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Modeling Non-Equilibrium Rarefied Gas Flows Past a Cross-Domain Reentry Unmanned Flight Vehicle Using a Hybrid Macro-/Mesoscopic Scheme

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Abstract: The cross-domain reentry unmanned flight vehicle passes through thin atmospheres and dense atmospheres when it comes across atmospheres in the near-space area. For the early transition regime, the classical macroscopic and mesoscopic approaches are either not accurate or computational too expensive. The hybrid macro-/mesoscopic method is proposed for simulating rarefied gas flows past a cross-domain reentry spheroid–cone unmanned flight vehicle in the present study. The R26 moment scheme is applied in the main flow from a macroscopic point of view, and the discrete velocity method (DVM) is used for solving the Boltzmann equation from a mesoscopic point of view. The simulation results show that the hybrid macro-/mesoscopic scheme is well-suited for non-equilibrium rarefied gas flows past a cross-domain reentry unmanned flight vehicle. The results obtained in this study are consistent with benchmark results, with a maximum density error of 9%. The maximum errors of the heat transfer coefficient and pressure coefficient are 2% and 4.6%, respectively. In addition, as the Knudsen number (Kn) becomes larger, the thickness of the shock layer at the head of the flight vehicle becomes thicker, and non-equilibrium effects become more critical for the aircraft. Since the Boltzmann–Shakhov equation has only been solved close to the wall of the spacecraft, the computational cost can be considerably saved.



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Keywords: cross-domain vehicle; reentry unmanned flight vehicle; rarefied gas flows; hybrid macro-/mesoscopic scheme; kinetic theory

1. Introduction

The near-space area bridges the atmospheric gap between the thin atmosphere and the dense atmosphere. Due to its unique airspace-location advantages, the adjacent space plays a crucial role in the struggle for air supremacy in modern warfare, high-precision rapid strikes, civilian communications, etc. The cross-domain reentry unmanned flight vehicle passes through free molecular, transition, slip, and continuum regimes when it comes across the atmosphere in the near-space area [1,2]. Due to the rarefied effect, the thermodynamics and the aerothermodynamics around space vehicles are totally different in various flow regimes [3], include high temperature, high pressure, high speed, etc. The traditional Navier–Stokes–Fourier (NSF) equation and the Fourier law based on the continuous flow assumption are not valid anymore. Additionally, solving the Boltzmann equation at the mesoscopic level is computational expensive and time-consuming [4].

Therefore, it is of great theoretical significance and engineering application prospect to develop a highly efficient and accurate numerical scheme to simulate the flow field and complex flow mechanism of aircraft in the whole flow domain.

Numerically, the flow field of the non-equilibrium rarefied gas flows past a cross-domain reentry unmanned flight vehicle can be described based on the mesoscopic or macroscopic method [4,5]. For the former, the direct simulation Monte Carlo (DSMC) method [6] and the discrete velocity method (DVM) [7] are the most popular schemes to solve the Boltzmann equation accurately. Specifically, the hypersonic non-equilibrium rarefied gas flows past the three-dimensional spacecraft have been widely simulated numerically [8,9]. For instance, the RAM-C II unmanned flight vehicle, Stardust reentry flight vehicle [8], and the Chinese lunar reentry capsule have been analyzed and evaluated against the benchmark data across various Knudsen and Mach numbers, and the DSMC method is able to present highly accurate results [9]. However, the DSMC scheme would suffer statistical noise for the low Mach or Knudsen numbers. For the deterministic approach, extensive works have been studied for simulating the rarefied gas flows based on nonlinear model Boltzmann equations, i.e., the Bhatnagar–Gross–Krook (BGK) model [10], the Shakhov model [11], the Ellipsoidal Statistical BGK model [12], and the Morse model [13]. The 1D shock-tube problems, rarefied gas flows past a 2D airfoil, and hypersonic flows past a 3D sphere–cone flight vehicle with different Kn , Re , and Ma numbers are simulated based on the high-order gas–kinetic unified algorithms (GKUA) [14]. Simulation results have indicated that the GKUA is able to provide accurate results for the reentry spacecraft application with a wide range of Kn numbers [14–16]. The Boltzmann equation’s complexity, particularly due to its multi-dimensional VDF and the intricate nonlinear collision term, makes its application to many real-world problems challenging. As a result, significant efforts have been made to develop alternative macroscopic modeling approaches to non-equilibrium flows.

The Navier–Stokes–Fourier (NSF) equations serve as the classical macroscopic framework for modeling gas flows. When supplemented with suitable velocity-slip and temperature-jump boundary conditions, they can capture key flow characteristics in weakly rarefied scenarios close to equilibrium, such as those in the slip regime. However, additional caution is required when thermal effects become significant [17]. To expand the use of macroscopic models into the transition regime, the G13 moment method was developed by Grad [18]. Using an approach similar to the Chapman–Enskog expansion, Struchtrup [19] regularized the G13 equations and obtained the R13 moment method. Gu and Emerson [20] and Struchtrup [21] derived the wall boundary conditions (WBCs) for the regularized 13 moment equations (R13) based on Maxwell’s kinetic WBC [22]. The R13 equations can accurately describe non-equilibrium effects for Knudsen numbers up to 0.25. Furthermore, Gu [23] expanded the moment scheme to develop the regularized 26 (R26) moment systems, which highlighted their capability as an effective engineering tool for analyzing non-equilibrium flows in the transition regime [24,25]. The moment method effectively connects traditional hydrodynamic models with kinetic models in the early transition regime, addressing the limitations of the NSF equations and the discrete velocity method (DVM), which are either insufficiently accurate or computationally inefficient. In this study, we integrated the R26 moment systems with the kinetic solver to model non-equilibrium rarefied gas flows around a cross-domain reentry unmanned flight vehicle.

The structure of the rest of this paper is outlined as follows. Section 2 provides a concise overview of the R26 moment scheme at the macroscopic level. The kinetic theory and the corresponding WBCs are described in Section 3. The hybrid macro-/mesoscopic scheme is also described in this section. Using the hybrid approach, Section 4 presents comprehensive results for simulating non-equilibrium rarefied gas flows around a cross-

domain reentry unmanned flight vehicle, followed by a discussion and conclusions in Section 5.

2. The R26 Moment Method

The governing equations of the R26 moment system are given as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0, \quad (1)$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} + \frac{\partial \sigma_{ij}}{\partial x_j} = -\frac{\partial p}{\partial x_i} \quad (2)$$

$$\frac{\partial \rho T}{\partial t} + \frac{\partial \rho u_i T}{\partial x_i} + \frac{2}{3R} \frac{\partial q_i}{\partial x_i} = -\frac{2}{3R} \left(p \frac{\partial u_i}{\partial x_i} + \sigma_{ij} \frac{\partial u_j}{\partial x_i} \right), \quad (3)$$

$$\frac{\partial \sigma_{ij}}{\partial t} + \frac{\partial u_k \sigma_{ij}}{\partial x_k} + \frac{\partial m_{ijk}}{\partial x_k} = -A_\sigma \frac{p}{\mu} \sigma_{ij} - 2p \frac{\partial u_{<i}}{\partial x_{j>}} - \frac{4}{5} \frac{\partial q_{<i}}{\partial x_{j>}} - 2\sigma_{k<i} \frac{\partial u_{j>}}{\partial x_k}, \quad (4)$$

$$\begin{aligned} \frac{\partial q_i}{\partial t} + \frac{\partial u_j q_i}{\partial x_j} + \frac{1}{2} \frac{\partial R_{ij}}{\partial x_j} = & -A_q \frac{p}{\mu} q_i - \frac{5}{2} p R \frac{\partial T}{\partial x_i} - \frac{7\sigma_{jk} R}{2} \frac{\partial T}{\partial x_k} - RT \frac{\partial \sigma_{jk}}{\partial x_k} + \frac{\sigma_{ij}}{\rho} \left(\frac{\partial p}{\partial x_j} + \frac{\partial \sigma_{jk}}{\partial x_k} \right) \\ & - \frac{2}{5} \left(\frac{7}{2} q_k \frac{\partial u_i}{\partial x_k} + q_k \frac{\partial u_k}{\partial x_i} + q_i \frac{\partial u_k}{\partial x_k} \right) - \frac{1}{6} \frac{\partial \Delta}{\partial x_i} - m_{ijk} \frac{\partial u_j}{\partial x_k}. \end{aligned} \quad (5)$$

$$\frac{\partial m_{ijk}}{\partial t} + \frac{\partial u_l m_{ijk}}{\partial x_l} + \frac{\partial \phi_{ijkl}}{\partial x_l} = -A_m \frac{p}{\mu} m_{ijk} - 3RT \frac{\partial \sigma_{<ij}}{\partial x_{k>}} - \frac{3}{7} \frac{\partial R_{<ij}}{\partial x_{k>}} + \mathfrak{M}_{ijk}, \quad (6)$$

$$\frac{\partial R_{ij}}{\partial t} + \frac{\partial u_k R_{ij}}{\partial x_k} + \frac{\partial \psi_{ijk}}{\partial x_k} = -A_{R1} \frac{p}{\mu} R_{ij} - \frac{28}{5} RT \frac{\partial q_{<i}}{\partial x_{j>}} - 2RT \frac{\partial m_{ijk}}{\partial x_k} - \frac{2}{5} \frac{\partial \Omega_{<i}}{\partial x_{j>}} + \mathfrak{R}_{ij}, \quad (7)$$

and

$$\frac{\partial \Delta}{\partial t} + \frac{\partial \Delta u_i}{\partial x_i} + \frac{\partial \Omega_i}{\partial x_i} = -A_{\Delta 1} \frac{p}{\mu} \Delta - 8RT \frac{\partial q_k}{\partial x_k} + \mathfrak{N}. \quad (8)$$

where m_{ijk} , R_{ij} , Δ , ϕ_{ijkl} , ψ_{ijk} , Ω_i are the high-order moments with no clear intuitive physical meaning; $\mathfrak{M}_{ijk}$, $\mathfrak{R}_{ij}$, $\mathfrak{N}$ are the nonlinear source terms. The constitutive relationships for high moments ϕ_{ijkl} , ψ_{ijk} , Ω_i and the expressions of the nonlinear source terms can be found in Gu and Emerson [23]. For the Shakhov collision model employed in the present study, the values of the collision constants in the moment equations A_σ , A_q , A_m , A_{R1} , A_{R2} , $A_{\Delta 1}$, $A_{\Delta 2}$, $A_{\phi 1}$, $A_{\phi 2}$, $A_{\psi 1}$, $A_{\psi 2}$, $A_{\psi 3}$, $A_{\Omega 1}$, $A_{\Omega 2}$, and $A_{\Omega 3}$ are listed in Table 1.

Table 1. Collision constants for the R26 moment scheme.

Notation	A_σ	A_q	A_m	A_{R1}, A_{R2}	$A_{\Delta 1}, A_{\Delta 2}$	$A_{\phi 1}, A_{\phi 2}$	$A_{\psi 1}, A_{\psi 2}, A_{\psi 3}$	$A_{\Omega 1}, A_{\Omega 2}, A_{\Omega 3}$
Shakhov	1	Pr	1	1,0	1,0	1,0	1,0,0	1,0,0

To close the R26 moment equations, Gu [23] derived the boundary conditions and the high order of moments for the R26 method. These equations of WBCs were obtained by an expansion of Hermite polynomials as well as Maxwell's kinetic WBCs [23]. Since the velocity distribution functions are approximated using Hermite polynomials, they become inaccurate at high Kn numbers.

3. Shakhov Model Equation and the WBCs

The governing equation of the Boltzmann model equation is given as follows:

$$\frac{\partial f}{\partial t} + C_i \frac{\partial f}{\partial x_i} = -\frac{1}{\tau} [f - f^{eq}], \quad (9)$$

where $f = f(t, x_i, C_i)$ represents the velocity distribution function and C_i is the molecular velocity. The mean relaxation time, τ , is evaluated as follows:

$$\tau = \frac{\mu}{p}. \quad (10)$$

For the Shakhov model [12], the equilibrium state of the velocity distribution function f^{eq} can be given by

$$f^{eq} = \frac{\rho}{(2\pi RT)^{3/2}} \exp\left(-\frac{(C_i - u_i)^2}{2RT}\right) \left[1 + (1 - \text{Pr}) \frac{(C_i - u_i)q_i}{5pRT} \left(\frac{(C_j - u_j)^2}{RT} - 5\right)\right]. \quad (11)$$

In Maxwell's WBCs [22], a certain proportion α of molecules undergo diffusive reflection, and the remaining part of the molecules undergo specular reflection, which can be expressed as

$$f^w = \begin{cases} \alpha f_M^w + (1 - \alpha)f(-C_i n_i), & C_i n_i \geq 0, \\ f(C_i n_i), & C_i n_i < 0, \end{cases} \quad (12)$$

in which f_M^w is the Maxwellian distribution function, and n_i is the unit vector normal to the wall:

$$f_M^w = \frac{\rho^w}{(\sqrt{2\pi RT^w})^3} \exp\left[-\frac{(C_i - u_i^w)^2}{2RT^w}\right], \quad (13)$$

in which u_i^w , ρ^w , and T^w are the velocity, density, and temperature of the wall, respectively.

The density ρ , velocity u_i , temperature T , and heat flux q_i can be calculated by

$$\rho = \int f d\mathbf{C}, \quad u_i = \frac{1}{\rho} \int C_i f d\mathbf{C}, \quad \frac{3}{2}\rho RT = \frac{1}{2} \int c^2 f d\mathbf{C} \quad \text{and} \quad q_i = \frac{1}{2} \int c^2 c_i f d\mathbf{C} \quad (14)$$

where c_i is the peculiar velocity:

$$c_i = C_i - u_i. \quad (15)$$

The pressure tensor p_{ij} can be separated by its trace and traceless part as follows:

$$p_{ij} = \int c_i c_j f d\mathbf{C} = p\delta_{ij} + \sigma_{ij}, \quad (16)$$

where δ_{ij} and σ_{ij} are the Kronecker delta and stress tensor, respectively. The expressions of the higher-order moments for the R26 moment system can also be obtained as

$$\begin{cases} m_{ijk} = \int c_{<i} c_j c_{>k} f d\mathbf{C}, \\ R_{ij} = \int c_{<i} c_j c_{>}^2 f d\mathbf{C} - 7RT\sigma_{ij}, \\ \Delta = \int c^4 f d\mathbf{C} - 15pRT, \end{cases} \quad \text{and} \quad \begin{cases} \phi_{ijkl} = \int c_{<i} c_j c_k c_{>l} f d\mathbf{C}, \\ \psi_{ijk} = \int c_{<i} c_j c_k c_{>}^2 f d\mathbf{C} - 9RTm_{ijk}, \\ \Omega_i = \int c^4 c_i f d\mathbf{C} - 28RTq_i. \end{cases} \quad (17)$$

The whole computational domain can be split into two regions: the kinetic layer near the wall and the main flow area. The kinetic theory and Maxwell's WBCs are adopted near the wall, and the macroscopic parameters can be calculated by solving the Boltzmann–Shakhov model equation. These macroscopic parameters can serve as the pseudo numerical boundaries for the R26 moment equations so that the R26 moment scheme can be solved. On the other hand, the velocity distribution functions can also be reconstructed by Hermite polynomials in the main flow area. And these VDFs serve as pseudo numerical boundaries for the DVM. Hence, the flow field can be solved numerically. Details of the numerical procedure of the hybrid macro-/mesoscopic scheme were summarized by Yang [4,5,26–28]. Since the distribution velocity function $f = f(t, x_i, C_i)$ is a 7-dimensional function, the discretization of the VDF should not only be implemented in the traditional physical

space x_i , but also in the phase space or velocity space C_i . Normally, the more the discrete velocity points selected, the higher the computational accuracy that can be achieved, but the computational cost also increases accordingly.

In the pseudo numerical boundaries, the VDFs can be reconstructed based on the Hermite polynomials:

$$f = f^{eq} \sum_{n=0}^{\infty} \frac{1}{n!} a_A^{(n)} H_A^{(n)} = f^{eq} \left(a^{(0)} H^{(0)} + a_i^{(1)} H_i^{(1)} + \frac{1}{2!} a_{ij}^{(2)} H_{ij}^{(2)} + \frac{1}{3!} a_{ijk}^{(3)} H_{ijk}^{(3)} + \dots \right) \quad (18)$$

where $H_A^{(n)}$ is the Hermite function, and $a_A^{(n)}$ are the corresponding coefficients. The expressions of these Hermite functions and the corresponding coefficients are listed in the Appendix. After some mathematical deductions, the fifth-order expansion of VDF in Hermite polynomials $f^{(5)}$ can be expressed as

$$f^{(5)} = f_M \left[1 + \frac{\sigma_{ij} c_i c_j}{2pRT} + \frac{c_i q_i}{pRT} \left(\frac{c^2}{5RT} - 1 \right) + \frac{m_{ijk} c_i c_j c_k}{6p(RT)^2} + \frac{\phi_{ijkl} c_i c_j c_k c_l}{24p(RT)^3} \right. \\ \left. + \frac{R_{ij} c_i c_j}{4p(RT)^2} \left(\frac{c^2}{7RT} - 1 \right) + \frac{\Delta}{8pRT} \left(\frac{c^4}{15(RT)^2} - \frac{2c^2}{3RT} + 1 \right) \right. \\ \left. + \frac{\psi_{ijk} c_i c_j c_k}{12p(RT)^3} \left(\frac{c^2}{9RT} - 1 \right) + \frac{c_i \Omega_i}{40p(RT)^2} \left(\frac{c^4}{7(RT)^2} - \frac{2c^2}{RT} + 5 \right) \right] \quad (19)$$

Once the VDF is calculated, the macroscopic parameters can be calculated based on Equations (14), (16), and (17), and the numerical procedure can be closed. The configuration of the hybrid macro-/mesoscopic scheme is shown in Figure 1.

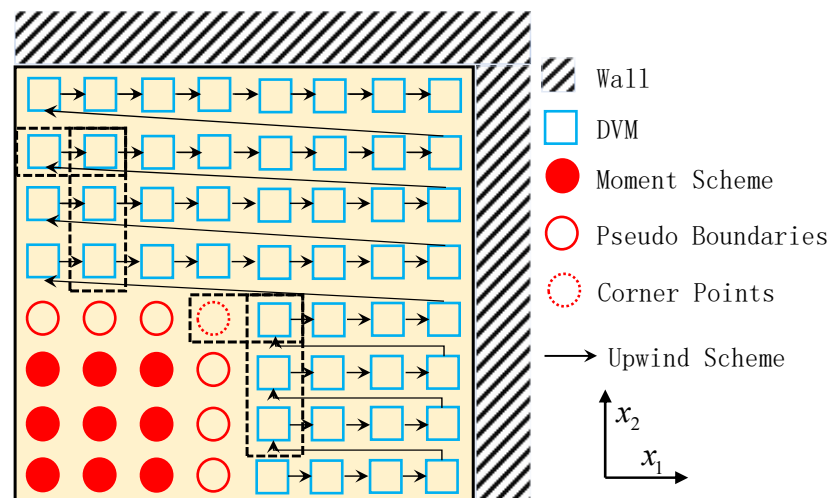


Figure 1. Configurational of the hybrid macro-/mesoscopic scheme.

4. Results and Discussion

4.1. Gas Dynamics of a Cross-Domain Reentry Spheroid–Cone Unmanned Flight Vehicle

The geometry of the cross-domain reentry spheroid-cone unmanned flight vehicle is shown in Figure 2. The length and the bottom radius of the unmanned flight vehicle are $L = 1.41$ m and $R = 0.5035$ m, respectively, and R is the reference length. Since the model has some experimental and DSMC simulation data, the calculation accuracy of the algorithm can be verified.

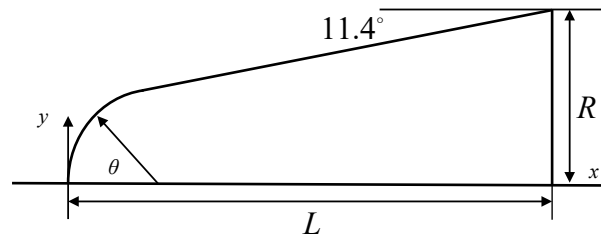


Figure 2. Configurational parameters of the cross-domain reentry spheroid-cone unmanned flight vehicle.

In this study, the gas dynamics of the incoming flow with $Ma = 2, 5$, and 10 and $Kn = 0.1$ and 1 were studied. When $Kn = 0.1$ and 1.0 , the flight height of the reentry vehicle was about $70\text{--}110$ km, and the physical space was discretized with $90,000$ and $10,000$ non-uniform tetrahedral grids, respectively, as shown in Figure 3. The velocity space was discretized with $44 \times 44 \times 24$ and $124 \times 64 \times 64$ points, respectively. The independence of the results with respect to the discretization of molecular velocity and physical space was tested and confirmed [29]. The non-uniform Newton–Cotes quadrature can be defined as

$$C_i = \frac{C_{\max}}{(N-1)^3} (1 - N + 2i)^3, i = 0, 1, \dots, N-1, \quad (20)$$

in which $C_{\max}$ and N represent the maximum discrete velocity and the number of the discrete velocities, respectively. The numerical quadrature of the velocity distribution function can be defined as

$$Y = \int f(C_i) dC_i = \frac{3C_{\max}}{(N-1)^3} \int f(C_i) a^2 da. \quad (21)$$

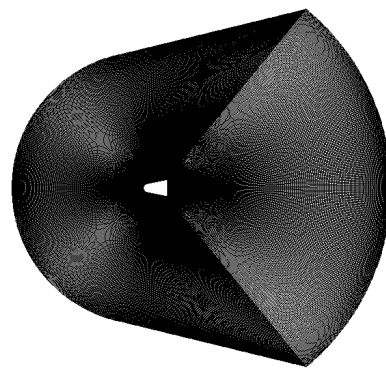


Figure 3. Physical space grid of the cross-domain reentry flight vehicle.

Since a is in the form of an arithmetic sequence, the integral with respect to a can be computed using the trapezoidal rule or the Newton–Cotes formula. Normally, the more the discrete velocity points selected, the higher the computational accuracy that can be achieved, but the computational cost also increases accordingly.

The flow field parameters of the non-equilibrium rarefied gas flows past a cross-domain reentry unmanned flight vehicle at $Ma = 2, Kn = 0.1$ and $Ma = 2, Kn = 1.0$ are shown in Figures 4a–f and 5a–f, respectively. When $Ma = 2$, 15 coupling layers are selected close to the wall of the spacecraft, and the ratios of the kinetic layers l to the reference length L_0 are $l/L_0 = 0.89$ for $Kn = 0.1$ and $l/L_0 = 1.81$ for $Kn = 1.0$, respectively. These coupling layers can produce accurate WBCs for the moment system.

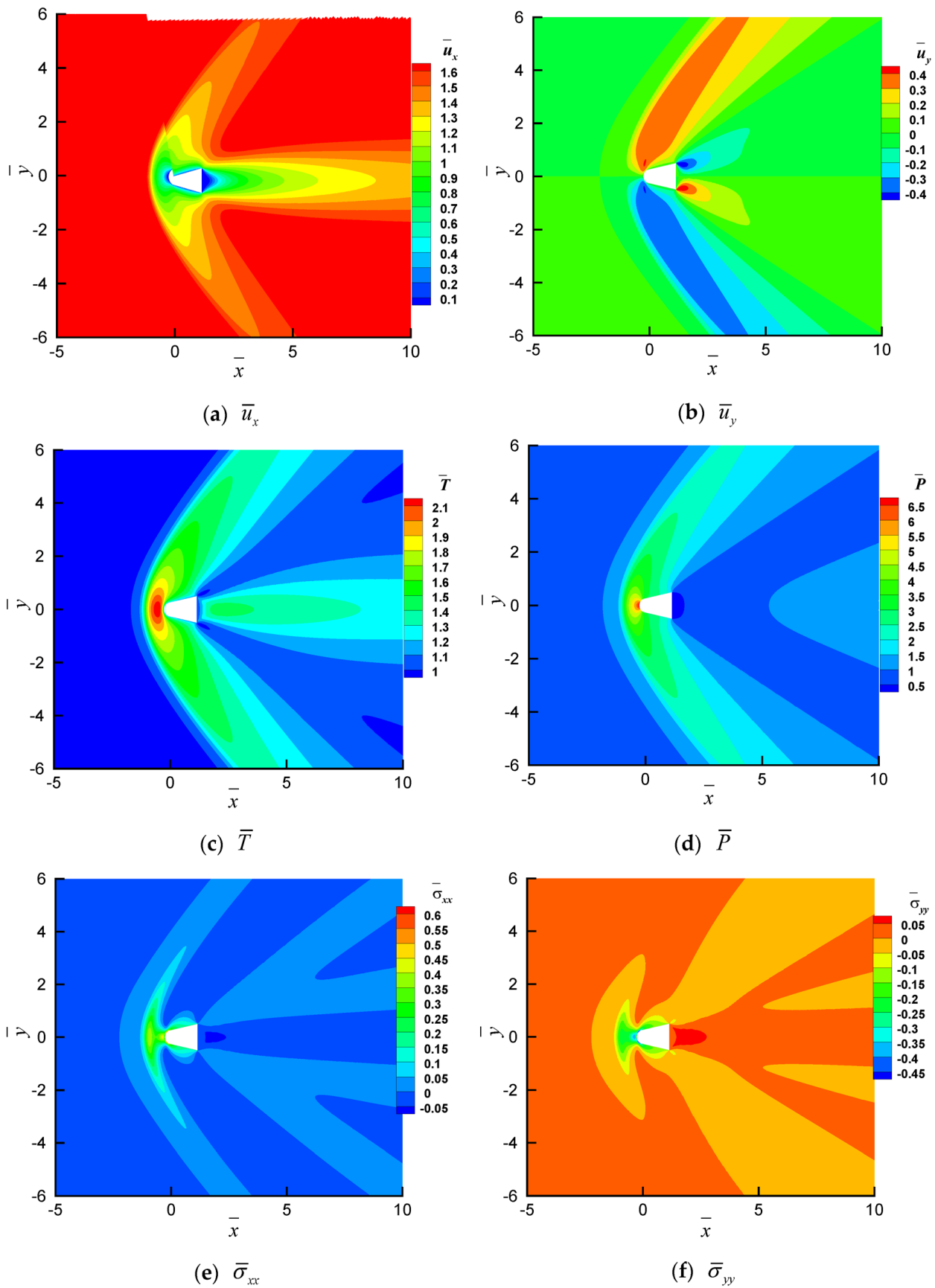


Figure 4. The flow field parameters of the non-equilibrium rarefied gas flows past a cross-domain reentry unmanned flight vehicle at $Ma = 2$, $Kn = 0.1$. (a) Velocity, $\bar{u}_x$; (b) velocity, $\bar{u}_y$; (c) temperature, $\bar{T}$; (d) pressure, $\bar{P}$; (e) stress tensor, $\bar{\sigma}_{xx}$; (f) stress tensor, $\bar{\sigma}_{yy}$.

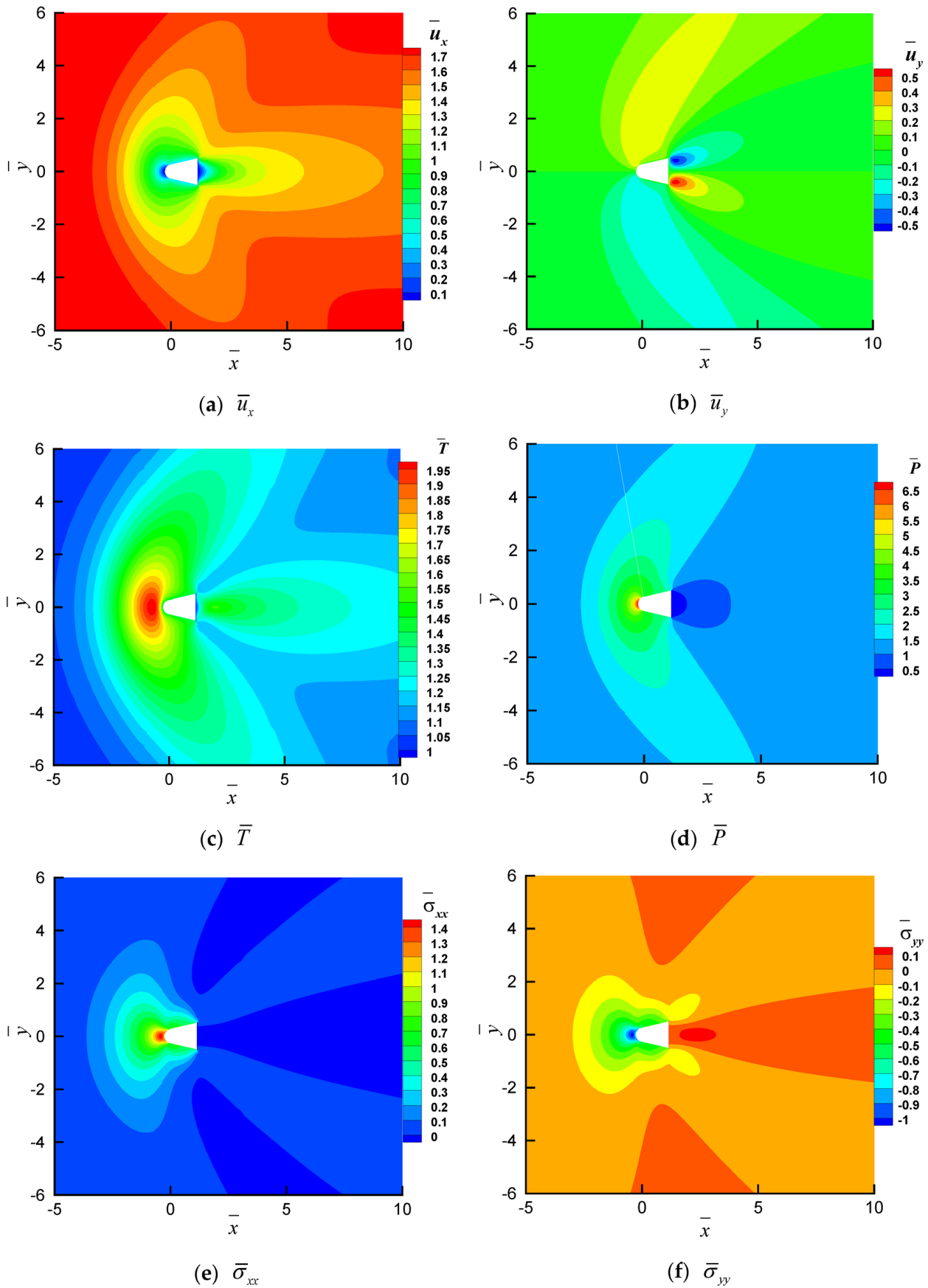


Figure 5. The flow field parameters of the non-equilibrium rarefied gas flows past a cross-domain reentry unmanned flight vehicle at $Ma = 2$, $Kn = 1.0$. (a) Velocity, $\bar{u}_x$; (b) velocity, $\bar{u}_y$; (c) temperature, $\bar{T}$; (d) pressure, $\bar{P}$; (e) stress tensor, $\bar{\sigma}_{xx}$; (f) stress tensor, $\bar{\sigma}_{yy}$.

The bow shock wave is clearly seen in front of the cone in Figure 4a, and the maximum dimensionless velocity $\bar{u}_x = 1.6$ and $\bar{u}_y = 0.4$. The maximum velocity appears in the expansion wave region on the upper right and lower right of the aircraft. Due to the strong shock compression, the temperature of the incoming flow rapidly rises when it approaches the front spherical vertebra of the aircraft, as shown in Figure 4c. Meanwhile, the pressure also rises rapidly with the compression of the air flow. The maximum dimensionless temperature and maximum dimensionless pressure are $\bar{T} = 2.1$, $\bar{P} = 6.5$, respectively. For $Ma = 2$, $Kn = 0.1$, the computational costs, in terms of computing memory and time, for the full DVM and the hybrid scheme are about 195.6 GB/1545 min and 3.84 GB/674 min, respectively.

Due to the non-equilibrium effect, it can be found from Figures 4a and 5a that the greater the Kn number, the greater the thickness of the shock wave. In addition, it can also be found that the maximum dimensionless velocity appears further away from the surface of the aircraft. This is because when $Kn = 1.0$, the average free path between molecules is longer, the collision frequency and interaction between molecules are greatly reduced, and the impact of the aircraft wall on gas molecules is reduced. Therefore, the location of the expansion wave appears farther away from the wall compared with a small Kn . From Figure 5c,d, it can be seen that when $Kn = 1.0$, the change in the gas molecular temperature and pressure is not as drastic as that of low Knudsen numbers, which is also caused by a reduction in the number of collisions between molecules. The maximum temperature and maximum dimensionless pressure are about $\bar{T} = 1.95$, $\bar{P} = 6.5$.

The flow field parameters of the non-equilibrium rarefied gas flows past a cross-domain reentry unmanned flight vehicle, as the Ma number increases to 5.0, are shown in Figures 6a–f and 7a–f. When $Kn = 0.1$, the strong shock structure can be clearly seen in front of the reentry flight vehicle, as shown in Figure 6a. When Kn is small and $Ma = 5.0$, the gas molecules are in the slip and early transition regimes. Hence, the compression effect of the shock wave is very strong, and the maximum dimensionless velocity is about $\bar{u}_x = 4.5$. In addition, it can be observed that the expansion wave of the aircraft tail becomes less obvious as the Ma number increases to 5.0.

For the hypersonic incoming flows, the temperature and pressure increase sharply due to the compression effect, as shown in Figure 6c. The maximum temperature appears near the front of the flight vehicle, and the value of the dimensionless maximum temperature and pressure are about $\bar{T} = 11$ and $\bar{P} = 36$, respectively. By comparing Figures 6a and 7a, it can be observed that as the Kn increases, the shock layer at the head of the flight vehicle becomes thicker, and non-equilibrium effects become more critical for the aircraft.

4.2. Comparison of the Computational Accuracy

The velocity, temperature, and density of the rarefied gas flows are represented by u_∞ , T_∞ , ρ_∞ , respectively. The Mach number Ma , Reynolds number Re , and Knudsen number Kn of the incoming flow can be expressed as follows:

- (i) The Ma number is defined as

$$Ma = \frac{u_\infty}{c_{\text{sound}}} \quad (22)$$

in which c_{sound} represents the speed of sound, which can be calculated as follows:

$$c_{\text{sound}} = \sqrt{\gamma \frac{kT_\infty}{m}} \quad (23)$$

in which k , m , and γ are the Boltzmann constant, molecular mass, and specific heat ratio, respectively.

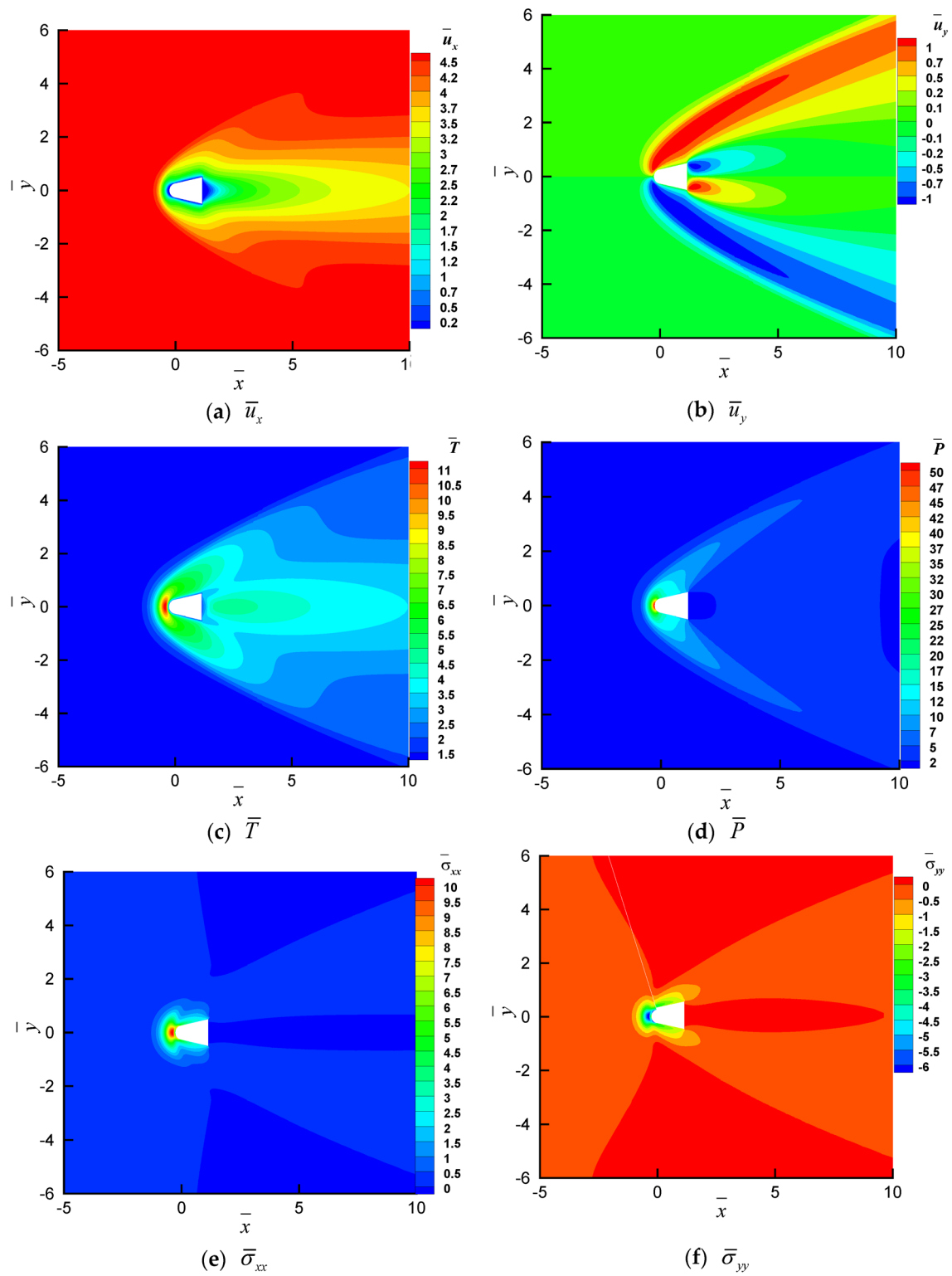


Figure 6. The flow field parameters of the non-equilibrium rarefied gas flows past a cross-domain reentry unmanned flight vehicle at $Ma = 5$, $Kn = 0.1$ (a) Velocity, $\bar{u}_x$; (b) velocity, $\bar{u}_y$; (c) temperature, $\bar{T}$; (d) pressure, $\bar{P}$; (e) stress tensor, $\bar{\sigma}_{xx}$; (f) stress tensor, $\bar{\sigma}_{yy}$.

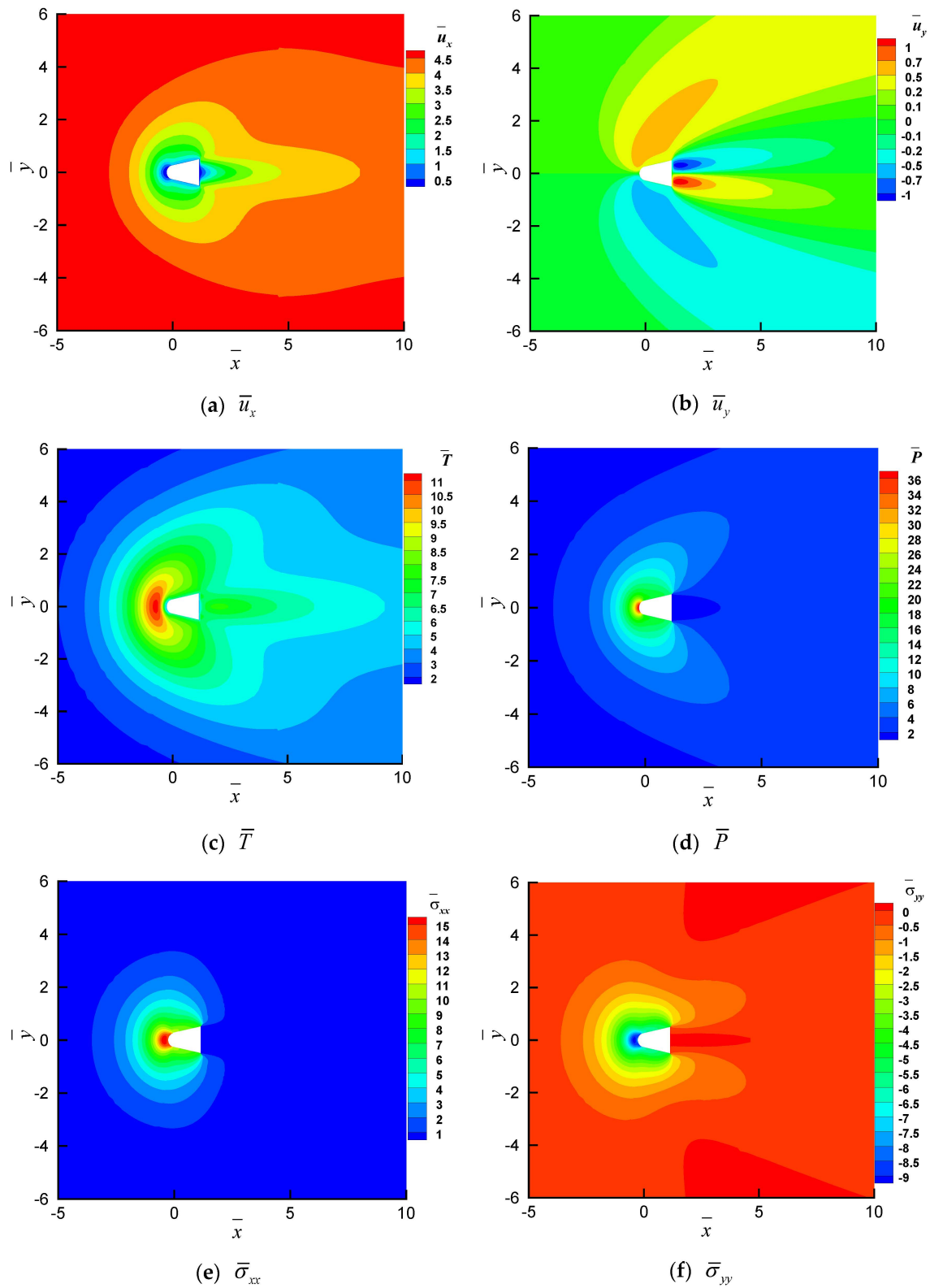


Figure 7. The flow field parameters of the non-equilibrium rarefied gas flows past a cross-domain reentry unmanned flight vehicle at $Ma = 5$, $Kn = 1.0$ (a) Velocity, $\bar{u}_x$; (b) velocity, $\bar{u}_y$; (c) temperature, $\bar{T}$; (d) pressure, $\bar{P}$; (e) stress tensor, $\bar{\sigma}_{xx}$; (f) stress tensor, $\bar{\sigma}_{yy}$.

(ii) Re and Kn are defined as follows:

$$Re = \frac{Ru_{\infty}\rho_{\infty}}{\mu}, \quad Kn = \frac{\lambda_{\infty}}{R} \quad (24)$$

in which R and μ represent the radius of the bottom and the viscosity of the gas, respectively.

(iii) The relationship between Re , Kn , and Ma is

$$Kn = \left(\gamma \frac{\pi}{2} \right)^{1/2} \frac{Ma}{Re} \quad (25)$$

(iv) The pressure coefficient C_p is expressed as follows:

$$C_p = \frac{P - P_\infty}{1/2 \rho_\infty u_\infty^2} \quad (26)$$

in which P and P_∞ represent the pressure of the aircraft and the pressure of the free flow, which follows the ideal gas law $P_\infty = \rho_\infty R T_\infty$.

(v) The heat transfer coefficient C_H is expressed as follows:

$$C_H = \frac{q_n}{1/2 \rho_\infty u_\infty^2} \quad (27)$$

where q_n represents the heat flux perpendicular to the wall.

Figure 8a,b show the temperature $\bar{T}$ and pressure $\bar{P}$ distributions along the center line in front of the reentry flight vehicle at $Ma = 5$, $Kn = 0.74$, and these results were compared with those obtained by Zhi-hui Li [30] and Felix Sharipov [31]. Figure 8c,d show the pressure coefficient C_p and the heat transfer coefficient C_H on the surface of the reentry flight vehicle at $Ma = 5$. The maximum dimensionless temperature in front of the flight vehicle is $\bar{T} = 11.3$, and the temperature decreases rapidly within a reference length. Similarly, the maximum dimensionless density in front of the flight vehicle is $\bar{\rho} = 6.8$, and the calculated results in this study are in good agreement with the benchmark results.

From Figure 8c,d, the calculation results obtained in this study are consistent with those obtained by Zhi-hui Li and Felix Sharipov. The maximum errors of heat transfer coefficient and pressure coefficient are 2.1% and 3%, respectively. In Figure 8c, it is evident that the maximum heat transfer coefficient C_H appears near the head of the aircraft, with a maximum value of $C_H = 0.82$. In addition, the heat transfer coefficient gradually decreases along the aircraft surface because the normal heat flow on the surface of the aircraft gradually decreases along the surface. In Figure 8d, the distribution of C_p is similar to the change in the heat transfer coefficient C_H , which reaches a maximum value of $C_p = 2.46$ at the head of the aircraft and then gradually decreases along the surface.

With a further increase in the Ma number, i.e., $Ma = 10.0$, the non-equilibrium effect becomes significant, and more discrete velocities are needed to grasp the non-equilibrium effect. Hence, a mesh of 10,000 non-uniform tetrahedral elements was used to discretize the physical space, and $124 \times 124 \times 64$ points were used to discretize the velocity space. The flow field parameters of the non-equilibrium flow past a cross-domain reentry unmanned flight vehicle at $Ma = 10$, $Kn = 0.1$ and $Ma = 10$, $Kn = 1.0$ are shown in Figure 9.

From Figure 9a, the curvature of the bow shock wave at $Ma = 10.0$ is greater than that of the low Mach numbers, and the expansion wave at the rear of the vehicle disappears. Meanwhile, it is clear to see from Figure 9a,c that as Kn increases, the thickness of the shock layer at the head of the flight vehicle becomes thicker, and non-equilibrium effects become more critical for the aircraft. The maximum velocities in the x and y directions are about $\bar{u}_x = 8.5$ and $\bar{u}_y = 1.8$, respectively. Due to the shock wave, there is a strong heating effect in front of the cone, with a maximum dimensionless temperature and a maximum dimensionless pressure of $\bar{T} = 32$ and $\bar{P} = 65$, respectively.

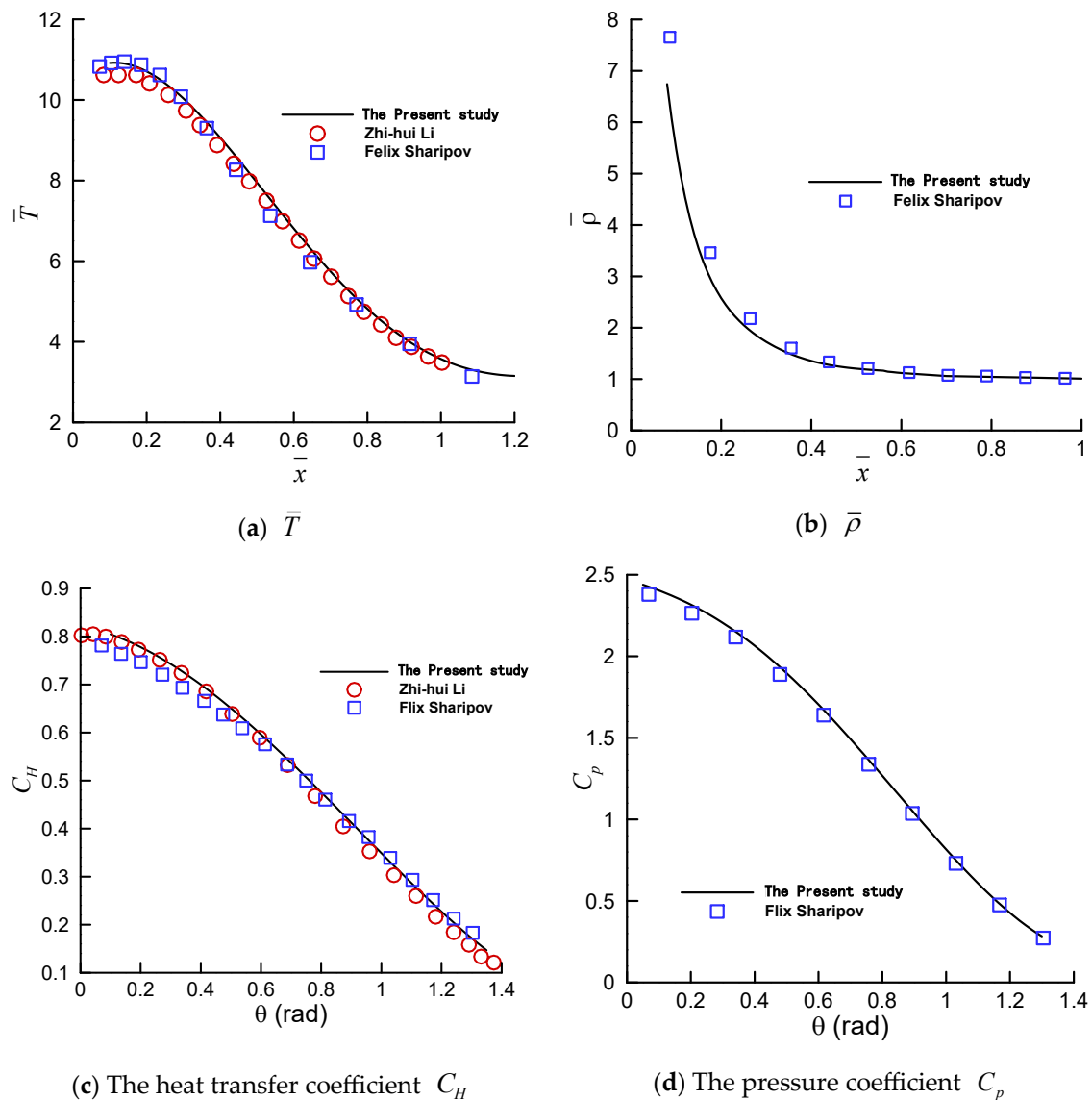


Figure 8. Flow parameters of the reentry flight vehicle. (a) Temperature, $\bar{T}$; (b) density, $\bar{\rho}$; (c) heat transfer coefficient, C_H ; (d) pressure coefficient, C_P , at $Ma = 5$, $Kn = 0.74$.

Figure 10a,b show the temperature $\bar{T}$ and pressure $\bar{P}$ distributions along the center line in front of the reentry flight vehicle at $Ma = 10$, and these results were compared with benchmark results. Figure 10c,d show the pressure coefficient C_P and heat transfer coefficient C_H of the surface of the reentry flight vehicle at $Ma = 10$.

It can be seen from Figure 10a that the temperature in front of the aircraft rapidly increases to $\bar{T} = 32$, and the maximum density is about $\bar{\rho} = 11$. The results obtained in this study are consistent with benchmark results, with a maximum density error of 9%. The maximum errors of the heat transfer coefficient and pressure coefficient are 2% and 4.6%, respectively. In addition, as the Mach number increases, the temperature and pressure ahead of the aircraft rise from $\bar{T} = 11$ to $\bar{T} = 32$ and from $\bar{\rho} = 6.8$ to $\bar{\rho} = 11$, respectively. The maximum heat transfer coefficient C_H and pressure coefficient C_P appear at the head of the aircraft, with values of $C_H = 1.0$ and $C_P = 2.23$, respectively.

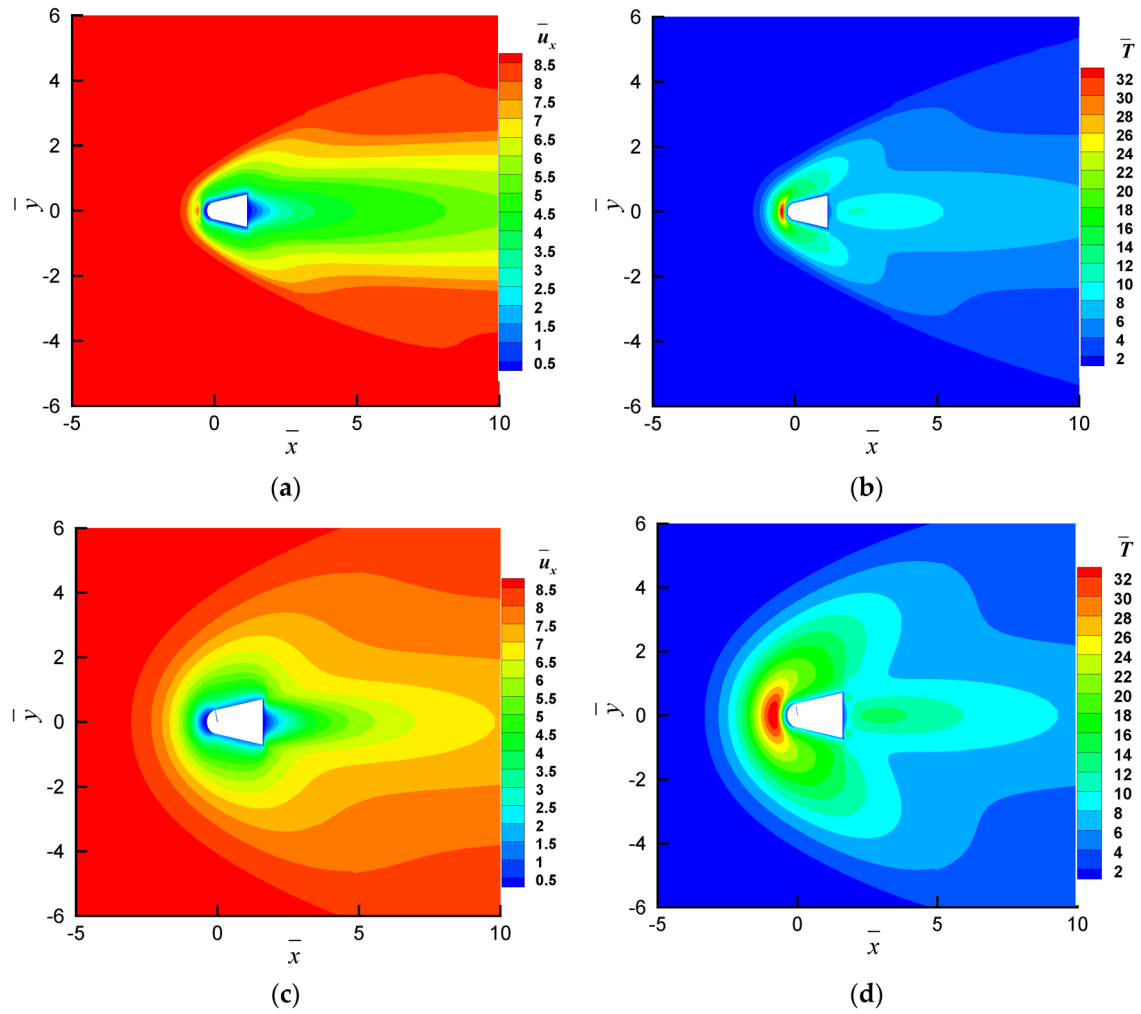


Figure 9. The flow field parameters of the non-equilibrium rarefied gas flows past a cross-domain reentry unmanned flight vehicle (a) $\bar{u}_x$ at $Ma = 10$, $Kn = 0.1$; (b) T at $Ma = 10$, $Kn = 0.1$; (c) $\bar{u}_x$ at $Ma = 10$, $Kn = 1.0$; (d) T at $Ma = 10$, $Kn = 1.0$.

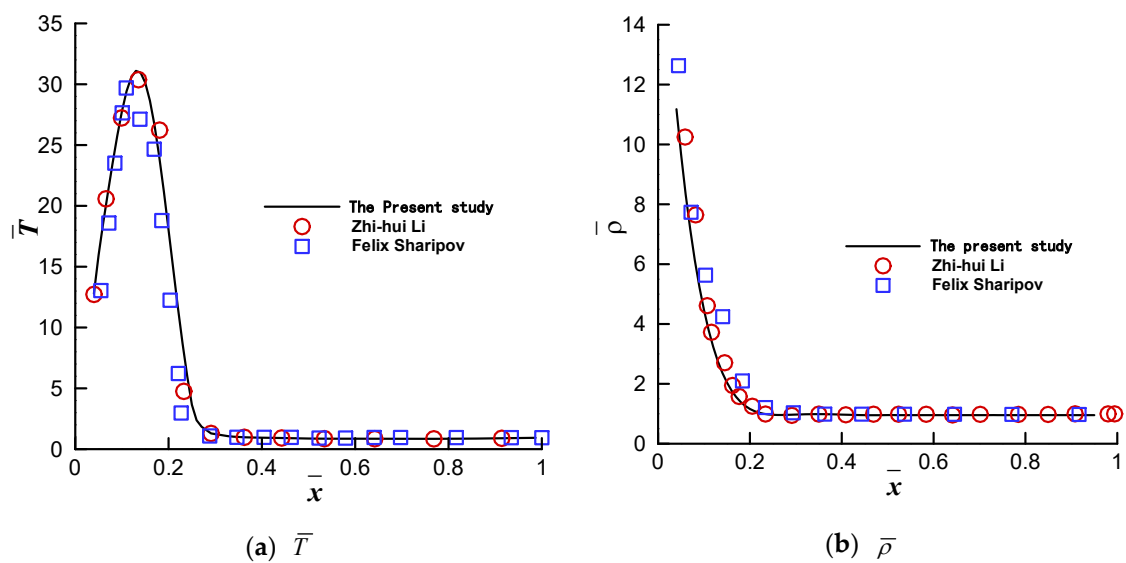


Figure 10. Cont.

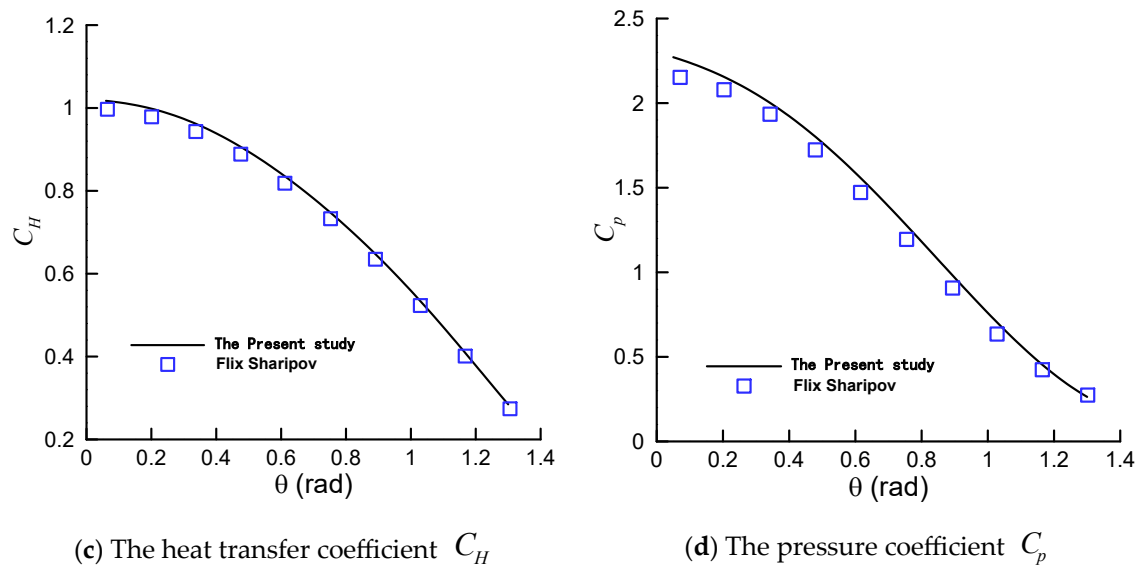


Figure 10. Flow parameters of the reentry flight vehicle. (a) Temperature; (b) density; (c) heat transfer coefficient; (d) pressure coefficient, at $Ma = 10$, $Kn = 0.004$.

5. Conclusions

A hybrid macro-/mesoscopic method is proposed for simulating rarefied gas flows past a cross-domain reentry unmanned flight vehicle in the present study. Gas dynamics of a typical cross-domain reentry spheroid-cone unmanned flight vehicle were investigated with different values of Kn and Ma , and the results were validated with those of Zhi-hui Li and Felix Sharipov. The simulation results showed that the hybrid macro-/mesoscopic scheme is well-suited for the non-equilibrium rarefied gas flows past a cross-domain reentry unmanned flight vehicle. Under the given conditions, the results obtained in this study are consistent with benchmark results, with a maximum density error of 9%. The maximum errors of the heat transfer coefficient and pressure coefficient were 2% and 4.6%, respectively. In addition, as the Knudsen number (Kn) becomes larger, the shock layer at the head of the flight vehicle becomes thicker, and non-equilibrium effects become more critical for the aircraft. Since the Boltzmann–Shakhov equation has only been solved close to the wall of a spacecraft, the computational cost can be considerably saved.

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