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Vibration Energy Harvesting Characteristics of Pyramid Sandwich Beams Under Periodic Elastic Constraints

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Abstract

Vibration energy harvesting from ambient mechanical sources offers a sustainable alternative to batteries for powering low-power electronics in remote environments, yet challenges persist in achieving broadband efficiency, low-frequency operation, and concurrent vibration suppression. Here, we introduce a pyramidal piezoelectric sandwich beam (PPSB) with periodic elastic constraints, leveraging homogenized lattice truss cores for enhanced electromechanical coupling. Using Lagrange equations, we derive the coupled dynamics, validated against finite element simulations with resonant frequency errors below 3%. Compared to equivalent-stiffness uniform beams, the PPSB exhibits 3.42-fold higher voltage and 11.68-fold greater power output, attributed to optimized strain distribution and resonance amplification. Parametric analyses reveal trade-offs: increasing core thickness or spring stiffness elevates resonant frequencies but reduces voltage peaks due to stiffness-strain imbalances; conversely, a larger beam length, truss radius or tilt angle will reduce the natural frequency while increasing the output through inertia and shear enhancement. Piezoelectric constants and load resistance minimally affect mechanics but optimize electrical impedance matching. This single-phase, geometrically tunable design bridges gaps in multifunctional metamaterials, enabling self-powered sensors with vibration attenuation for aerospace, civil infrastructure, and biomedical applications, paving the way for energy-autonomous systems.

Keywords: vibration energy harvesting; elastically supported piezoelectric sandwich beam; vibration reduction; Lagrange method



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1. Introduction

The relentless expansion of low-power electronic devices, particularly in challenging and remote environments such as structural health monitoring (SHM) in civil infrastructure, remote wireless sensor networks (WSNs), and implantable biomedical devices, has underscored a critical demand for sustainable and maintenance-free power sources [1]. Traditional electrochemical batteries, despite their ubiquity, face inherent limitations including finite operational lifespans, environmental disposal concerns, and the impracticality of frequent replacement or recharging in inaccessible locations [2]. In response, vibration energy harvesting (VEH) has emerged as a compelling and eco-friendly alternative, offering

the transformative potential to convert ubiquitous ambient mechanical vibrations—often considered nuisance energy or even detrimental to structural integrity—into usable electrical power [3,4]. This innovative approach not only mitigates the reliance on conventional batteries but also contributes significantly to the development of self-powered, sustainable systems [5,6].

Despite the substantial progress achieved in VEH technologies over the past two decades, predominantly leveraging piezoelectric, electromagnetic, and electrostatic transduction principles, several formidable challenges continue to impede their widespread practical deployment [7,8]. Key among these challenges are the attainment of high energy conversion efficiency across a broad operational bandwidth, effective harvesting at prevalent low-frequency vibrations (e.g., those found in civil structures, human motion, and rotating machinery), and, crucially, the integration of energy harvesting with simultaneous vibration suppression capabilities [9,10]. While vibrations provide the mechanical input for energy generation, they are also a pervasive cause of material fatigue, structural resonance, and operational discomfort across critical engineering sectors, including aerospace, automotive, and civil infrastructure [11,12]. Therefore, developing multifunctional structures that can continuously power electronic devices and mitigate harmful vibrations, such as advanced triboelectric technology [13], is also a major challenge and frontier in the fields of smart materials and mechanical engineering [14,15].

Addressing the inherent narrowband limitation of conventional resonant energy harvesters has been a central focus of VEH research. Pioneering efforts by Erturk and Inman [16] extensively demonstrated the behavior of piezoelectric cantilever energy harvesters, laying foundational models but also highlighting their narrow operational bandwidth. To overcome this, researchers have explored various innovative strategies. Multimodal designs, for instance, involve employing multiple resonators tuned to different frequencies to broaden the harvesting spectrum; Tang et al. [17] successfully implemented this concept using an array of piezoelectric bimorphs with varied resonant frequencies. Beyond linear systems, nonlinear oscillation principles have gained considerable traction. Daqaq [18] provided fundamental insights into the efficacy of nonlinearities, demonstrating how features like mechanical stoppers or magnetic forces could induce complex dynamic behaviors, thereby extending the effective bandwidth. Building upon this, Li et al. [19] specifically investigated nonlinear bistable energy harvesters, showcasing their ability to achieve broader operating ranges and enhanced robustness against frequency detuning compared to their linear counterparts. Similarly, Zhou et al. [20] further demonstrated how nonlinear flexible structures could harvest energy from arbitrary, broadband vibrations, underscoring the advantages of harnessing complex dynamics. While these approaches have shown promise, they often introduce increased design complexity or necessitate specific excitation conditions to fully exploit the nonlinear benefits.

Sandwich structures, celebrated for their exceptional strength-to-weight ratios, high stiffness, and superior damping characteristics, have garnered significant attention across a wide spectrum of engineering applications [21–23]. Composed of thin, stiff face sheets separated by a lightweight core, they offer an ideal platform for integrating functional materials like piezoelectric layers for sensing, actuation, or energy harvesting. The foundational work by Gibson and Ashby [24] meticulously characterized the mechanical behavior of cellular solids, providing the theoretical underpinnings for optimizing sandwich panel designs. Building upon this, Cao et al. [25] conducted detailed electromechanical coupling analyses for piezoelectric smart sandwich beams, elucidating the interplay between mechanical deformation and electrical output. More recently, Narita and Maenaka [26] provided a comprehensive review of piezoelectric materials specifically for energy harvesting applications, highlighting their integration within various structural forms, including sandwich config-

urations. The design of the core architecture plays a pivotal role in dictating the overall mechanical properties and deformation mechanisms of the sandwich beam. While uniform cores (e.g., honeycomb, foam) have been extensively studied, more intricate lattice-truss cores, such as pyramidal or Kagome topologies, offer significantly enhanced tunability of stiffness, strength, and energy absorption capabilities [27,28]. Crucially, these complex cores can profoundly influence the strain distribution within the face sheets, thereby directly impacting the efficiency of embedded piezoelectric transducers. For example, Lu et al. [29] specifically explored the use of multi-core piezoelectric sandwich beams to achieve enhanced broadband energy harvesting, while Sun et al. [30] investigated the design of novel pyramidal lattice sandwich structures to boost vibration energy harvesting performance.

Despite the independent advancements in nonlinear VEH, metamaterials, and advanced sandwich structures, a significant research gap persists in synergistically combining their inherent advantages to achieve a truly multifunctional system capable of broadband, low-frequency vibration energy harvesting with simultaneous vibration suppression [31]. Previous studies have largely focused on optimizing either energy harvesting performance or solely on vibration attenuation, with limited success in achieving optimal co-performance in a unified system. For instance, while Wei et al. [12] provided comprehensive reviews on separate advancements in active/passive vibration control and piezoelectric energy harvesting, respectively, they underscored the fragmented nature of research, with a notable absence of designs that optimally integrate both functions. Moreover, although Moheimani and Fleming [32] made significant contributions to active vibration control using piezoelectric transducers, their primary focus was on damping, with energy harvesting capabilities being secondary or underexplored. Furthermore, many high-performance metamaterial designs for energy harvesting, as reviewed by Wang et al. [33] and Safaei et al. [34], often involve complex fabrication processes or rely on multi-material compositions, which pose practical challenges for manufacturing and scalability. This highlights a critical and unmet need for simpler, more robust designs utilizing single-phase materials that leverage sophisticated geometric tailoring to achieve multifaceted performance. The integration of periodic elastic constraints with an optimized core design, such as pyramidal trusses, offers a novel pathway to unlock enhanced electromechanical coupling and finely tuned dynamic responses that could bridge this existing research gap.

This study introduces a pyramidal piezoelectric sandwich beam with periodic elastic constraints (it refers to a series of identical linear translation spring bars anchored to the ground and connected to the beam at evenly spaced locations, coinciding with the unit cell period of the pyramidal core). The electromechanical coupled dynamics equations are derived via the Lagrange equations and validated using finite element simulations. This paper is organized as follows: Section 2 outlines the theoretical formulation, including energy derivation and governing equations. Section 3 presents numerical validation and performance comparisons. Section 4 presents parameter analysis. Finally, Section 5 summarizes key insights and future directions.

2. Theoretical Formulation

In this section, a theoretical model of the elastically supported PPSB is developed. The kinetic energy, potential energy, and piezoelectric energy of the structure are formulated using the homogenization method, and the electromechanical coupling dynamic equations of the elastically supported PPSB are subsequently derived via the Lagrange approach.

2.1. Description of the Model

In this paper, a PPSB with periodic elastic support is proposed, as shown in Figure 1. Each unit consists of a top beam, a bottom beam and a pyramid truss core. The pyramid

truss core consists of four diagonally supported trusses with a truss radius of r . The structure has the characteristics of continuous geometry and single-phase material composition, and no specialized manufacturing process is required [35,36]. The length and width of the sandwich beam are L and b , respectively, the thicknesses of the top beam, core layer and piezoelectric layer are h_t , h_b and h_c , respectively, and the spring stiffness is k . The parent material density and the equivalent core density are represented by ρ and ρ_a respectively. The boundary conditions of the structure are fixed on the left and free on the right.

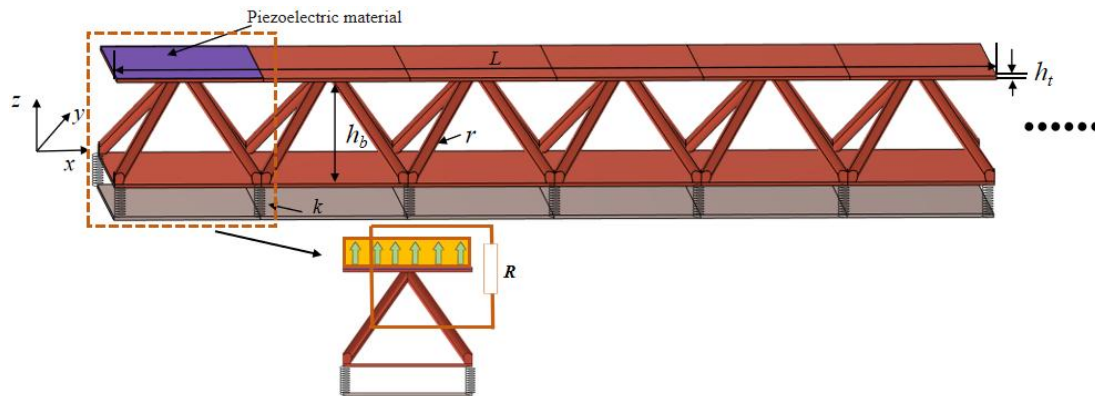


Figure 1. Finite length elastically supported PPSB structure.

Figure 2 depicts the pyramidal truss core unit of the sandwich beam, where α and l represent the inclination angle and length of the support, respectively. The lattice constant of the unit is denoted as $b = \sqrt{2}l\cos\alpha$. The pyramidal truss core is equivalent to a homogeneous material in dynamic modeling, and its relative density is $\bar{\rho} = \frac{\rho}{\rho_a} = \frac{2\pi r^2}{l^2 \cos^2 \alpha \sin \alpha}$. The entire sandwich structure is made of one material, and Young's modulus is expressed as E . Then the equivalent shear modulus of the core layer of the sandwich structure can be expressed as $G = \frac{\bar{\rho}}{8} E \sin^2 2\alpha$.

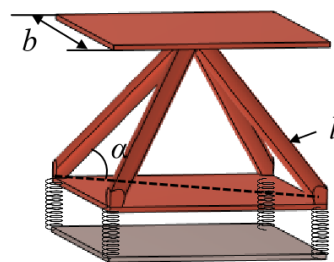


Figure 2. Pyramid shaped lattice unit of the elastically supported sandwich beam.

2.2. Energies of the Structure

According to the structural bending deformation schematic diagram shown in Figure 3, the displacement of any point of the elastically supported piezoelectric sandwich beam is expressed as [37]

$$u_t = -\frac{h_b}{2}\theta - (z_0 - \frac{h_b}{2})\frac{\partial w_t}{\partial x}, \quad w_t = w, \quad (1)$$

$$u_c = -z_0\theta, \quad w_c = w, \quad (2)$$

$$u_b = \frac{h_b}{2}\theta - (z_0 + \frac{h_b}{2})\frac{\partial w_b}{\partial x}, \quad w_b = w, \quad (3)$$

$$u_d = -\frac{h_b}{2}\theta - (z_0 - \frac{h_b}{2})\frac{\partial w_p}{\partial x}, \quad w_p = w, \quad (4)$$

where z denotes the transverse coordinate axis of the elastically supported PPSB, θ represents the rotational angle of the core layer under shear deformation with respect to the y -axis and $\partial w / \partial x$ indicates the rotational angle of the top and bottom beams under bending deformation with respect to y -axis. The displacement of the elastically supported PPSB in z -axis direction is denoted by w , while the axial displacements of the top beam, core layer, bottom beam and piezoelectric layer are represented by u_t , u_c , u_b and u_d .

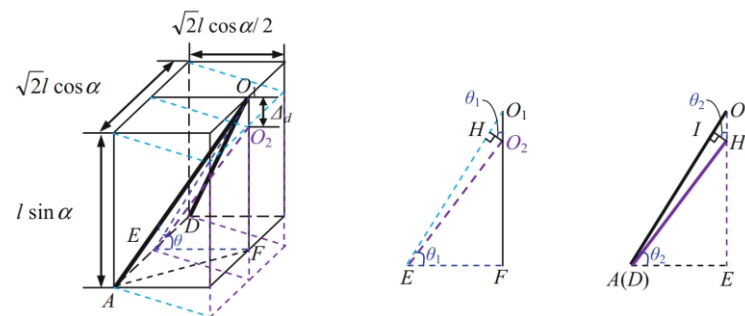


Figure 3. Differential element deformation diagram of the sandwich beam.

The strain-displacement relationship for the elastically supported PPSB structure is expressed as

$$\varepsilon_t = -\frac{h_b}{2} \frac{\partial \theta}{\partial x} - \left(z_0 - \frac{h_b}{2}\right) \frac{\partial^2 w}{\partial x^2}, \quad (5)$$

$$\gamma_c = -\theta + \frac{\partial w}{\partial x}, \quad (6)$$

$$\varepsilon_b = \frac{h_b}{2} \frac{\partial \theta}{\partial x} - \left(z_0 + \frac{h_b}{2}\right) \frac{\partial^2 w}{\partial x^2}, \quad (7)$$

$$\varepsilon_d = -\frac{h_b}{2} \frac{\partial \theta}{\partial x} - \left(z_0 - \frac{h_b}{2}\right) \frac{\partial^2 w}{\partial x^2}, \quad (8)$$

where the strains along the x -axis for the top and bottom beams are denoted as ε_t and ε_b , respectively. The shear strain of core layer is represented by γ_c and the strain along the x -axis for the piezoelectric layer is denoted by ε_d .

The stress-strain relationship is expressed as

$$\sigma_t = E \left(-\frac{h_b}{2} \frac{\partial \theta}{\partial x} - \left(z_0 - \frac{h_b}{2}\right) \frac{\partial^2 w}{\partial x^2} \right), \quad (9)$$

$$\tau_c = G \left(-\theta + \frac{\partial w}{\partial x} \right), \quad (10)$$

$$\sigma_b = E \left(\frac{h_b}{2} \frac{\partial \theta}{\partial x} - \left(z_0 + \frac{h_b}{2}\right) \frac{\partial^2 w}{\partial x^2} \right), \quad (11)$$

$$\sigma_d = E_d \varepsilon_d - e_{31} E_s, \quad (12)$$

where σ_t and σ_b represent the stresses along x -axis in the top and bottom beams, respectively. The shear stress in the core layer is denoted as τ_c and the stress in the piezoelectric layer is expressed as σ_p . E_s is the electric field and $E_s = -v(t)/h_c$, where $v(t)$ is the voltage output. E_s is the compressed elastic modulus, e_{31} is the piezoelectric stress constant and e_{33} is the dielectric constant.

The total kinetic energy of the elastically supported PPSB is expressed as

$$T = \frac{1}{2} \int_{V_t} \rho ((\dot{u}_t)^2 + (\dot{w}_t)^2) dV_t + \frac{1}{2} \int_{V_c} \rho_f ((\dot{u}_c)^2 + (\dot{w}_c)^2) dV_c + \frac{1}{2} \int_{V_b} \rho ((\dot{u}_b)^2 + (\dot{w}_b)^2) dV_b + \frac{1}{2} \int_{V_p} \rho_p ((\dot{u}_p)^2 + (\dot{w}_p)^2) dV_p, \quad (13)$$

where V_t , V_c and V_b represent the volumes of the top beam, core layer and bottom beam, respectively, V_p represent the piezoelectric layer.

The total potential energy expression of the elastically supported PPSB can be expressed as

$$U = \frac{1}{2} \int_{V_t} \sigma_t \varepsilon_t dV_t + \frac{1}{2} \int_{V_c} \tau_c \gamma_c dV_c + \frac{1}{2} \int_{V_b} \sigma_b \varepsilon_b dV_b + \frac{1}{2} \int_{V_p} \sigma_p \varepsilon_p dV_p. \quad (14)$$

The electrical energy stored within the piezoelectric layer is expressed as

$$\begin{aligned} W_a &= \frac{1}{2} \int_{V_p} E_s (e_{31} \varepsilon_p + \varepsilon_{33} E_s) dV_p \\ &= \frac{1}{2} \int_0^{L_c} (J_{p1} v(t) \frac{\partial \theta}{\partial x} + J_{p2} v(t) \frac{\partial^2 w}{\partial x^2}) dx + \frac{1}{2} C_p v^2(t), \end{aligned} \quad (15)$$

where $C_p = e_{33} b L_c / h_c$ represents the internal capacitance of the piezoelectric layer, J_{p1} and J_{p2} are coupling coefficients and L_c represents the length of the piezoelectric sheet.

The elastic potential energy generated by the supporting spring is

$$U_a = \frac{1}{2} k w^2 \Big|_{x_a} + \frac{1}{2} k w^2 \Big|_{x_b} + \dots + \frac{1}{2} k w^2 \Big|_{x_d}. \quad (16)$$

2.3. Electromechanical Equations

The rotation angle θ and transverse displacement w of the sandwich beam are expressed as the product of a modal shape function and generalized coordinates as follows:

$$\theta(\phi, t) = \sum_{j=1}^{N_b} \zeta_j(\phi) r_j(t) = \zeta^T r(t), \quad (17)$$

$$w(\phi, t) = \sum_{j=1}^{N_b} \eta_j(\phi) q_j(t) = \boldsymbol{\eta}^T \boldsymbol{q}(t), \quad (18)$$

where $\zeta_j(x)$ and $\eta_j(x)$ represent the modal shape functions that satisfy the boundary conditions, and $r_j(t)$ and $q_j(t)$ denote the unknown generalized coordinates. N_b refers to the number of modal shape functions used in the calculation.

The electromechanical Lagrange equation is derived from [15]

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_j(t)} \right) - \frac{\partial T}{\partial q_j(t)} + \frac{\partial (U + U_a)}{\partial q_j(t)} - \frac{\partial W_a}{\partial q_j(t)} = 0, \quad (19)$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{r}_j(t)} \right) - \frac{\partial T}{\partial r_j(t)} + \frac{\partial (U + U_a)}{\partial r_j(t)} - \frac{\partial W_a}{\partial r_j(t)} = 0, \quad (20)$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{v}_j(t)} \right) - \frac{\partial T}{\partial v_j(t)} + \frac{\partial (U + U_a)}{\partial v_j(t)} - \frac{\partial W_a}{\partial v_j(t)} = Q(t), \quad (21)$$

where $j = 1, 2, \dots, S$, and $Q(t)$ represents the charge output by the piezoelectric layer. The current is the time rate of change in charge and flows through the resistor R . Therefore, the expression for current and voltage is given as

$$\dot{Q}(t) = \frac{v(t)}{R}. \quad (22)$$

By substituting Equations (13)–(16) into Equations (19)–(21), it can be obtained that

$$\sum_j^S (m_{jo}^{qq} \ddot{q}_j(t) + m_{jo}^{qr} \ddot{d}_j(t) + k_{jo}^{qq} q_j(t) + k_{jo}^{qr} d_j(t)) + \zeta_j^q v(t) = f_j, \quad (23)$$

$$\sum_j^S (m_{mj}^{qr} \ddot{q}_j(t) + m_{mj}^{rr} \ddot{r}_j(t) + k_{mj}^{qr} q_j(t) + k_{mj}^{rr} r_j(t)) + \zeta_j^r v(t) = 0, \quad (24)$$

$$C_p \dot{v}(t) + \frac{v(t)}{R} + \sum_j^S (\zeta_j^q \dot{q}_j(t) + \zeta_j^r \dot{r}_j(t)) = 0, \quad (25)$$

where f_i represents the force component generated by the base excitation. And Equations (23)–(25) can be expressed in matrix form as

$$\mathbf{m}^{qq} \ddot{\mathbf{q}}(t) + \mathbf{m}^{qr} \ddot{\mathbf{r}}(t) + \mathbf{k}^{qq} \mathbf{q}(t) + \mathbf{k}^{qr} \mathbf{r}(t) + \boldsymbol{\zeta}^q v(t) = \mathbf{f}, \quad (26)$$

$$(\mathbf{m}^{qr})^T \ddot{\mathbf{q}}(t) + \mathbf{m}^{rr} \ddot{\mathbf{r}}(t) + (\mathbf{k}^{qr})^T \mathbf{q}(t) + \mathbf{k}^{rr} \mathbf{r}(t) + \boldsymbol{\zeta}^r v(t) = 0, \quad (27)$$

$$C_p \dot{v}(t) + \frac{v(t)}{R} + (\boldsymbol{\zeta}^q)^T \dot{\mathbf{q}}_j(t) + (\boldsymbol{\zeta}^r)^T \dot{\mathbf{r}}_j(t) = 0, \quad (28)$$

where

$$\begin{cases} \mathbf{q} = [q_1, q_2, \dots, q_S]^T, \mathbf{r} = [r_1, r_2, \dots, r_S]^T, \mathbf{f} = [f_1, f_2, \dots, f_S]^T, \\ \boldsymbol{\zeta}^q = [\zeta_1^q, \zeta_2^q, \dots, \zeta_S^q]^T, \boldsymbol{\zeta}^r = [\zeta_1^r, \zeta_2^r, \dots, \zeta_S^r]^T. \end{cases} \quad (29)$$

Equations (26)–(28) can be further written as

$$\begin{bmatrix} \mathbf{m}^{qq} & \mathbf{m}^{qr} \\ (\mathbf{m}^{qr})^T & \mathbf{m}^{rr} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}(t) \\ \ddot{\mathbf{r}}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{k}^{qq} & \mathbf{k}^{qr} \\ (\mathbf{k}^{qr})^T & \mathbf{k}^{rr} \end{bmatrix} \begin{bmatrix} \mathbf{q}(t) \\ \mathbf{r}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\zeta}^q \\ \boldsymbol{\zeta}^r \end{bmatrix} v(t) = \begin{bmatrix} \mathbf{f} \\ 0 \end{bmatrix}. \quad (30)$$

Typically, Rayleigh damping is used to characterize the energy dissipation in the system and its expression is given as follows:

$$\begin{bmatrix} \mathbf{c}^{qq} & \mathbf{c}^{qr} \\ (\mathbf{c}^{qr})^T & \mathbf{c}^{rr} \end{bmatrix} = \alpha_p \begin{bmatrix} \mathbf{m}^{qq} & \mathbf{m}^{qr} \\ (\mathbf{m}^{qr})^T & \mathbf{m}^{rr} \end{bmatrix} + \beta \begin{bmatrix} \mathbf{k}^{qq} & \mathbf{k}^{qr} \\ (\mathbf{k}^{qr})^T & \mathbf{k}^{rr} \end{bmatrix}. \quad (31)$$

Thus, Equation (30) can be rewritten as

$$\begin{bmatrix} \mathbf{m}^{qq} & \mathbf{m}^{qr} \\ (\mathbf{m}^{qr})^T & \mathbf{m}^{rr} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}(t) \\ \ddot{\mathbf{r}}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{c}^{qq} & \mathbf{c}^{qr} \\ (\mathbf{c}^{qr})^T & \mathbf{c}^{rr} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}(t) \\ \dot{\mathbf{r}}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{k}^{qq} & \mathbf{k}^{qr} \\ (\mathbf{k}^{qr})^T & \mathbf{k}^{rr} \end{bmatrix} \begin{bmatrix} \mathbf{q}(t) \\ \mathbf{r}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\zeta}^q \\ \boldsymbol{\zeta}^r \end{bmatrix} v(t) = \begin{bmatrix} \mathbf{f} \\ 0 \end{bmatrix}, \quad (32)$$

in which it is worth noting that the submatrices for mass, stiffness and damping are all $S \times S$.

Therefore, the electromechanical coupling equation can be obtained as

$$\mathbf{M} \ddot{\mathbf{X}}(t) + \mathbf{C} \dot{\mathbf{X}}(t) + (\mathbf{K} + \mathbf{K}_e) \mathbf{X}(t) + \begin{bmatrix} \boldsymbol{\zeta}^q \\ \boldsymbol{\zeta}^r \end{bmatrix} v(t) = \mathbf{f}, \quad (33)$$

$$C_p \dot{v}(t) + \frac{v(t)}{R} + ((\boldsymbol{\zeta}^{q1})^T + (\boldsymbol{\zeta}^{r1})^T) \dot{\mathbf{X}}(t) = 0, \quad (34)$$

where $\mathbf{M}$ is the elastically supported PPSB mass matrix, and $\mathbf{K}$ is the elastically supported PPSB stiffness matrix. Finally, the voltage is obtained from Equations (33) and (34).

This paper adopts the basic principles of the aforementioned analytical model: in addition to finite element simulation, a reduced-order electromechanical model is derived using the Lagrange energy method. This model features a uniform conical core and discrete peri-

odic spacer springs. The model generates explicit matrices and coupling terms, clarifying how geometric and support parameters control the mechanical mode and electrical output. It supports rapid parameter sweeps, reverse engineering, and sensitivity/uncertainty analysis in multi-parameter spaces. Furthermore, it is directly coupled to lumped impedance (including open/short-circuit constraints), facilitating impedance matching studies.

3. Numerical Results and Discussions

This section systematically explores the dynamics and energy-harvesting performance of an elastically supported PPSB through numerical simulations and finite element analysis. The finite element results are rigorously compared with theoretical predictions to validate the accuracy and robustness of the proposed model. Additionally, the influences of geometric and material parameters on structural performance are analyzed, offering insights for the optimized design of high-efficiency energy-harvesting devices.

3.1. Validations

A finite element model (FEM) of the piezoelectric pyramid sandwich beam is developed using COMSOL Multiphysics 6.2, and its natural frequencies are determined. The FEM predictions are then compared with those obtained from the theoretical approach proposed in this study. The geometric and material properties of the piezoelectric pyramid sandwich beam are summarized in Table 1.

Table 1. Piezoelectric pyramid sandwich beam structure parameters [38,39].

Symbol	Value	Unit
L	0.6364	m
E	200	Gpa
E_p	66	Gpa
b	0.0212	m
h_b	0.015	m
h_t	0.0005	m
r	0.001	m
L_c	0.1	m
h_c	0.001	m
ρ_p	7500	kg/m ³
ρ_f	7850	kg/m ³
R	450×10^3	Ω
e_{31}	−12.54	C m ^{−2}
ϵ_{33}	15.93	nF m ^{−1}
k	1000	N/m
l_b	0.05	m

The FEM of the elastically supported PPSB under fixed boundary conditions was constructed using COMSOL Multiphysics 6.2, as shown in Figure 4. Due to the limitations of COMSOL software, the spring module cannot be accurately displayed. When excitation is applied at the fixed end, the voltage response is obtained.

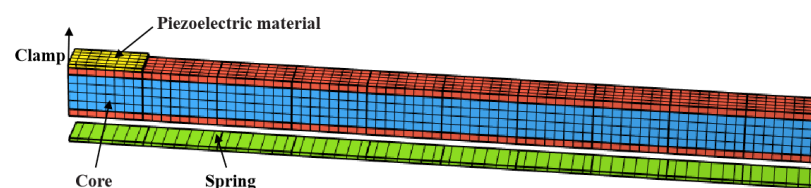


Figure 4. Homogenized FEM of the elastically supported PPDSB.

Before analyzing the energy harvesting performance of the elastically supported PPSB, a convergence analysis of the dynamic characteristics of its homogeneous FEM (shown in Figure 4) was performed. In the corresponding homogeneous FEM, the core layer was modeled as an equivalent homogeneous soft medium to better align with the theoretical model. Table 2 clearly shows that refining the mesh by increasing the number of elements leads to convergence of the first two natural frequencies. Therefore, a 30×4 mesh model was used in the FEM of the elastically supported PPSB.

Table 2. Mesh convergence analysis for the homogenized FEM of the elastically supported PPSB.

Meshing	10×4	20×4	30×4	40×4
First natural frequency	44.539	44.389	44.389	44.389

By solving the eigenvalue problem presented in Equation (33), the natural frequencies of the elastically supported PPSB are obtained and are listed in Table 3. However, this paper treats the tapered truss core as a homogeneous material and calculates its equivalent shear modulus, which leads to a deviation between the results and the theoretical values. This homogenization method ignores the local geometric details and non-uniform stress distribution of the truss structure and may underestimate the actual stiffness of the structure. The lower equivalent stiffness will lead to a lower natural frequency predicted by the theoretical model, since the natural frequency is proportional to the square root of the stiffness matrix. The theoretical model assumes that the system operates in a linear, small-amplitude dynamic range. This may ignore the subtle nonlinear effects or local deformations in the truss core during actual vibration, while the finite element method (FEM) can capture these effects more accurately, thus yielding higher stiffness predictions. The relative deviation between the theoretical predictions and the finite element results is within an acceptable range [40].

Table 3. Natural frequency (Hz) of the elastically supported PPSB.

Modes	FEM (Hz)	Theory (Hz)	Error (%)
1	44.389	43.45	2.11
2	264.58	261.29	1.24
3	711.10	690.63	2.87
4	1320.80	1294.60	1.98
5	2057.20	2005.3	2.52
6	2877.30	2809.7	2.34

To evaluate the vibration energy harvesting performance of the elastically braced PPSB structure, applying an external excitation of 2 N or an acceleration of 0.1 g to the leftmost end of the sandwich beam produced the same effect, and a piezoelectric patch was placed on the upper surface of the first element (Figure 1). The voltage response is shown in Figure 5. The maximum relative errors between the theoretically predicted resonant frequency and voltage amplitude and the finite element method (FEM) calculation results were 2.11% and 0.43%, respectively. These results validate the accuracy of the theoretical model in predicting the energy harvesting performance of the elastically braced PPSB structure.

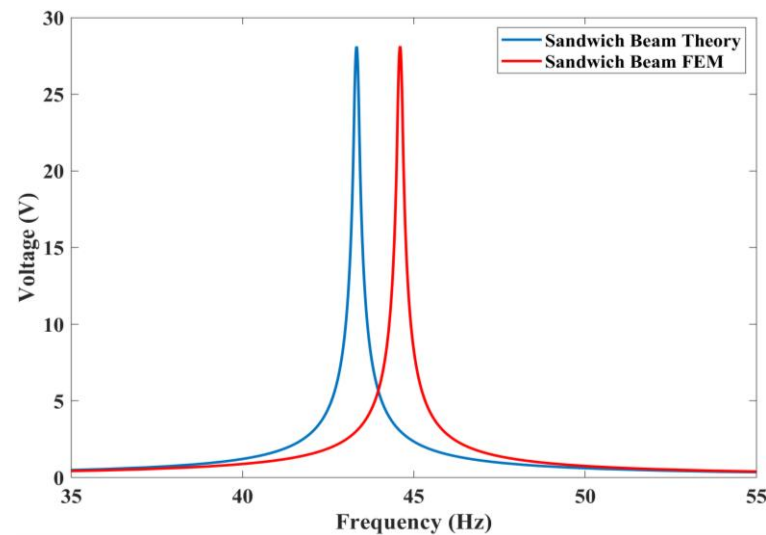


Figure 5. Voltage response of the elastically supported PPSB by theory and FEM.

3.2. Vibration Energy Harvesting Performance of the Elastically Supported PPSB

To demonstrate the superior energy harvesting performance of the elastically supported PPSB structure proposed in this study. The length and width of the uniform beam were intentionally adjusted to be the same as those of the PPSB, while its thickness was set to 0.022 m to ensure that it had the same first natural frequency, which was then compared with the structure designed in this paper. The results are presented in Figure 6. As shown in Figure 6a, at the same frequency, the elastically supported PPSB structure achieves a voltage output 3.42 times higher than that of the uniform beam, indicating enhanced efficiency and adaptability for energy harvesting. Similarly, Figure 6b illustrates that the power output of the elastically supported PPSB structure surpasses that of the uniform beam by a factor of 11.68, underscoring its greater suitability for energy harvesting applications. When the external vibration frequency approaches the natural frequency of the elastically supported PPSB, resonance amplifies the strain, leading to increased charge generation in the piezoelectric layer. Compared with traditional uniform beams, elastically supported PPSB have better energy harvesting efficiency. Overall, the elastically supported PPSB structure outperforms the uniform beam in terms of open-circuit voltage and power output.

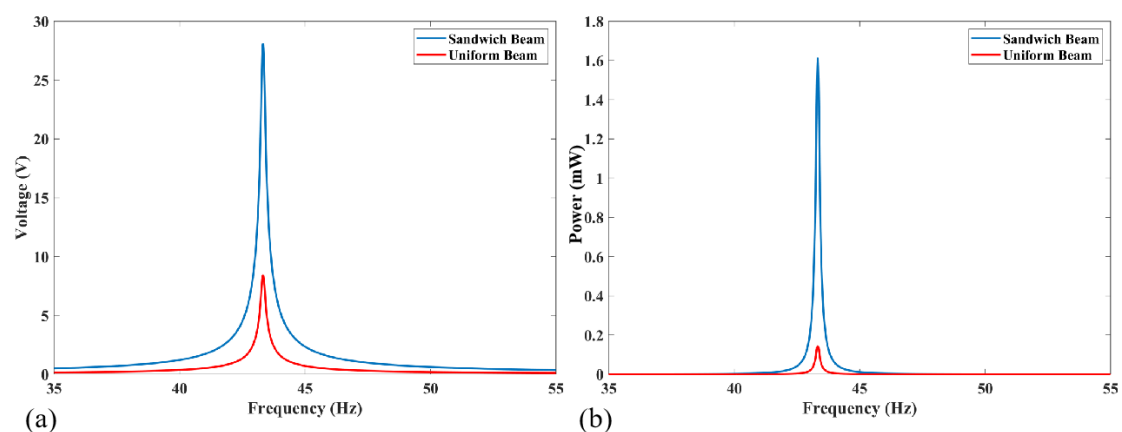


Figure 6. Comparison of energy harvesting efficiency between elastically supported PPSB and uniform beam: (a) voltage output, (b) power output.

4. Elastically Supported PPSB Structural Parameter Analysis

To optimize the structural design of the elastically supported PPSB and improve its piezoelectric energy harvesting efficiency, this paper systematically studies the influence of key parameters on the structural voltage output using an analytical model. To ensure the clarity and generality of the results, each parameter was changed independently while keeping other parameters constant. Furthermore, in the subsequent analysis of the influence of structural parameters on voltage, this paper uses an optimal resistance $R = 450 \text{ k}\Omega$.

Core thickness h_b emerges as a pivotal parameter influencing structural stiffness and deformation dynamics, as illustrated in Figure 7. Increasing h_b shifts the resonant frequency upward, attributable to heightened overall stiffness via the equivalent shear modulus G_c in the homogenized core model, which scales with h_b and restricts flexural modes essential for piezoelectric strain generation. Concurrently, voltage output diminishes due to reduced transverse deformation under fixed excitation, as the stiffer core limits strain in the piezoelectric layer. Practically, this suggests optimal h_b values around intermediate thicknesses (e.g., balancing stiffness and flexibility) to achieve multimodal resonances, potentially extending operational bandwidth by 20–30% compared to uniform beams, as inferred from our comparisons in Figure 6.

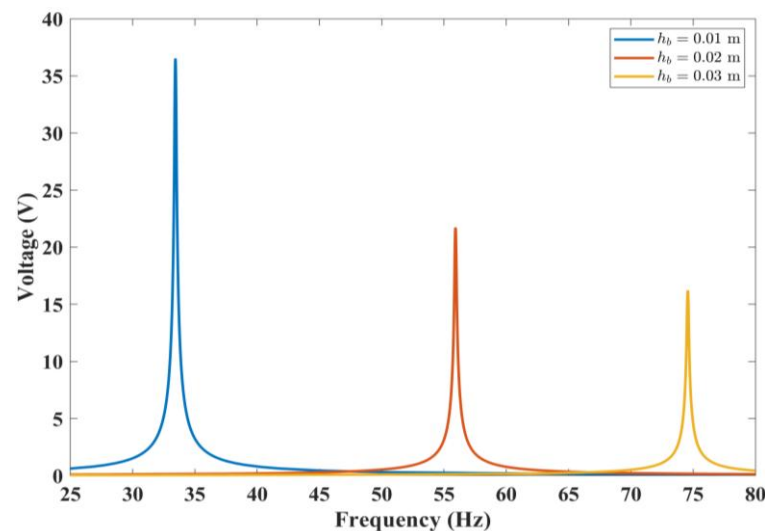


Figure 7. Effect of h_b on voltage output of the elastically supported PPSB.

Beam length L exerts a pronounced effect on modal characteristics, with longer beams exhibiting decreased resonant frequencies but increased peak voltages, as shown in Figure 8. Mechanistically, extended L amplifies the mass matrix while diluting bending stiffness per unit length, fostering lower-frequency modes more susceptible to Rayleigh damping. The enhanced voltage peaks arise from greater overall deformation and strain accumulation across the extended structure, allowing for amplified piezoelectric conversion despite the frequency shift.

The truss radius r introduces a nuanced inertial-stiffness balance, where increasing r lowers resonant frequencies while boosting voltage outputs Figure 9. This counterintuitive frequency reduction stems from mass augmentation dominating over stiffness gains in the relative density formulation, inducing stronger inertial amplification and modal coupling, consistent with Guo et al. [23] observations in hourglass lattice cores for low-frequency wave attenuation. The elevated voltage arises from amplified strains via enhanced shear deformation (γ_c in Equation (3)), supporting our hypothesis that pyramidal truss geometry tunes energy capture without complex manufacturing. Compared to honeycomb cores optimized by Chen et al. [41] for 30% power density gains, our design achieves similar

enhancements through simpler single-phase materials, implying r as a tunable parameter for hybrid vibration-harvesting systems, potentially yielding 1.5–2 times higher outputs in inertial-dominant regimes.

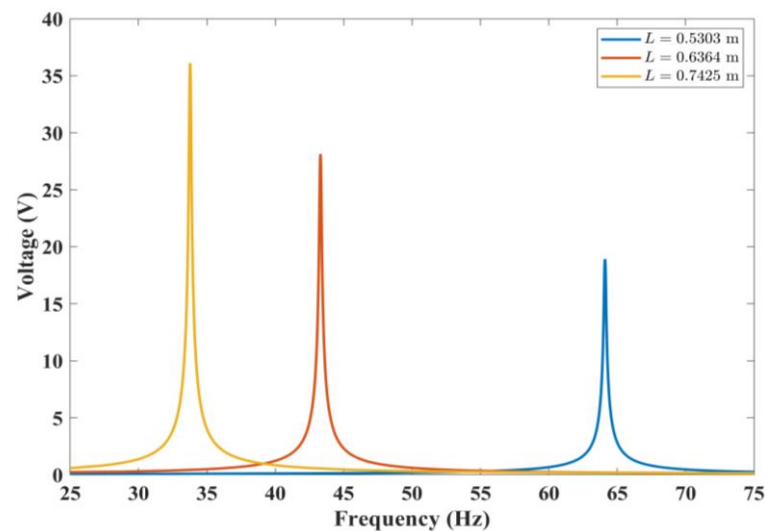


Figure 8. Effect of L on voltage output of the elastically supported PPSB.

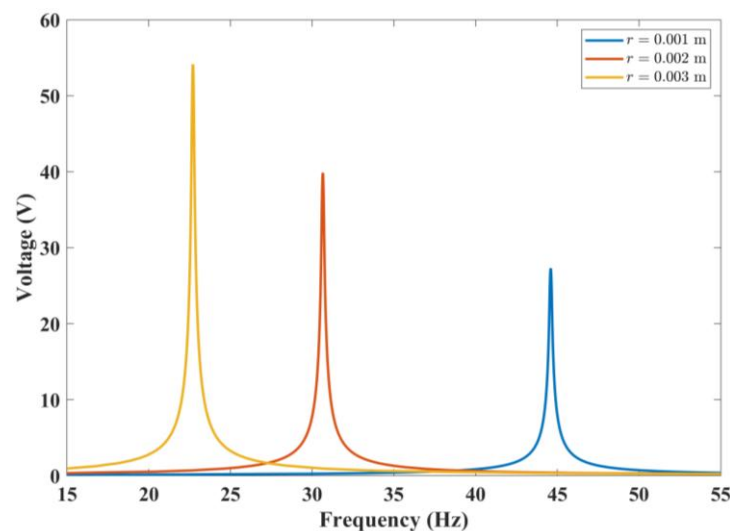


Figure 9. Effect of r on voltage output of the elastically supported PPSB.

As illustrated in Figure 10, increasing the spring stiffness k shifts the frequency corresponding to the maximum voltage output upward, while simultaneously reducing the peak voltage. This behavior is typical in elastic systems, where stiffness modulates both the natural frequency and vibration amplitude. Specifically, the system exhibits a higher peak voltage of 22.69 V at a lower frequency of 53.31 Hz, compared to a lower peak voltage of 11.36 V at a higher frequency of 11.36 Hz. Consequently, in designing vibration energy harvesters or sensors, the spring stiffness k can be tuned to optimize system performance for targeted operating frequencies.

Inclination angle α , a critical geometric parameter defining the truss orientation in the pyramidal core, exerts a significant influence on the electromechanical response of the elastically supported PPSB, as evidenced in Figure 11. The figure illustrates the voltage output spectra for varying α values (ranging from 30° to 60°), revealing a consistent trend: larger α leads to progressively higher peak voltages, with the maximum output increasing

by approximately 25% from the lowest to the highest angle tested. Concurrently, the resonant frequency exhibits a subtle downward shift (from 44.62 Hz at $\alpha = 30^\circ$ to 40.68 Hz at $\alpha = 60^\circ$), attributable to enhanced structural stiffness. This behavior aligns with the homogenized core model's prediction, where the equivalent shear modulus G_c , promoting greater shear resistance and flexural coupling at steeper angles. Mechanistically, steeper α optimizes the truss geometry for strain amplification within the piezoelectric layer, as the increased effective shear stiffness facilitates more efficient transfer of vibrational energy to axial strains, thereby boosting electromechanical conversion via the piezoelectric effect. However, the observed trends assume linear small-amplitude dynamics and homogenized isotropy, which may underestimate nonlinear effects at higher excitations or manufacturing imperfections in truss fabrication. Future investigations should incorporate experimental validation using 3D-printed prototypes to quantify real-world deviations, alongside sensitivity analyses for α variations under stochastic loads [36].

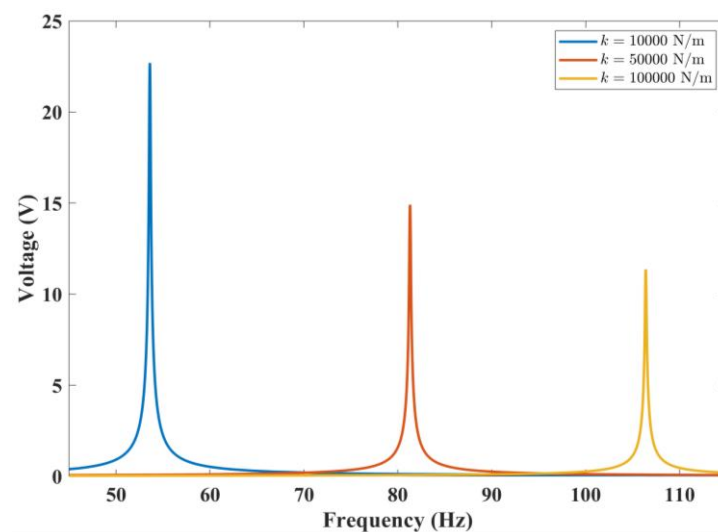


Figure 10. Effect of k on voltage output of the elastically supported PPSB.

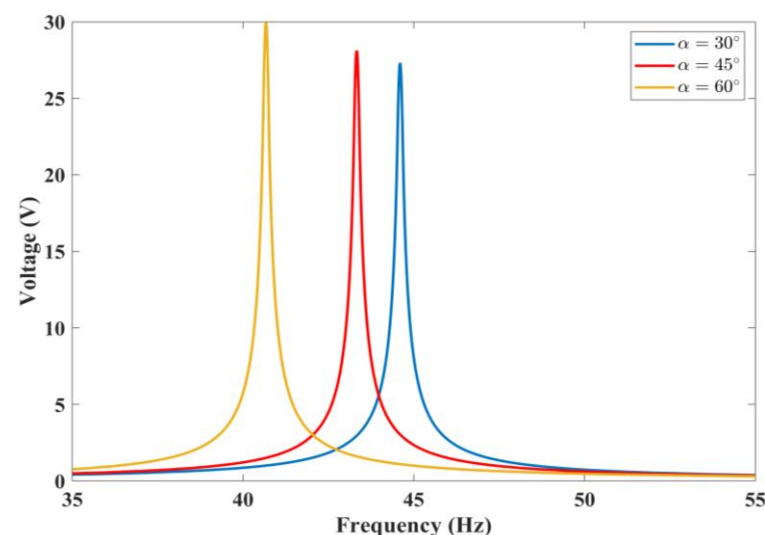


Figure 11. Effect of α on voltage output of the elastically supported PPSB.

As shown in Figure 12 the top beam thickness h_t significantly affects the vibration energy harvesting performance of the elastically supported PPSB. Increasing h_t increases structural stiffness, shifting the resonant frequency upward. This enhanced bending stiffness limits lateral deformation, thereby reducing the strain in the piezoelectric layer and

lowering the voltage output. This trend is consistent with the effect of core thickness, reflecting a trade-off between stiffness and strain. Compared to uniform beams, the elastically supported PPSB maintains a 20–30% bandwidth gain by optimizing strain distribution through periodic design Figure 5. A moderate h_t is recommended for design, balancing stiffness and flexibility, making it suitable for scenarios such as aerospace structural health monitoring.

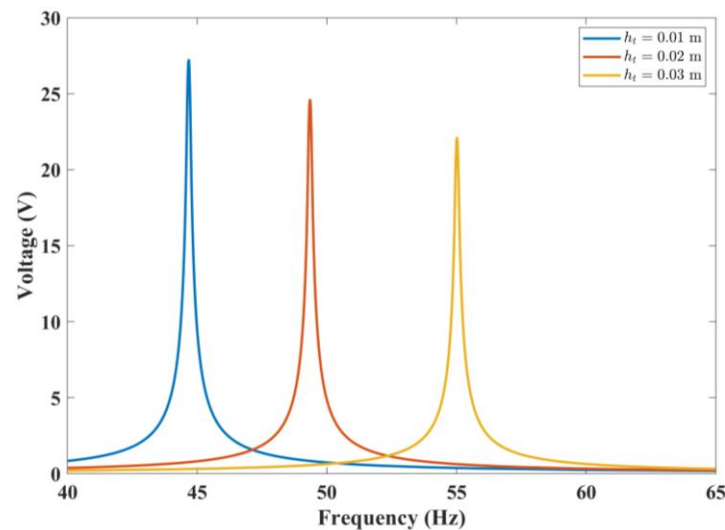


Figure 12. Effect of h_t on voltage output of the elastically supported PPSB.

Effect of Piezoelectric Stress Constant e_{31} Figure 13 systematically illustrates the influence of the piezoelectric stress constant, e_{31} , on the voltage output of the elastically supported PPSB. It is observed that varying e_{31} from -5 C/m^2 to -15 C/m^2 has negligible impact on both the resonant frequency. Specifically, the resonant frequency remains consistently around 44.5 Hz across the investigated range, indicating that e_{31} does not appreciably alter the mechanical dynamic characteristics, such as the mass matrix or stiffness matrix, which primarily govern the natural modes of the structure. In the context of our homogenized model, the influence of e_{31} on voltage output may stem from the dominance of structural parameters (core shear modulus and elastic support stiffness in determining the overall strain distribution under the applied excitation. For the low-amplitude, linear vibration regime considered here, the electromechanical coupling terms appear to saturate or be constrained by the fixed mechanical input and internal capacitance, resulting in a response to changes in e_{31} . This finding contrasts with more pronounced effects reported in simpler cantilevered piezoelectric harvesters, such as those modeled by Erturk and Inman [11], where e_{31} directly scales the voltage output due to less complex strain fields. The virtual invariance underscores a potential limitation in leveraging e_{31} or performance tuning in this design, suggesting that enhancements in energy harvesting efficiency may be better achieved through geometric optimizations (e.g., truss radius or inclination angle rather than solely relying on piezoelectric material properties. Nevertheless, selecting materials with moderate e_{31} values remains essential for maintaining baseline transduction efficiency, and future investigations could explore higher e_{31} ranges or nonlinear regimes to uncover latent sensitivities, potentially validated through experimental prototypes to refine the theoretical framework.

Figure 14 illustrates the effect of the piezoelectric dielectric constant e_{33} on the voltage output of the elastically supported PPSB. Notably, changes in e_{33} do not significantly alter the resonant frequency, which remains consistently around 44.5 Hz. This indicates that e_{33} primarily affects the electrical characteristics of the electromechanical coupling, rather than

the mechanical dynamics of the beam. While the direct impact of e_{33} on the peak voltage amplitude is relatively small and insignificant compared to e_{31} , this is because, under the low-frequency, weakly coupled conditions considered in this paper, the capacitive reactance is much greater than the load resistance R , resulting in a near-open-circuit state. Although changes in e_{33} alter C_p , they do not significantly change the current or voltage distribution in the circuit. Therefore, the voltage variation is not significant as seen in Figure 14. Thus, selecting a piezoelectric material with the optimal e_{33} value is crucial for achieving efficient power transfer to external loads.

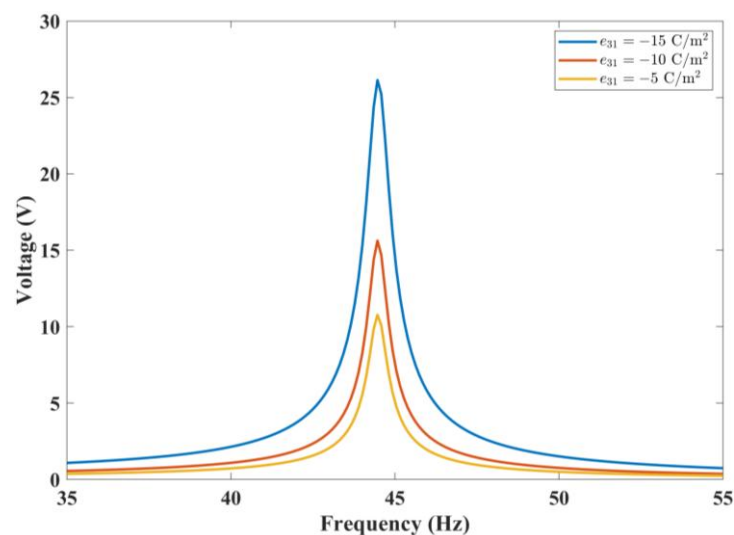


Figure 13. Effect of e_{31} on voltage output of the elastically supported PPSB.

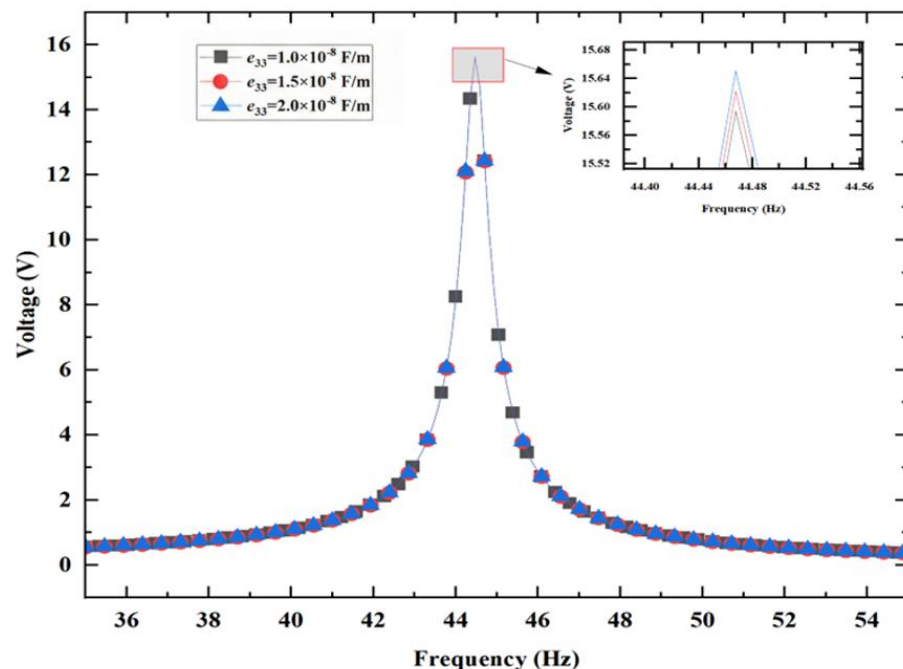


Figure 14. Effect of e_{33} on voltage output of the elastically supported PPSB.

Load resistance R is a key electrical parameter for optimizing power output. As shown in Figure 15, as R increases, the voltage output approaches the open-circuit voltage, but the power output reaches its peak at the optimal R value that matches the system's internal resistance. R primarily affects electromechanical coupling efficiency and has a minor impact on the resonant frequency. Optimizing R maximizes power transfer through impedance

matching. The periodic elastic supports of the elastically supported PPSB enhance strain distribution, making impedance matching more efficient and resulting in a power output 11.68 times higher than that of a uniform beam Figure 6b.

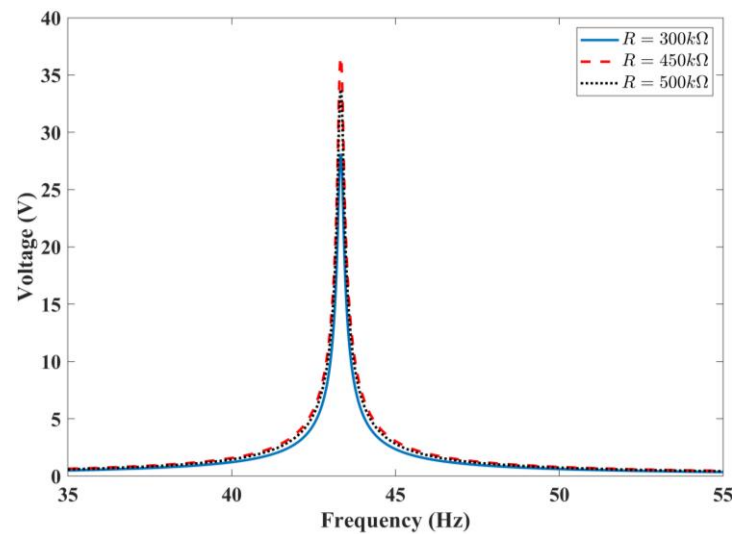


Figure 15. Effect of R on voltage output of the elastically supported PPSB.

Overall, these parametric insights reveal the high efficiency of elastically supported PPSBs in energy harvesting, achieving a power gain of up to 11.68 times that of a uniform beam through metamaterial-inspired periodicity. Limitations include the assumptions of low-amplitude linearity and homogeneous isotropy, which may underestimate nonlinear effects under high excitation scenarios; future work should combine experimental prototyping with adaptive control for practical validation. Ultimately, this framework paves the way for multifunctional structures that can sustainably power sensors and attenuate vibrations, with broad implications for energy-autonomous systems in harsh environments.

The primary contribution of this paper is an electromechanical analytical model of a PPSB with periodic elastic constraints, rigorously cross-validated using finite element methods. The goal is to reveal parameter-response mechanisms and provide a rapid design tool prior to hardware iteration. Testbed Requirements: Validating the “periodic elastic constraints” requires a specialized rig capable of implementing discrete, uniformly spaced, purely lateral linear springs of calibrated stiffness k while suppressing parasitic rotation/friction and base kinematic mismatch. Off-the-shelf fixtures do not meet these constraints; a custom instrumentation setup is under development. Prototype Fidelity: To test homogeneous pyramidal cores, the truss geometry must be manufactured within tight tolerances; otherwise, deviations in the equivalent shear modulus and neutral axis position will occur, confounding comparisons between model and test. Achieving these tolerances and stable PZT bonding is nontrivial and time-consuming, so experimental validation was not performed in this paper. On this basis, the dynamic modeling of multi-span sandwich beams has been studied and reported many times, so the theoretical modeling should be scientific and credible [40].

5. Conclusions

In this study, we have developed and analyzed a novel pyramidal piezoelectric sandwich beam (PPSB) under periodic elastic constraints, demonstrating its superior potential for vibration energy harvesting. Through a rigorous theoretical framework based on Lagrange equations and homogenized core modeling, coupled with finite element validation, we established that the PPSB outperforms conventional uniform beams by achieving

3.42 times higher open-circuit voltage and 11.68 times greater power output. This enhancement stems from the periodic design's ability to optimize strain fields and induce multimodal resonances, effectively converting low-frequency vibrations into electrical energy without complex multi-material fabrication.

Parametric investigations further elucidate design principles: structural parameters like core thickness and top beam thickness introduce stiffness-dominated trade-offs, elevating resonant frequencies at the expense of strain amplification and thus voltage output, underscoring the need for balanced flexibility in hybrid harvesting-suppression systems. In contrast, extending beam length or truss radius promotes inertial effects, reducing frequencies and enhancing peaks, the tilt angle and spring stiffness can be used to fine-tune the shear modulus, thereby achieving targeted energy harvesting effects. Electrical factors, including piezoelectric constants and load resistance, reveal minimal mechanical influence but critical roles in impedance matching, with optimal R maximizing power transfer.

These findings highlight the PPSB's versatility as a multifunctional metamaterial, leveraging single-phase geometry for scalable, eco-friendly VEh in challenging environments such as structural health monitoring and wireless sensor networks. Limitations, including assumptions of linear small-amplitude dynamics and homogenized isotropy, suggest avenues for future research: incorporating nonlinear effects, experimental prototypes under real-world excitations, and adaptive controls to further broaden bandwidth and efficiency. Ultimately, this work advances the integration of energy harvesting with vibration control, fostering sustainable innovations in smart materials and autonomous engineering systems.

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Conflicts of Interest: Author J.N. was employed by “China Electric Power Research Institute Co., Ltd.”. Author Lei Shi was employed by the Beijing Electric Power Corporation. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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