



Article

A Rapid Separation Method of Spindle Radial Error Motion Under Cutting Conditions

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Abstract

Measuring the radial error motion of a spindle during the cutting process is important for monitoring the machining quality of the workpiece and spindle running condition. The problem of long separation time encountered in the application of traditional methods is analyzed under cutting conditions. A rapid separation strategy based on step separation is studied. Combining the advantages of the bidirectional measurement and multi-probe methods, a rapid separation method for spindle radial error motion is proposed. Based on the characteristics of the first harmonic components at high and low spindle speeds, an extraction method for the installation eccentricity is developed. The spindle built-in encoder acquisition technology and angle position resampling method are used to reduce the influence of cutting conditions on error separation. Finally, an actual cutting experiment is conducted to verify the effectiveness of the proposed method, and it is shown that the proposed method cannot only achieve rapid separation but also obtain better separation accuracy under cutting conditions.

Keywords: cutting process; rapid separation method; radial error motion; built-in sensor; error separation

1. Introduction

As a key component of a machine tool, the spindle is directly related to the machining accuracy of the machine tool and the machining quality of the workpiece. The radial error motion of a spindle is an important characterization of the spindle running state and is often used in machine tool design, accuracy testing and monitoring [1,2]. In the cutting process, the radial error motion of the spindle does not only provide the basis for monitoring and studying the dynamic characteristics of the spindle, but can also be used for workpiece quality evaluation and control, such as for some rotary parts in the micromachine and aerospace fields [3].

The spindle radial error motion is usually measured when the machine tool is not cutting, in order to obtain a smaller interference and available arrangement space. However, when it is necessary to obtain the operating state of the spindle under the action of the cutting force, especially an unstable cutting force, the measurement results under the non-cutting state can provide a rough judgment but not an accurate judgment. In the actual cutting process, the dynamic kinematic characteristics of the machine tool, weight and installation accuracy of the workpiece or tool, and dynamic cutting force all cause the spindle error motion to change [4,5]. This change cannot be ignored and is even important for studying the dynamic characteristics of the spindle under an external force



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and monitoring the machining error of the workpiece. Therefore, it is necessary to study a spindle error motion measurement method under the cutting conditions.

The spindle radial error motion is measured directly by a displacement sensor measuring against a reference artifact [6]. The displacement sensor signal includes the spindle radial error motion, artifact roundness, and eccentricity error. Therefore, it is necessary to remove these undesired components from the sensor signal and present the true radial error motion [7]. Various measurement methods can be divided into two categories according to the selected reference artifact. First, a precision sphere or cylinder is used as a reference artifact attached to the spindle. Because the artifact roundness is at nanometer accuracy, its contribution in sensor signal is considered negligible [8]. The other, an imprecision cylindrical surface on the spindle is selected as the reference artifact. An error separation technique (EST) is required to separate the radial error motion from the undesired component [9]. In addition, there are also some new methods being explored, such as adaptive Bayesian harmonic-comb reconstruction method and pure vision-based measuring radial error of rotating spindles [10,11]. There are also control methods for regulating radial errors during the machining process after radial error separation [12].

Since 1960, many studies have been conducted on EST, and a variety of separation methods have been proposed. The three classes of error separation techniques have been commonly used, including reversal, multi-step, and multi-probe methods. Donaldson [13] proposed an inverse measure method with the advantages of easy adjustment and a simple algorithm. Since both the probe and artifact need to be rotated in the reverse direction for the next step, its separation accuracy is affected by the angle repositioning errors of the two and the artifact eccentricity.

The multi-step method and reverse method both use a displacement sensor; the difference is that only the sensor or the artifact needs to be rotated in the multi-step method. By rotating the spindle at different starting angles, the combined signals of the spindle error motion and the roundness error with equally spaced initial phase angles are obtained. Fourier transform is used to separate these two types of errors. The problem with this method is that harmonic suppression occurs at the first harmonic component and the harmonic components of integer multiples the number of steps, and the errors of this order cannot be separated, resulting in incomplete error separation. Buajarern et al. [14] studied the influence of the number of steps in the multi-step method on the separation of roundness errors. Haitjema [15] proposed a combined step to enhance error separation and reduce the number of measurement steps required. Increasing the number of measurement steps can reduce the number of harmonic suppressions in the lower order range, but it will introduce more disturbance errors owing to the increase in the number of inversions. In addition, for the reverse and multi-step method, an additional precision indexing table driving the artifact or sensors is necessary, which limits the use of these methods under cutting conditions.

The most notable advantage of the multi-probe method is the realization of on-machine measurements. Much research has been carried out on multi-probe error separation techniques and the three-probe method, which is a typical representative of the multi-probe method and in wide use. Meanwhile some issues related to the method are discussed. The multi-probe method suffers from harmonic suppression, which means that some harmonic components of the roundness error and error motion cannot be effectively separated. It is common to optimize the installation angles of the three sensors to obtain a set of optimized installation angles to mitigate harmonic suppression within an effective range [16]. Wei proposed a hybrid multipoint method, which uses a combination of two displacement sensors and an angle sensor to reduce the occurrence of harmonic suppression. However, this method can only separate one directional component of the spindle radial error motion [17].

Zhang proposed a four-point method to reduce the number of harmonic suppressions by increasing the number of displacement sensors [18]. In addition to the study of the suppressed harmonics, Shi analyzed and classified the non-suppressed harmonic components through the weight function; and proposed a hybrid three-probe method to select the harmonic components far from the suppression from the combination of multiple sets of installation angles to ensure the separation accuracy of each order harmonic component and improve the accuracy of error separation [19]. By adopting a new integrated fixture and establishing an in situ evaluation method, Xu verified the effectiveness of the three-point method in ultra precision spindle radial error measurement [20]. The harmonic order of harmonic suppression can be changed by selecting the appropriate sensor installation angle or increasing the number of sensors, but it cannot be completely avoided. The feasible method is to control no suppression within a certain harmonic range. However, the first-order harmonic suppression problem is inevitable, which means that the first-order harmonic component of the spindle error motion and the eccentricity of the artifact cannot be separated. In the ISO [6] and ANSI/ASME [4] standards, the first-order harmonic component of the spindle error motion is considered to be non-existent, while the first-order harmonic component of the displacement signal is considered to be caused by the installation eccentricity of the measured part. Lu analyzed the generation and existence of the first-order harmonic component of the spindle radial error motion from both theoretical and experimental aspects and proposed a method that can partially separate the first-order harmonic component [21].

Compared with the other two methods, the multi-point method is the only method that can realize online measurement; however, there are few examples of its application in the cutting process. Because there are still many problems when this method is used in the actual cutting process. The first is the calculation time of the separation algorithm. In the monitoring and control of workpiece errors, the measurement procedure is time-consuming and generates a large volume of data. If the error separation algorithm takes a long time and the error separation time greatly exceeds the cutting time, the purpose of quality monitoring and out-of-tolerance warning cannot be achieved. Secondly, under the cutting conditions, the error separation faces the problems of uneven displacement sampling angle interval, small number of periodic sampling points, and inaccurate data interception. When the multi-point method is directly applied, the separation error will increase, and even the result will be wrong. These special factors in the cutting process will hinder the application of the traditional method under the cutting conditions.

Therefore, aiming at these questions, this paper studies the rapid separation method of spindle radial error motion under the cutting conditions. According to the composition of the displacement sensor signal, the separation method and problem of the interference information are analyzed. Aiming at the problem of separation time of traditional error separation method under the cutting conditions, a fast separation strategy is proposed. Combined with the principle of bidirectional method and multi-probe method, a fast separation method of radial rotation error is proposed. The separation method of installation eccentricity error is studied. In order to reduce the influence of cutting force, spindle speed fluctuations, and other factors on the error separation accuracy, the spindle built-in encoder signal acquisition technology and angle position resampling method are adopted. The feasibility and effectiveness of the proposed method is verified by an actual cutting experiment.

The remainder of this paper is organized as follows. In Section 2, the radial error motion of spindle separation method is analytically investigated. In Section 3, aiming at the problem of long separation time of traditional methods, a fast error separation strategy is proposed. In Section 4, a fast separation method of spindle radial rotation error is

proposed, and the key problems such as the separation method of installation eccentricity, the signal acquisition technology of spindle built-in encoder, the angle position resampling method, and the sensor installation angle are studied. In Section 5, cutting experiments are performed to validate this method. Section 6 gives the conclusions.

2. Spindle Radial Error Motion Separation Method Analysis

2.1. Displacement Sensor Measurement Data

Usually when the reference coordinate system is determined, the displacement sensor should be installed in this reference frame to measure the relative motion between the rotor-installed artifact and the reference frame. Figure 1 shows the information included in the displacement sensor measurement data. There are important points. They are the artifact measurement cross-section's center O_e , the rotation center O_r , and the average point O_a . Point O_a is fixed on the reference coordinate plane XY and point O_e and O_r are fixed on the artifact. The axis of rotation radial error motion is the vector from O_a to O_r . The vector from O_r to O_e has constant length and rotates at the same speed and direction as the artifact rotation. The vector from O_e to P represents the roundness deviation of the artifact. When the constant information of indicator measurement is removed, sensor measurement includes both the radial motion of the artifact's geometric center O_e and the roundness deviation. The radial motion of the geometric center O_e consists of a combination of the radial error motion of rotor and the rotation of geometric center relative to the rotation center as artifact rotating. In order to extract the radial error motion of rotor, the artifact's roundness and the eccentricity of the geometric center to rotation center need to be separated from displacement sensor measurement data and removed.

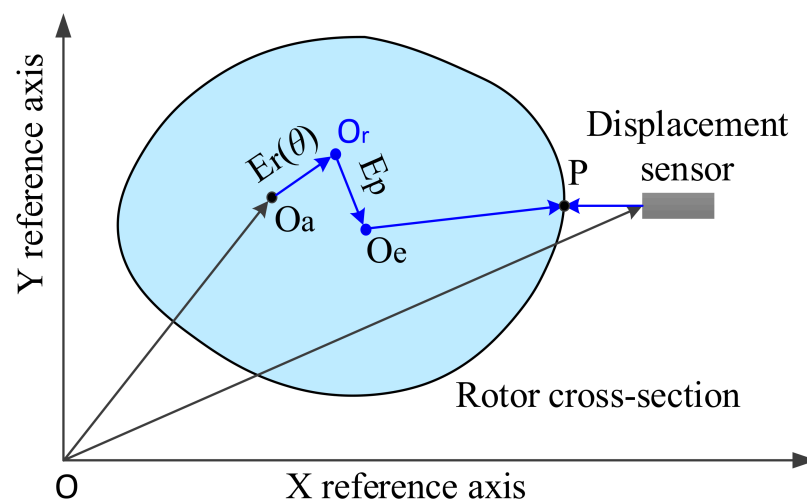


Figure 1. Decomposition of displacement sensor measurement data.

2.2. Artifact's Roundness Separation Method

The three-probe method, shown in Figure 2, uses three displacement sensors arranged along the circumferential direction that simultaneously measure the distance changes in artifact surface in three directions. Sensors S_2 and S_3 are positioned with respect to sensor S_1 at angles φ and τ , respectively. The center lines of the three sensors intersect at a point that coincides with the average point O_a . Meanwhile the coordinate origin of reference frame is set at the average point.

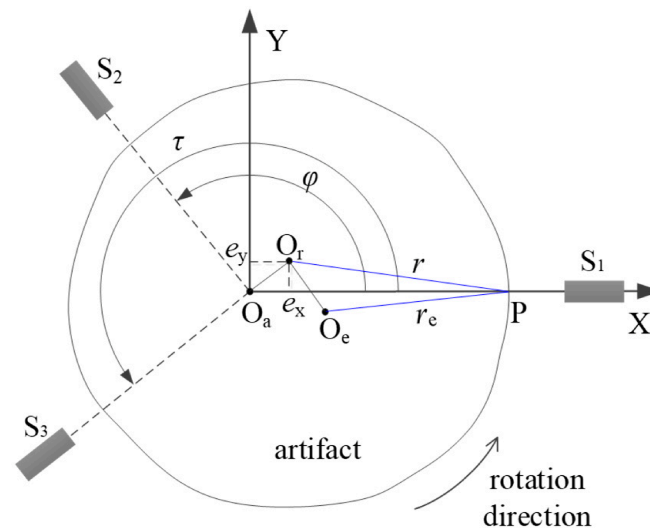


Figure 2. Three-probe method schematic diagram.

When the rotor drives the artifact to rotate, the rotation angle of the artifact is θ and:

$$s_1(\theta) = r(\theta) + e_x(\theta) \quad (1)$$

$$s_2(\theta) = r(\theta - \varphi) + e_x(\theta) \cos \varphi + e_y(\theta) \sin \varphi \quad (2)$$

$$s_3(\theta) = r(\theta - \tau) + e_x(\theta) \cos \tau + e_y(\theta) \sin \tau \quad (3)$$

Then, by selecting specified weight coefficient, the radial error motion components of rotor $e_x(\theta)$ and $e_y(\theta)$ can be eliminated in weighted summation of the three sensor measurement data.

$$s(\theta) = k_1 s_1(\theta) + k_2 s_2(\theta) + k_3 s_3(\theta) \quad (4)$$

$$\begin{aligned} s(\theta) &= k_a r(\theta) + k_2 r(\theta - \varphi) + k_3 r(\theta - \tau) \\ &+ (k_1 + k_2 \cos \varphi + k_3 \cos \tau) e_x(\theta) \\ &+ (k_1 + k_2 \sin \varphi + k_3 \sin \tau) e_y(\theta) \end{aligned} \quad (5)$$

Setting

$$\begin{cases} k_1 = 1 \\ k_2 = -\sin \tau / \sin(\tau - \varphi) \\ k_3 = \sin \varphi / \sin(\tau - \varphi) \end{cases} \quad (6)$$

Then the coefficients of $e_x(\theta)$ and $e_y(\theta)$ in Equation (4) can be zero.

$$\begin{cases} k_1 + k_2 \cos \varphi + k_3 \cos \tau = 0 \\ k_1 + k_2 \sin \varphi + k_3 \sin \tau = 0 \end{cases} \quad (7)$$

The combined signal $s(\theta)$ can be expressed as:

$$s(\theta) = r(\theta) + k_2 r(\theta - \varphi) + k_3 r(\theta - \tau) \quad (8)$$

The discrete Fourier transformation (DFT) is used to transform the combined signal $s(\theta)$ from spatial domain to the harmonic domain. Equation (8) becomes:

$$S(k) = R(k) + k_2 e^{-jk\varphi} R(k) + k_3 e^{-jk\tau} R(k) \quad (9)$$

To extract the artifact roundness, a transfer function of $S(k)$ to $R(k)$ is used, which indicates how well each harmonic k can be recovered. The transfer function $W(k)$ can be expressed as:

$$W(k) = 1 + k_2 e^{-jk\varphi} + k_3 e^{-jk\tau} \quad (10)$$

Then, $r(\theta)$ is determined through the inverse discrete Fourier transformation:

$$R(k) = \frac{S(k)}{W(k)} \quad (11)$$

$$r(\theta) = IDFT(R(k)) \quad (12)$$

The three-probe method is insensitive to some harmonic components of the error to be separated.

This phenomenon is known as harmonic suppression. Harmonic suppression includes the first harmonic suppression and the other harmonic suppression. The first harmonic suppression makes the first harmonic component of the spindle radial rotation error unable to be separated from the installation eccentricity of the measured surface, while the other harmonic suppression makes a certain harmonic component of the roundness error unable to be separated. The first harmonic suppression is inevitable. By optimizing the angle position of the sensor and selecting a reasonable harmonic range, the harmonic suppression can be avoided in a certain harmonic range.

2.3. Installation Eccentricity Separation Method

If the geometric center coincides with the rotation center, the eccentricity is zero. Otherwise, the error motion caused by deviation of the two center points will be mixed into calculation process of the radial error motion and cause an incorrect separated result. This eccentricity is caused by a variety of reasons, such as misalignment in installation process of the artifact, thermal shift, load changes, preload changes and the speed changes. Eccentricity caused by installation is considered to be the main factor. In a sense, eccentricity always exists; therefore, the interference induced from the eccentricity should be removed.

Assume that there is eccentricity between the two centers, and the radial error and roundness error are zero. The contribution of the deviation in the sensor measurement data will be analyzed. There is eccentricity between the rotation center of the rotor and its geometric center, as shown in Figure 3. The eccentricity vector is p , the size is ρ , and the initial angle is δ .

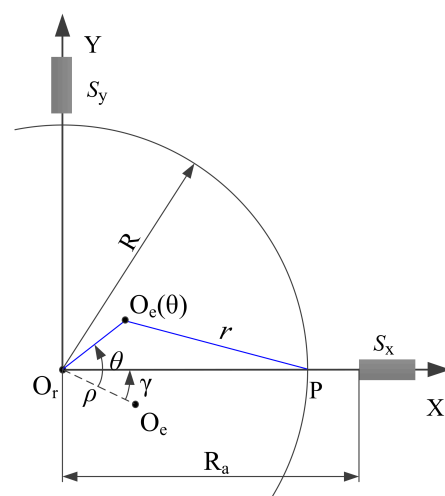


Figure 3. Characteristic analysis of eccentricity signal.

The output data of the sensor arranged along the X axis is:

$$s_x = R_a - \rho \cos(\theta + \gamma) - \sqrt{R^2 - \rho^2 \sin^2(\theta + \gamma)} \quad (13)$$

Because

$$\sqrt{1-x} \approx 1 - \frac{x}{2} \quad (14)$$

s_x can be simplified as:

$$s_x \approx R_a - R - \rho \cos(\theta + \gamma) + \frac{\rho^2 \sin^2(\theta + \gamma)}{2R} \quad (15)$$

After removing the constant value, s_x can be shown:

$$s_x \approx -\rho \cos(\theta + \gamma) - \frac{\rho^2}{4R} \cos(2\theta + 2\gamma) \quad (16)$$

Similarly, the output data of the sensor arranged along the Y axis is:

$$s_y \approx -\rho \sin(\theta + \gamma) + \frac{\rho^2}{4R} \cos(2\theta + 2\gamma) \quad (17)$$

When the ratio of deviation to artifact radius is small enough, the second harmonic component caused by the deviation will be ignored, and the main part of the interference is the first harmonic component. The plural form of interference caused by deviation is shown as:

$$\Delta s = -\rho \cos(\theta + \gamma) - i\rho \sin(\theta + \gamma) = -\rho e^{i(\theta + \gamma)} \quad (18)$$

It can be seen from the above equation that the deviation of the rotation center from geometric center will bring +1 harmonic component to the measured data of the sensors. It represents these vectors rotating at the same speed of the axis of rotation in the same direction.

It is generally believed that the first harmonic component in the sensor signal is caused by the installation eccentricity of the measurement surface, and there is no first harmonic component in the radial rotation error of the spindle, but this view lacks a reliable basis. For example, when the bearing support stiffness caused by the non-circular error surface of the spindle bearing stator is inconsistent in the direction, or when the support structure stiffness of the spindle is inconsistent in the direction, the first harmonic motion of the spindle will be generated under the excitation of the unbalanced force. Through the kinetic analysis of rotation axis, it can be clearly seen that the fundamental radial error motion can be exhibited under the unbalanced excitation.

As shown in Figure 4, when the center of gravity of the rotor is offset from the geometric center and the rotor rotates at an angular velocity ω , the unbalanced force causes the rotation center of the rotor to deviate from the stator center. The trajectory can be described as:

$$\begin{cases} \Delta x = \frac{m\delta\omega^2}{k_x - m\omega^2} \cos \theta \\ \Delta y = \frac{m\delta\omega^2}{k_x - m\omega^2} \sin \theta \end{cases} \quad (19)$$

The motion error is transformed into a complex expression which is:

$$e = \Delta x + j\Delta y = M_a(k_y + k_x - 2m\omega^2)e^{j\theta} + M_a(k_y - k_x)e^{-j\theta} \quad (20)$$

$$M_a = m\delta\omega^2/2(k_x - m\omega^2)(k_y - m\omega^2) \quad (21)$$

It can be seen that the rotor generates a first harmonic motion, including +1 harmonic component and -1 harmonic component. The radial stiffness of the spindle bearing or support structure is inevitably different in the two vertical directions, and this difference produces the -1 harmonic component.

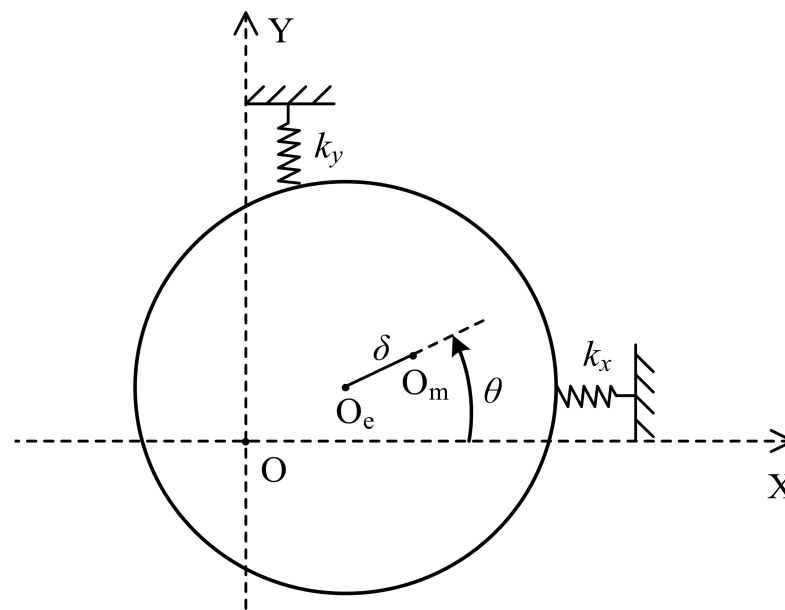


Figure 4. Spindle rotor kinetic analysis.

In the current standards of spindle metrology, the fundamental component of the sensor radial motion data is considered to be the eccentricity entirely. This method is obviously not desirable. It is necessary to study the method of eccentricity separation according to the performance characteristics of the first-order harmonic component of eccentricity and spindle rotation error in the sensor signal.

3. Rapid Separation Strategy of Spindle Rotation Error

In principle, the traditional three-probe method can realize the online measurement of the spindle radial error motion without the influence of the measured surface. However, there are problems in the real-time and rapidity of error separation. There are two ways to implement the three-point method in the cutting process of the machine tool. The first way is to collect the data of the entire cutting process and then to separate the error after the cutting. Owing to the large amount of data and long calculation time, when the workpiece is out of tolerance, it cannot be warned in time. This method belongs to the measurement after cutting and cannot achieve real-time and fast separation. Another approach is to intercept and separate data into segments during the cutting process. Thus, owing to the small amount of data involved in the calculation, the error separation speed and the response speed to abnormal conditions are faster than the first method. However, each data processing content repeatedly includes data interception, roundness and eccentricity error separation, and rotation error separation. Consequently, the interference information is repeatedly separated, which affects the rapidity of spindle rotation error separation.

As shown in Figure 5, the sampling cycle time of the cutting data is t_1 , and the error separation time is t_2 . After n sampling cycles, the error separation delay time is n times t_{d1} . As the cutting time increases, the error separation delay time becomes longer and longer.

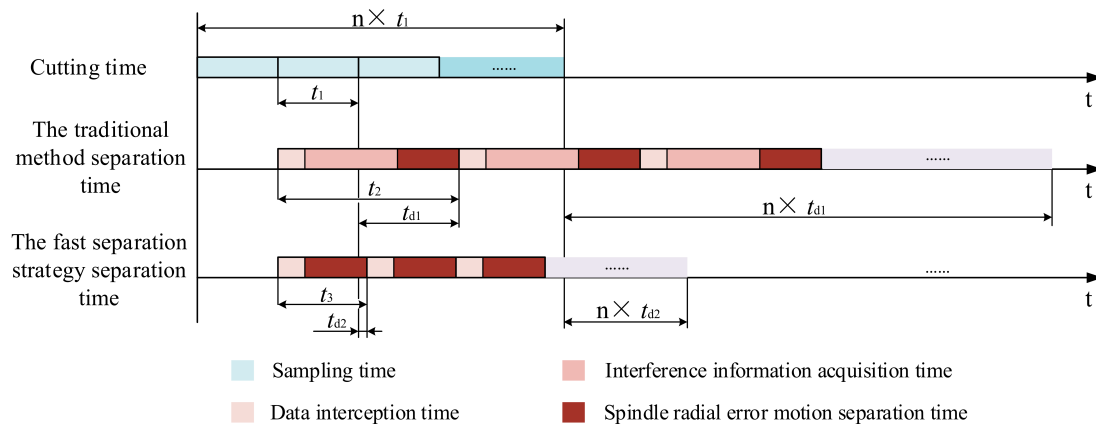


Figure 5. Comparison of calculation time between traditional error separation and fast separation strategy. The ellipsis in the figure represents the next cutting and separation cycle.

Because the interference information composed of the artifact roundness and installation eccentricity is only related to the measured surface, it does not change under different cutting conditions. Therefore, the interference information can be separated before cutting. In the cutting process, it is only necessary to separate the radial error motion by removing the interference information from the measured signal. The separation time is shortened. The difference t_{d2} between the error separation time t_3 and the cutting time t_1 of the sampling period decreases, and the real-time and rapidity of the error separation improve.

Owing to the presence of cutting force changes, speed fluctuations, etc., the spindle rotation angle under cutting conditions is used as a benchmark for error. According to the rapid separation strategy of spindle radial rotation error, the separation process is divided into two steps as shown in Figure 6. The first step is to separate the interference information before cutting. Based on the three-probe method and eccentricity separation method, the interference information at certain angle positions of the rotor can be obtained. By selecting a higher sampling frequency and a lower rotor speed, the amount of interference information cycle data can be increased. More detailed data is beneficial for the separation of spindle radial error motion. The second step is to remove the interference information from the displacement signal of the sensor and separating the radial error motion of the spindle.

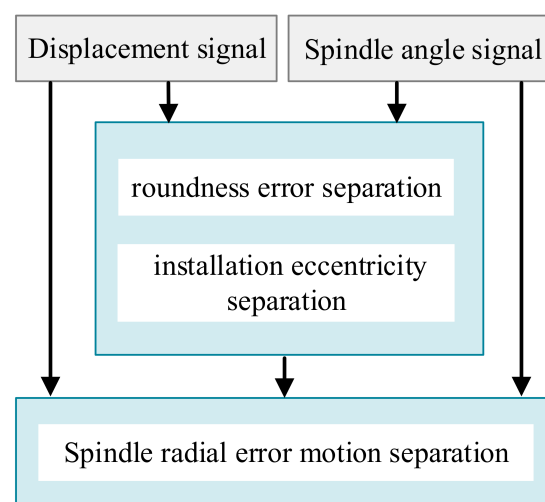


Figure 6. Rapid separation strategy procedure.

The measurement conditions such as speed and sampling frequency can be selected according to the requirements to improve the accuracy and data density of the interference information separation results. Based on high-density interference data, a lookup table and interpolation calculation are used to quickly obtain interference information and remove it from the sensor signals. This method greatly improves the accuracy and rapidity of the rotation error separation.

4. The Rapid Separation Method for Spindle Radial Error Motion

4.1. The Principle of Fast Error Separation Based on the Combination of Bidirectional Measurement Method and Three-Point Method

The three-probe method has the advantage of online measurement due to its tolerance of the selection of the measured parts. From the separation principle, the three-point method has harmonic suppression, and cannot separate the installation eccentricity and the first-order harmonic component. In other words, the radial rotation error of the spindle cannot be completely separated by the three-point method. In addition, when it is applied in the cutting process, the accuracy of separation is affected by the external conditions of cutting, which requires the support of other technologies. The three-probe method can be used to extract the artifact roundness by optimizing the sensor arrangement angle.

The bidirectional measurement method uses two displacement sensors arranged vertically to measure the radial error in X and Y directions respectively, and the roundness error of the artifact is considered zero because a standard ball is used as a perfect artifact surface. The advantage of this method is that the radial error motion in two directions can be calculated rapidly. The idea of the bidirectional measurement method is borrowed in the rapid separation method. According to the fast separation strategy, after the interference information is obtained, it is directly removed from the two vertical sensor signals based on the spindle encoder signal under the cutting condition, and the radial error motion of the spindle is directly separated.

$$\begin{cases} e_x(\theta) = s_x(\theta) - r(\theta) \\ e_y(\theta) = s_y(\theta) - r(\theta - 90^\circ) \end{cases} \quad (22)$$

In addition, the vertical arrangement of two displacement sensors in the separation of installation eccentricity can make the calculation faster and more convenient. In the first step, the three-probe method is used to separate the artifact roundness and the installation eccentricity is separated by the sensors arranged perpendicular to each other. The second step involved separating the radial error motion of the spindle using the principle of bidirectional measurement. All separation processes need to be based on the rotation angle of the spindle. In all separation processes, regardless of interference information or spindle radial error motion, the spindle rotation angle must be used as a reference.

The first harmonic component in the radial rotation error of the spindle included +1 and −1 components. The eccentricity is mainly reflected as the +1 first harmonic component in the sensor signal. Therefore, some scholars consider the −1 harmonic component of the sensor signal as the radial rotation error of the spindle, and the +1 harmonic component as the eccentricity. Although it is more accurate than the method of considering all the first harmonic components as eccentricity, the method still has errors because the eccentricity separation result contains the +1 harmonic component of the spindle error.

The first harmonic component of the spindle rotation error is generated by an internal or external unbalanced force acting on the spindle system with inconsistent stiffness. The value of the first harmonic component is related to the angular velocity of spindle. The higher the angular velocity, the greater is the offset between the rotation center and stator

center. Therefore, according to the influence of angular velocity on the first harmonic component of the spindle, a new installation eccentricity separation method is proposed.

Figure 7 shows the separation principle. At low speed, the fundamental frequency signal of the sensor contains the installation eccentricity information and the +1 and −1 harmonic components of the spindle error. Owing to the low speed, the spindle error value is small, and it can be considered that the sensor fundamental frequency signal is mainly caused by the installation eccentricity. Therefore, the +1 harmonic component at low speed is extracted as the installation eccentricity of the artifact. Under the cutting speed, the fundamental frequency signal of the sensor contained the workpiece installation eccentricity and the first harmonic component of the spindle error under the cutting conditions. The first harmonic component of the radial error motion under the cutting conditions can be separated by removing the eccentricity. The proposed eccentricity separation method has a certain error because it contains the +1 harmonic component of the radial rotation error of the spindle at low speed, but the advantage is that the method is simple and easy to implement.

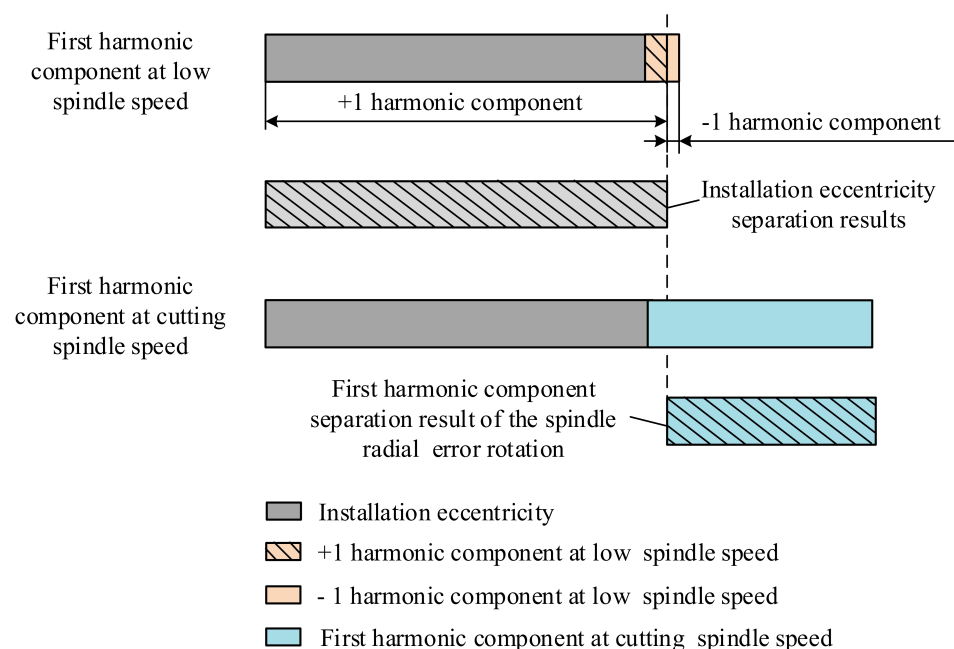


Figure 7. Separation principle of installation eccentricity.

4.2. Synchronous Acquisition Technology of Spindle Built-In Encoder and Displacement Sensor Signal

At present, the equal angular position interval sampling is usually replaced by the equal time interval sampling in the measurement process because of practical implementation difficulties. In cutting process, error separation based on equal time interval sampling data will face challenges of data angle in homogeneity caused by cutting force changes and speed fluctuations, especially when the cutting process is unstable and the force is hard.

In CNC machine tools, the built-in encode of the spindle as a feedback element provides real-time angular position for the motion control system. In this study, a three-way connector was used to shunt the angle signal from the built-in sensor to the spindle driver without affecting the normal operation of the machine. Using a self-developed signal acquisition instrument, the angle signal of the spindle and radial displacement signal of the artifact are acquired simultaneously. The acquisition principle is shown in Figure 8.

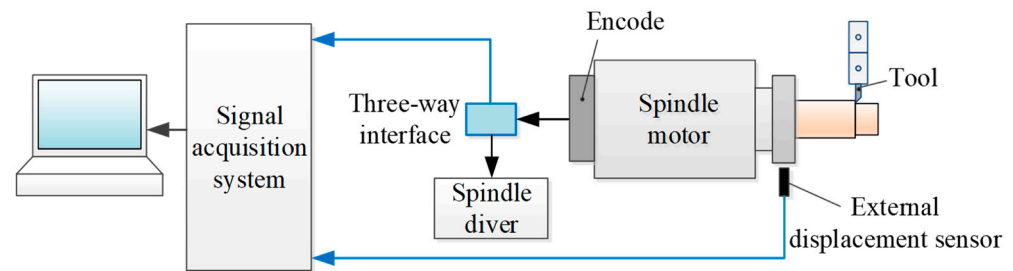


Figure 8. Sensor signal acquisition technology.

The angle position resampling method is as follows. The angle and displacement signal acquired are $\theta = [\theta_1, \theta_2, \theta_3, \dots, \theta_n]$ and $\mathbf{s} = [s_1, s_2, s_3, \dots, s_n]$ through the signal acquisition instrument under a sampling frequency. Based on the sampling frequency and the spindle speed, a suitable angular interval value $\Delta\theta$ is calculated and angle series are constructed $\theta\mathbf{n} = \theta_0 + i\Delta\theta \ i = 1, 2, 3, \dots, m$. The displacement signal corresponding to the angle position can be shown as

$$\mathbf{sn} = [sn_1, sn_2, sn_3, \dots, sn_m] \quad (23)$$

The displacement signal can be resampled by linear interpolation.

$$sn_i = s_k + \frac{(\theta n_i - \theta_k)(s_{k+1} - s_k)}{(\theta_{k+1} - \theta_k)}, \quad \theta n_i \in [\theta_k, \theta_{k+1}] \quad (24)$$

Through the above calculation, the displacement signal was resampled in an angular interval $\Delta\theta$ and used to extract the interference information.

4.3. The Position Angle of the Sensor

According to the principle of fast separation, if the position angle of one sensor is 0° and the other is 90° , the X- and Y-axis components of the rotation error can be directly separated. Therefore, the positions of the two displacement sensors are 0° and 90° respectively.

In the extraction of roundness error, if the transfer function $W(k)$ approaches a very low value, i.e., zero, the corresponding k harmonic component of roundness error cannot be distinguished and the harmonic suppression occurs. The value of $W(k)$ is determined by the harmonic k and sensor angular position. The harmonic suppression can be avoided if the sensor angular position and the harmonic order of the roundness error are well considered, and the transfer function is far away from zero for any harmonic k in the range of interest. Figure 9 indicates the minimal value of $|W(k)|$ for the 2nd to 50th harmonics under all possible sensor position angle combinations. The dark blue regions depict that the value of $|W(k)|$ is equal to zero at least once in the above range of harmonic and the harmonic component will occur in separation process. These combinations of φ and τ must be avoided. The dark red regions depict that the value of $|W(k)|$ is very high or even non-existent, where φ and τ are approximately equal or 180 degrees apart. So, these combinations are also avoided because of high computational cost. Choosing angle combinations should avoid the dark blue and dark red regions. From the detailed image near $\varphi = 90^\circ$, it is clear that the angle combinations resulting in harmonic suppression are densely distributed. However, there are still large areas that do not produce harmonic suppression near some angle combinations.

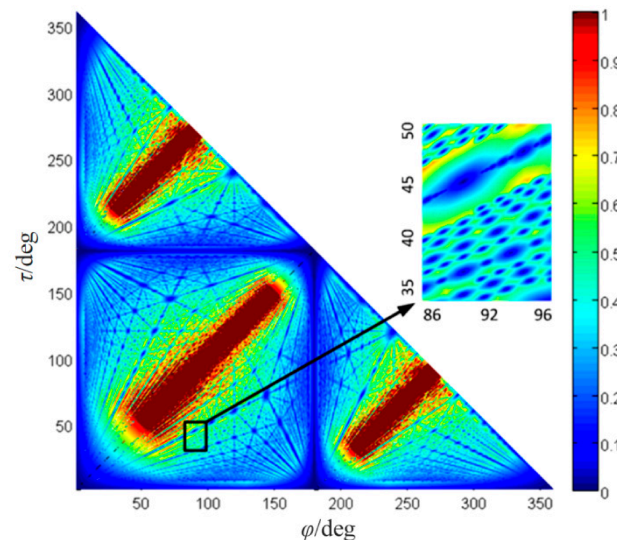


Figure 9. The minimal value of $|W(k)|$.

In measurement process, angle installation error of sensors is inevitable. To ensure that harmonic suppression does not occur even when an installation error exists, an appropriate safe region around the angle combination must be considered in choosing the values of φ and τ . According to the above principles, the optimal angle of the third sensor is 44.5° . Angular mounting deviation of sensors is a vital factor impairing the accuracy of roundness error separation of the measured workpiece. Two approaches are proposed to alleviate the negative effects caused by sensor mounting errors. First, the optimized angular combination is selected by taking the harmonic suppression performance under angular deviation into account. Second, the actual mounting angles of sensors are measured using angular signals from the spindle encoder and are used instead of the designed angles in the error separation measurement. The two proposed approaches reduce the impact of sensor angular deviation on the accuracy of error separation [22].

5. Experimental Verification

The effectiveness of the spindle radial error motion rapid separation method under cutting conditions was verified using a workpiece cylindrical turning experiment. The proposed method is analyzed and verified by comparing the spindle radial error motion measurement results with the roundness error of the machined surface of the workpiece.

5.1. Experimental Arrangement

In the experiment, a YOUJIAFTC-20 CNC (Hangzhou Good-Friend Precision Machinery Co., Ltd., Hangzhou, China) lathe was used, and its CNC system was SINUMERIK840D (Siemens AG., Nuremberg, Germany). The processing content is the cylindrical turning of the workpiece. The signals collected in the experiment included the radial displacement signal of the measuring section at the chuck end and the angle signal of the built-in encoder of the spindle. The optimal angle (0° , 90° , 44.5°) was selected as the arrangement angle of the displacement sensor. The displacement sensor adopts MICRO-EPSILONDT3010-U1-A-C3 (MICRO-EPSILON Messtechnik GmbH & Co., KG., Ortenburg, Germany) eddy current sensor with a range of 1 mm and a resolution of $0.1 \mu\text{m}$. The spindle angle signal is obtained from the position feedback port of the spindle drive through the three-way interface. The machine tool signal acquisition instrument developed by the research group was used to acquire two types of signals synchronously, and the sampling frequency was 2 kHz, as shown in Figure 10.

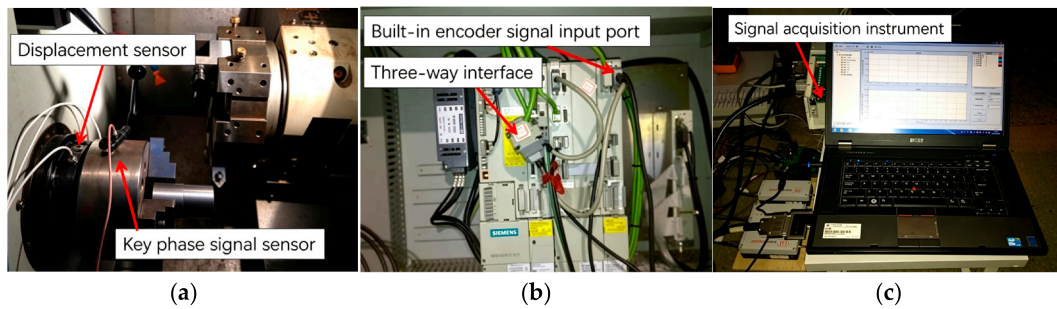


Figure 10. Experimental setup. (a) Displacement sensor arrangement. (b) Built-in encode signal interception. (c) Machine tool signal acquisition instrument.

5.2. Experimental Procedure

According to the rapid separation method of rotation error under the cutting conditions, the experiment is divided into two steps. The first step is to run the machine tool without machining at a spindle speed of 30 r/min, which is used to obtain the interference information, artifact roundness and installation eccentricity. At lower spindle speed, more sampling points and more detailed interference information can be obtained. The second step is the normal cutting process of the workpiece. The spindle speed is 600 r/min, tool feed speed is 100 mm/min, the cutting depth is 1 mm, which is used to separate the radial error motion of the spindle under cutting conditions.

The sensitivity analysis of the fast separation method to sensor noise, encoder resolution, and cutting process vibration is as follows. Sensor noise: The eddy current sensor used has a resolution of 0.1 μm and a noise level of $<0.05\%$ FSO. The impact is negligible compared to the $\sim 7 \mu\text{m}$ error motion. Encoder resolution: The SINUMERIK840D built-in encoder provides 1024 pulses/rev, which, combined with our 2 kHz sampling and angular resampling, yields an angular resolution of $\sim 0.35^\circ$, sufficient for the 50th harmonic analysis. Cutting variation: The angular resampling method inherently compensates for speed fluctuations, making the method robust to moderate variations in cutting conditions.

5.3. Experimental Results and Analysis

5.3.1. Interference Information Separation Results

From the three displacement sensor signals and the angle signal of the spindle encoder, the data of a rotation period is intercepted and the number of points is 4000. Based on the spindle speed and sampling frequency, the rotor angle sequence of ideal equal angle interval sampling is calculated, and then the displacement signal is resampled at the angle position. The obtained displacement signal is shown in Figure 11.

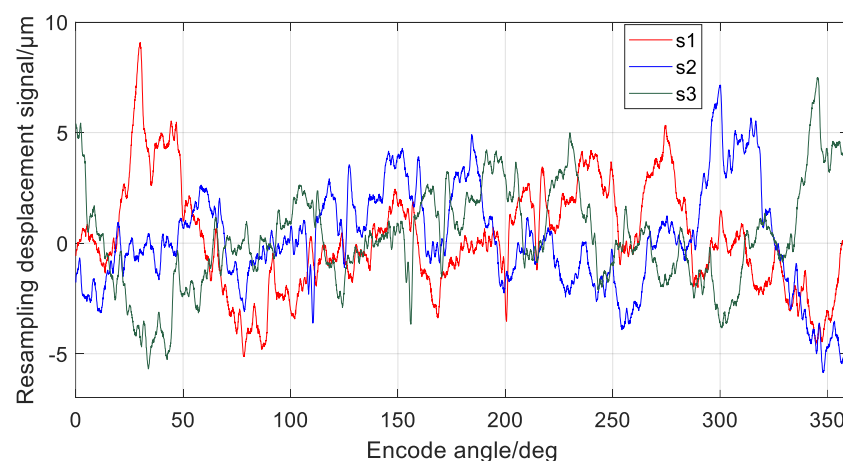


Figure 11. Displacement signal after angle position resampling.

The first harmonic components in X and Y directions are extracted from the sensors s1 and s2 arranged perpendicularly to each other. The combination curve is shown in Figure 12, and the curve is elliptical. The first harmonic component curve is divided into +1 harmonic component and −1 harmonic component, as shown in Figure 13. The +1 harmonic component is extracted as the installation eccentricity of the measured circular surface. The eccentricity radius is 0.85 μm and the eccentricity angle is 76.58°.

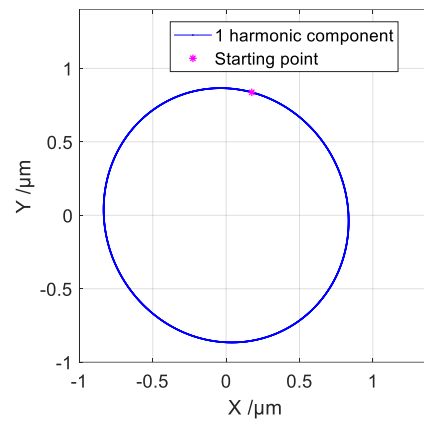


Figure 12. The first harmonic component combination curve.

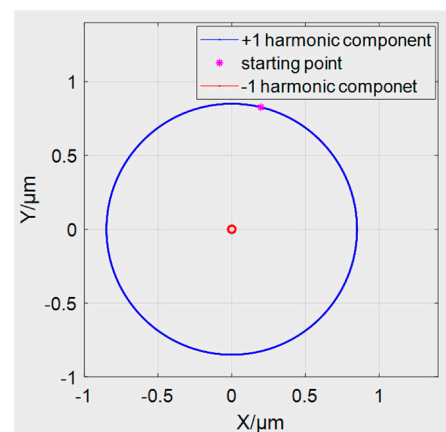


Figure 13. +1 harmonic component and −1 harmonic component.

Using the displacement signal after resampling at the angular position, the roundness of the measured surface is separated using the three-point method as shown in Figure 14. The interference information based on the angle is obtained, including the roundness error and installation error of the measured surface.

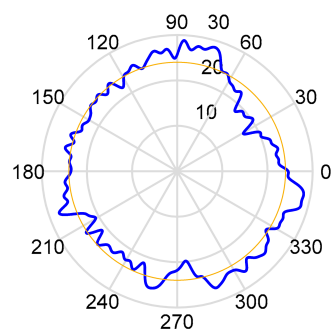


Figure 14. Artifact roundness separation result.

5.3.2. The Radial Error Motion of Spindle Separation Result

During the cylindrical turning process of the workpiece, the radial error motion of the spindle is separated from the two displacement signals arranged perpendicularly to each other. The displacement signal is intercepted when the turning tool cuts to a certain axial position. The signal contains five rotation cycles, and the data of the first and fifth cycles are not complete periodic data. Based on the angle of the spindle encoder, the intercepted displacement signals in the X and Y directions are shown in Figure 15.

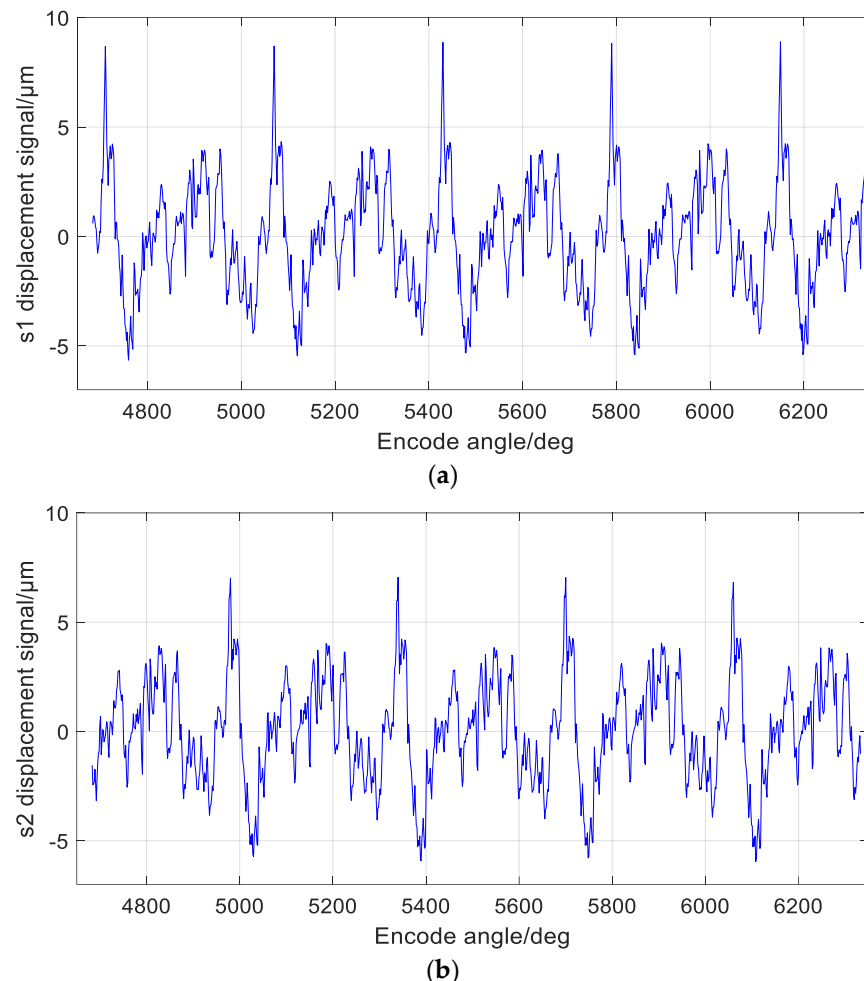


Figure 15. Displacement signal in cutting (a) X-axis direction displacement signal, (b) Y-axis direction displacement signal.

Owing to the influence of speed fluctuation and cutting force in the cutting process, the angle signal of the spindle encoder is not equal to the angle interval. The interference information at the corresponding angle position is extracted and removed from the displacement signals in the two vertical directions. The separated spindle radial error motion is illustrated in Figure 16. Affected by the influence of the cutting state and real-time change in cutting, there are some small differences in the results of each cycle.

In the experiment, the pre-cutting interference separation takes about 2.4 s for 4000 data points using a standard MATLAB R2025a script. In the cutting process, the online calculation only involves table lookup and subtraction of the pre-stored interference values, which takes less than 5 milliseconds per rotation cycle. This is significantly faster than the traditional approach, which would require re-running the three-probe DFT algorithm for every data segment (normally taking 0.6–0.9 s per cycle).

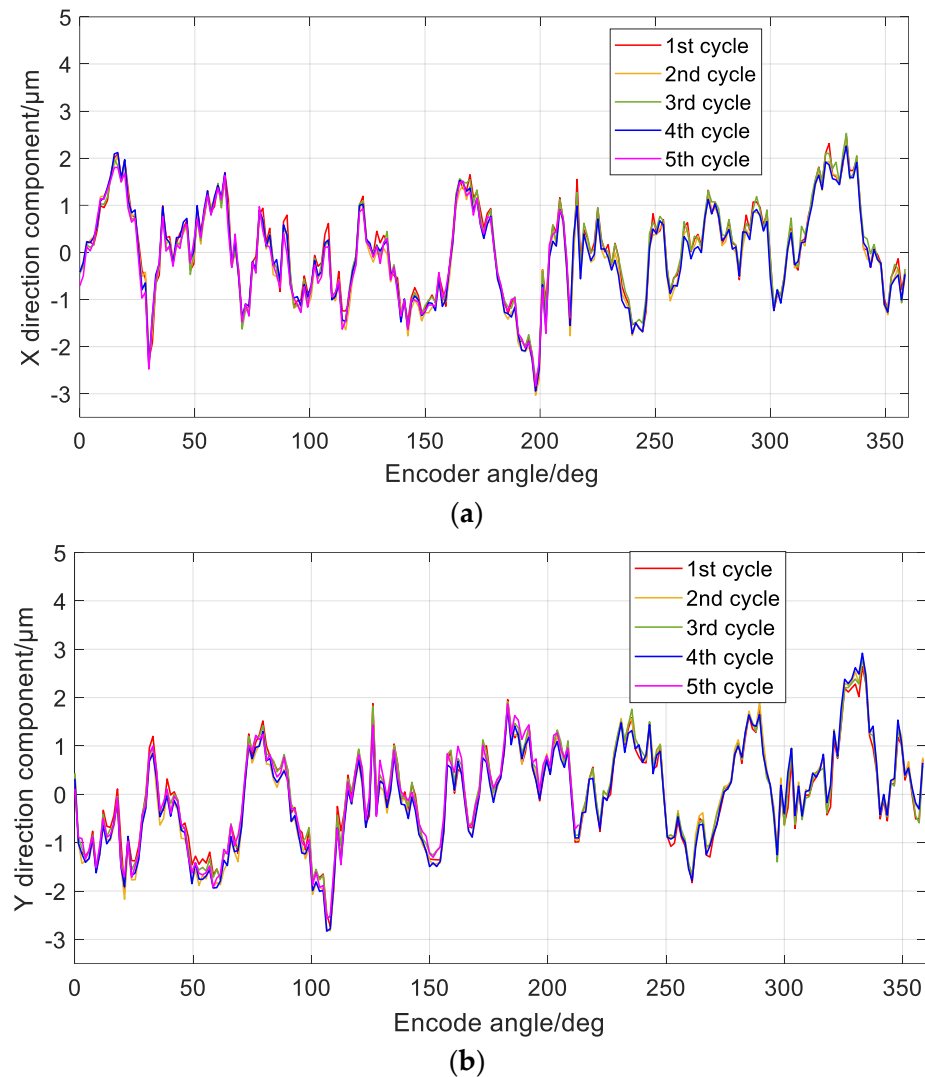


Figure 16. Separation result of the spindle radial error motion in a sampling period (a) X-axis component, (b) Y-axis component.

The accuracy of the radial error motion of spindle separation results under the cutting conditions cannot be directly tested. During the turning process, the radial error motion of the spindle is reflected in the contour error of the workpiece by the position offset of the tool relative to the workpiece. However, the chuck and workpiece possess multiple vibration modes, resulting in inconsistent vibration responses at the two locations, particularly when the workpiece has a complex structure, clamping scheme and supporting configuration. In this case, the radial vibration measured at the chuck cannot represent the vibration at the workpiece. Therefore, it is essential to establish a dynamic model for the workpiece system and build the correlation between the radial vibration at the chuck and the workpiece vibration. In this experiment, complex workpiece systems are avoided, and short, simple workpieces are adopted to reduce the vibration discrepancy between the workpiece end and the chuck end. Therefore, the radial error motion separation result can be verified using the measurement results of the workpiece roundness error. According to the motion model of turning tool, the separation result of spindle radial error motion is transformed into the roundness of workpiece surface by coordinate transformation. The result is shown in Figure 17, and the contribution of spindle radial error motion to the roundness error of workpiece is 6.7 μm.

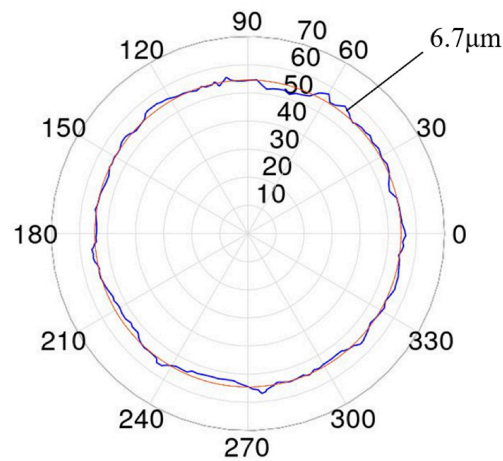


Figure 17. The workpiece roundness calculated by the spindle radial error motion.

On the other hand, the error of the machined surface of the workpiece is measured. The HuiZhi YD200 cylindricity meter (Luoyang Huizhi Measurement & Control Technology Co., Ltd., Luoyang, China) is used to measure the roundness error of this section, as shown in Figure 18. Figure 19 shows the graphical display interface of the workpiece roundness measurement results. The first 50 harmonic components of the roundness are used for construction. The measurement result is $7.57\ \mu\text{m}$.

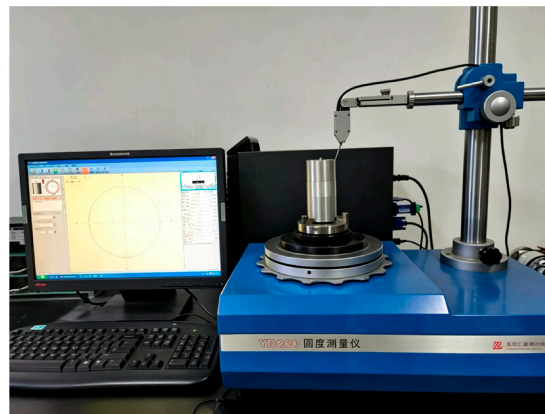


Figure 18. Workpiece roundness measurement.

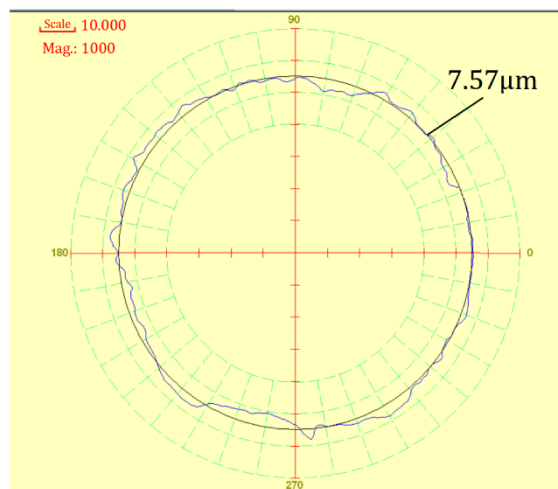


Figure 19. Workpiece roundness measurement result.

The roundness of the workpiece constructed by the radial rotation error of the separated spindle is $6.7\text{ }\mu\text{m}$, and the actual roundness measurement value of the workpiece is $7.57\text{ }\mu\text{m}$. The results are relatively close, which verifies the accuracy of the fast separation method in separating the radial error motion of the spindle. There are some differences between the two results. These differences may be caused by the following reasons. One is the error in the measurement process of the two methods, and the other is that the influence of the shape of the tool tip and the adjacent tool marks is not considered when the spindle radial error motion is converted into the workpiece roundness.

6. Conclusions

To achieve online measurement of spindle radial error motion under cutting conditions, this study proposes a spindle radial error motion fast separation method based on the bidirectional measurement method and multi-probe method. Based on the error separation principle, the proposed method adopts a segmented separation strategy, which suppresses redundant interference signals generated by repeated separation processes, shortens the error separation calculation time under cutting loads, and improves the computational efficiency of spindle radial error motion extraction. The built-in signal acquisition module of the machine tool is employed to collect spindle encoder signals, which provide angular references for the interception and processing of sensor displacement signals. An angular position resampling algorithm is applied to mitigate the variation in sampling angular intervals induced by dynamic cutting conditions, thereby improving the extraction precision of interference error components during cutting. The separation process based on spindle encoder angular signals enhances the extraction accuracy of spindle radial error motion under cutting environments and enables reliable online and fast error separation in turning processes. The extracted spindle radial error motion data are transformed into the workpiece coordinate system and compared with the off-machine roundness measurement results of machined surfaces. The comparison results show that the separated spindle error data achieve favorable consistency with the offline-measured workpiece roundness profiles. Under actual cutting conditions, the proposed method realizes efficient extraction of spindle radial error motion with competent measurement precision.

This study lays a fundamental foundation for future research on the dynamic characteristics of spindles under dynamic cutting conditions. Combined with in-depth studies of the chuck–workpiece vibration correlation model, the proposed method enables real-time monitoring of radial eccentricity at the cutting position. It supports real-time monitoring and early warning of machining errors and provides feedback signals to optimize cutting parameters in intelligent manufacturing. These promising aspects represent the core research orientations for our follow-up investigations.

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