designed to estimate multivariate Ripley's K-function for one-dimensional data.

K1D computes the one-dimensional simplification of the multivariate Ripley K-function (e.g., time or line transects). K1D computes the dependence between two or more ordered one-dimensional event categories [1]. The motivation for using this method is to test whether timestamps of two processes (addition and deletion in our case of study) co-occur more than would be expected by chance (S7). A comprehensive overview of the K-function and the bivariate K-function can be found in [2].

K1D has three key characteristics[1]: a) Determination of the bivariate K-function and its transform (L) utilising an edge correction. The K function can be determined in one of three methods. b) Calculating confidence envelopes using one of three methods: 1) randomization of events (perhaps weighted by an intensity function), 2) a 'circular' ('toroidal' in two dimensions) shift of records relative to one another, (we choose this method for our simulation) or 3) a shuffling of events (i.e., randomization without replacement). c) The calculation of smoothed event frequencies using a predetermined window width.

S1   One-dimensional bivariate multivariate K-function

In other words, for each event in record A, the bivariate K-function returns the fraction of events in record B that occurred within t time units (the "temporal window") of that event. The bivariate K-function is written as following [1]:

$$\tilde{K}_{AB}(t) = \frac{T}{n_A n_B} \sum_{i=1}^{n_A} \sum_{j=1}^{n_B} w(\mathbf{A}_i, \mathbf{B}_j) I(|A_i - B_j| < t), \tag{S1}$$

where T is the length of the record (transect), n_A is the number of events in record A, and n_B is the number of events in record B. In one dimension, A_i and B_j are the timestamps of events, I is the identity function, and $w(A_i, B_j)$ is an edge correction, set to 2 if $\|A_i - B_j\|$ is greater than the distance of A_i to the nearest "edge" of the record, and otherwise set to 1. K values greater than $2t$ indicate attraction or synchronicity between A and B within the time window t. K values close to $2t$ indicate that A and B have no relationship or are independent, whereas K values less than $2t$ indicate repulsion or asynchrony. The edge correction causes a slight difference between $\tilde{K}_{AB}(t)$ and $\tilde{K}_{BA}(t)$ (i.e., whether distances are measured from A to B or vice versa). Then the transform L-function is defined as: $\tilde{L}_{AB}(t) = \tilde{K}_{AB}(t) - t$. We use a package named 'K1D' which implements the L-function in R. The program interprets the first column of data as record 'A' and the second column as record 'B'. Therefore, for 'forward selection,' the K-function looks for 'B' events that follow 'A' events. K-function searches for 'B' events that precede 'A' events in

the 'backward selection' method. With both the forward and reverse selection methods, only two records are allowed.

Confidence envelope for the bivariate K-function: Simulations are utilized to create a confidence envelope. There are three ways of simulating such interval which we select 'circular' shifts that works as follow: Each randomization consists of shifting each record a random number of time unit (seconds in our case) and wrapping events from the end to the beginning of the record. This test examines only the dependence between two records and maintains the first-order properties (frequency) of each record during randomization [2]. Percentiles of the distribution of the simulated K-function are used to determine the confidence envelopes. Where $\tilde{K}_{AB}(t)$ is greater than, within, or less than the confidence envelope, statistically significant synchrony (attraction), independence, or statistically significant asynchrony (repulsion) in a window of t is indicated.

In our simulation, we chronologically ordered the events regardless of the nodes involved. All timestamps are converted to seconds because the program handles integer data more efficiently, and timestamps are all shifted to the origin of the minimum time in the monitoring time span. We limited T to a maximum of three hours to improve the resolution of events. We considered the following cases in our simulation: a) Sequences of addition and deletion types of events occurring in one day, b,c) Sequences of addition and deletion types of successive events occurring in a two-day interval, d) Sequence of successive addition type of events occurring within a day, e) Sequence of addition types of events occurring within two consecutive days, f) Sequences of deletion types of events occurring within two consecutive days. Figures S8-S13 demonstrates the L-function (transform of the K-function) for the events in the upper figure with 95% confidence envelopes (red lines) for all 6 cases. In Figure. S8 The L-function lies within the confidence envelopes, indicating the independence of the underlying addition and deletion events that occur within a 3-hour period. It should be noted that this may not generalize to all the events happening in a 24-hour period. Since the addition of nodes or links may compensate for the removal of nodes in some cases. In Figures S9 and S10 the L-function mostly exceeds the lower confidence envelope indicating strong repulsion between the events. Indeed the behaviour change at around $t \approx 30000$ in Figure. S10 and turns into no synchrony/Independence in a smaller window. The L-function, as shown in Figure. S11, exhibits dispersion between occurrences of the same type (addition) occurring within a day. The area between the lower confidence envelope and L-function is even larger in the case of addition and deletion events occurring in two successive days as illustrated in Figures. S12 and S14, respectively. The behaviour does not exhibit a consistent trend in Figure. S13; we see no synchrony at the beginning, followed by a small interval of attraction, then no synchrony again until $t \approx 33000$ after which the repulsion occurs. Given all the observations, we conclude that the events of same/the other type mainly exhibit independence from or repulsion toward each other depending on the arrival rates of the events which the events in the latter case may inhibit the occurrence of each other.

References

[1] Daniel G Gavin. "K1D: Multivariate Ripley's K-function for one-dimensional data". In: *University of Oregon* 80 (2010).

[2] Thorsten Wiegand and Kirk A. Moloney. "Rings, circles, and null-models for point pattern analysis in ecology". In: *Oikos* 104.2 (2004), pp. 209–229.
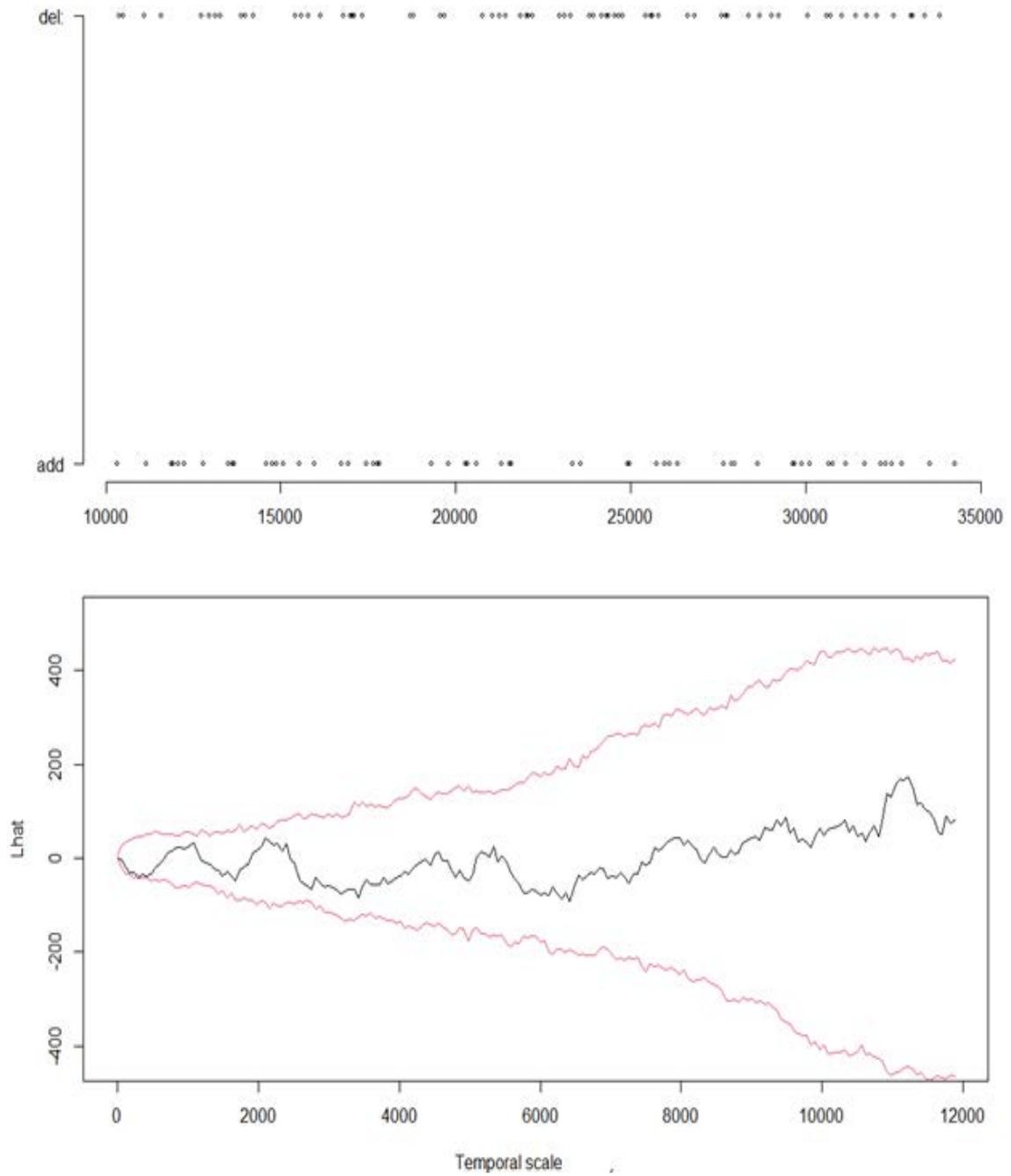
**Figure S8:** A. Sequences of addition and deletion types of events occurring in one day
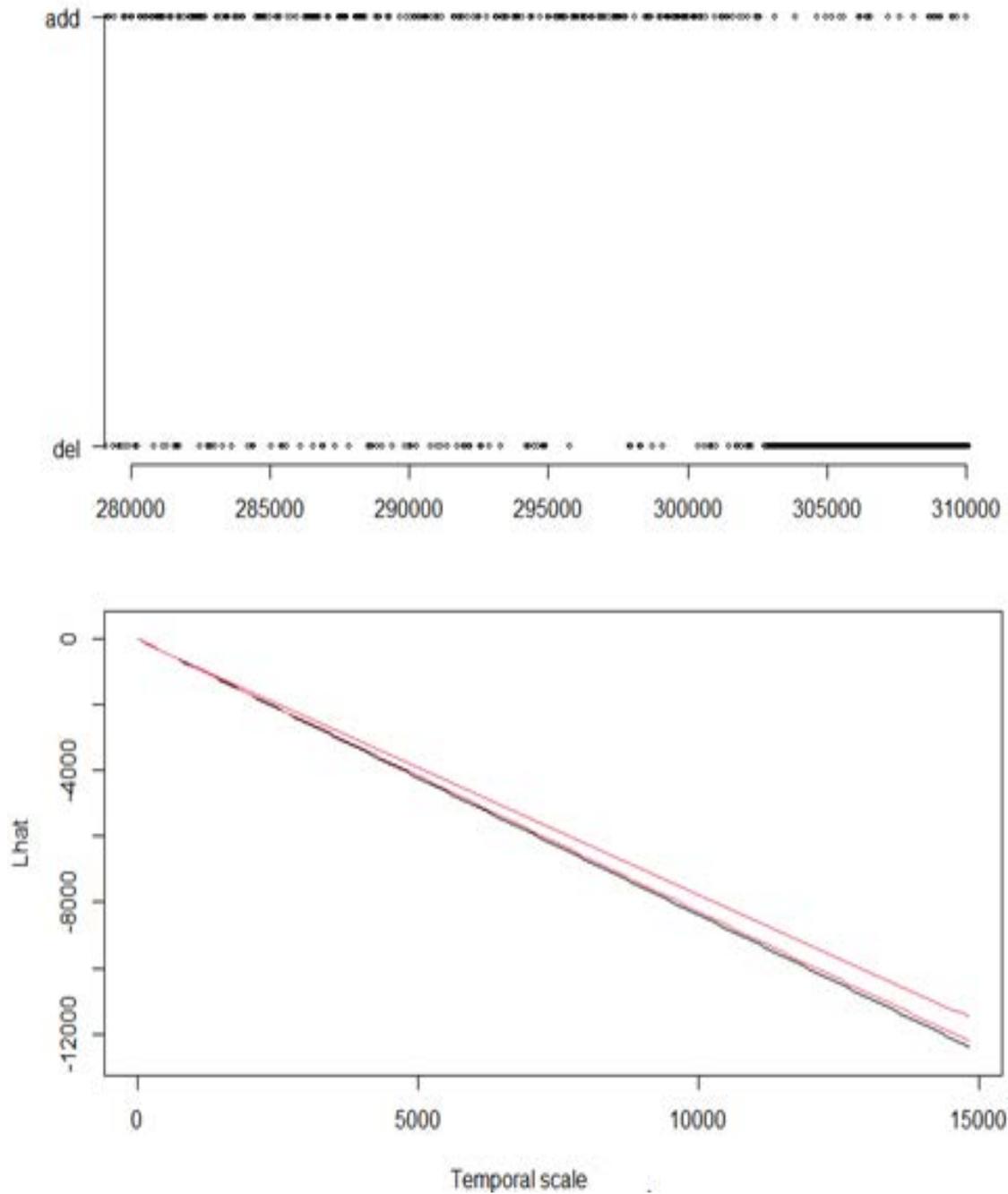
Figure S9: B. Sequences of addition and deletion types of successive events occurring in a two-day interval
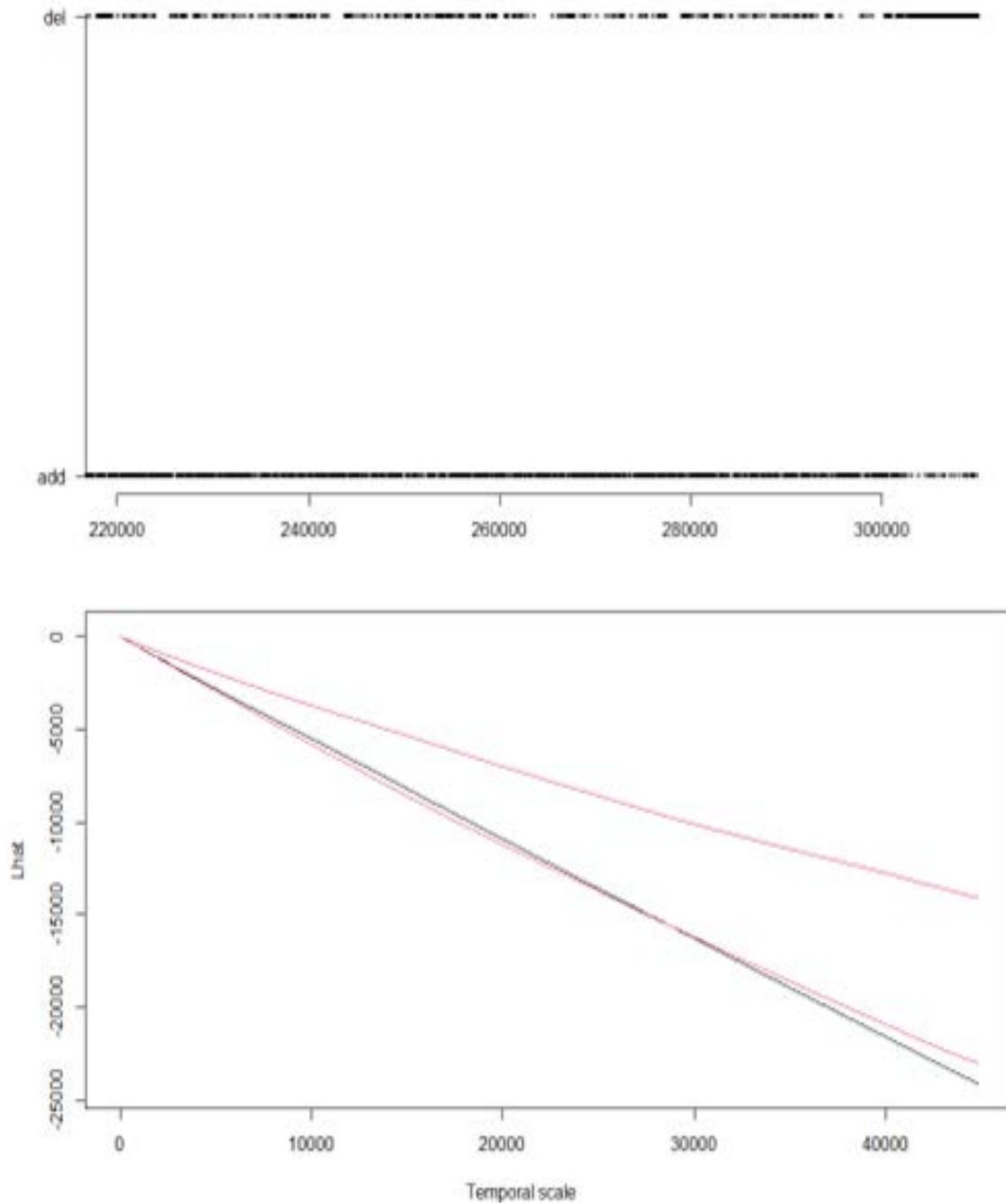
Figure S10: C. Sequences of addition and deletion types of successive events occurring in a two-day interval
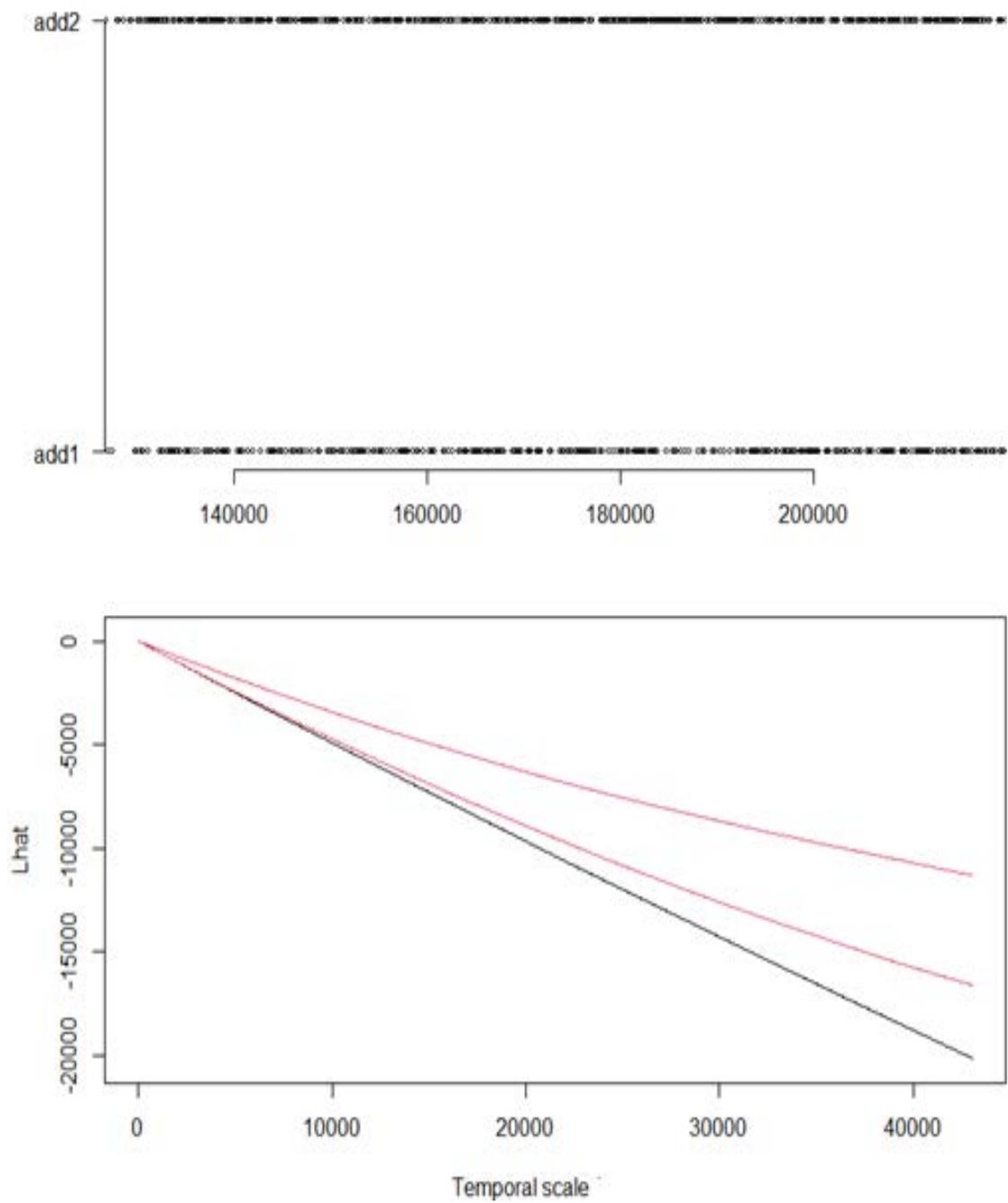
Figure S11: D. Sequence of successive addition type of events occurring within a day
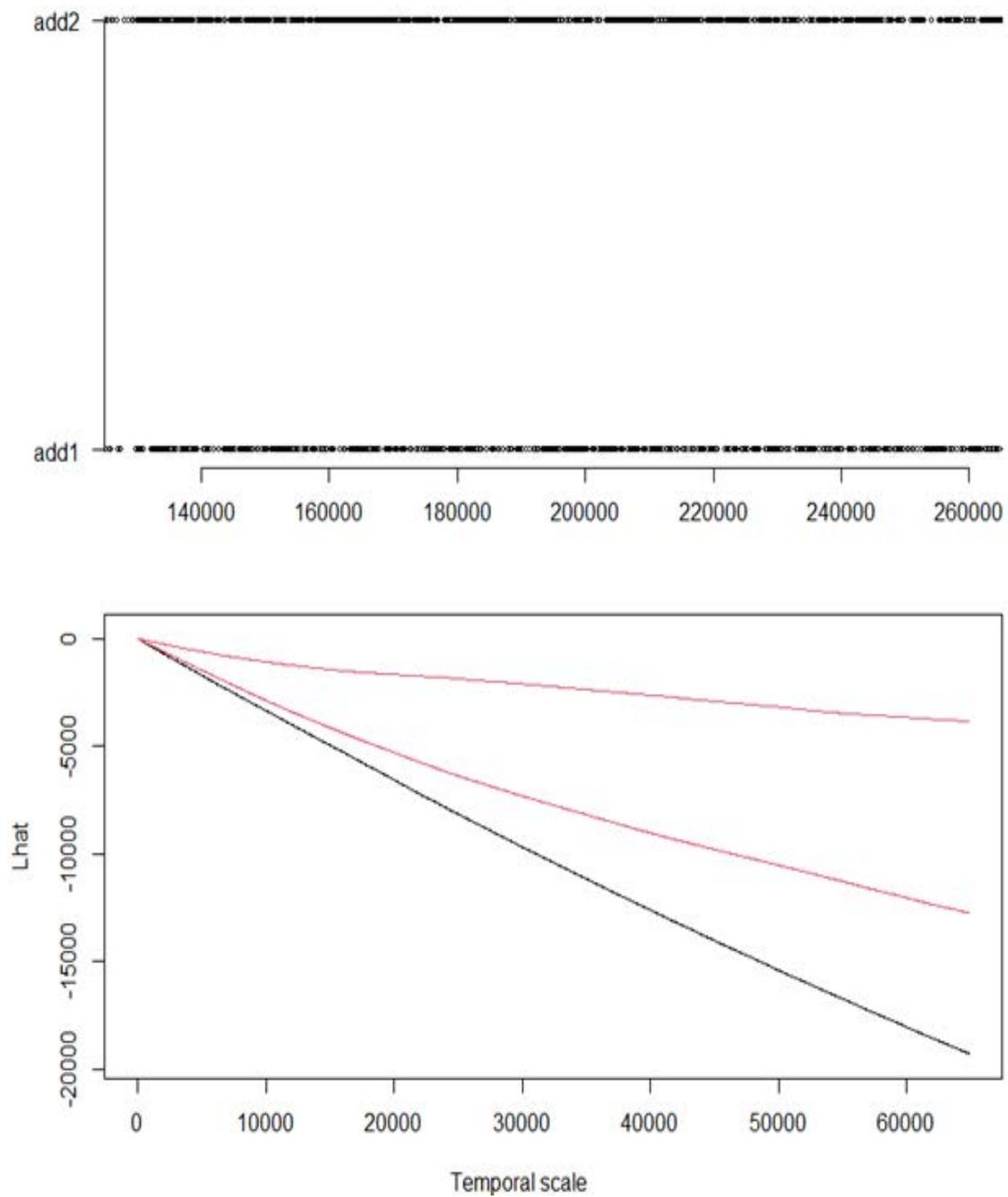
**Figure S12:** E. Sequence of addition types of events occurring within two consecutive days
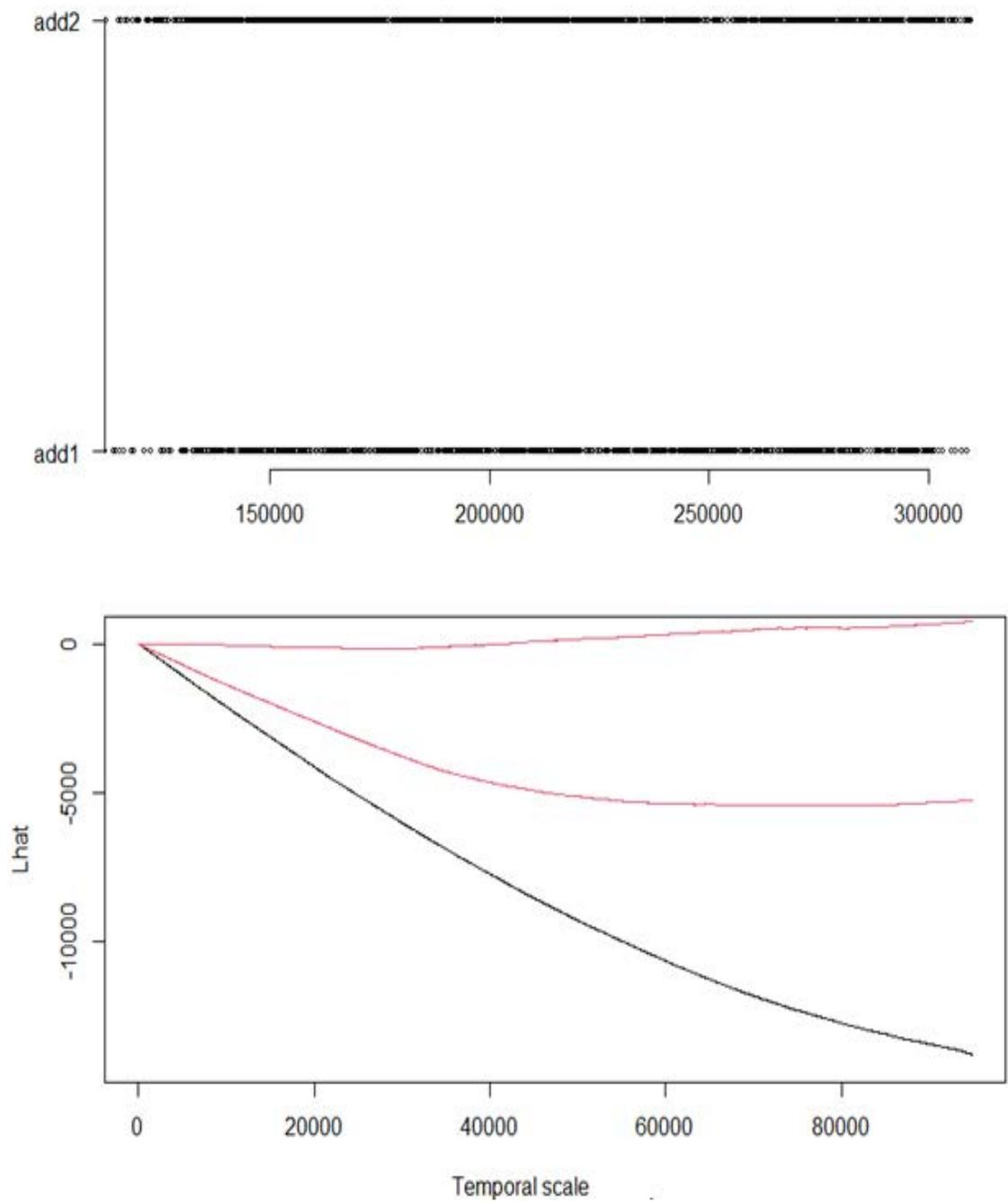
**Figure S13:** F. Sequences of deletion types of events occurring within two consecutive days
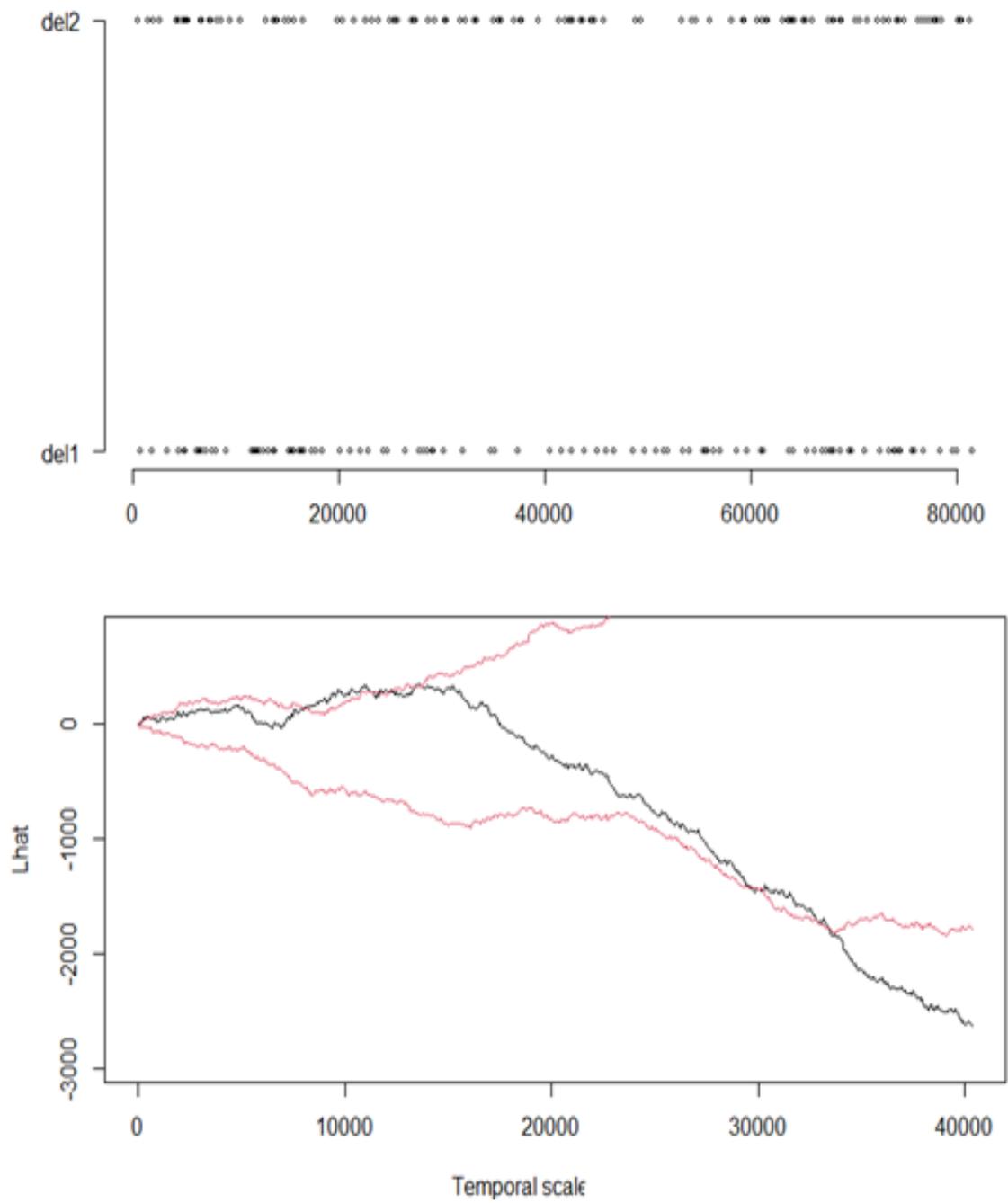
Figure S14: G. Sequences of deletion types of events occurring within a day