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A Unified Three-Dimensional Micromechanical Framework for Coupled Inelasticity and Damage Evolution in Diverse Composite Materials

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Abstract

This study introduces a comprehensive three-dimensional micromechanical framework to capture the nonlinear mechanical behavior of diverse composite materials, including coupled elastic degradation, inelastic strain evolution, and phenomenological failure in their constituents. The primary objective is to integrate a generalized elastic degradation–inelasticity (EDI) model into the parametric high-fidelity generalized method of cells (PHFGMC) micromechanical approach, enabling accurate prediction of nonlinear responses and failure mechanisms in multi-phase composites. To achieve this, a unified three-dimensional orthotropic EDI modeling formulation is developed and implemented in the PHFGMC. Grounded in continuum mechanics, the EDI employs scalar field variables to quantify material damage and defines an energy potential function. Thermodynamic forces are specified along three principal directions, decomposed into tensile and compressive components, with shear failure accounted for across the respective planes. Inelastic strain evolution is modeled using incremental anisotropic plasticity theory, coupling damage and inelasticity to maintain generality and flexibility for diverse phase behaviors. The proposed model offers a general, unified framework for modeling damage and inelasticity, which can be calibrated to operate in either coupled or decoupled modes. The PHFGMC micromechanics framework then derives the overall (macroscopic) nonlinear and damage responses of the multi-phase composite. A failure criterion can be applied for ultimate strength evaluation, and a crack-band type theory can be used for post-ultimate degradation. The method is applicable to different types of composites, including polymer matrix composites (PMCs) and ceramic matrix composites (CMCs). Applications demonstrate predictions of monotonic and cyclic loading responses for PMCs and CMCs, incorporating inelasticity and coupled damage mechanisms (such as crack closure and tension–compression asymmetry). The proposed framework is validated through comparisons with experimental and numerical results from the literature.

Keywords: 3D composites; micromechanics; PHFGMC; inelasticity; damage softening



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1. Introduction

Composite structures can be analyzed using macromechanical methods as well as multiscale micromechanical approaches. The macromechanical modeling approach treats the composite material as a homogenized anisotropic medium, where the properties of its individual constituents are averaged. The latter method is more suitable for linear elastic

analysis, assuming the absence of nonlinear effects and ultimate failure stress states in regions of stress concentration [1].

Alternatively, a micromechanical analysis approach preserves the composite's inherent architecture rather than homogenizing it, making it essential for accurately capturing nonlinear material behavior after damage initiation and inelastic strain evolution while resolving distinct damage characteristics. It effectively accounts for stress concentrations and enables analysis beyond the structural load-bearing capacity. To achieve this, micromechanical modeling provides a precise representation of the local mechanical interactions between fiber and matrix constituents, offering a more accurate alternative to homogenized nonlinear anisotropic models [2–5]. It is worth noting that while the computational cost of micromechanical analysis is higher than that of the homogenization approach, advancements in computing power continue to minimize this limitation.

The high-fidelity generalized method of cells (HFGMC) micromechanical theory can be applied for detailed nonlinear analysis of composites with multiple phases [6]. It serves as a feasible alternative to conventional numerical approaches, such as finite element, finite-volume, and finite difference methods. This is due to its direct specialization for multiphase composites, with a formulation rooted in micromechanical variables essential for determining the elastic and inelastic concentration tensors of the phases, along with detailed local field distributions. The HFGMC method has developed from earlier approaches, including the method of cells (MOC) and the generalized method of cells (GMC) [7–9], accordingly. The HFGMC incorporates a higher-order displacement expansion, while both the MOC and GMC are confined to a linear displacement expansion.

There have been several recent updates to the HFGMC formulation. Ref. [10] developed an iterative procedure to reduce residual errors and ensure that the governing equations of HFGMC are satisfied in their entirety. Ref. [11] developed a finite strain HFGMC formulation to analyze composites under large deformations, requiring conjugate stress and strain measures alongside the deformation gradient tensor. The HFGMC method has seen widespread application over the past two decades (cf. [12–17]). However, the HFGMC formulation relies on orthogonal arrays of cells to model the phase geometry. This constraint necessitates a significant number of cells to precisely capture intricate geometric details.

To overcome this HFGMC deficiency, ref. [2] proposed a linear and parametric geometric mapping (PHFGMC) that can be flexibly applied to general phase geometries, utilizing quadrilateral cell shapes that are converted into an auxiliary uniform square configuration. It is crucial to emphasize that in the latter approach (i.e., PHFGMC), although linear geometric mapping is used, a quadratic displacement expansion, akin to the original HFGMC, is still carried out, but within the auxiliary space coordinate system. The latter PHFGMC formulation was developed for doubly periodic composites, where it was later generalized by [3] for the triply periodic multiphase composites with a periodic microstructure, where general hexahedral subcells were introduced to represent triply-periodic composites. This triply periodic PHFGMC represents the most general case, with the previously discussed doubly periodic PHFGMC serving as a special case. Thus, in the present study, the nonlinear triply periodic PHFGMC formulation is employed to model inelastic deformation in composites and their damage.

A thorough understanding of composite damage processes and the development of accurate predictive models is crucial for enhancing their effectiveness and reliability. This is particularly challenging due to the complex damage mechanisms and interactions between damage and inelastic deformation. Therefore, a versatile model is needed to capture both elastic degradation and inelastic deformation. Ideally, the model should be free from predefined material assumptions and capable of modeling damage in tension,

compression, and shear in all three directions. Additionally, it should account for the coupling between damage mechanisms and allow for the evolution of inelastic strains across all damage modes, which can be tailored to specific failure cases. In the present study, a robust three-dimensional model is developed that is implemented in the general PHFGMC formulation. The damage is based on continuum damage mechanics (CDM) and inelastic strain evolution is based on the incremental plasticity theory.

Continuum damage mechanics (CDM), based on phenomenological approaches, often utilize the principles of irreversible thermodynamics, employing internal state variables to characterize the material's loss of load-bearing capacity, with material parameters calibrated through macroscopic experiments [18–21]. The primary difference between continuum damage mechanics (CDM) and fracture mechanics lies in how they handle displacement fields. CDM employs a standard continuum framework with smooth displacement fields, facilitating straightforward application. On the other hand, fracture mechanics deals with discontinuous displacement fields, necessitating specialized approaches such as remeshing or extended finite element methods to represent the discontinuities effectively [22].

Ref. [23] developed a model to characterize the damage behavior of an elementary ply in a fibrous-composite laminate, utilizing continuum damage mechanics (CDM) to represent the degradation of composite laminates. Their model also combines plasticity to describe permanent inelastic strain with isotropic hardening function. Although their model accounts for the distinction between tension and compression, damage is only considered in the transverse tension and in-plane shear directions. Ref. [24] proposed a model to characterize the mechanical behavior of plain-woven C/SiC composites, focusing on orthotropic damage modes. Through a series of experimental tests [25], they calibrated their model, which incorporates both material degradation and the evolution of plastic strain. This model is broader than that of [23] as it accounts for damage in the fiber direction. However, it is limited to in-plane loading scenarios and is more applicable to the specific material under consideration. Ref. [26] proposed a computational framework for modeling impact-induced damage in composite structures, incorporating intraply failure, plastic deformation, and interply delamination. Their approach builds on the work of Ladeveze and collaborators [23,27], which was implemented for in-plane failure within a commercial explicit finite element software. Interface degradation is handled using the method described by [28], where stresses are systematically reduced via an interface damage mechanism once a critical strain is surpassed. In [29] an elastoplastic-damage framework is developed to incorporate irreversible deformations arising from plastic behavior and material deterioration. The evolution of inelastic strains is described using plasticity theory, implemented through a return mapping algorithm. The model has been validated using two types of laminates: T300/1034-C carbon/epoxy composites and AS4/PEEK composites. Their results indicate that the proposed methodology effectively predicts the failure loads of carbon fiber-reinforced composite laminates. However, the model remains limited to a two-dimensional framework. Additionally, their study employs a one-parameter plastic potential formulation, specifically adapted for plane stress conditions, to characterize the irreversible strains, which are confined to transverse and/or shear loading. Ref. [30] introduced a framework for studying the behavior of fiber-reinforced ceramic matrix composites (CMCs) under multiaxial loading, with a focus on materials exhibiting orthotropic fibrous reinforcement. This model extends the formulation pioneered by [31] for concrete, where damage progression is linked to changes in the compliance tensor under applied loads. Experimental tests conducted on 2-D woven composites were utilized to calibrate the model, demonstrating its effectiveness in capturing the nonlinear response under various on-axis and off-axis uniaxial loading scenarios. One of the limitations of the model is its inability to incorporate hysteretic effects associated with frictional sliding, which may become significant under

certain non-proportional loading conditions [30]. Building on the work of [32,33] proposed a single-parameter flow rule for orthotropic plasticity to capture the nonlinear behavior of fiber-reinforced composites, under the assumption that plastic deformation along the longitudinal fiber direction is negligible. This model was tested and validated using experimental data from boron/aluminum and graphite/epoxy composites. Ref. [34] presented a damage model tailored for elastic–brittle fiber-reinforced composites for C/C–SiC laminates. The model employs five damage parameters to describe the reduction in elastic properties. Using Hashin’s criteria [35,36] within the effective stress space, the damage surface is defined. The progression of each damage parameter, corresponding to a specific failure mode, is influenced by the effective stress acting on the relevant failure plane. Ref. [37] proposed a damage model to capture the degradation of in-plane elastic properties within an idealized matrix phase, represented by a single scalar variable. The model does not account for residual deformation after unloading. It considers damage in fibers and the matrix, with its progression governed by the in-plane stress state of the matrix. This evolution is described using functions based on the square root of the Tsai–Wu criterion, applied in the effective stress framework. Ref. [38] developed a three-dimensional model to capture the mechanical behavior of composite laminates subjected to impact. Their damage framework, grounded in continuum damage mechanics (CDM), accounts for degradation in the two in-plane directions corresponding to normal stresses and in all shear directions, while assuming no damage occurs in the out-of-plane normal direction. Although the model shares certain characteristics with that of [23], it is less parameter-intensive. Furthermore, the damage variables reach their upper limit of unity when the material can no longer bear shear loads. Ref. [39] introduced a constitutive model designed specifically to capture the anisotropic behavior of 3D needled C/C–SiC composites. In this framework, the inelastic deformation of the material is described through the principles of plasticity theory. To represent the deterioration of the material’s stiffness, three independent scalar damage variables were defined. The model also incorporates a distinctive plastic potential function, which includes adjustable parameters to account for the directional dependence of plastic deformation. Furthermore, an exponential damage function was developed, grounded in the Weibull statistical distribution of material strength, to effectively describe the reduction in stiffness along each principal material direction. Although designed to investigate the behavior of 3D needled C/C–SiC composites, the model was developed within a two-dimensional framework, and its plastic potential function does not incorporate stress decomposition, thereby restricting its applicability to certain composite materials. Ref. [40] introduced a two-dimensional framework aimed at modeling the behavior of fiber-reinforced ceramics with porous matrices. This approach incorporates mechanisms to simulate damage under both tensile and shear stress conditions. To characterize the material’s plastic deformation, Hill’s criterion [41] was utilized as the plastic potential.

The present study introduces a robust three-dimensional generalized framework for characterizing the mechanical behavior of fibrous composites, implemented within the PHFGMC. The formulation incorporates Continuum Damage Mechanics (CDM) to model damage evolution in both the matrix and the fibers. Inelastic strain evolution is captured using the incremental plasticity theory, with a local Newton–Raphson iterative scheme and a return-mapping algorithm implemented to ensure robust numerical integration. To incorporate a strength criterion, the three-dimensional form of the Tsai–Wu failure criterion is introduced, enabling a comprehensive assessment of failure onset within the orthotropic fibrous composite under multiaxial loading conditions. It should be emphasized that the proposed micromechanical model does not impose any *a priori* assumptions regarding the presence or absence of damage in specific directions. In contrast to previous approaches, where damage is typically considered in a single direction within a two-

dimensional framework, the present investigation defines damage using nine variables. These variables correspond to all normal and shear directions, effectively capturing both tensile and compressive damage mechanisms. The evolution of these damage variables occurs under all stress states and is determined through the solution of the boundary-value problem, with their magnitudes resolving to either finite values or zero based on the computed response and material properties. On the other hand, the inelastic strain induces plastic deformation in all directions, incorporating adjustable parameters across all stress states to account for the directional dependence of inelastic strain evolution. Applications are presented for polymeric matrix composites (PMCs) and ceramic matrix composites (CMCs). The analysis of PMCs is validated against phase-field simulations conducted by [42], while CMC results are verified against NASA data by [43].

It should be mentioned that plasticity in composite materials arises from distinct mechanisms depending on the matrix type. In PMCs, plasticity stems from matrix deformation, interfacial effects, and progressive damage. Interfacial shear, fiber debonding, and crack bridging contribute to energy dissipation, while microcracking and void growth further enhance plasticity under high strain [44]. In CMCs, plasticity primarily results from toughening mechanisms that prevent brittle failure [45]. Weak fiber–matrix interfaces enable debonding and frictional sliding, while crack deflection, fiber bridging, and microcracking enhance energy dissipation. Additional mechanisms such as phase transformations and grain boundary sliding contribute to plastic-like behavior, particularly under high temperatures or dynamic loading [46,47]. These mechanisms collectively improve damage tolerance, making PMCs ideal for lightweight applications and CMCs suitable for extreme environments like aerospace and thermal protection systems [48].

This paper is organized as follows. In Section 2, the PHFGMC formulation is briefly presented. Section 3 elaborates on the mathematical framework of the developed coupled material degradation and plasticity model. Section 4 discusses the numerical methodology employed in this study, including its implementation. Benchmark simulations are detailed in Section 5, followed by conclusions in Section 6.

2. PHFGMC Micromechanical Model for 3D Multi-Phase Composites

The PHFGMC method is formulated here in its most general three-dimensional form with a nonlinear framework. It is important to note that only a brief overview of the formulation is provided, while further details can be found in [5]. The parametric HFGMC adopted in this study is highly suitable for capturing nonlinear and damage behaviors, as noted by [4]. The primary objective of the PHFGMC micromechanical approach is to estimate the effective properties and to model behavior of periodic composite materials. A schematic representation depicts a triply-periodic multiphase material system with a global coordinate framework is shown in Figure 1a, where a global Cartesian coordinate system (x_1, x_2, x_3) is employed to define the periodic composite and the three-dimensional repeating unit cell (RUC) is characterized using a local coordinate system (y_1, y_2, y_3) as in Figure 1b.

The displacement field is decomposed into two distinct terms: one represented by a linear polynomial in the global coordinate system, where the average strain acts as the coefficient, and another described by a quadratic polynomial in the local RUC coordinate system, incorporating micro-scale displacement variables as coefficients. Within the 3D PHFGMC framework, the RUC is discretized into a three-dimensional arrangement of hexahedral subcells with arbitrary shapes as shown in Figure 1b. Figure 1c displays an individual hexahedral subcell, defined within its physical coordinate system (y_1, y_2, y_3) .

This subcell is then mapped onto a standardized parametric coordinate system (r, s, t) (Figure 1d) as

$$y_i(r, s, t) = \sum_{k=1}^8 H_k(r, s, t) y_{ki}, \quad i = 1, 2, 3 \quad (1)$$

where $y_i(r, s, t)$ represents the coordinates of an arbitrary point within the subcell domain, while y_{ki} specifies the coordinates corresponding to the k -th vertex of the subcell. The shape functions $H_k(r, s, t)$ are defined as:

$$H_k(r, s, t) = \frac{1}{8}(1 + r_k r)(1 + s_k s)(1 + t_k t) \quad (2)$$

where $r_k, s_k, t_k \in \{-1, 1\}$ are the natural coordinates of the k -th node in the reference element.

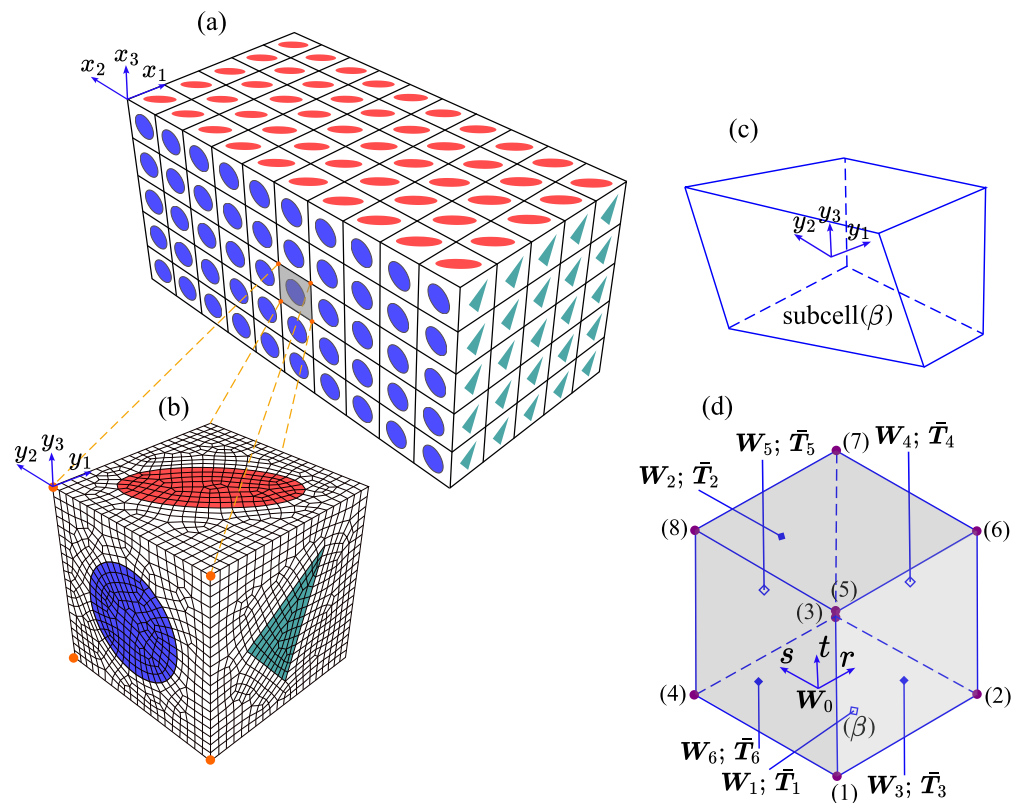


Figure 1. A schematic depiction of a triply periodic structure within the global (x_1, x_2, x_3) coordinate system of a multiphase composite (a), and the repeating unit cell (RUC) comprises hexahedral subcells of arbitrary shapes, defined in relation to the local (y_1, y_2, y_3) coordinate system (b); a hexahedral subcell (β) depicted in both its physical (c) and natural (d) coordinate systems, with the face numbering indicated. The average traction and displacement vectors corresponding to each face are also shown.

It is important to highlight that, although the parametric transformation is linear, the displacement field is inherently quadratic. The displacement expansion within each subcell is expressed in its full quadratic form as:

$$\begin{aligned} u = & u^0 + W_0 + \frac{1}{2}(W_4 - W_6)r + \frac{1}{2}(W_5 - W_3)s + \frac{1}{2}(W_2 - W_1)t \\ & + \frac{1}{4}(W_4 + W_6 - 2W_0)(3r^2 + rs + rt - 1) + \frac{1}{4}(W_3 + W_5 - 2W_0)(3s^2 + rs + st - 1) \\ & + \frac{1}{4}(W_1 + W_2 - 2W_0)(3t^2 + rt + st - 1), \end{aligned} \quad (3)$$

where the externally applied macroscopic displacement field is given by $\mathbf{u}^0 = \boldsymbol{\epsilon}^0 \cdot \mathbf{x}$. The face-average displacement vectors at the six subcell faces are denoted by $\mathbf{W}_i$ for $i = 1, \dots, 6$, while $\mathbf{W}_0$ represents an internal displacement variable.

The subcell (β)-averaged virtual work balance is given by

$$\delta \mathbf{U}_0^{(\beta),T} \mathbf{f}_U^{(\beta)} + \delta \mathbf{W}^{(\beta),T} \mathbf{f}_W^{(\beta)} = \int_V \delta \boldsymbol{\epsilon}^{(\beta),T} \boldsymbol{\sigma}^{(\beta)} dV = \int_V \left(\delta \mathbf{U}_0^{(\beta),T} \mathbf{I}_B^{(\beta),T} + \delta \mathbf{W}^{(\beta),T} \mathbf{B}_W^{(\beta),T} \right) \boldsymbol{\sigma}^{(\beta)} dV, \quad (4)$$

where $\{\delta \mathbf{U}_0^{(\beta)}, \delta \mathbf{W}^{(\beta)}\}$ denote the global displacement gradients and the locally averaged micro-displacement gradient vectors, respectively. The internal resisting vectors $\{\mathbf{f}_U^{(\beta)}, \mathbf{f}_W^{(\beta)}\}$ characterize the subcell's contribution to the overall stress averaged over the RUC and its local stress contribution, respectively. The stress tensor $\boldsymbol{\sigma}^{(\beta)}$ is defined as

$$\boldsymbol{\sigma}^{(\beta)} = \mathbf{C}^\beta : \boldsymbol{\epsilon}^{(\beta)}, \quad (5)$$

based on the generalized Hooke's law, the mechanical behavior of the subcell is characterized by $\mathbf{C}^\beta$, which defines its material stiffness. The terms $\{\mathbf{I}_B^{(\beta)}, \mathbf{B}_W^{(\beta)}\}$ are formulated as a direct global strain-gradient matrix and a matrix that correlates the spatial strain components with the micro-displacement variables, respectively. For a detailed formulation of these matrices, the interested reader is referred to [5]. Collectively, the total dimensions of $\{\mathbf{U}_0^{(\beta)}, \mathbf{W}^{(\beta)}\}$ encompass all displacement variables for subcell β .

The two components of the generalized internal force vector $\{\mathbf{f}_U, \mathbf{f}_W\}$ are conjugate to the global displacement gradients and to local variations, respectively, defined as

$$\mathbf{f}_U = \int_V \mathbf{I}_B^T \mathbf{C} (\mathbf{I}_B \mathbf{U}_0 + \mathbf{B}_W \mathbf{W}) dV, \quad \mathbf{f}_W = \int_V \mathbf{B}_W^T \mathbf{C} (\mathbf{I}_B \mathbf{U}_0 + \mathbf{B}_W \mathbf{W}) dV, \quad (6)$$

where the findings are expressed without the β notation, as the resulting formulations are valid for both individual RUC subcells and the complete assembly.

The stiffness matrix is determined by differentiating the generalized internal force vectors with respect to the subcell variables $\mathbf{W}$ and $\mathbf{U}_0$ as

$$\mathbf{K} = \begin{bmatrix} \int_V \mathbf{B}_W^T \mathbf{C} \mathbf{B}_W dV & \int_V \mathbf{B}_W^T \mathbf{C} \mathbf{I}_B dV \\ \int_V \mathbf{I}_B^T \mathbf{C} \mathbf{B}_W dV & \int_V \mathbf{I}_B^T \mathbf{C} \mathbf{I}_B dV \end{bmatrix}. \quad (7)$$

The stiffness matrix in the PHFGMC maintains a symmetric structure. Within its formulation, traction continuity between adjacent subcells is approximated through a weak formulation approach. Displacement compatibility is achieved by averaging and linking the micro-variables corresponding to shared interfaces in the governing equations. As a result, periodic boundary conditions for the RUC are enforced by matching the displacements on opposing faces. The equilibrium residual of the complete RUC system is formulated in a generalized way, incorporating interface continuity and periodicity conditions on the displacement micro-variables as follows

$$\mathbf{R} \equiv \mathbf{P} - \mathbf{f} = \left\{ \begin{matrix} \mathbf{P}_W \\ \mathbf{P}_U \end{matrix} \right\} - \left\{ \begin{matrix} \int_V \mathbf{B}_W^T \boldsymbol{\sigma} \\ \int_V \mathbf{I}_B^T \boldsymbol{\sigma} \end{matrix} \right\} = \{0\}, \quad (8)$$

where $\mathbf{P}$ and $\mathbf{f}$ are external and internal load vectors, respectively. Typically, in an RUC configuration, $\mathbf{P}_W$ is not explicitly defined, as the outer surfaces are generally governed by periodic boundary conditions applied to the corresponding conjugate micro-variables, $\mathbf{W}$. However, an exception occurs when internal surfaces are free and subjected to pressure-type loading. The generalized force $\mathbf{P}_U$ that is applied corresponds to the average stress

within the RUC with a unit volume. At a given time t , the system of equations governing the incremental response can be expressed as follows

$$\begin{bmatrix} \int_V \mathbf{B}_W^T \mathbf{C} \mathbf{B}_W dV & \int_V \mathbf{B}_W^T \mathbf{C} \mathbf{I}_B dV \\ \int_V \mathbf{I}_B^T \mathbf{C} \mathbf{B}_W dV & \int_V \mathbf{I}_B^T \mathbf{C} \mathbf{I}_B dV \end{bmatrix}^{(t)} \begin{Bmatrix} \Delta \mathbf{W} \\ \Delta \mathbf{U}_0 \end{Bmatrix}^{(t+\Delta t)} = \{\mathbf{R}\}^{(t+\Delta t)}, \quad (9)$$

where t denotes the time at the beginning of the increment, and Δt represents the time increment.

3. Coupled Elastic Degradation Damage—Plasticity Model

We employ a comprehensive three-dimensional framework that accounts for both damage progression and inelastic strain evolution. Initially, we outline some key characteristics, as identified through observations from the literature. The developed model aims to simulate these behaviors, tailored to each material, based on its specific parameters:

The material demonstrates notable non-linear behavior in its stress-strain response, attributed to the initiation and evolution of damage and inelastic deformation [49]

Under cyclic loading, in the absence of damage progression and with only permanent deformation occurring, the stress diminishes along a trajectory that remains parallel to the elastic slope [50]

Under compressive loading following tension, the closure of cracks leads to a partial recovery of stiffness [51]

Under cyclic loading, when only damage is present without the evolution of inelastic strain, the material's stress-strain response crosses the origin [52]

The width of the damaged zone is a material property that correlates with the amount of energy dissipated during the fracture process [53]

The sharpness of transition on the stress-strain curve following complete damage governs the extent of the regularization zone [54]

The crack driving force is non-decreasing, in accordance with the second law of thermodynamics [55]

Building on these observed mechanical responses, the material model will be constructed using continuum damage mechanics and the incremental plasticity.

3.1. Damage Evolution

The strain energy per unit volume of the damaged material, denoted as $\rho\psi_e$, is expressed as a function of the nominal stress σ and the associated damage variables, as follows

$$\rho\psi_e = \frac{1}{2} \left[\frac{\langle \sigma_1 \rangle^2}{E_1^0(1-d_{1t})} + \frac{\langle -\sigma_1 \rangle^2}{E_1^0(1-d_{1c})} + \frac{\langle \sigma_2 \rangle^2}{E_2^0(1-d_{2t})} + \frac{\langle -\sigma_2 \rangle^2}{E_2^0(1-d_{2c})} + \frac{\langle \sigma_3 \rangle^2}{E_3^0(1-d_{3t})} + \frac{\langle -\sigma_3 \rangle^2}{E_3^0(1-d_{3c})} + \frac{\sigma_4^2}{G_{12}^0(1-d_{12})} - \frac{2\nu_{12}^0}{E_1^0} \sigma_1 \sigma_2 + \frac{\sigma_5^2}{G_{13}^0(1-d_{13})} - \frac{2\nu_{13}^0}{E_1^0} \sigma_1 \sigma_3 + \frac{\sigma_6^2}{G_{23}^0(1-d_{23})} - \frac{2\nu_{23}^0}{E_2^0} \sigma_2 \sigma_3 \right], \quad (10)$$

where ρ is the mass density, ψ_e is damaged material strain energy per unit mass, $\langle \cdot \rangle$ are the Macaulay brackets defined as $\langle x \rangle = (x + |x|)/2$. This formulation decomposes the energy contributions from the positive and negative components of normal stress, facilitating a more precise characterization of damage mechanisms under tensile and compressive loading conditions. The shear stress component is intentionally left unsplit to preserve the consistency and integrity of the thermodynamic formulation.

The model utilizes nine damage variables to represent the progressive degradation of material properties under tensile, compressive, and shear loading. Specifically, d_{1t} and d_{1c} characterize damage in the fiber direction under tension and compression, respectively,

while d_{2t} and d_{2c} describe damage in the first transverse direction. Similarly, d_{3t} and d_{3c} describe damage in the second transverse direction. Additionally, $\{d_{12}, d_{13}, d_{23}\}$ are introduced to capture shear-related damage. These damage variables are mathematically defined as

$$\begin{aligned} d_{1t} &= 1 - \frac{E_1}{E_1^0}, & d_{1c} &= 1 - \frac{E_1}{E_1^0}, & d_{2t} &= 1 - \frac{E_2}{E_2^0}, & d_{2c} &= 1 - \frac{E_2}{E_2^0}, & d_{3t} &= 1 - \frac{E_3}{E_3^0}, & d_{3c} &= 1 - \frac{E_3}{E_3^0}, \\ d_{12} &= 1 - \frac{G_{12}}{G_{12}^0}, & d_{13} &= 1 - \frac{G_{13}}{G_{13}^0}, & d_{23} &= 1 - \frac{G_{23}}{G_{23}^0}, \end{aligned} \quad (11)$$

here, E_1, E_2, E_3 and G_{ij} represent the degraded elastic moduli in the fiber, first transverse, second transverse and shear directions, respectively, while E_1^0, E_2^0, E_3^0 and G_{ij}^0 denote the corresponding initial, undamaged elastic properties.

The non-negative thermodynamic forces Y_i , which drive the evolution of the damage variables, are derived from the damaged material strain energy as follows

$$Y_i = \rho \frac{\partial \psi_e}{\partial d_i}, \quad (i = 1t, 1c, 2t, 2c, 3t, 3c, 12, 13, 23) \quad (12)$$

Thermodynamic forces function similarly to energy-release rates, controlling the evolution of damage in the same manner that energy-release rates control crack growth.

Substitution of (10) in (12) yields the conjugate quantities $\{Y_{1t}, Y_{1c}, Y_{2t}, Y_{2c}, Y_{3t}, Y_{3c}, Y_{12}, Y_{13}, Y_{23}\}$, which are associated with damage variables $\{d_{1t}, d_{1c}, d_{2t}, d_{2c}, d_{3t}, d_{3c}, d_{12}, d_{13}, d_{23}\}$, respectively

$$\begin{aligned} Y_{1t} &= \frac{\langle \sigma_1 \rangle^2}{2E_1^0(1-d_{1t})^2}, & Y_{1c} &= \frac{\langle -\sigma_1 \rangle^2}{2E_1^0(1-d_{1c})^2}, & Y_{2t} &= \frac{\langle \sigma_2 \rangle^2}{2E_2^0(1-d_{2t})^2}, & Y_{2c} &= \frac{\langle -\sigma_2 \rangle^2}{2E_2^0(1-d_{2c})^2}, \\ Y_{3t} &= \frac{\langle \sigma_3 \rangle^2}{2E_3^0(1-d_{3t})^2}, & Y_{3c} &= \frac{\langle -\sigma_3 \rangle^2}{2E_3^0(1-d_{3c})^2}, & Y_{12} &= \frac{\sigma_4^2}{2G_{12}^0(1-d_{12})^2}, & Y_{13} &= \frac{\sigma_5^2}{2G_{13}^0(1-d_{13})^2}, \\ Y_{23} &= \frac{\sigma_6^2}{2G_{23}^0(1-d_{23})^2}. \end{aligned} \quad (13)$$

To account for the interaction between different damage mechanisms [24], coupled damage force norms are introduced:

$$\begin{aligned} Y_{1t}^* &= Y_{1t} + \Lambda_{12}(Y_{2t})^{\lambda_{12}} + \Lambda_{13}(Y_{3t})^{\lambda_{13}} + \gamma_{1,12}Y_{12}H(\tilde{\sigma}_1 + \tilde{\sigma}_2) + \gamma_{1,13}Y_{13}H(\tilde{\sigma}_1 + \tilde{\sigma}_3) \\ Y_{1c}^* &= Y_{1c} \\ Y_{2t}^* &= Y_{2t} + \Lambda_{21}(Y_{1t})^{\lambda_{21}} + \Lambda_{23}(Y_{3t})^{\lambda_{23}} + \gamma_{2,12}Y_{12}H(\tilde{\sigma}_1 + \tilde{\sigma}_2) + \gamma_{2,23}Y_{23}H(\tilde{\sigma}_2 + \tilde{\sigma}_3) \\ Y_{2c}^* &= Y_{2c} \\ Y_{3t}^* &= Y_{3t} + \Lambda_{31}(Y_{1t})^{\lambda_{31}} + \Lambda_{32}(Y_{2t})^{\lambda_{32}} + \gamma_{3,13}Y_{13}H(\tilde{\sigma}_1 + \tilde{\sigma}_3) + \gamma_{3,23}Y_{23}H(\tilde{\sigma}_2 + \tilde{\sigma}_3) \\ Y_{3c}^* &= Y_{3c} \\ Y_{12}^* &= Y_{12} + \gamma_{12,1}Y_{1t} + \gamma_{12,2}Y_{2t} - g_c \left(\langle Z_{12} - Z_{12}^0 \rangle \right) \langle Z_{12} - Z_{12}^0 \rangle \\ Y_{13}^* &= Y_{13} + \gamma_{13,1}Y_{1t} + \gamma_{13,3}Y_{3t} - g_c \left(\langle Z_{13} - Z_{13}^0 \rangle \right) \langle Z_{13} - Z_{13}^0 \rangle \\ Y_{23}^* &= Y_{23} + \gamma_{23,2}Y_{2t} + \gamma_{23,3}Y_{3t} - g_c \left(\langle Z_{23} - Z_{23}^0 \rangle \right) \langle Z_{23} - Z_{23}^0 \rangle \end{aligned} \quad (14)$$

where

$$\begin{aligned} Z_{12} &= \frac{\langle -(\sigma_1 + \sigma_2) \rangle^2}{E_1^0 + E_2^0}, & g_c(\langle Z_{12} - Z_{12}^0 \rangle) &= a_{c12} \left(1 - e^{-\langle Z_{12} - Z_{12}^0 \rangle / b_{c12}} \right) \\ Z_{13} &= \frac{\langle -(\sigma_1 + \sigma_3) \rangle^2}{E_1^0 + E_3^0}, & g_c(\langle Z_{13} - Z_{13}^0 \rangle) &= a_{c13} \left(1 - e^{-\langle Z_{13} - Z_{13}^0 \rangle / b_{c13}} \right) \\ Z_{23} &= \frac{\langle -(\sigma_2 + \sigma_3) \rangle^2}{E_2^0 + E_3^0}, & g_c(\langle Z_{23} - Z_{23}^0 \rangle) &= a_{c23} \left(1 - e^{-\langle Z_{23} - Z_{23}^0 \rangle / b_{c23}} \right) \end{aligned} \quad (15)$$

Here, Λ_{ij} , λ_{ij} denote the interaction effects of primary damage mechanisms along direction j influencing direction i , whereas $\gamma_{i,jk}$ and $\gamma_{ij,k}$ characterize the coupling effects of shear and primary damages, with shear along direction jk affecting direction i , and tensile effect along k influencing direction ij , respectively. The Heaviside function H is incorporated to suppress any tensile damage induced by shear stresses under the combined influence of compression and shear loads. Additionally, the parameter Z is introduced to account for the inhibitory influence of compressive stress on the progression of shear damage. In this study, the coupling effects are omitted to simplify the analysis, with $\{Y_{1t}^* = Y_{1t}, Y_{2t}^* = Y_{2t}, Y_{3t}^* = Y_{3t}, Y_{12}^* = Y_{12}, Y_{13}^* = Y_{13}, Y_{23}^* = Y_{23}\}$.

The evolution of damage is governed by logistic functions [24] expressed in terms of the norms of the damage driving forces

$$d_i = f_i(\langle Y_{i,max}^* - Y_i^0 \rangle) = a_i - \frac{a_i}{1 + (\langle Y_{i,max}^* - Y_i^0 \rangle / b_i)^{c_i}}, \quad (i = 1t, 1c, 2t, 2c, 3t, 3c, 12, 13, 23), \quad (16)$$

where the subscript “max” represents the maximum value of the variable recorded in the loading history to ensure compliance with the positive dissipation condition $\dot{d}_i \geq 0$; Y_i^0 are the damage initiation thresholds, and a_i , b_i , c_i are material parameters controlling the shape of the damage evolution curves. The parameter a can have values from 0 to 1 and defines the upper limit of the damage variable. For instance, if calibrated to 0.8, the material experiences a maximum degradation of 80%. In contrast, a value of 1 allows for complete softening. The parameter b is adjusted based on experimental data to match the material’s maximum strength, where increasing its value raises the material’s strength. The parameter c controls the sharpness of the transition to material failure on the stress-strain curve, defining how abruptly the material loses strength. It is worth noting that the logistic functions facilitate a continuous and smooth progression of damage, eliminating the requirement for discrete damage activation criteria typically employed in earlier models. The logistic function was selected for the damage evolution law due to its sigmoid shape, which ensures a smooth, bounded progression from 0 to 1, effectively capturing the gradual accumulation and saturation of damage in composites, unlike linear functions that oversimplify nonlinearity or exponential functions that may lead to abrupt or unbounded changes.

The partial derivatives of $\rho\psi_e$ with respect to the stress tensor σ yield the elastic strain tensor ε^e

$$\varepsilon^e = \rho \frac{\partial \psi_e}{\partial \sigma} = \varepsilon - \varepsilon^p, \quad (17)$$

where ε is the total strain tensor and ε^p is the inelastic strain tensor. Following the derivation, we can write

$$\varepsilon^e = C_d : \sigma, \quad (18)$$

where $(\cdot)$ symbol designates an inner product, C_d is the damaged compliance tensor, where its matrix representation is given by

$$C_d = \begin{bmatrix} \frac{1}{E_1^0(1-d_{1(t,c)})} & -\frac{\nu_{12}^0}{E_1^0} & -\frac{\nu_{13}^0}{E_1^0} & 0 & 0 & 0 \\ -\frac{\nu_{12}^0}{E_1^0} & \frac{1}{E_2^0(1-d_{2(t,c)})} & -\frac{\nu_{23}^0}{E_2^0} & 0 & 0 & 0 \\ -\frac{\nu_{13}^0}{E_1^0} & -\frac{\nu_{23}^0}{E_2^0} & \frac{1}{E_3^0(1-d_{3(t,c)})} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}^0(1-d_{12})} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}^0(1-d_{13})} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{23}^0(1-d_{23})} \end{bmatrix}, \quad (19)$$

in which $\{\nu_{12}^0, \nu_{13}^0, \nu_{23}^0\}$ are Poisson's ratios of the undamaged material, and the effective damage variables $d_{1(t,c)}$, $d_{2(t,c)}$, and $d_{3(t,c)}$ are selected based on the sign of the corresponding normal stress

$$d_{i(t,c)} = \begin{cases} d_{it} & \text{if } \sigma_i \geq 0 \\ d_{ic} & \text{if } \sigma_i < 0 \end{cases}, \quad i = 1, 2, 3 \quad (20)$$

Equation (18) can be written as

$$\sigma = S_d : \varepsilon^e, \quad (21)$$

with $S_d = C_d^{-1}$ is the damaged stiffness tensor.

The effective stress tensor is fundamental to the formulation of the plasticity model, as detailed in the following sections. The effective stress tensor $\tilde{\sigma}$, which characterizes the stress transmitted through the undamaged portion of the material, is introduced to formalize the strain equivalence principle, which states that *the stress–strain behavior of a damaged material under nominal stress is considered identical to that of an undamaged material under the corresponding effective stress* [56]

$$\tilde{\sigma} = S_0 : \varepsilon^e, \quad (22)$$

where S_0 is the undamaged stiffness tensor. The effective stress is related to the nominal stress through the damage effect tensor M

$$\tilde{\sigma} = M : \sigma, \quad (23)$$

with M reads

$$M = S_0 : C_d, \quad (24)$$

following [24,29,34,57], M can be approximated as

$$M = \begin{bmatrix} \frac{1}{1-d_{1(t,c)}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{1-d_{2(t,c)}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{1-d_{3(t,c)}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{1-d_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{1-d_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{1-d_{23}} \end{bmatrix}. \quad (25)$$

3.2. Inelastic Strain Growth

Capturing the nonlinear or inelastic behavior in the developed model is crucial. This inelastic strain, while governed by mechanisms distinct from the plastic strain in ductile metals, can be effectively characterized using the macroscopic framework of plasticity [33,58]. Thus, the inelastic behavior of the composite is modeled within the framework of incremental plasticity. A yield function f is defined following the work of [23] that includes contributions from the effective stress components as (The yield function introduced by [23] is a two-dimensional function incorporating contributions from shear stress and normal stress in the 2-direction. In contrast, the function proposed in this work is a generalized three-dimensional anisotropic model that accounts for contributions from all stress sources.)

$$f(\bar{\sigma}, \bar{\varepsilon}^p) = \sqrt{H(\bar{\sigma}_1)m_{1t}^2\bar{\sigma}_1^2 + H(-\bar{\sigma}_1)m_{1c}^2\bar{\sigma}_1^2 + H(\bar{\sigma}_2)m_{2t}^2\bar{\sigma}_2^2 + H(-\bar{\sigma}_2)m_{2c}^2\bar{\sigma}_2^2 + H(\bar{\sigma}_3)m_{3t}^2\bar{\sigma}_3^2 + H(-\bar{\sigma}_3)m_{3c}^2\bar{\sigma}_3^2 + m_{12}^2\bar{\sigma}_4^2 + m_{13}^2\bar{\sigma}_5^2 + m_{23}^2\bar{\sigma}_6^2} - \kappa(\bar{\varepsilon}^p) = 0, \quad (26)$$

where m_{1t} , m_{1c} , m_{2t} , m_{2c} , m_{3t} , m_{3c} , m_{12} , m_{13} , and m_{23} are weighting coefficients, and $\kappa(\bar{\varepsilon}^p)$ is an isotropic hardening function expressed in terms of the equivalent inelastic strain $\bar{\varepsilon}^p$

$$\kappa(\bar{\varepsilon}^p) = \bar{\sigma}(\bar{\varepsilon}^p) = R_0 + \beta(\bar{\varepsilon}^p)^\gamma, \quad (27)$$

where R_0 is the initial threshold stress of inelastic strain; β , and γ are material constants that specify the shape of the hardening function; $\bar{\sigma}$ is the equivalent stress associated with equivalent inelastic strain $\bar{\varepsilon}^p$.

We define the rate of the inelastic strain via the associated flow rule as follows

$$\dot{\varepsilon}^p = \dot{\lambda}^p \frac{\partial f}{\partial \bar{\sigma}}, \quad (28)$$

where $\dot{\lambda}^p$ is the plastic multiplier, determined from the consistency condition $\dot{f} = 0$, satisfying the Kuhn–Tucker conditions

$$\dot{\lambda}^p \geq 0, \quad f \leq 0, \quad \dot{\lambda}^p f = 0. \quad (29)$$

The integration of an expanded yield function facilitates the accurate representation of plasticity under multiaxial stress conditions, while enabling its interaction with damage mechanisms. The introduction of weighting coefficients m_{1t} , m_{1c} , m_{2t} , m_{2c} , m_{3t} , m_{3c} , m_{12} , m_{13} , and m_{23} allows precise calibration of the model against experimental data, thereby capturing the anisotropic yield behavior under tensile, compressive, and shear loading scenarios.

Differentiating (26) wrt the effective stress and substitute it in (28), we get

$$\begin{bmatrix} \dot{\varepsilon}_1^p \\ \dot{\varepsilon}_2^p \\ \dot{\varepsilon}_3^p \\ \dot{\varepsilon}_4^p \\ \dot{\varepsilon}_5^p \\ \dot{\varepsilon}_6^p \end{bmatrix} = \dot{\lambda}^p \begin{bmatrix} (H(\bar{\sigma}_1)m_{1t}^2\bar{\sigma}_1 + H(-\bar{\sigma}_1)m_{1c}^2\bar{\sigma}_1) / \bar{\sigma} \\ (H(\bar{\sigma}_2)m_{2t}^2\bar{\sigma}_2 + H(-\bar{\sigma}_2)m_{2c}^2\bar{\sigma}_2) / \bar{\sigma} \\ (H(\bar{\sigma}_3)m_{3t}^2\bar{\sigma}_3 + H(-\bar{\sigma}_3)m_{3c}^2\bar{\sigma}_3) / \bar{\sigma} \\ m_{12}^2\bar{\sigma}_4 / \bar{\sigma} \\ m_{13}^2\bar{\sigma}_5 / \bar{\sigma} \\ m_{23}^2\bar{\sigma}_6 / \bar{\sigma} \end{bmatrix}, \quad (30)$$

$$\text{where } \bar{\sigma} = \sqrt{H(\bar{\sigma}_1)m_{1t}^2\bar{\sigma}_1^2 + H(-\bar{\sigma}_1)m_{1c}^2\bar{\sigma}_1^2 + H(\bar{\sigma}_2)m_{2t}^2\bar{\sigma}_2^2 + H(-\bar{\sigma}_2)m_{2c}^2\bar{\sigma}_2^2 + H(\bar{\sigma}_3)m_{3t}^2\bar{\sigma}_3^2 + H(-\bar{\sigma}_3)m_{3c}^2\bar{\sigma}_3^2 + m_{12}^2\bar{\sigma}_4^2 + m_{13}^2\bar{\sigma}_5^2 + m_{23}^2\bar{\sigma}_6^2}.$$

Using rate of plastic work $\{\dot{W}^P = \bar{\sigma} : \dot{\epsilon}^p = \bar{\sigma} \dot{\epsilon}^p\}$ with (30), we find

$$\dot{\lambda}^p = \dot{\epsilon}^p. \quad (31)$$

3.3. Tsai–Wu Failure Criterion for 3D Stress State

In many situations, composite structures experience intricate multiaxial loading scenarios, where each stress component influences the ultimate strength, and their interactions play a significant role. Consequently, it is essential to establish an appropriate failure criterion for accurately predicting the structural strength. The Tsai–Wu failure criterion [59] is a simple and widely used failure theory for anisotropic materials, which will be adopted in the current work. For a general three-dimensional stress state, the failure criterion is expressed as:

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j \leq 1, \quad (32)$$

where $i, j = 1, 2, 3, 4, 5, 6$, corresponding to the normal $(\sigma_1, \sigma_2, \sigma_3)$ and shear $(\sigma_4, \sigma_5, \sigma_6)$ stress components in Voigt notation. A material point is considered to have failed when the flag variable in (32) is equal to or greater than 1. The expanded form of the criterion is:

$$F_1 \sigma_1 + F_2 \sigma_2 + F_3 \sigma_3 + F_{11} \sigma_1^2 + F_{22} \sigma_2^2 + F_{33} \sigma_3^2 + 2F_{12} \sigma_1 \sigma_2 + 2F_{13} \sigma_1 \sigma_3 + 2F_{23} \sigma_2 \sigma_3 + F_{44} \sigma_4^2 + F_{55} \sigma_5^2 + F_{66} \sigma_6^2 \leq 1, \quad (33)$$

the coefficients in the criterion are derived from the material's strength properties with linear terms defined as

$$F_1 = \frac{1}{X_t} - \frac{1}{X_c}, \quad F_2 = \frac{1}{Y_t} - \frac{1}{Y_c}, \quad F_3 = \frac{1}{Z_t} - \frac{1}{Z_c}, \quad (34)$$

where $X_t, X_c, Y_t, Y_c, Z_t, Z_c$ are the tensile and compressive strengths in the 1, 2, and 3 directions, respectively. Quadratic terms read

$$F_{11} = \frac{1}{X_t X_c}, \quad F_{22} = \frac{1}{Y_t Y_c}, \quad F_{33} = \frac{1}{Z_t Z_c}, \quad (35)$$

interaction terms are

$$F_{12}, \quad F_{13}, \quad F_{23}, \quad (36)$$

these terms are determined empirically and represent the interaction between normal stresses in different directions. Their accurate determination poses significant challenges. While biaxial tension tests are commonly recommended for their evaluation [59], they require specialized specimens and sophisticated testing apparatus. Alternatively, off-axis tension tests, as described in [25], generate combined stress states. However, their applicability is limited, as non-uniform stress distributions arise near the specimen ends due to constraint effects. In light of these challenges, we follow [24] and consider various plausible values for F_{12} , F_{13} , and F_{23} under generalized tension and compression stress states

$$F_{12} = \begin{cases} F_{12}^t & (\sigma_1 + \sigma_2) \geq 0 \\ F_{12}^c & (\sigma_1 + \sigma_2) < 0 \end{cases}, \quad F_{13} = \begin{cases} F_{13}^t & (\sigma_1 + \sigma_3) \geq 0 \\ F_{13}^c & (\sigma_1 + \sigma_3) < 0 \end{cases}, \quad F_{23} = \begin{cases} F_{23}^t & (\sigma_2 + \sigma_3) \geq 0 \\ F_{23}^c & (\sigma_2 + \sigma_3) < 0 \end{cases}. \quad (37)$$

The quadratic terms (shear stresses) defined as

$$F_{44} = \frac{1}{S_{12}^2}, \quad F_{55} = \frac{1}{S_{13}^2}, \quad F_{66} = \frac{1}{S_{23}^2}, \quad (38)$$

where S_{12}, S_{13}, S_{23} are the shear strengths in the 1–2, 1–3, and 2–3 planes, respectively.

It is worth mentioning that although the PHFGMC-EDI framework is general in its formulation and applicability to a wide range of composite materials exhibiting coupled damage and inelasticity, the specific constitutive parameters for any new material must first be determined through targeted experimental characterization.

4. Numerical Implementation

4.1. Integration Algorithm

The backward Euler implicit integration method is employed, with the integration algorithm tailored to the material model. The time step is divided into several increments, each spanning the interval $[t_n, t_{n+1}]$. During each increment, ABAQUS performs Newton iterations until convergence is achieved, defined by a residual falling below a specified tolerance threshold. Integration algorithm is defined as

$$\begin{aligned}\epsilon_{n+1} &= \epsilon_n + \Delta\epsilon_{n+1}, \\ \epsilon_{n+1}^p &= \epsilon_n^p + \Delta\lambda_{n+1}^p \left. \frac{\partial f}{\partial \tilde{\sigma}} \right|_{n+1}, \\ \bar{\epsilon}_{n+1}^p &= \bar{\epsilon}_n^p + \Delta\lambda_{n+1}^p, \\ \tilde{\sigma}_{n+1} &= S_0 : (\epsilon_{n+1} - \epsilon_{n+1}^p), \\ f_{n+1}(\tilde{\sigma}_{n+1}, \bar{\epsilon}_{n+1}^p) &\leq 0, \\ \sigma_{n+1} &= S_{d,n+1} : (\epsilon_{n+1} - \epsilon_{n+1}^p),\end{aligned}\quad (39)$$

where ϵ_n and $\Delta\epsilon_{n+1}$ are known; $\{\Delta\lambda_{n+1}^p, \epsilon_{n+1}^p, \bar{\epsilon}_{n+1}^p, \tilde{\sigma}_{n+1}, S_{d,n+1}, \sigma_{n+1}\}$ are solution-dependent variables. $\Delta\lambda_{n+1}^p$ is determined from the yield function such that

$$f(\tilde{\sigma}(\Delta\lambda_{n+1}^p), \bar{\epsilon}^p(\Delta\lambda_{n+1}^p)) = 0, \quad (40)$$

the Newton–Raphson is employed locally—at integration points—to find the value for $\Delta\lambda_{n+1}^p$ for iteration $k+1$ correcting value at previous iteration k as follows

$$\Delta\lambda_{n+1}^{p,(k+1)} = \Delta\lambda_{n+1}^{p,(k)} - \frac{f(\Delta\lambda_{n+1}^{p,(k)})}{f'(\Delta\lambda_{n+1}^{p,(k)})}, \quad (41)$$

the iterations are stopped once the yield function is less than or equal a tolerance (10^{-6}) as

$$f(\tilde{\sigma}(\Delta\lambda_{n+1}^{p,(k+1)}), \bar{\epsilon}^p(\Delta\lambda_{n+1}^{p,(k+1)})) \leq 10^{-6}. \quad (42)$$

The subroutine procedure is described in Algorithm 1.

Algorithm 1 Subroutine Algorithm

Input: Material properties, solution-dependent variables from the previous increment
 Calculate total strain $\epsilon_{n+1} = \epsilon_n + \Delta\epsilon_{n+1}$
 Compute effective strain by subtracting accumulated plastic strain
 Compute undamaged stiffness S_0 and compliance C_0
 Compute effective stress $\tilde{\sigma}$ from S_0
 Compute yield function $f(\tilde{\sigma}, \bar{\epsilon}^p)$

Algorithm 1 *Cont.*

```

if  $f_{n+1} \leq 0$  then
    Elastic response assumed correct
else
    Initialize  $\Delta\bar{\epsilon}^p = 0$ 
    for  $k = 1$  to max iterations do
        Update  $\tilde{\sigma}$  by subtracting inelastic strain increments
        Recompute  $f$  and  $\frac{\partial f}{\partial \tilde{\sigma}}$ 
        Solve for plastic multiplier increment
        Update equivalent plastic strain
        if  $|f| < \text{tolerance}$  then
            Break loop
        end if
    end for
    if  $|f| \geq \text{tolerance}$  then
        Cutback and reduce time step
    else
        Update plastic strain with converged values
    end if
end if
Select active damage variables
Compute damaged stiffness  $S_d$  and compliance  $C_d$ 
Update effective strain based on plastic strains
Compute nominal stress  $\sigma$  using  $S_d$ 
Compute thermodynamic forces and update stored values
Update damage variables  $d_i$ 
if Tsai–Wu failure criterion met then
    Pass failure stress and small Jacobian
end if
Compute consistent tangent matrix
Update state variables for the next increment

```

4.2. Consistent Tangent Tensor

The consistency condition ensures that the stress state continuously satisfies the yield criterion during the entire loading process. This condition is mathematically enforced by the following set of equations

$$\begin{aligned} f(\tilde{\sigma}_{ij}, \bar{\epsilon}^p) &= 0, \\ f(\tilde{\sigma}_{ij} + d\tilde{\sigma}_{ij}, \bar{\epsilon}^p + d\bar{\epsilon}^p) &= 0, \end{aligned} \quad (43)$$

therefore, we obtain

$$df = 0 \Rightarrow \frac{\partial f}{\partial \tilde{\sigma}_{ij}} d\tilde{\sigma}_{ij} + \frac{\partial f}{\partial \kappa} \frac{\partial \kappa}{\partial \bar{\epsilon}^p} d\bar{\epsilon}^p = 0. \quad (44)$$

Defining the plastic modulus $\{H_p = \frac{\partial \kappa}{\partial \bar{\epsilon}^p} = \beta \gamma (\bar{\epsilon}^p)^{\gamma-1}\}$ and substitution of (22), (26)–(28) and (31) in (44), we have

$$\frac{\partial f}{\partial \tilde{\sigma}_{ij}} : S_0 : d\epsilon - \frac{\partial f}{\partial \tilde{\sigma}_{ij}} : S_0 : d\epsilon^p - H_p d\lambda^p = 0, \quad (45)$$

after some algebraic manipulation, we can write

$$d\lambda^p = \frac{\frac{\partial f}{\partial \tilde{\sigma}_{ij}} : S_0 : d\epsilon}{\frac{\partial f}{\partial \tilde{\sigma}_{ij}} : S_0 : \frac{\partial f}{\partial \tilde{\sigma}} + H_p}. \quad (46)$$

Using the incremental form of (21) and (46), we have

$$\begin{aligned}
 d\sigma &= S_d : d\epsilon - S_d : d\lambda^p \frac{\partial f}{\partial \bar{\sigma}} \\
 &= S_d : d\epsilon - S_d : \frac{\frac{\partial f}{\partial \bar{\sigma}_{ij}} : S_0 : d\epsilon}{\frac{\partial f}{\partial \bar{\sigma}_{ij}} : S_0 : \frac{\partial f}{\partial \bar{\sigma}} + H_p} \frac{\partial f}{\partial \bar{\sigma}} \\
 &= \left(S_d - \frac{\left(S_d : \frac{\partial f}{\partial \bar{\sigma}_{ij}} \right) \otimes \left(S_0 : \frac{\partial f}{\partial \bar{\sigma}} \right)}{\frac{\partial f}{\partial \bar{\sigma}_{ij}} : S_0 : \frac{\partial f}{\partial \bar{\sigma}} + H_p} \right) : d\epsilon \\
 &= S_{dp} : d\epsilon,
 \end{aligned} \tag{47}$$

where S_{dp} is the consistent tangent tensor. The numerator of S_{dp} in (47) is symmetrized to increase convergence rate, so we rewrite it to have the symmetric jacobian matrix J_{dp} as follows

$$J_{dp} = S_d - \frac{\left(S_d : \frac{\partial f}{\partial \bar{\sigma}_{ij}} \right) \otimes \left(S_0 : \frac{\partial f}{\partial \bar{\sigma}} \right) + \left(S_0 : \frac{\partial f}{\partial \bar{\sigma}} \right) \otimes \left(S_d : \frac{\partial f}{\partial \bar{\sigma}_{ij}} \right)}{2 \left(\frac{\partial f}{\partial \bar{\sigma}_{ij}} : S_0 : \frac{\partial f}{\partial \bar{\sigma}} + H_p \right)}. \tag{48}$$

To avoid having zero Jacobian for failed elements, $J_{dp} = 0.01S_0$ for such elements.

5. Numerical Experiments

This section presents the validation of our proposed **PHFGMC-EDI** model, where *EDI* denotes the coupling of *Elastic Degradation and Inelasticity*. Henceforth, the model proposed in the present study will be referred to as PHFGMC-EDI. Validation is carried out for two distinct composite systems: IM7/977-3 and C/C-SiC. For IM7/977-3, the damage predictions obtained from PHFGMC-EDI are compared against results from a phase field approach [42]. The model parameters were calibrated using experimental data from [60,61] for damage and from [1] for incremental plasticity.

In the case of C/C-SiC, the elastic response predicted by PHFGMC-EDI is validated against the results reported in [43]. Damage and plasticity parameters were adopted from the data by [24,25]. To rigorously assess the performance of the PHFGMC-EDI model, the C/C-SiC RUC is subjected to cyclic loading to simulate crack localization, incorporating coupled elastic degradation and plasticity, while damage evolution is simultaneously tracked in both the carbon and silicon carbide matrices to accurately capture the underlying failure mechanisms.

5.1. IM7/977-3: RUC Modeling and Validation

This subsection presents the numerical modeling and analysis of the IM7/977-3 composite. The geometric configuration of the RUC is shown in Figure 2a,b, which correspond to two mesh resolutions: a coarse mesh (C1) and a fine mesh (C2). The IM7 carbon fibers are represented in green, while the 977-3 epoxy matrix is shown in grey.

The fiber direction is aligned with the 1-axis, and the 2- and 3-axes denote the transverse directions. To enable direct comparison with the phase-field modeling results reported by [42], the same geometric parameters are adopted in the current study. Specifically, each fiber is assigned a diameter of 7 μm , and the overall fiber volume fraction is maintained at 60%. The domains for C1 and C2 are discretized into 3480 and 43,348 hexahedral subcells, respectively, via unstructured mesh. This approach highlights one of the key advantages of the PHFGMC framework over the traditional HFGMC, as it allows the use of randomly arranged subcells for domain discretization, offering greater flexibility in representing complex microstructures. Employing two mesh resolutions also establishes a basis for assessing

the mesh sensitivity of the proposed model in accurately capturing the mechanical response of the composite.

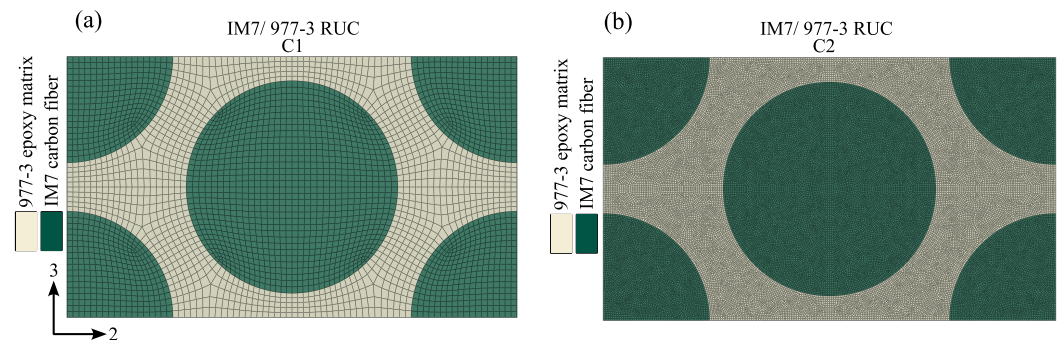


Figure 2. IM7/977-3 representative unit cell (RUC) discretized with two mesh configurations: (a) coarse mesh (C1) and (b) refined mesh (C2). IM7 carbon fibers are shown in green color, while the 977-3 epoxy matrix is shown in grey color. (See the online version of this article for a full-color presentation of this figure.)

The material properties of the IM7 carbon fiber and 977-3 epoxy matrix are listed in Tables 1 and 2, respectively. The model is first evaluated for damage behavior without accounting for inelastic strain evolution. The IM7/977-3 RUC is subjected to transverse tension along the 2-direction for both mesh configurations, C1 and C2.

The corresponding stress–strain responses obtained using the proposed PHFGMC-EDI model are presented in Figure 3a. The results indicate that the model exhibits negligible mesh dependency, as the responses for both the coarse mesh (PHFGMC-EDI/C1) and fine mesh (PHFGMC-EDI/C2) almost match. Moreover, the simulation results show good agreement with the experimental data reported by [60]. For comparison, results from the phase-field-based model (PHFGMC-PF) are also included, where it can be observed that the PHFGMC-EDI formulation exhibits a more stable post-peak response.

Table 1. Material properties of IM7 carbon fiber [1].

Property	Symbol	Value
Longitudinal modulus	E_{11} [GPa]	273.0
Transverse modulus	$E_{22} = E_{33}$ [GPa]	16.0
Shear modulus (1–2, 1–3)	$G_{12} = G_{13}$ [GPa]	13.0
Shear modulus (2–3)	G_{23} [GPa]	6.25
Poisson's ratio (1–2, 1–3)	$\nu_{12} = \nu_{13}$	0.31
Poisson's ratio (2–3)	ν_{23}	0.28

Table 2. Material properties of 977-3 epoxy matrix [1].

Property	Symbol	Value
Young's modulus	E [GPa]	3.3
Poisson's ratio	ν	0.35
Tension damage initiation thresholds	$Y_{1t}^0 = Y_{2t}^0 = Y_{3t}^0$ [MPa]	0.7
Compression damage initiation thresholds	$Y_{1c}^0 = Y_{2c}^0 = Y_{3c}^0$ [MPa]	11.5
Shear damage initiation thresholds	$Y_{12}^0 = Y_{13}^0 = Y_{23}^0$ [MPa]	2.0
Logistic evolution law parameter a	a_i ($i = 1t, 1c, 2t, 2c, 3t, 3c, 12, 13, 23$)	1.0
Logistic evolution law parameter b	b_i ($i = 1t, 1c, 2t, 2c, 3t, 3c, 12, 13, 23$) [MPa]	0.47
Logistic evolution law parameter c	c_i ($i = 1t, 1c, 2t, 2c, 3t, 3c, 12, 13, 23$)	1.0
Initial threshold stress of inelastic strain	R_0 [MPa]	15.0
Weighting coefficients	$m_{1t} = m_{1c} = m_{2t} = m_{2c} = m_{3t} = m_{3c}$	0.5
Weighting coefficients	$m_{12} = m_{13} = m_{23}$	1.0
Hardening parameter	β [MPa]	75.0
Hardening parameter	γ	0.29

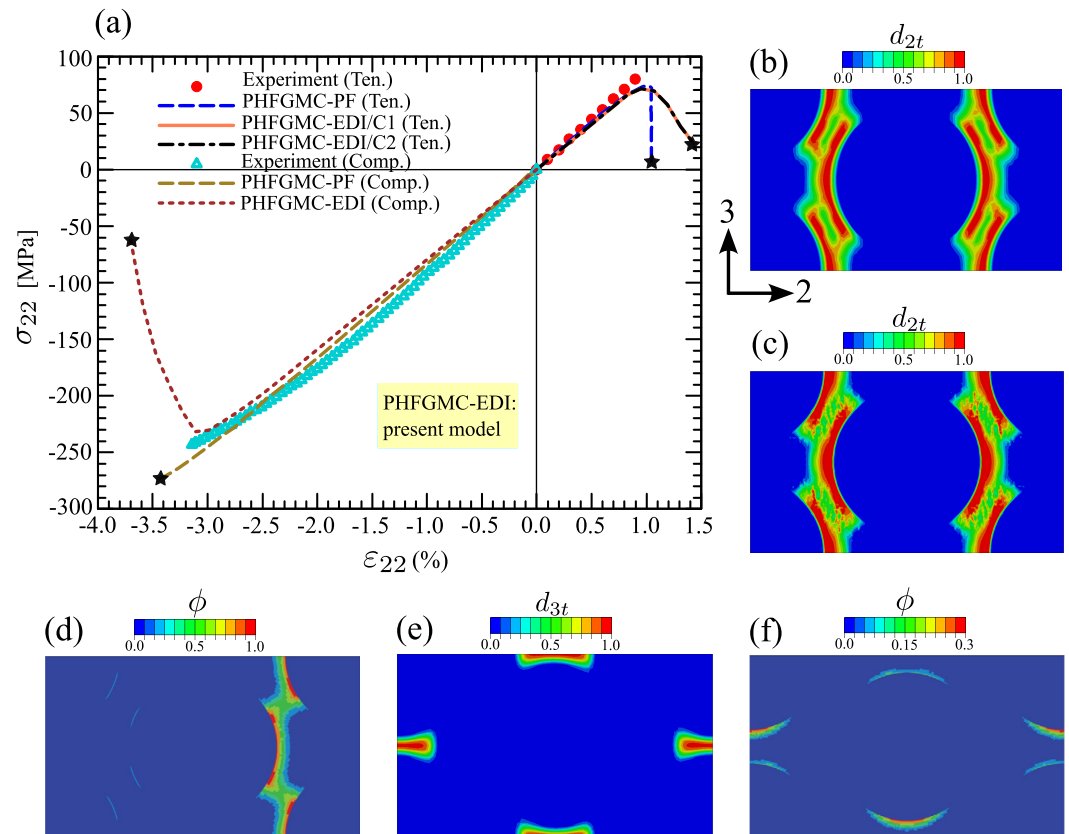


Figure 3. IM7/977-3 RUC, transverse tension (2-dir.): (a) Stress–strain responses by PHFGMC-EDI (both meshes) with exp. data [60] and PHFGMC-PF predictions [42]. (b,c) PHFGMC-EDI damage contours (C1, C2). (d) PHFGMC-PF damage contours. Black stars mark snapshot locations. Transverse compression (2-dir.): (a) PHFGMC-EDI stress–strain vs. exp. [61] and PHFGMC-PF [42]; (e) PHFGMC-EDI damage contours; (f) PHFGMC-PF phase-field contours.

The damage variable d_{2t} contours for the C1 and C2 mesh configurations are shown in Figure 3b,c, respectively, at the locations indicated by stars in Figure 3a. In this context, a damage value of $d_{2t} = 1$ corresponds to complete material failure, while $d_{2t} = 0$ indicates an undamaged state. Accordingly, the damaged region exhibits a continuum of values between 0 and 1, reflecting the gradual evolution of material degradation.

The predicted damage patterns appear largely mesh-independent, with both C1 and C2 exhibiting similar crack trajectories and consistent crack widths. This mesh consistency demonstrates the robustness and convergence of the proposed PHFGMC-EDI model under mesh refinement. Furthermore, the results align well with those obtained from the phase-field-based model, as depicted in Figure 3d.

The IM7/977-3 RUC is analyzed under transverse compression for C1 case, with the resulting stress–strain responses presented in Figure 3a. Predictions from the proposed PHFGMC-EDI model are compared with experimental results reported by [61] and numerical results obtained using the phase-field (PHFGMC-PF) model from [42]. A strong agreement among all three datasets confirms the accuracy and reliability of the proposed model. The corresponding damage contours predicted by the PHFGMC-EDI and PHFGMC-PF models are shown in Figure 3e,f, respectively. The damage localization paths are notably close, indicating consistency between the two modeling approaches. Importantly, the PHFGMC-EDI model demonstrates a notably more stable post-peak response compared to the PHFGMC-PF formulation. In the latter, the phase-field variable fails to reach full evolution, likely due to the sharp and unstable softening behavior encountered in the post-peak regime.

Figure 4a illustrates the stress–strain response of the IM7/977-3 RUC subjected to transverse tension in the 3-direction. The results from both mesh configurations (C1 and C2) show excellent agreement, indicating consistent mechanical behavior. The corresponding damage variable d_{3t} contours are presented in Figure 4b,c for the C1 and C2 meshes, respectively.

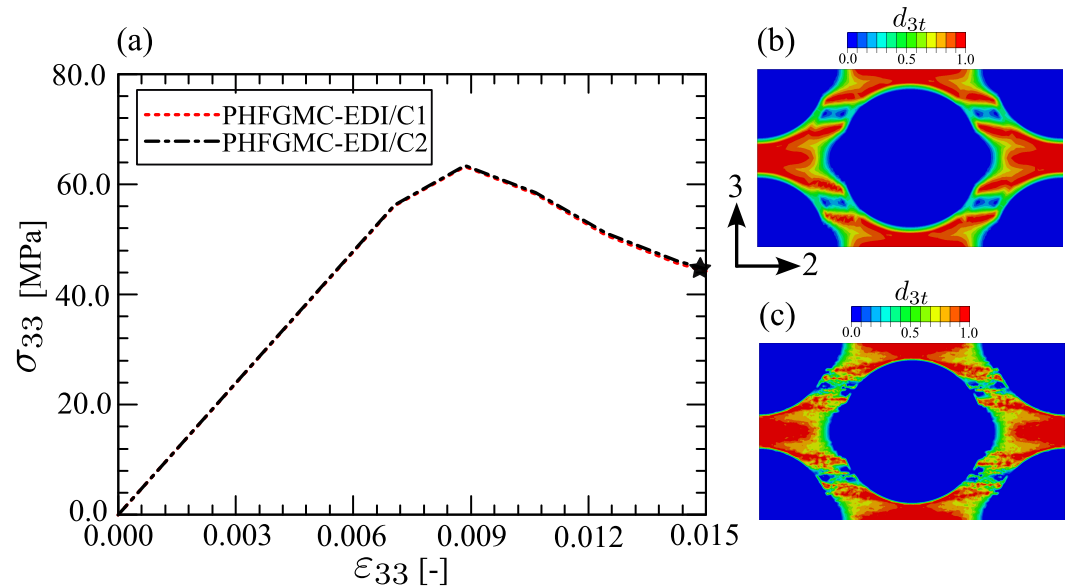


Figure 4. IM7/977-3 RUC subjected to transverse tension along the 3-direction: (a) stress–strain responses obtained using the PHFGMC-EDI model for both C1 and C2 mesh configurations, (b) damage contours for the coarse mesh (C1), and (c) damage contours for the refined mesh (C2). Black stars indicate the locations along the stress–strain curve where the damage contour snapshots were captured.

Notably, the regularization zone remains nearly unchanged with mesh refinement, demonstrating the mesh-independent nature of the PHFGMC-EDI model. Furthermore, both meshes exhibit nearly identical crack paths. The observed damage trajectories reflect delamination behavior in the composite, as evidenced by crack branching patterns.

Figure 5 presents the response of the RUC under shear loading in the 1–2 plane. The stress–strain curves are shown alongside both the experimental results and the predictions from the PHFGMC-PF model. The damage variable d_{12} contours, displayed in Figure 5b, exhibit symmetric crack paths around the central fiber, closely matching the patterns reported by [42] using the PHFGMC-PF approach. As shown in Figure 5c, where the interacting cracks merge completely upon contact.

Figure 6 illustrates the incorporation of plasticity effects within the PHFGMC-EDI framework. In Figure 6a, the stress–strain behavior of the 977-3 epoxy matrix is calibrated based on the data of the Romberg-Osgood deformation theory of plasticity documented by [1], demonstrating an accurate fit. This calibrated plasticity model is then applied to the IM7/977-3 RUC under transverse tension along the 2-direction, as shown in Figure 6b.

The resulting stress–strain response captures the expected nonlinear behavior, validating the effectiveness of the plasticity formulation. Moreover, the distribution of equivalent plastic strain, presented in Figure 6c, reveals localized zones of inelastic deformation that correlate well with regions of high damage accumulation. These observations confirm that the PHFGMC-EDI model can robustly simulate the coupled evolution of plasticity and damage in composite microstructures.

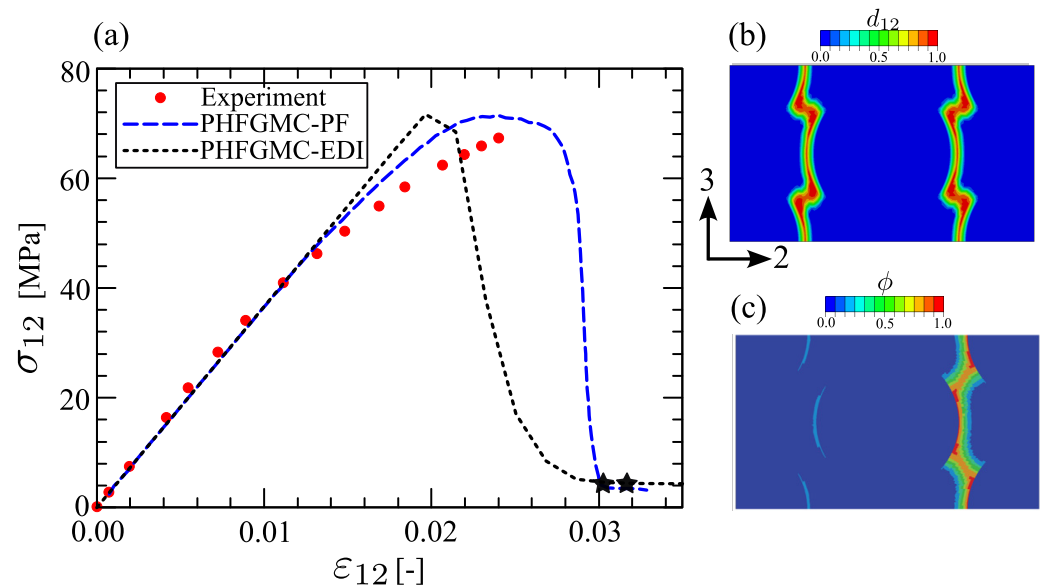


Figure 5. IM7/977-3 RUC subjected to shear along the 1–2 direction: (a) stress–strain responses predicted by the PHFGMC-EDI model, compared with experimental results from [60] and numerical results from the PHFGMC-PF model [42]; (b) damage contours from the PHFGMC-EDI model; (c) phase-field variable contours from the PHFGMC-PF model. Black stars mark the locations on the stress–strain curve corresponding to the damage contour snapshots.

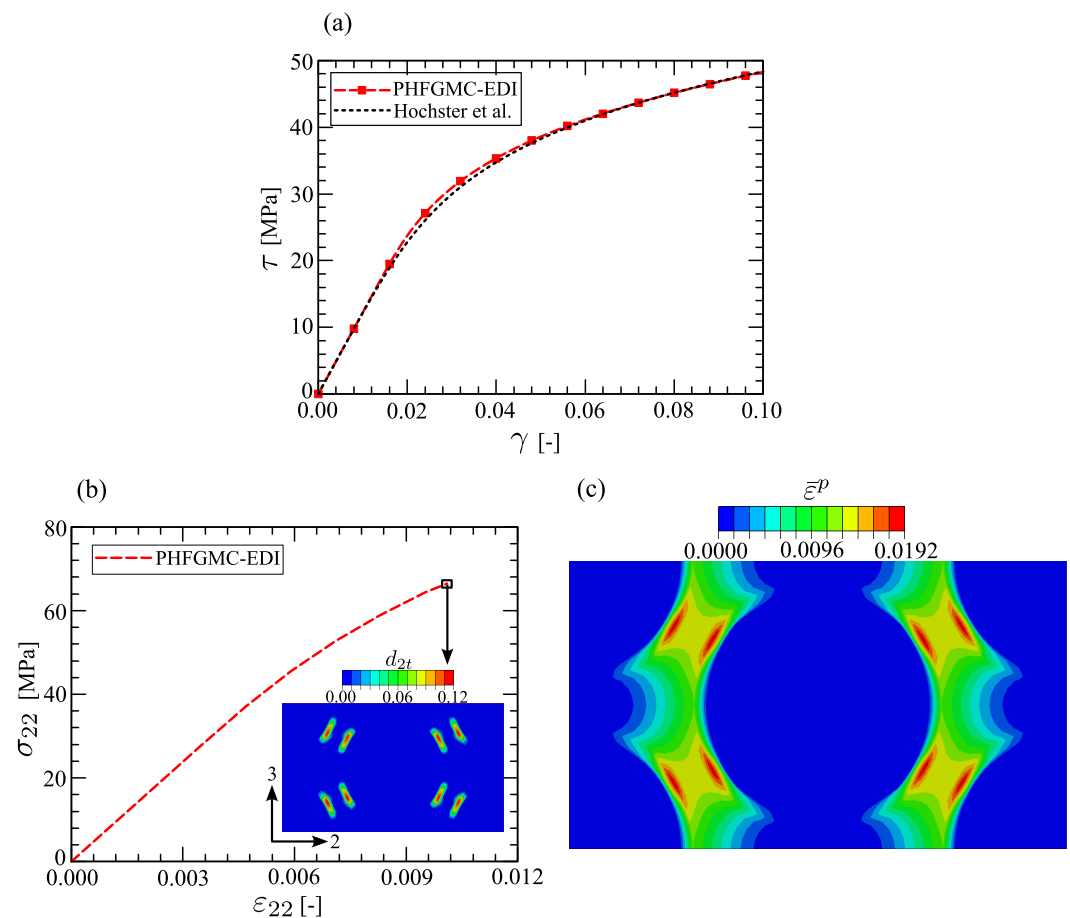


Figure 6. Calibration and response of the IM7/977-3 system incorporating plasticity: (a) Calibration of the 977-3 epoxy matrix to the Romberg–Osgood plasticity model based on data from [1]; (b) Stress–strain response of the IM7/977-3 RUC under transverse tension in the 2-direction; (c) Equivalent plastic strain contours predicted by the PHFGMC-EDI model.

The equivalent plastic strain contours in Figure 6c highlight the onset and evolution of inelastic deformation within the epoxy matrix prior to the development of macroscopic damage. The contours show that plastic deformation is highly localized near the fiber–matrix interfaces, where stress concentrations are typically highest due to the mechanical mismatch between constituents.

Figure 7a shows an SEM image of the IM7/977-3 system microstructure with a fiber volume fraction of 0.6. This system is modeled using the present approach under transverse tension in the 2-direction. It is worth mentioning that while the SEM image in Figure 7a depicts a real microstructure with inherent randomness in fiber distribution, the PHFGMC-EDI framework approximates this as a periodic arrangement within the RUC to enable efficient micromechanical analysis. For truly random microstructures, the model provides a reasonable approximation using a statistically representative RUC, though full stochastic extensions could be explored in future work to enhance accuracy. The corresponding damage contours are displayed in Figure 7b. It can be observed that the damage localizes perpendicular to the loading direction and initiates at locations of stress concentration.

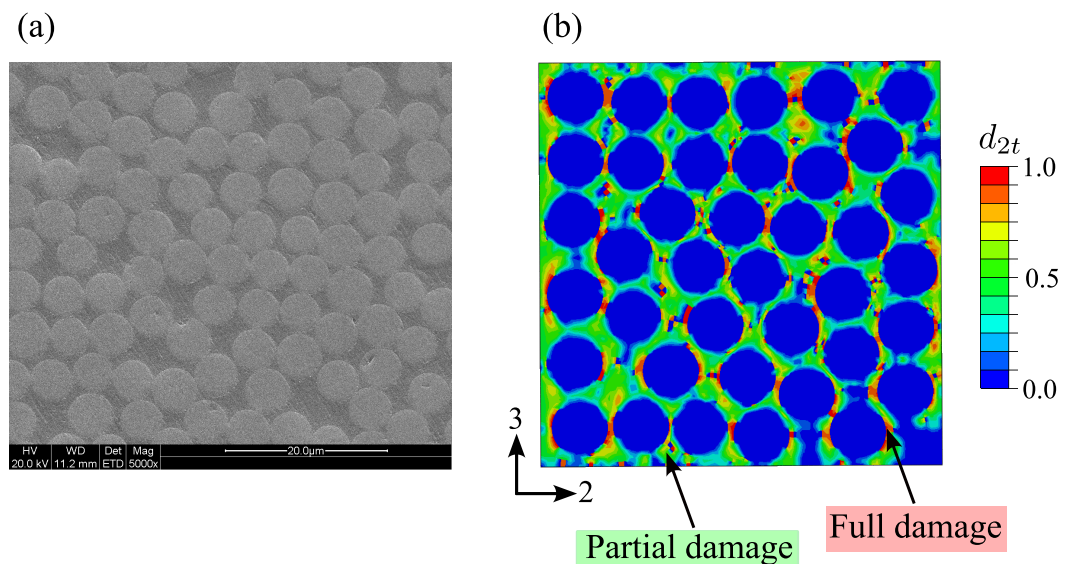


Figure 7. (a) SEM image of the IM7/977-3 system microstructure (fiber volume fraction = 0.6); (b) damage contours.

5.2. C/C-SiC: RUC Modeling and Validation

C/C-SiC composites are composed of three primary constituents—carbon fibers, an amorphous carbon matrix, and a silicon carbide (SiC) matrix—distributed across multiple length scales. To model the microstructure of the filament-wound C/C-SiC composite using a representative unit cell (RUC), it is essential to recognize, simplify, and quantify the main microstructural characteristics. Figure 8a illustrates the geometric representation of the simplified RUC derived from the micrograph of [62] following the work of [43,63].

We aimed to closely follow the geometrical dimensions and arrangement of fibers provided in the work of [43]. However, due to the enhanced flexibility of the PHFGMC modeling approach, we adjusted the fiber diameter and nearby features. This allowed us to better preserve the phase volume fractions reported in the work of [43]. In the model, carbon fibers (green) are embedded within an amorphous carbon matrix (grey), together forming what is designated as the C/C bundle, while the surrounding silicon carbide matrix is shown in red. The principal parameters of the C/C-SiC RUC, as illustrated in Figure 8a, are detailed in Table 3. A three-dimensional modeling framework is employed, with the thickness of the RUC along the fiber direction (1-direction) set to 0.015.

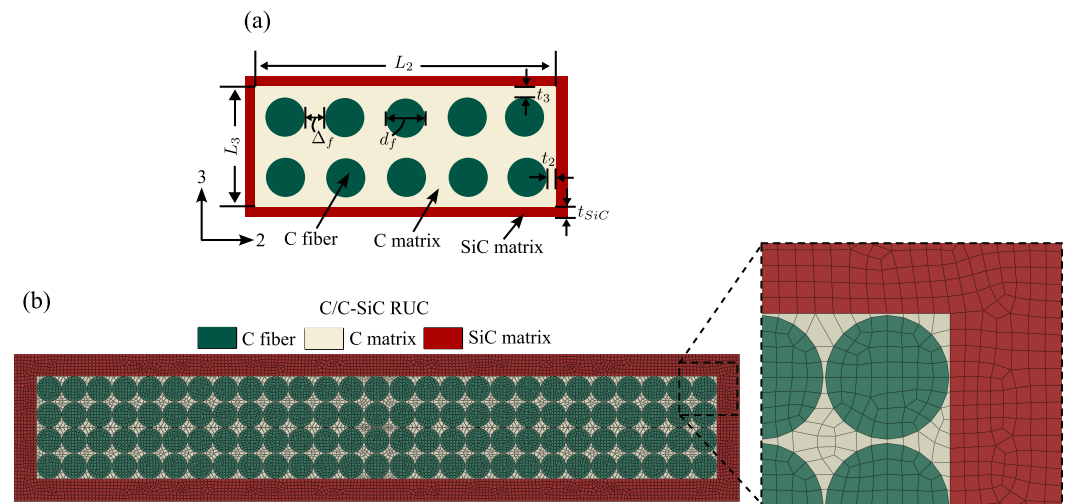


Figure 8. (a) Geometric representation of the C/C-SiC RUC, highlighting the carbon fibers (green), amorphous carbon matrix (grey), and SiC matrix (red). Refer to Table 3 for details on the corresponding parameters. (b) Meshed full PHFGMC RUC (number of hexahedral subcells: 36,360).

Table 3. Summary and definitions of essential microstructural parameters employed in constructing the C/C-SiC RUC.

Description	Symbol	Value
Dimension of C/C bundle along 2-direction	L_2	1.0
Dimension of C/C bundle along 3-direction	L_3	0.15
C fiber diameter	d_f	0.0364
Spacing between fibers	Δ_f	0.0006
Thickness of SiC matrix	t_{SiC}	0.033
Cover of C/C bundle along 2-direction	t_2	0.0003
Cover of C/C bundle along 3-direction	t_3	0.0005
Number of fibers in 2-direction	n_2	27
Number of fibers in 3-direction	n_3	4

The RUC of the C/C-SiC microstructure is shown in Figure 8b, with the corresponding subcell grid displayed and magnified. Due to the flexibility of the PHFGMC, the domain can be discretized into randomly arranged hexahedral subcells, as shown in the zoomed-in inset on the right. The RUC domain is discretized into 36,360 hexahedral subcells, with three subcells along the out-of-plane direction. The elastic properties of the C fiber, amorphous C matrix, and SiC matrix materials were adapted from [64], which was also used by [43], and are presented in Table 4.

Table 4. Elastic properties of C fiber, amorphous C, and SiC matrices (adapted from [64]).

C Fiber	Value	C Matrix	Value	SiC Matrix	Value
E_{11} [GPa]	294.0	E [GPa]	62.6	E [GPa]	15.0
E_{22} [GPa]	18.4	ν	0.3	ν	0.3
ν_{12}	0.2	G [GPa]	24.0	G [GPa]	5.77
ν_{23}	0.25				
G_{12} [GPa]	8.96				

5.2.1. Elastic Properties

The elastic properties of the C/C-SiC RUC were predicted using the PHFGMC-EDI model and validated against results from the generalized method of cells (GMC) employed by [43]. The RUC in this study utilized the same volume fraction data as that of [43]. The results presented in Table 5 show that the effective properties predicted by the PHFGMC-EDI

are comparable to the previously published results by [43]. The differences between the two approaches are justifiable. It is important to note that the current study employed a three-dimensional framework, in contrast to the two-dimensional framework used by [43], which may account for some of the observed differences in the results.

Table 5. Prediction of the elastic properties of the C/C-SiC RUC using the PHFGMC-EDI and GMC models.

Property	PHFGMC-EDI	GMC [43]	% Difference
E_{11} [GPa]	157.25	160.0	1.71
E_{22} [GPa]	22.61	21.0	7.66
E_{33} [GPa]	21.85	20.5	6.58
ν_{12}	0.26	0.251	3.58
ν_{13}	0.26	0.255	1.96
ν_{23}	0.32	0.312	2.56
G_{12} [GPa]	9.50	8.96	6.02
G_{13} [GPa]	8.73	8.34	4.67
G_{23} [GPa]	7.72	7.45	3.62

5.2.2. Tension Cycles

This section aims to rigorously model damage and plasticity within the C/C-SiC RUC. Despite the structural and material relevance of this system, experimental data specific to its damage behavior remain unavailable. To overcome this challenge and enable meaningful simulations—while also showcasing the high capabilities of our proposed model—the damage and plasticity parameters are adopted from the work of Li et al. [24,25], which investigated a two-dimensional C/SiC composite composed of closely related constituent materials. The parameters for the carbon matrix, essential for capturing its inelastic response, are listed in Table 6. In cases where data were not reported—specifically, the compression damage initiation thresholds—reasonable estimates were introduced to ensure completeness of the model. It should be emphasized that the following analysis serves as a demonstration of the PHFGMC-EDI model’s capabilities in simulating damage and plasticity for C/C-SiC composites, rather than a direct validation, as exact parameters were unavailable and were instead adapted from data on a similar C/SiC material [24,25].

Table 6. Damage and plasticity parameters of the carbon matrix (adapted from [24]).

Property	Symbol	Value
Tension damage initiation thresholds	$Y_{1t}^0 = Y_{2t}^0 = Y_{3t}^0$ [MPa]	1.35×10^{-3}
Compression damage initiation thresholds	$Y_{1c}^0 = Y_{2c}^0 = Y_{3c}^0$ [MPa]	6.0
Shear damage initiation thresholds	$Y_{12}^0 = Y_{13}^0 = Y_{23}^0$ [MPa]	2.17×10^{-3}
Logistic evolution law parameter a	a_i ($i = 1t, 1c, 2t, 2c, 3t, 3c, 12, 13, 23$)	1.0
Logistic evolution law parameter b	b_i ($i = 1t, 1c, 2t, 2c, 3t, 3c, 12, 13, 23$) [MPa]	0.47
Logistic evolution law parameter c	c_i ($i = 1t, 1c, 2t, 2c, 3t, 3c, 12, 13, 23$)	1.0
Initial threshold stress of inelastic strain	R_0 [MPa]	18.79
Weighting coefficients	$m_{1t} = m_{1c} = m_{2t} = m_{2c} = m_{3t} = m_{3c}$	1
Weighting coefficients	$m_{12} = m_{13} = m_{23}$	1.54
Hardening parameter	β [MPa]	1.31×10^5
Hardening parameter	γ	0.86

A key objective of this section is to capture the complete damage evolution of the C/C-SiC RUC, including the post-peak behavior. To achieve this, the parameters of the logistic damage evolution law were intentionally modified from those originally proposed by [24]. These modifications were necessary, as the original parameter choices in [24] limited the damage variable to a maximum of 0.87, thereby preventing the model from fully reaching the failure regime. Specifically, the use of an a parameter set to 0.87 and c value less than 1 constrained the model’s ability to capture the post-peak softening response. In contrast,

the adjustments made in the present work enable the damage variable to evolve toward complete failure, providing a more comprehensive and physically realistic representation of the material behavior under progressive loading.

In the following analysis, specific cases are explored by selectively activating either damage or plasticity, and then both mechanisms simultaneously. It is important to note that damage and plasticity are effectively deactivated by assigning sufficiently high values to the damage initiation thresholds and the initial yield stress for inelastic deformation, respectively.

Figure 9 presents the response of the C/C–SiC RUC under transverse tension cyclic loading applied in the 2-direction. The locations labeled P1, P2, and P3 correspond to snapshots of the evolving damage contours. In this analysis, damage is activated exclusively in the carbon matrix, while both the SiC matrix and carbon fibers are modeled as undamaged and purely elastic. As plasticity is not active, the stress–strain response exhibits a progressive reduction in stiffness with each cycle, yet returns to the origin after unloading, indicating no residual strain or permanent deformation. This behavior is consistent with a damage-driven degradation mechanism in the absence of inelastic strain accumulation.

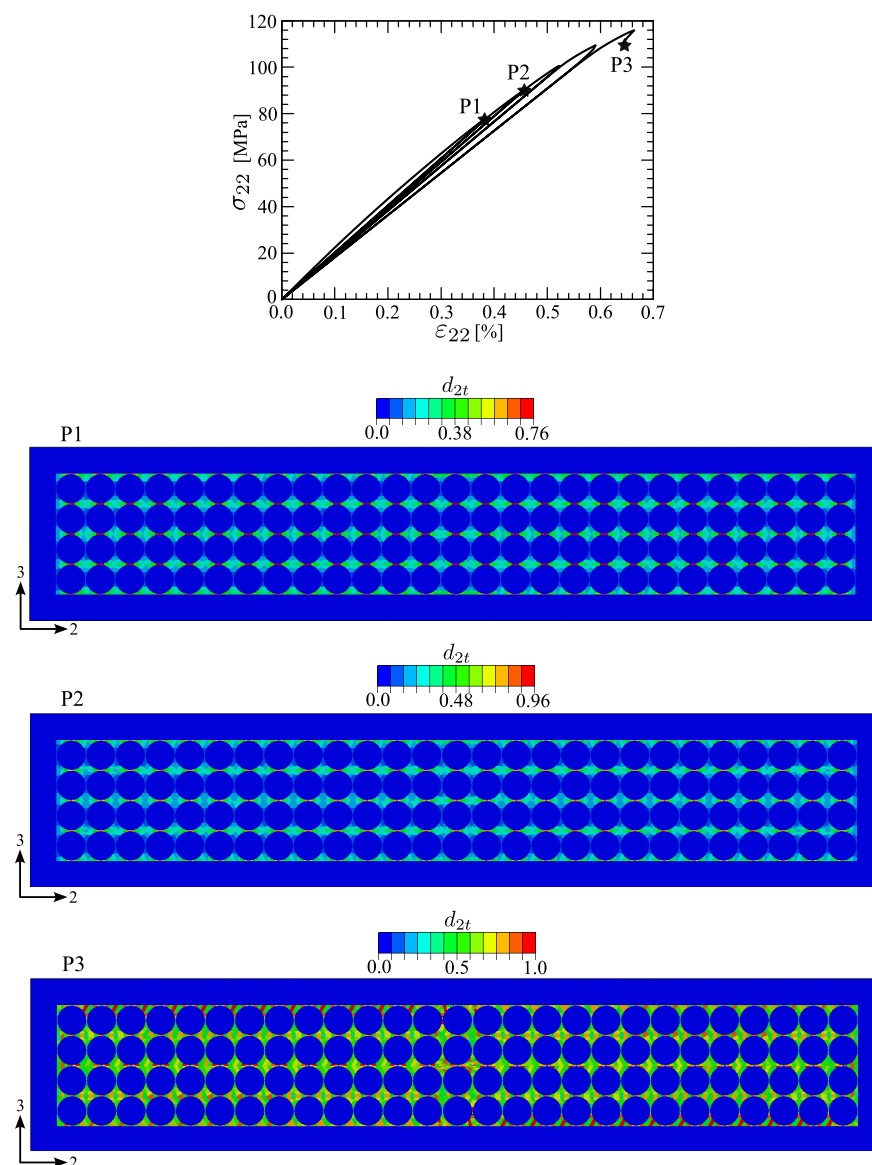


Figure 9. Stress–strain response of the C/C–SiC RUC under tension cycles with damage activated in the carbon matrix, accompanied by corresponding damage contour snapshots.

Another case explored in this study involves the activation of damage in both the carbon matrix and the SiC matrix, while all material phases are otherwise assumed to remain elastic. The damage parameters for the SiC matrix are taken to be equal to those of the carbon matrix, in the absence of experimental calibration data. This assumption enables a qualitative assessment of the role of matrix interactions in the overall degradation of the composite. The corresponding mechanical response is shown in Figure 10.

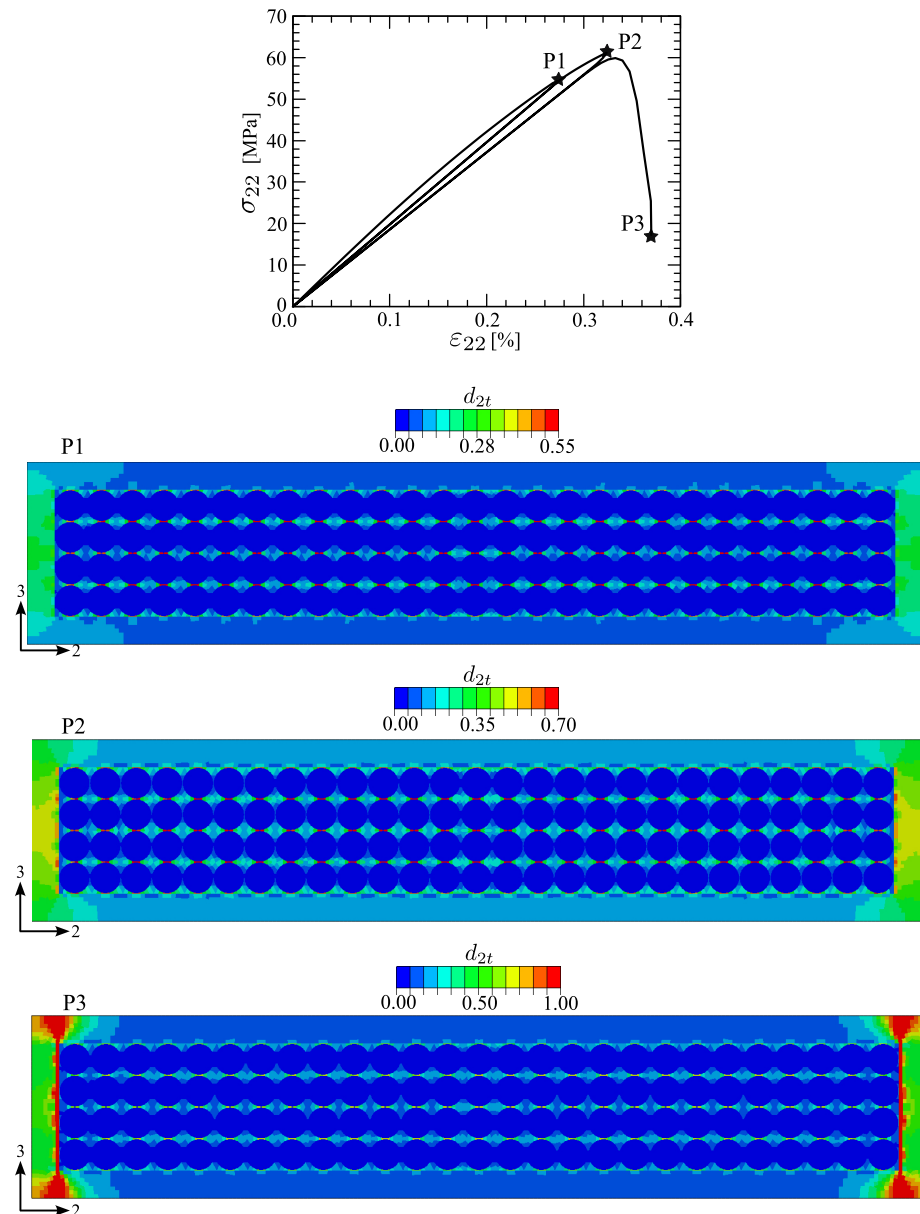


Figure 10. Stress–strain response of the C/C–SiC RUC under tension cycles with damage activated in both the carbon and SiC matrices, accompanied by corresponding damage contour snapshots.

In this scenario, damage evolves progressively in both matrices during cyclic transverse tension applied in the 2-direction. With each loading cycle, microstructural degradation accumulates, resulting in a noticeable reduction in structure's stiffness. However, since no plastic deformation is considered, the unloading paths consistently return to zero strain, indicating the absence of residual deformation. The mechanical softening is thus attributed purely to stiffness degradation caused by damage accumulation.

Notably, damage initiation is first observed in the carbon matrix, primarily due to stress concentrations induced by the surrounding fibers. As loading progresses, the damage

begins to propagate into the adjacent SiC matrix, driven by stress redistribution between the two phases. This interaction between the damageable matrices triggers a cascading failure mechanism that ultimately results in the complete degradation of the representative unit cell.

Next, the case in which damage is deactivated and only plasticity is active in the C/C–SiC RUC is investigated under transverse tension cycles. In this scenario, the carbon matrix is assumed to exhibit inelastic behavior, with plasticity parameters defined in Table 6. The corresponding stress–strain response is shown in Figure 11a, with a magnified view presented in Figure 11b.

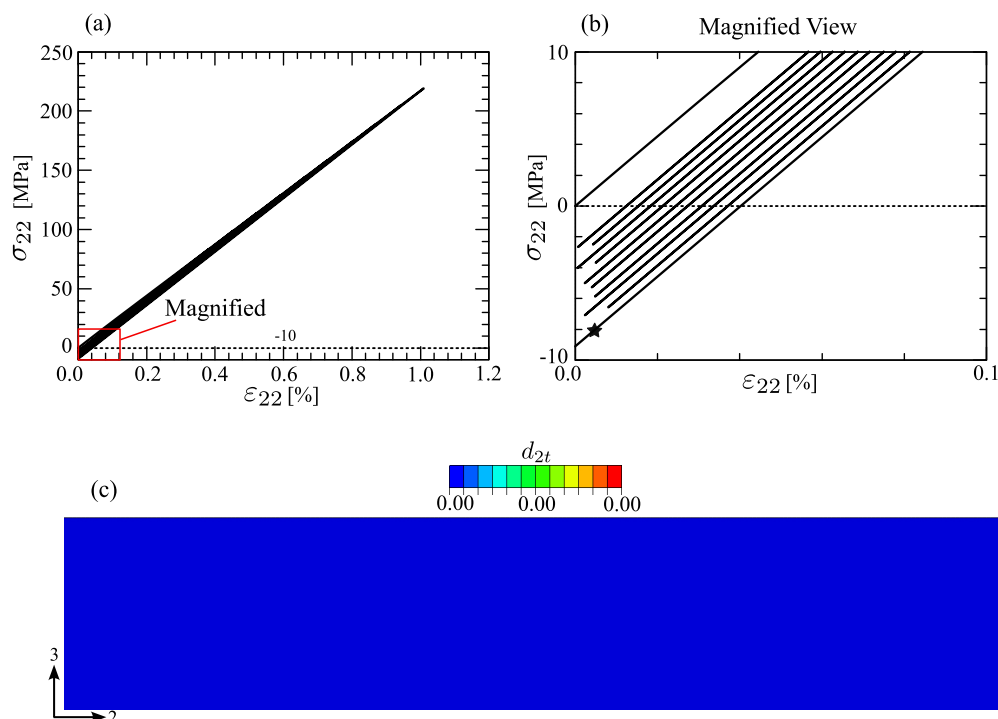


Figure 11. Response of the C/C–SiC RUC under transverse tension cycles with plasticity activated in the carbon matrix: (a) overall stress–strain response, (b) magnified view of the cyclic behavior, and (c) damage contour snapshot confirming the absence of damage at the end of the final cycle.

As no damage is permitted in any of the constituent phases, the overall stiffness of the RUC remains constant across loading cycles, with unloading paths retaining a parallel slope. However, due to inelastic deformation in the carbon matrix, permanent strain accumulates after each cycle, as indicated by the residual strain offset upon unloading. Figure 11c further verifies the absence of damage, showing uniform damage contours consistent with the modeling assumptions.

It is also noteworthy that although the RUC is subjected to tension-only loading, the stress response exhibits negative values during unloading. This behavior is attributed to the internal stress redistribution and inelastic strain accumulation driven by inelastic flow in the matrix phase.

Figure 12 presents the response of the carbon matrix under the combined activation of damage and inelasticity, evaluated at a representative material point. As these mechanisms evolve, the material exhibits progressive stiffness degradation with each loading cycle, accompanied by the accumulation of permanent strain—preventing the response from returning to the origin. A distinct crack closure phenomenon is observed: during compression, the material stiffness recovers to its original undamaged level. Interestingly, compressive stresses emerge even though only tension-to-zero stress loading is applied, a behavior attributed to the inelastic deformation pathway and its directional evolution.

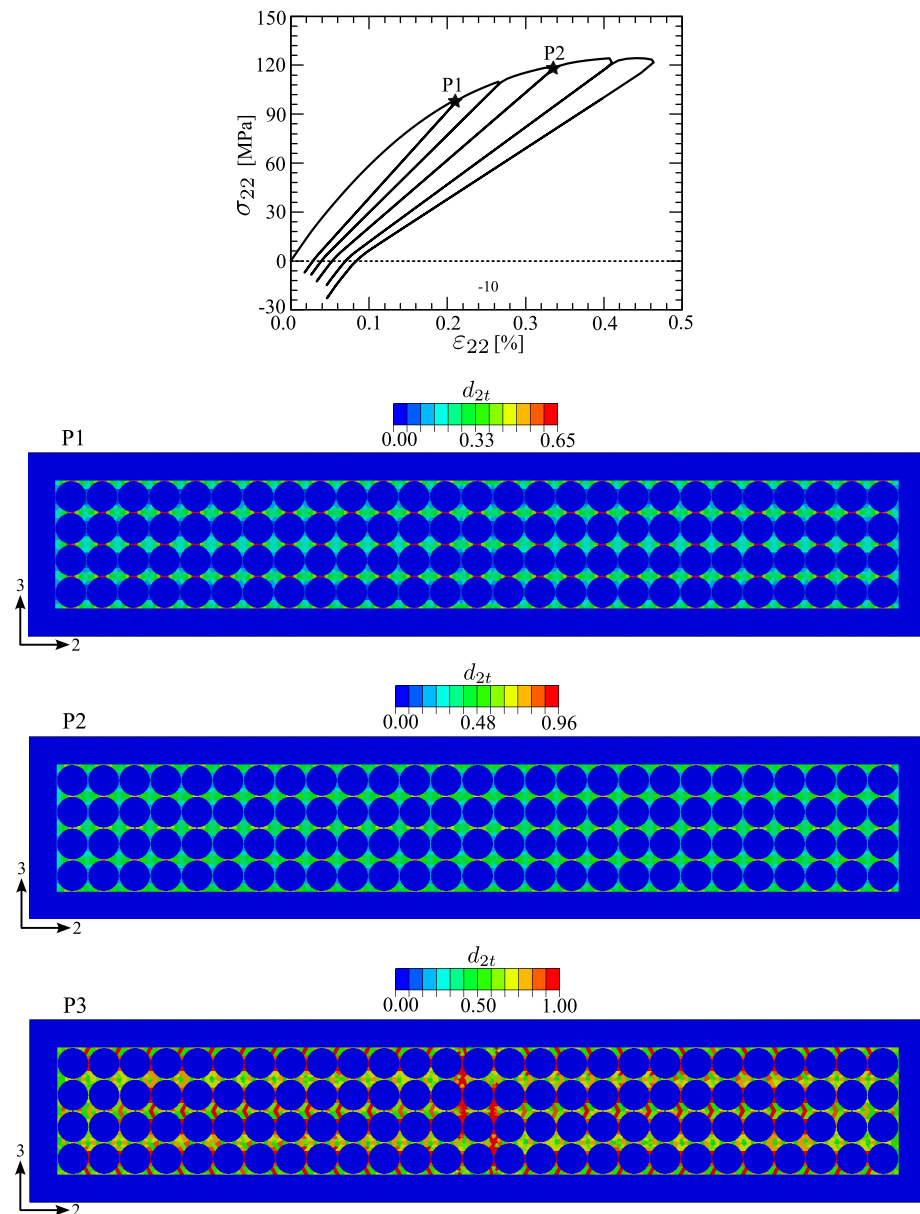


Figure 12. Stress–strain response at a representative point in the carbon matrix under cyclic loading with both damage and inelasticity activated, accompanied by corresponding damage contour snapshots. The material exhibits progressive stiffness degradation and permanent strain accumulation with each cycle. The crack closure phenomenon is evident, as the stiffness in compression returns to its original undamaged value.

These results underscore the high versatility and predictive capabilities of the proposed model, which can accurately capture complex material behavior under various loading conditions. For brevity, only a subset of representative scenarios is presented, though the model is applicable to a broad range of mechanical responses.

5.2.3. Tension–Compression Cycles

The C/C-SiC RUC is modeled under tension–compression cyclic loading applied in the 2-direction. The damage initiation thresholds for tension and compression are presented in Table 6. Specific cases are tested to demonstrate the high capabilities of the developed model. The first case involves damage activation in the carbon matrix, where Figure 13 illustrates the corresponding stress–strain curve.

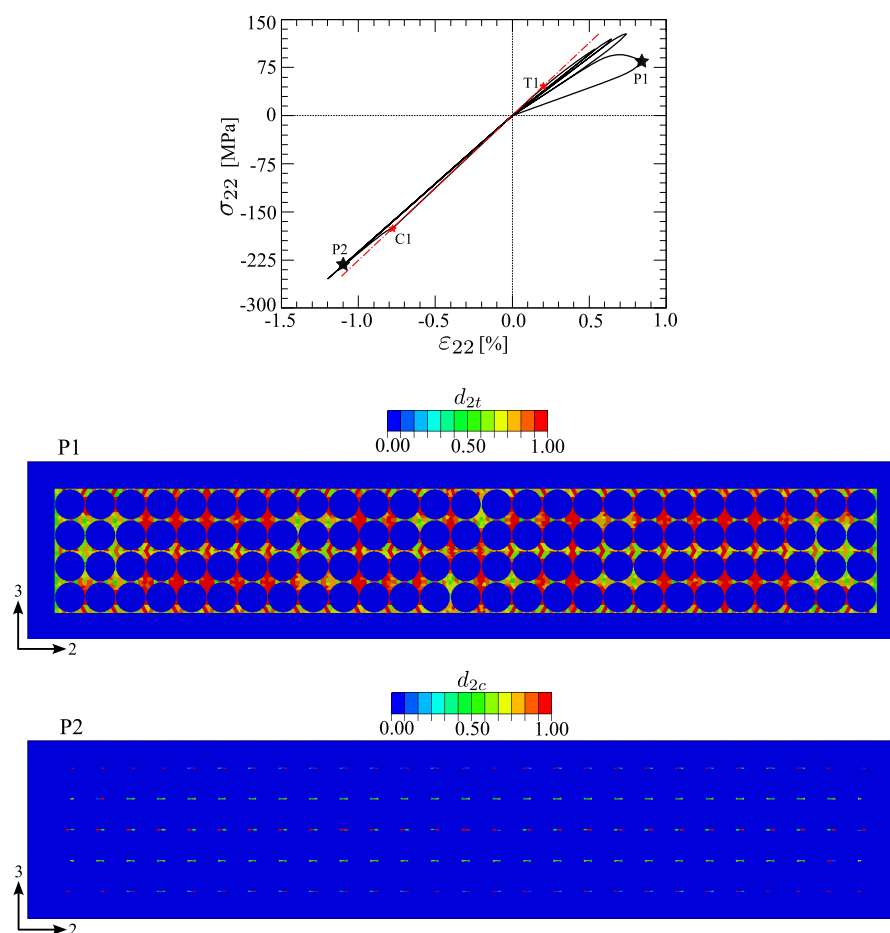


Figure 13. Stress–strain response of the C/C–SiC RUC under tension–compression cyclic loading with damage localized in the carbon matrix. Points T1 and C1 indicate the onset of damage under tension and compression, respectively. Damage contour snapshots at P1 and P2 illustrate the crack closure behavior and the evolution of tensile and compressive stiffness.

T1 and C1 represent critical points at which damage begins to localize within the carbon matrix under tension and compression, respectively. Specifically, T1 marks the onset of damage initiation during the tension cycle, while C1 indicates the point at which damage begins to localize under compression. Damage contours, denoted as d_{1t} and d_{1c} , are captured at points P1 and P2, providing a visual representation of the evolving damage states.

The behavior of the material during the loading cycles reveals a clear pattern of crack closure. After the tension cycle is completed, the stress–strain curve exhibits a return to the origin; however, this return is characterized by a noticeable reduction in tensile stiffness. This reduction is a direct consequence of the damage accumulation, which impairs the material’s ability to resist further tensile loading.

In the subsequent compression cycle, the material initially exhibits stiffness identical to that of the undamaged response, indicating no immediate damage initiation. This is attributed to the higher threshold for damage initiation under compression compared to tension. Consequently, the compression cycle remains largely unaffected by damage until the stress exceeds the initiation threshold C1, beyond which damage begins to localize, and the material’s behavior transitions accordingly.

The RUC is subsequently tested for inelastic deformation evolution in the carbon matrix without damage under cyclic tension–compression loading. Figure 14a shows the response when the tensile weighting coefficient is active, while the compressive weighting

coefficient is inactive. As observed, inelastic deformation is induced primarily by tension, with the inelastic deformation direction (IDD) aligned along the tensile loading axis.

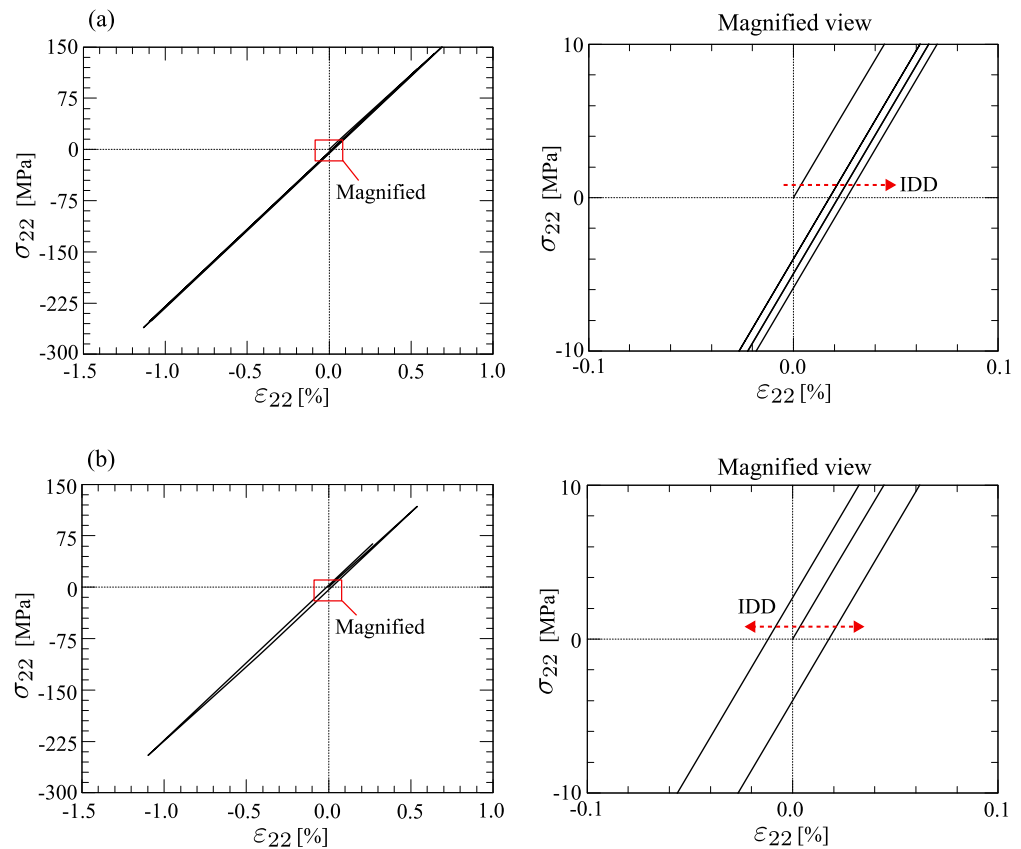


Figure 14. Stress–strain response of the C/C–SiC RUC under cyclic tension–compression loading without damage. (a) Inelastic deformation induced primarily by tension, with the tensile weighting coefficient active and the compressive weighting coefficient inactive. (b) Inelastic deformation aligned with both tension and compression directions when both weighting coefficients are active.

When both the tensile and compressive weighting coefficients are active, the IDD aligns with both the tensile and compressive loading directions, as shown in Figure 14b. Since damage is not active, the slope of the stress–strain response remains constant throughout the loading cycles, as observed in both Figure 14a,b.

The high capabilities of the EDI model are clearly demonstrated. Damage can be activated independently along any of the orthogonal directions under either tension or compression, as well as along shear planes. Furthermore, inelastic deformation can be modeled to be induced selectively by tension, compression, or shear, or by a calibrated combination of all three mechanisms.

The response of the RUC with both damage and plasticity activated in the carbon matrix is shown in Figure 15a, with magnified views of the tensile and compressive regions provided in Figures 15b and 15c, respectively. The corresponding tensile and compressive damage contours are also illustrated. As the damage initiation threshold in tension is significantly lower than in compression, tensile damage clearly dominates the RUC response. Plastic deformation is primarily induced by tensile stresses, as evidenced by the permanent strain accumulation shown in Figure 15b.

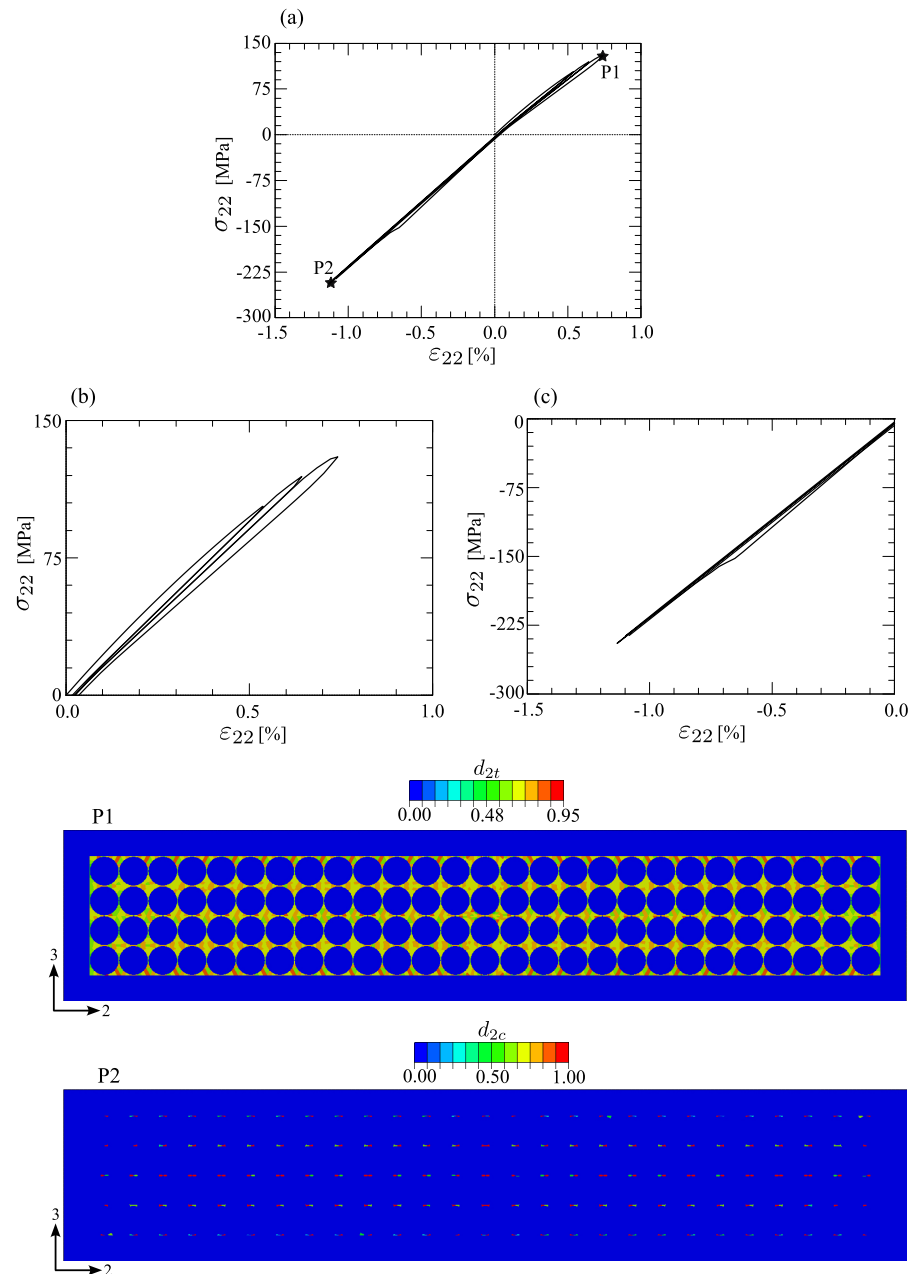


Figure 15. Response of the RUC under cyclic tension–compression loading with both damage and plasticity activated in the carbon matrix. (a) Overall stress–strain response. (b) Magnified view of the tensile region, showing permanent deformation due to plasticity and dominant tensile damage. (c) Magnified view of the compressive region. Snapshots of the corresponding tensile and compressive damage contours are shown at the bottom.

These examples collectively demonstrate the high versatility and predictive power of the proposed PHFGMC-EDI model in capturing complex inelastic and damage behaviors under cyclic loading. The model allows independent activation of damage and inelasticity in tension, compression, and shear, with full control over their directional sensitivity and interaction. Despite presenting only a subset of possible scenarios, the selected cases highlight the model’s ability to reproduce key physical phenomena such as stiffness degradation, permanent deformation, and crack closure. This flexibility makes the proposed framework a powerful tool for simulating the nonlinear response of advanced composite materials under realistic loading conditions.

Figure 16 compares the cyclic response predicted by the Li et al. model [24] and the present model for a plain-woven homogenized C/SiC composite. An excellent agreement

is obtained in the tensile regime. Under compression, both models successfully reproduce the crack-closure phenomenon. However, in the model of Li et al. [24], the stiffness recovers gradually during unloading due to the use of an exponential function for compression, whereas in the present model it returns in a step-like manner.

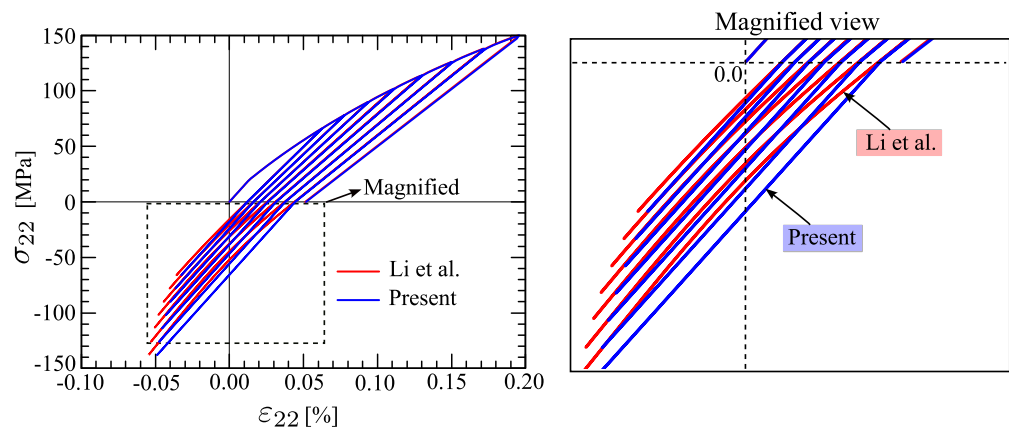


Figure 16. Response of the plain-woven C/SiC composite representative unit cell (RUC) under cyclic tension–compression loading, comparing results from the Li et al. model [24] and the current model, with both damage and plasticity activated.

6. Conclusions and Discussion

This study introduced a comprehensive three-dimensional micromechanical framework to describe the nonlinear mechanical behavior of composite materials, accounting for coupled elastic degradation, inelastic strain evolution, and phenomenological failure mechanisms. The generalized elastic degradation–inelasticity (EDI) model was implemented within the parametric high-fidelity generalized method of cells (PHFGMC), enabling robust simulation of nonlinear response and failure mechanisms in composite systems.

The formulation is grounded in continuum mechanics, using a unified orthotropic damage model based on scalar field variables governed by an energy potential function. Thermodynamic forces are defined along three orthogonal directions—each decomposed into tensile, compressive, and shear components, with shear failure modeled on each principal plane. An incremental anisotropic plasticity model couples damage evolution with inelastic strain, ensuring generality and flexibility across diverse composite constituents. While a failure criterion is used for ultimate strength evaluation, the framework also permits implementation of a crack-band theory to simulate post-ultimate degradation.

The simulated examples demonstrated the predictive capability and flexibility of the proposed framework in modeling the complex behavior of two high-performance composite systems: IM7/977-3 and C/C–SiC. These systems pose distinct challenges in microstructural architecture, constituent behavior, and damage evolution, yet the model effectively captured the key mechanisms governing their mechanical response.

For IM7/977-3, the framework reproduced the nonlinear stress–strain behavior in agreement with experimental data, capturing matrix-dominated damage, stiffness degradation, and inelastic strain accumulation. The modularity of the framework allowed damage and plasticity mechanisms to be selectively activated, offering insight into the underlying physics. Its computational efficiency further enabled parametric studies to guide material optimization. Specifically, the computational efficiency of the PHFGMC-EDI framework arises from its micromechanical formulation, solving a compact system of $21 \times N_c$ governing equations per RUC, where N_c is the number of subcells (typically 1000–5000 for complex triply periodic composites). This yields far fewer degrees of freedom than equivalent FE models with comparable microstructural resolution. Benchmarks indicate that PHFGMC

runtime makes up at most 50% of displacement-based FE simulations under nonlinear loading while preserving accuracy in local fields and responses [5]. Efficiency is boosted by parametric mapping, enabling flexible arbitrary phase geometries with fewer subcells than orthogonal HFGMC, thus cutting assembly and solution times.

For the C/C–SiC composite, the complex three-phase microstructure—comprising carbon fibers, amorphous carbon, and SiC matrix—was explicitly modeled using a representative unit cell informed by microstructural data and literature. The model successfully captured progressive mechanical degradation and crack closure behavior under cyclic loading. Importantly, it also captured tension–compression asymmetry, a hallmark of such composites. The use of a logistic damage evolution law facilitated a smooth, continuous transition to failure.

Results showed convergence under mesh refinement—an essential aspect of damage simulation. Unlike phase-field methods that require energy decomposition to isolate tensile damage, the PHFGMC–EDI approach models tension-driven failure directly by calibrating compression thresholds.

In summary, the PHFGMC–EDI framework provides a versatile and high-fidelity tool for simulating the nonlinear behavior of advanced composite materials. Its generalized 3D formulation, integration of scalar-field damage modeling with energy potential-based thermodynamics, orthotropic representation, and selectively activated inelasticity mechanisms make it well-suited for capturing complex multi-phase, multi-mechanism responses. Future work will focus on extending the framework to high-temperature applications, incorporating viscoplastic effects, and applying it to novel architected composite systems.

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