

Review

Non-Acoustic Detection and Localization of Large Underwater Targets for Unmanned Platforms: A Review of Wake-Based, Magnetic, and Gravity Anomaly Methods

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Highlights

What are the main findings?

- Wake-based detection, magnetic anomaly detection (MAD), and gravity anomaly detection (GAD) provide complementary non-acoustic routes for large underwater target detection and localization on unmanned platforms.
- Wake-based methods are more suitable for wide-area cueing and trajectory inference, MAD is relatively mature for short-range confirmation and localization, while GAD remains promising but is limited by weak signals and strict platform-stability requirements.

What are the implications of the main findings?

- Future unmanned-platform-based non-acoustic detection systems should integrate wake, magnetic, electromagnetic, and gravity-related signatures through multi-source fusion and multi-platform collaboration.
- Key future directions include realistic target modeling, weak-signal extraction, background suppression, onboard sensor integration, platform stability, autonomous mission planning, and cooperative operation among aerial, surface, and underwater unmanned platforms.

Abstract

The detection and localization of large underwater targets are important for maritime security, marine resource exploration, and underwater situational awareness, while the increasing acoustic stealth of underwater vehicles has limited conventional acoustic methods. This review provides a systematic overview of non-acoustic detection and localization technologies for large underwater targets, with emphasis on their relevance to unmanned aerial, surface, and underwater platforms. Wake-based detection, magnetic anomaly detection (MAD), and gravity anomaly detection (GAD) are reviewed as three representative non-acoustic routes. A bibliometric analysis is first conducted to summarize research trends, major contributors, and emerging hotspots. Wake-based methods are discussed in terms of wake signatures, modeling approaches, sensing platforms, and localization potential. MAD is analyzed from the perspectives of magnetic dipole modeling, target-based detection, noise-based detection, artificial intelligence (AI)-based detection, and magnetic localization. GAD is discussed with respect to physical feasibility, gravity-gradient target modeling, inversion methods, and engineering constraints. The review shows that wake-based methods are suitable for wide-area search and trajectory inference, MAD is relatively mature for short-range confirmation and localization, and GAD remains promising but less mature.



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Future research should focus on onboard sensors, platform stability, weak-signal extraction, background suppression, quantitative evaluation metrics, multi-source fusion, autonomous mission planning, and multi-platform collaboration.

Keywords: non-acoustic sensing; underwater object detection and localization; unmanned platforms; autonomous underwater vehicle; unmanned underwater vehicle; wake-based detection; magnetic anomaly detection; gravity anomaly detection; multi-source fusion

1. Introduction

The detection and localization of large underwater targets are important for maritime security, ocean resource exploration, underwater situational awareness, and marine scientific research [1–3]. Sonar has traditionally been the dominant technology for this task [4–7]. Active sonar detects targets by transmitting acoustic waves and receiving their echoes, whereas passive sonar relies on acoustic signals radiated by the targets. These acoustic methods have made substantial progress and remain indispensable in many underwater detection scenarios.

However, acoustic detection faces increasing challenges. Active sonar may expose the position of the detection platform because it requires acoustic emission. Passive sonar relies heavily on target-radiated acoustic signals and becomes less effective when the target is quiet. In recent years, low-noise propulsion systems, vibration isolation techniques, anechoic coatings, and integrated acoustic stealth designs have substantially reduced the acoustic signatures of modern underwater vehicles. In some cases, the radiated noise of advanced underwater vehicles may approach, or even fall below, the ambient ocean noise level. Although advanced vector hydrophones, fiber-optic hydrophones, large-aperture arrays, and signal-processing methods have improved the ability of sonar systems to detect weak acoustic signals [8–13], the increasing acoustic stealth of underwater targets highlights the need for complementary non-acoustic detection and localization technologies.

From the perspective of unmanned platforms, non-acoustic detection and localization are particularly important because unmanned aerial vehicles (UAVs), unmanned surface vehicles (USVs), autonomous underwater vehicles (AUVs), and unmanned underwater vehicles (UUVs) can provide flexible observation geometries and complementary sensing capabilities. UAVs can support wide-area remote sensing using optical, infrared, light detection and ranging (LiDAR), or synthetic aperture radar (SAR) payloads. USVs can carry magnetic, optical, and hydrodynamic sensors for regional monitoring. AUVs and UUVs can perform short-range confirmation, passive sensing, and relative localization using magnetic, pressure, flow, or gravity-related measurements. Therefore, non-acoustic underwater object detection and localization (UODL) is not only a sensing task, but also a platform-oriented problem that involves onboard sensor integration, navigation accuracy, platform stability, communication constraints, mission planning, and multi-platform collaboration.

Large underwater targets inevitably disturb the surrounding ocean environment and generate various non-acoustic signatures during navigation. These signatures include wake-related disturbances, such as hydrodynamic wakes, free-surface disturbances, internal waves, turbulent wakes, bubble wakes, thermal anomalies; environmental or biogeochemical responses, such as bioluminescent responses and chemical traces; and target-related physical-field anomalies, such as magnetic anomalies, electromagnetic fields, and gravity or gravity-gradient anomalies [14–20].

Compared with acoustic signals, non-acoustic signatures are generated by different physical mechanisms and are affected by different environmental conditions. They therefore provide complementary information for underwater target detection and localization. Among these signatures, wake-based detection, magnetic anomaly detection (MAD), and gravity anomaly detection (GAD) have attracted increasing attention because they represent three distinct information sources: target-induced environmental disturbances, target-related magnetic signatures, and target-induced mass-density perturbations. This review focuses on these three routes for three main reasons. First, they cover complementary physical mechanisms and sensing principles. Second, they are closely related to large underwater targets and can potentially be supported by unmanned aerial, surface, and underwater platforms. Third, they span different levels of technical maturity, from relatively mature MAD to feasibility-oriented gravity-gradient detection. Other signatures, including direct optical detection, radiation-related sensing, chemical traces, electric-field detection, and some electromagnetic responses, are discussed only as complementary or extension mechanisms because their applicability is usually more constrained by target exposure, environmental transparency, source specificity, or operational conditions.

Although this review focuses on large underwater targets, the reviewed principles may also be extended to smaller UUV systems. Large targets are emphasized because their wake, magnetic, and gravity-related signatures are generally stronger and more detectable. For smaller UUVs, these signatures are usually weaker and decay over shorter ranges. As a result, higher sensor sensitivity, closer platform-target proximity, and stronger background suppression are required.

Wake-based detection relies on the indirect observation of target-induced environmental disturbances. These disturbances may include Bernoulli humps, Kelvin waves, internal wakes, turbulent wakes, bubble wakes, thermal wakes, and biological wakes. Wake signatures may cover a much larger spatial region than the target itself and may persist after the target has passed. Therefore, wake-based detection is useful for wide-area search, historical trajectory inference, and target motion estimation [21,22]. With advances in SAR, optical imaging, infrared imaging, laser backscattering, pressure sensing, and bio-inspired flow sensors, this field has gradually evolved from qualitative wake observation to quantitative wake feature extraction and localization-oriented analysis. However, wake signatures are strongly affected by sea state, stratification, turbulence, background currents, water clarity, thermal exchange, and biological activity. Their persistence can improve detection probability, but it also weakens the real-time relationship between the observed wake and the current target position. Therefore, wake-based localization usually requires wake-age estimation, trajectory reconstruction, environmental correction, and multi-wake feature fusion.

MAD is one of the most widely studied non-acoustic detection technologies. Most large underwater targets contain ferromagnetic materials or magnetic components, which can disturb the geomagnetic field and generate magnetic anomaly signals. When the sensor-target distance is sufficiently large, the target magnetic field can often be approximated by an equivalent magnetic dipole model [2,23,24]. This approximation provides the theoretical basis for many MAD and magnetic localization methods. Existing MAD algorithms can be broadly divided into target-based, noise-based, and artificial intelligence (AI)-based methods. Target-based methods, such as the orthogonal basis function (OBF) method, exploit the characteristic waveform of the magnetic anomaly signals [25–29]. Noise-based methods, including minimum entropy detection (MED), empirical mode decomposition (EMD), higher-order crossing (HOC), and stochastic resonance (SR), detect anomalies by analyzing statistical or structural differences between background noise and signal-containing observations [30–33]. AI-based methods further improve weak-signal extraction,

magnetic-source localization, and classification by learning discriminative features from measured or simulated magnetic signals [34–37]. Nevertheless, MAD still faces major challenges, including weak-anomaly extraction under low signal-to-noise ratio (SNR) conditions, colored geomagnetic noise, platform magnetic interference, magnetic stealth technologies, and limited generalization of data-driven algorithms.

Gravity and gravity-gradient anomaly detection provide another potential route for non-acoustic underwater target detection. Even when an underwater vehicle is approximately neutrally buoyant, its internal mass distribution, hull geometry, equipment layout, and the offset between the center of gravity and the center of buoyancy may generate weak gravity or gravity-gradient perturbations. Gravity-related signals are passive, non-emitting, and difficult to shield in the conventional sense, giving GAD unique potential for covert detection [38]. Early studies on gravity-based underwater vehicle detection were limited by instrument sensitivity. However, recent advances in gravity gradiometers, atomic interferometry, and high-precision inertial measurement have renewed interest in this direction. Existing studies mainly include inversion algorithms based on simplified particle models [39–43] and forward modeling studies based on underwater-vehicle-like geometric models [19,38,44]. However, GAD remains at a feasibility-oriented stage. Target-induced gravity-gradient signals are extremely weak and can be easily affected by platform acceleration, vibration, attitude variation, ocean tides, seafloor topography, geological density variations, and instrument noise. More importantly, a methodological gap remains between particle-model-based inversion and geometry-consistent forward modeling for realistic underwater vehicles.

Although many valuable studies have addressed underwater target detection and localization, existing reviews usually focus on acoustic detection, a single non-acoustic modality, or a specific class of detection algorithms. A systematic comparison of wake-based detection, MAD, and GAD remains insufficient, especially from the perspectives of physical mechanisms, modeling assumptions, sensing platforms, detection algorithms, localization capabilities, and engineering limitations. Such a comparison is necessary because these three methods differ in observable signatures, operational ranges, and technical maturity. Wake-based methods are suitable for wide-area search and trajectory inference, but they are sensitive to environmental conditions and provide limited real-time localization capability. MAD is relatively mature and can support both detection and localization, but it is constrained by weak magnetic-anomaly extraction, platform interference, and magnetic stealth. GAD is attractive because gravity-related signals are difficult to eliminate or shield, but its practical application remains limited by extremely weak signals, strict platform-stability requirements, and insufficient experimental validation. To clarify the novelty and scope of this review, Table 1 compares the present review with related reviews and representative study directions in underwater target detection and localization.

To address these issues, this review examines non-acoustic detection and localization technologies for large underwater targets, with particular emphasis on wake-based detection, MAD, and GAD. The main contributions of this review are summarized as follows.

- (1) A bibliometric analysis of non-acoustic UODL is conducted to identify the research status, major contributors, and emerging research hotspots in this field.
- (2) Wake-based detection is reviewed in terms of physical wake signatures, wake modeling and characterization methods, sensing technologies, localization potential, and engineering challenges. Representative wake signatures, including hydrodynamic wakes, bubble wakes, thermal wakes, and biological wakes, are compared according to their generation mechanisms and observability.

- (3) MAD is systematically reviewed from the perspectives of physical modeling, target-based detection, noise-based detection, AI-based detection, and magnetic localization. Particular attention is paid to the relationship between magnetic dipole modeling, weak anomaly extraction, magnetic gradient tensor (MGT)-based localization, and extensions beyond conventional static MAD.
- (4) GAD is analyzed in terms of physical feasibility, gravity-gradient target modeling, localization and inversion methods, and practical constraints. The review highlights the gap between particle-model-based inversion and geometry-consistent forward prediction for large neutrally buoyant underwater vehicles.
- (5) The roles of unmanned platforms in non-acoustic UODL are discussed, including UAV-based wide-area wake observation, USV-based regional sensing, AUV- and UUV-based close-range confirmation, and multi-platform cooperative detection.
- (6) Major technical challenges and future directions are summarized from a platform-oriented perspective, including onboard sensor integration, weak-signal extraction, background-field suppression, navigation and attitude stability, sensor fusion, mission planning, target simulation, multi-target localization, and autonomous multi-platform collaboration.

Table 1. Comparison between related reviews/studies and the present review.

Review Focus	Representative Scope	Main Limitation Relative to This Review	Difference of this Review
Acoustic detection reviews [45–47]	sonar, passive acoustic recognition, hydrophone arrays	mainly acoustic; less attention to non-acoustic sensing	focuses on non-acoustic detection and localization
Wake or wake-flow studies/reviews [48,49]	wake physics, SAR/optical/infrared wake observation	limited discussion of localization and unmanned-platform integration	links wake signatures with platform-based cueing and trajectory inference
MAD reviews [23,50]	magnetic anomaly detection, magnetic localization	usually focus on magnetic methods alone	compares MAD with wake-based and GAD routes
Gravity- or gravity-gradient-based detection studies [19,38,39,41,51]	gravity-gradient forward modeling, inversion, and feasibility analysis	scattered theoretical studies; lack of dedicated platform-oriented review	systematically evaluates GAD from physical feasibility, target modeling, inversion methods, sensor requirements, and platform constraints
General underwater detection reviews [52,53]	broad underwater sensing technologies	lack unified task chain for unmanned platforms	builds a platform–sensor–signal–algorithm–output framework

The remainder of this paper is organized as follows. Section 2 presents the bibliometric analysis of underwater object detection and localization. Section 3 reviews wake-based detection and localization methods. Section 4 discusses magnetic anomaly detection and localization. Section 5 analyzes gravity and gravity-gradient anomaly detection and localization. Section 6 summarizes the technical challenges and future research directions. Section 7 concludes the paper. The unified platform–sensor–signal–algorithm–output framework for non-acoustic UODL on unmanned platforms is shown in Figure 1.

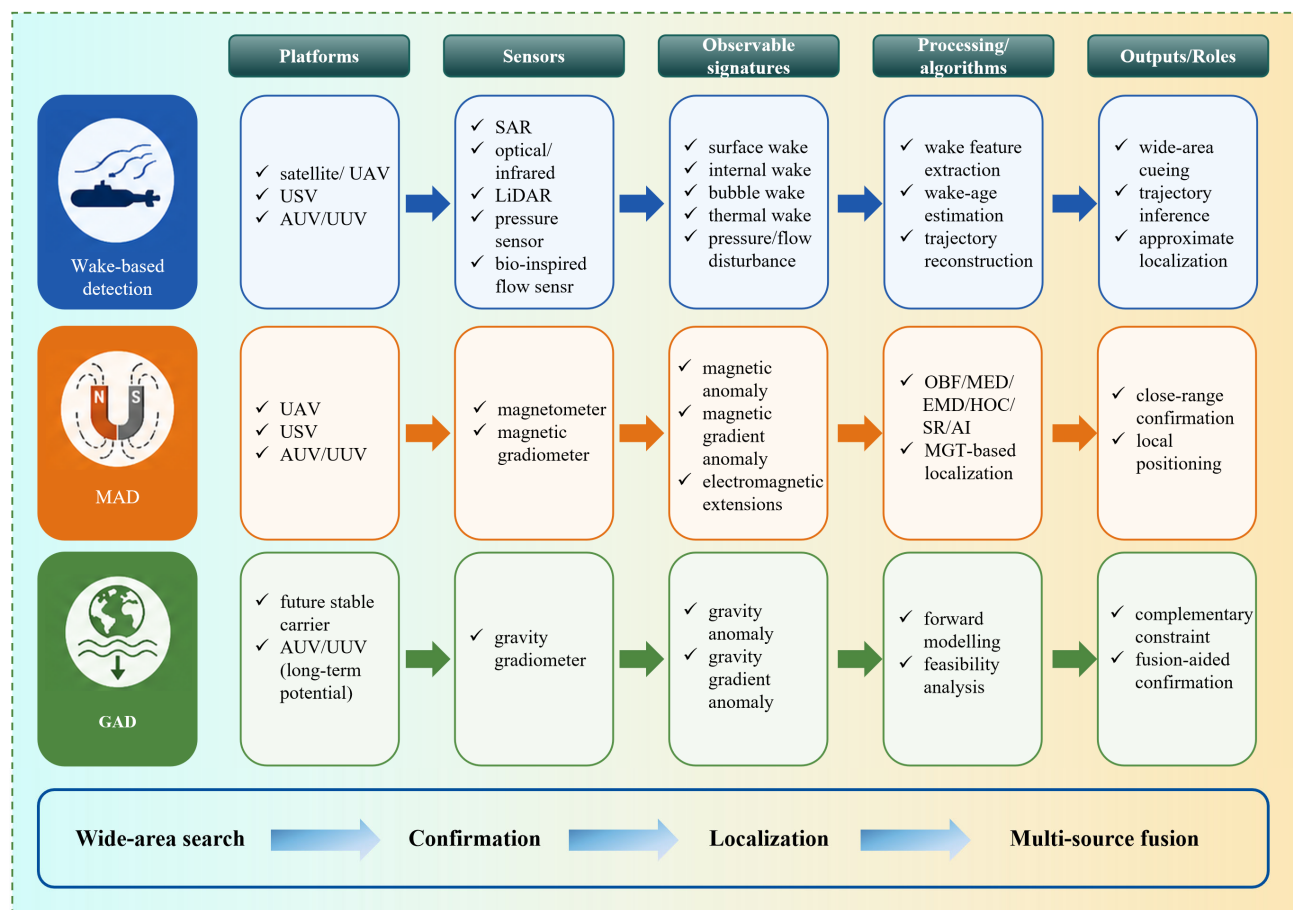


Figure 1. Unified platform-sensor-signal-algorithm-output framework for non-acoustic UODL on unmanned platforms.

2. Bibliometric Analysis

To provide an overview of the research landscape of non-acoustic underwater object detection and localization, a bibliometric analysis was performed using the Web of Science Core Collection. Considering the scope of this review, the retrieval strategy was designed using three groups of terms: underwater target terms, detection/localization task terms, and non-acoustic physical-signature terms. The underwater target terms included “underwater target”, “underwater object”, “underwater vehicle”, “submarine”, “submersible”, “AUV”, and “UUV”. The task-related terms included “detection”, “localization”, “positioning”, “tracking”, “recognition”, “classification”, and “inversion”. The non-acoustic-signature terms covered wake-related signatures, magnetic anomalies, gravity anomalies, electromagnetic fields, thermal signatures, optical backscattering, pressure fields, and bio-inspired or biological sensing. The complete search query is provided in the Supplementary Materials.

The search scope was set to “Topic”, and the time span was set from 2000 to 2026. Articles, review articles, and proceedings papers were retained. After retrieval, duplicate records and irrelevant studies were removed manually. Studies focusing only on acoustic sonar, underwater image enhancement without detection or localization relevance, underwater communication, general robot navigation, or marine biology without target-detection relevance were excluded. Finally, 1483 records were retained for bibliometric analysis. CiteSpace 7.0.0 was used to analyze collaboration networks, keyword bursts, and knowledge evolution, while VOSviewer 1.6.20 was used to visualize bibliometric networks and clustering results. Figures 2 and 3 summarize the country distribution, institution

distribution, keyword burst analysis, and keyword clustering results. Figure 4 shows the annual and cumulative publication trends from 2000 to 2025.

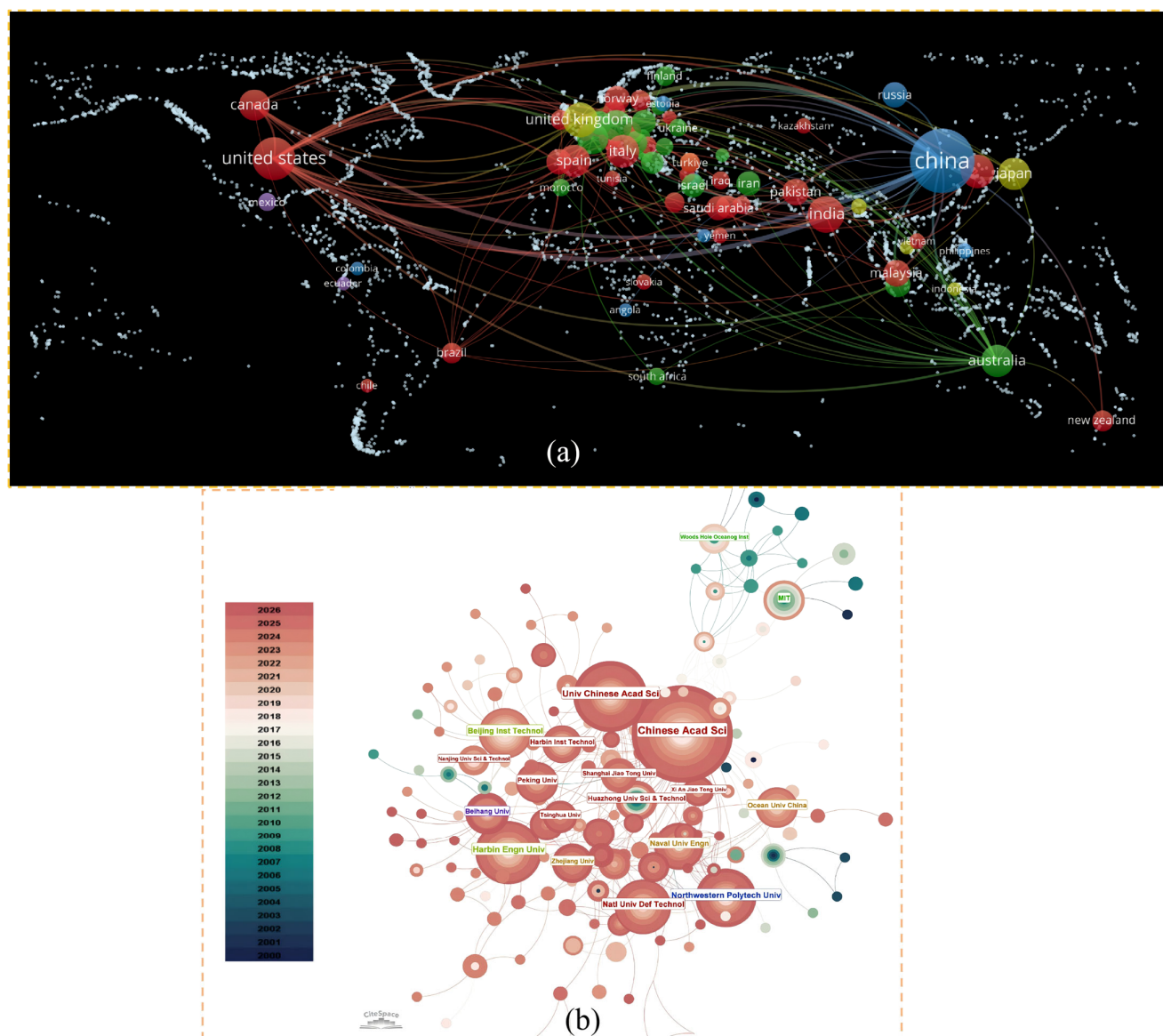


Figure 2. Bibliometric analysis of non-acoustic UODL: (a) country collaboration network; (b) institution collaboration network.



Figure 3. Keyword analysis of non-acoustic UODL: (a) keyword burst analysis; (b) keyword clustering and evolution.

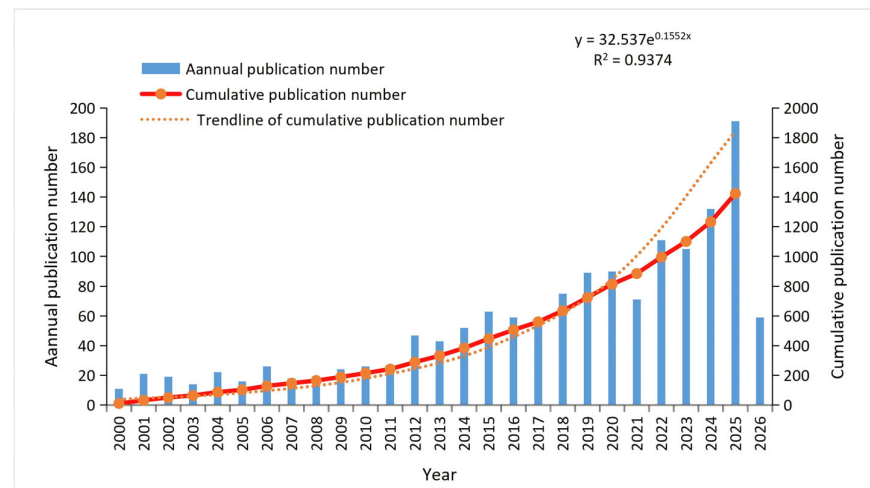


Figure 4. Annual and cumulative publication trends from 2000 to 2025.

This bibliometric dataset was constructed as a core dataset for non-acoustic UODL. Some general MAD and GAD method papers do not explicitly mention underwater targets, but they provide transferable algorithms for magnetic anomaly detection, magnetic dipole localization, MGT inversion, or gravity-gradient inversion. These method-oriented papers were screened manually and included in the qualitative review sections when relevant, but they were not merged into the bibliometric dataset. This strategy ensures that the bibliometric results reflect the research landscape of non-acoustic UODL, rather than broader studies on geophysical exploration, pipeline detection, mine detection, or other non-underwater-target studies.

2.1. Research Subjects of UODL

Figure 2a shows the country-level collaboration network in non-acoustic UODL. The node size represents publication output, and the links between nodes indicate international collaboration. China shows the largest node in the retrieved dataset, suggesting a high level of publication activity in this field. The United States, India, the United Kingdom, South Korea, Italy, Australia, Japan, Spain, and several other countries also contribute actively. The collaboration network further shows that China has established cooperation with countries such as the United States, Canada, the United Kingdom, Australia, and Japan. The United States also maintains collaborative links with China, Italy, South Korea, India, and Singapore. These results indicate that UODL has become an internationally active research topic rather than a regionally isolated research direction.

Figure 2b presents the institution-level distribution and collaboration network. The Chinese research institutions show high publication output and strong network centrality. Representative institutions include the Chinese Academy of Sciences, Harbin Engineering University, Northwestern Polytechnical University, Beijing Institute of Technology, Harbin Institute of Technology, Zhejiang University, Ocean University of China, and Dalian Maritime University. These institutions cover multiple research directions, including underwater vehicles, remote sensing, hydrodynamic wakes, magnetic sensing, intelligent detection, and underwater localization. This distribution reflects the interdisciplinary nature of non-acoustic UODL, which involves ocean engineering, fluid mechanics, geophysics, electromagnetics, sensor technology, signal processing, and artificial intelligence.

2.2. Research Hotspots of UODL

The keyword burst results in Figure 3a reveal the evolution of research hotspots over time. Early studies mainly focused on fundamental detection, localization, sensor net-

works, and underwater vehicle-related topics. With advances in remote sensing, intelligent algorithms, and non-acoustic sensing technologies, recent studies have increasingly emphasized wake detection, electromagnetic signatures, AI, feature extraction, and multi-source information processing. These bursts indicate that UODL research has gradually shifted from conventional detection and localization toward complex-environment perception, weak-signal extraction, and intelligent decision-making.

The keyword clustering results in Figure 3b further show that UODL is an interdisciplinary field involving several closely related research themes. The major clusters include remote sensing detection, cooperative systems, target localization, pattern recognition, underwater vehicles, underwater signals, synthetic aperture techniques, and intelligent detection. These clusters are consistent with the technical scope of this review. Wake-based detection is closely related to remote sensing, surface signatures, wake features, and underwater vehicle hydrodynamics. MAD is associated with magnetic anomaly signals, weak-signal extraction, magnetic localization, and intelligent classification. GAD is linked to gravity or gravity-gradient anomaly modeling, inversion, and feasibility analysis. Therefore, the bibliometric results support the selection of wake-based detection, MAD, and GAD as the three major non-acoustic detection and localization routes reviewed in this paper.

Figure 4 shows that the annual publication number generally increased from 2000 to 2025, while the cumulative publication number followed an approximately exponential growth trend. This pattern indicates that UODL has received growing attention over the past two decades. The increase has become particularly evident in recent years and can be attributed to several factors. First, the increasing acoustic stealth of underwater targets has promoted the development of complementary non-acoustic sensing methods. Second, rapid advances in remote sensing, high-sensitivity sensors, and autonomous underwater platforms have expanded the available observation routes. Third, machine learning and deep learning have provided new tools for weak-signal detection, feature extraction, target recognition, and multi-source fusion.

Overall, the bibliometric analysis shows that non-acoustic UODL has evolved from a relatively scattered research topic into a rapidly developing interdisciplinary field. The main contributors include countries and institutions with strong research activities in ocean engineering, underwater vehicles, remote sensing, sensor technology, and intelligent information processing. The research hotspots have gradually shifted toward wake signatures, magnetic anomalies, electromagnetic and gravity-related signals, artificial intelligence, and multi-platform sensing. These trends are consistent with the structure of this review, which focuses on wake-based detection, MAD, and GAD, and further discusses the future development of realistic target modeling, weak-signal extraction, background-field suppression, high-precision sensors, multi-source fusion, and multi-platform collaboration.

3. Wake-Based Detection and Localization of Large Underwater Targets

Building on the overview in Section 1, this section examines wake-based UODL as an indirect sensing route based on target-induced environmental disturbances. The discussion focuses on how representative wake features are generated, modeled, observed, and converted into target-related information. The section is organized around five aspects: physical wake signatures, wake modeling and characterization, wake sensing and localization technologies, wake-to-target information extraction, and engineering challenges. In the unified framework shown in Figure 1, wake-based detection mainly supports wide-area cueing, trajectory reconstruction, and approximate localization.

3.1. Physical Basis and Wake Signatures

Figure 5 illustrates representative wake signatures generated by moving underwater vehicles. The following discussion focuses on the wake signatures most relevant to UODL, including Bernoulli and Kelvin wakes, internal wakes, turbulent wakes, bubble wakes, thermal wakes, and biological wakes. Because these signatures differ in generation mechanism, persistence, and observability, they require different sensing configurations and interpretation strategies.

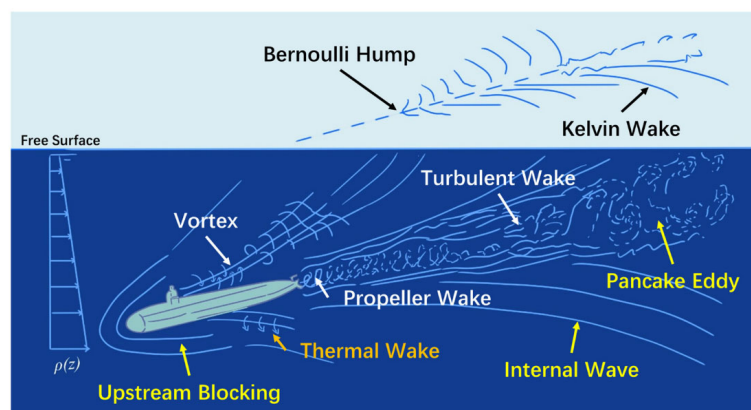


Figure 5. Sketch of submerged vehicle wakes in stratified fluids [48].

Bernoulli humps are free-surface elevations caused by pressure variations when an underwater vehicle moves near the sea surface. Tunaley modeled the underwater target as a Rankine oval composed of sources and sinks and analytically derived the Bernoulli hump distribution [54]. Sudharsun investigated the Bernoulli hump generated by submerged vehicle models through numerical and experimental methods [17]. The results showed that the hump height is related to the target size, speed, and submergence depth. Therefore, Bernoulli humps can theoretically provide information about the target depth and motion state, but their detectability decreases rapidly as diving depth increases.

Kelvin wakes are V-shaped wave patterns generated by moving bodies on or below the free surface. For underwater targets, the observable Kelvin wake depends on target speed, submerged depth, hull size, and sea-surface conditions. A classical Kelvin wake consists of transverse waves, divergent waves, and cusp waves [55]. Although Kelvin wakes have been widely studied for surface ships, those generated by submerged targets are usually weaker and more difficult to observe [56]. Recently, Chen developed a KelvinPointNet-based velocity inversion method for ship Kelvin wakes in polarization images [57]. By detecting key points on wake skeletons, the method extracted Kelvin wake features and estimated ship velocity and heading, suggesting that wake geometry can provide useful motion information for wake-based detection and localization.

Internal wakes are generated when an underwater vehicle disturbs the density stratification of seawater [58,59]. Hull motion and propeller jets may induce oscillatory motions in stratified water, and these disturbances can propagate over relatively long distances. Internal and turbulent wakes can modulate capillary-gravity waves on the sea surface, thereby changing sea-surface roughness. Because radar backscatter is sensitive to such roughness variations, internal wakes may be detected indirectly from remote sensing images under suitable sea conditions.

Turbulent wakes originate from viscous flow separation and vortex shedding around the hull and propulsion system. As the Reynolds number increases, the flow behind an underwater vehicle becomes increasingly unstable and contains multi-scale vortical structures. These wakes are closely related to hydrodynamic pressure fields, propeller

signatures, wake-induced mixing, and bubble transport. They provide a physical basis for pressure-array sensing, bio-inspired flow sensing, and wake-field reconstruction.

Bubble wakes are mainly caused by propeller cavitation, pump-jet propulsion, rough hull-water interaction, and gas entrainment. Small bubbles in the wake may persist for a relatively long time under the combined effects of turbulence, dissolution, buoyancy, and surface contamination. Because microbubbles have strong optical scattering characteristics, bubble wakes can be detected using optical imaging or laser backscattering methods.

Thermal wakes are generated by heat discharge, cooling-water release, and thermocline disturbance. For nuclear-powered underwater vehicles, thermal discharge may produce local temperature anomalies in the surrounding seawater. In addition, when a submerged vehicle moves through a stratified ocean, it may bring colder or warmer water toward the sea surface and form a detectable thermal trace. Infrared imaging can be used to detect these weak thermal signatures, but its effectiveness is strongly affected by atmospheric conditions, air–sea heat exchange, and background temperature variability.

Biological wakes arise from the response of marine organisms to hydrodynamic disturbance, light stimulation, heat discharge, or chemical changes. Rohr investigated wake-induced bioluminescence and discussed the feasibility of wake detection using bioluminescent organisms [60]. However, biological wake detection remains at an early stage because environmental bioluminescence noise, organism distribution, light absorption, and suspended particles may strongly affect the reliability of this approach. The main characteristics of representative wake signatures are summarized in Table 2. The table compares their physical origins, observable features, useful target information, and major limitations.

Table 2. Comparison of wake signatures for large underwater target detection.

Wake Signature	Physical Origin	Observable Feature	Useful Information	Main Limitation
Bernoulli and Kelvin wakes	Pressure variation and free-surface wave making	Surface elevation, V-shaped wave pattern	Speed, heading, shallow-depth indication	Rapid attenuation with depth and sea-state sensitivity
Internal wakes	Disturbance of density stratification	Internal waves and surface roughness modulation	Trajectory and passage trace	Dependence on stratification and background currents
Turbulent wakes	Flow separation, vortex shedding, propulsion jet	Velocity fluctuation, pressure disturbance, vortical structures	Relative position and wake tracking	Fast evolution and strong environmental mixing
Bubble wakes	Cavitation, gas entrainment, bubble transport	Optical or acoustic scattering from microbubbles	Activity trace and local confirmation	Affected by bubble lifetime, water clarity, and turbulence
Thermal and biological wakes	Heat discharge, thermocline disturbance, organism response	Temperature anomaly or bioluminescence change	Passage indication under suitable conditions	Strong background interference and low universality

3.2. Wake Modeling and Characterization Methods

Wake modeling and characterization provide the physical basis for wake-based detection and localization. Existing studies can be broadly classified into theoretical analysis, experimental measurement, and computational fluid dynamics (CFD).

Theoretical methods are usually based on idealized flow assumptions and simplified target geometries. Early studies mainly considered simple sources, thin bodies, or canonical shapes moving in homogeneous or stratified fluids. Hudimac derived the velocity potential of a moving source in a two-layer ocean and obtained the wave-making resistance of a thin ship under free-surface and internal-wave conditions [61]. Keller analyzed wakes

generated by wave sources in dispersive media and discussed far-field wake behavior in layered fluids [62]. Miles and Long studied internal waves generated by simple bodies in stratified flows [63–68]. Tuck simulated free-surface wakes of underwater vehicles using potential-flow theory [66,67]. Zhang calculated the sea-surface wake amplitude of underwater vehicles under typical depth and velocity conditions [68]. Xue further analyzed Bernoulli humps, Kelvin wakes, and internal wakes generated by underwater vehicles and proposed wake-based velocity and submergence-depth estimation methods [69].

Experimental methods are essential for validating theoretical models and obtaining wake data under controlled conditions. Early experiments mainly focused on wakes behind simple geometries such as spheres, dipoles, and ellipsoids [70–77]. Spedding investigated the evolution of initially turbulent bluff-body wakes in a stably stratified fluid and identified three temporal stages of wake evolution, namely the initial three-dimensional turbulent regime, the transitional non-equilibrium regime, and the late quasi-two-dimensional regime [78]. Meunier further introduced buoyancy-controlled and viscous three-dimensional regimes [79]. Later studies gradually shifted toward ship-like bodies and underwater vehicle models. Gilreath studied internal wakes generated by a self-propelled body in stratified fluids [80]. Beigi measured propeller-induced wakes using a five-hole pressure probe [81]. Yao developed surface particle image velocimetry to measure the wake generated by the DARPA SUBOFF benchmark model in a stratified tank [59]. Several studies have investigated thermal wake characteristics using experimental platforms and infrared measurement systems [82–85].

CFD has become an important tool for analyzing complex wake fields. Direct numerical simulation (DNS) can resolve all flow scales and provide detailed turbulence information, but its computational cost is prohibitively high for most engineering-scale underwater vehicle problems. Reynolds-averaged Navier–Stokes (RANS) methods are computationally efficient and suitable for engineering applications, but they rely on turbulence closure models and may fail to capture detailed unsteady wake structures. Large eddy simulation (LES) provides a compromise between DNS and RANS by resolving large-scale eddies and modeling sub-grid-scale effects. Early CFD studies mainly focused on canonical bodies such as spheres, disks, cylinders, and ellipsoids in stratified fluids [86–109]. More recent studies have extended CFD methods to realistic underwater vehicle wakes. RANS, detached eddy simulation (DES), and hybrid methods have been used to study wake structures, pressure fields, and flow evolution around underwater vehicles [56,110–116]. Li recently modeled the hydrodynamic pressure field and wake field of an advancing underwater vehicle and analyzed their correlation, providing useful support for pressure-field-based and wake-field-based detection [117].

3.3. Wake Sensing, Detection, and Localization Technologies

Wake-based detection technologies can be classified according to sensing platform and observable wake feature. Existing methods mainly include remote-sensing, optical and thermal wake detection, pressure-field and hydrodynamic sensing, and bio-inspired wake sensing.

SAR is mainly used to detect hydrodynamic wakes through sea-surface roughness modulation. Because radar backscatter is sensitive to capillary-gravity waves and surface roughness, internal and turbulent wakes generated by underwater vehicles may appear as bright or dark streaks in SAR images. SAR offers wide-area coverage, all-weather capability, and long-range observation. However, SAR wake detection is affected by sea state, wind speed, surface currents, imaging geometry, and wake-target ambiguity. In addition, SAR imaging and interpretation require sophisticated processing, and the detected wake may correspond to a past trajectory rather than the current target position.

LiDAR and laser backscattering methods are mainly used to detect shallow targets or bubble wakes. Blue-green lasers can penetrate seawater more effectively than many other optical wavelengths, making them useful for underwater target and bubble wake sensing. In bubble wakes, microbubbles produce strong Mie scattering, and the backscattered optical signal can be used to infer the presence of wake traces. Yang proposed an extreme cross-correlation-based laser backscattering method to enhance bubble wake detection under turbulence-coupled conditions, improving the signal-to-background ratio and wake feature capture probability [118]. However, LiDAR-based wake detection is still affected by water clarity, suspended particles, laser power, and platform altitude.

Infrared imaging is mainly used to detect thermal wakes. Thermal wakes may appear as weak sea-surface temperature contrasts caused by heat discharge or thermocline disturbance. Infrared imaging offers passive operation, high spatial resolution, and fast imaging. However, the thermal signature of a submerged target is usually weak and can be easily masked by air–sea heat exchange, solar radiation, weather conditions, and background temperature variations. Therefore, infrared wake detection is more suitable for favorable environmental conditions and usually needs to be combined with other information sources.

Spectral and hyperspectral imaging can detect changes in the optical properties of water caused by shallow targets, biological wakes, or wake-induced variations in suspended particles and ocean color. Multispectral systems are relatively simple and can be optimized for specific bands, whereas hyperspectral systems provide richer spectral information and better adaptability to different environments. However, spectral imaging is strongly affected by illumination, water clarity, atmospheric correction, and sea-surface conditions. Its application to large underwater target detection is therefore usually limited to shallow-water targets or wake signatures that can modify the optical properties of the upper ocean.

Pressure-field and hydrodynamic sensing provide another important route for wake-based detection. The motion of an underwater vehicle induces hydrodynamic pressure disturbances and wake velocity fluctuations, which can be measured using pressure sensors, velocity sensors, or distributed arrays. Compared with airborne remote sensing, hydrodynamic sensing is more suitable for short-range detection and relative localization. Recent studies have shown a correlation between the pressure field and wake field of an underwater vehicle, suggesting that pressure measurements may provide useful information for identifying target motion and wake evolution [117].

Bio-inspired wake sensing has developed rapidly in recent years. Inspired by fish lateral lines, cilia, and seal whiskers, bio-inspired sensors can measure weak flow disturbances, vortex-induced vibrations, and wake-induced pressure variations. Eberhardt designed a wake information detection and tracking system inspired by seal whiskers [119]. Wang developed a self-powered biomimetic tactile sensor based on triboelectric nanogenerators and seal-whisker-like structures [120,121]. Hu proposed a cilium-inspired wake sensor for passive detection and positioning of AUV wakes. By extracting the characteristic frequency of the propeller and establishing a mapping between spectral amplitude and wake velocity, this method can estimate the possible relative position between the sensor and the AUV propeller [122]. Zhao designed a micro-electromechanical systems (MEMS) differential pressure sensor inspired by the fish lateral line system and successfully identified propeller wake characteristics with different blade numbers [123]. These studies indicate that wake-based detection is gradually shifting from large-scale remote sensing toward short-range passive localization and tracking based on bio-inspired flow sensors.

The main characteristics of wake sensing and detection technologies are summarized in Table 3. As shown in Table 3, remote sensing is suitable for wide-area search, whereas

pressure-field and bio-inspired sensors are more suitable for short-range detection, relative localization, and wake tracking.

Table 3. Comparison of wake sensing and detection technologies.

Technology	Platform	Detectable Feature	Main Role	Main Limitation
SAR	Satellite, airborne	Surface roughness caused by internal or turbulent wakes	Wide-area screening and track inference	Sea state, wind, currents, and wake-target ambiguity
Optical and polarization imaging	Airborne, spaceborne	Kelvin wake geometry, surface streaks, optical contrast	Wake skeleton extraction and motion estimation	Illumination, clouds, sea surface, and water clarity
Infrared imaging	Airborne, spaceborne	Sea-surface temperature anomaly	Thermal wake detection and local confirmation	Air–sea heat exchange and weak thermal contrast
LiDAR and laser backscattering	Airborne, shipborne, laboratory	Bubble wake scattering and shallow-water features	Bubble wake detection and enhancement	Suspended particles, attenuation, and platform altitude
Pressure and bio-inspired flow sensing	Underwater arrays, AUV, fixed nodes	Pressure fluctuation, vortex signal, lateral-line response	Short-range tracking and relative localization	Limited coverage and sensor calibration requirement

3.4. Wake-to-Target Information Extraction for Unmanned Platforms

For applications on unmanned platforms, wake-based detection is useful only when the observed wake signatures can be converted into target-related information. In general, the wake-to-target information extraction process includes four steps: wake observation, wake feature extraction, wake-age estimation, and trajectory or position inference. Satellite and UAV platforms usually provide wide-area observations of surface wake patterns, thermal traces, optical contrasts, or bubble-related scattering features. USVs, AUVs, UUVs, and fixed underwater nodes can provide closer-range measurements of pressure fluctuations, velocity disturbances, or bio-inspired lateral-line responses. These observations do not directly provide the real-time target position, but they can constrain the target trajectory, heading, speed, passage time, and possible current location.

Different wake signatures provide different types of target information. Kelvin or free-surface wake geometry can be used to infer target heading and speed when the wake skeleton, wake angle, or wake symmetry can be extracted reliably. Internal and turbulent wakes may provide passage traces and trajectory constraints, especially when they modulate sea-surface roughness and become observable in SAR or optical images. Bubble and thermal wakes are more suitable for recent activity indication or local confirmation, because their persistence and observability depend strongly on bubble lifetime, thermal diffusion, water clarity, and background environmental conditions. Pressure-field and bio-inspired wake sensing can provide short-range relative localization by measuring wake-induced fluctuation frequencies, vortex patterns, or amplitude distributions.

A key difficulty is that the detected wake may correspond to a historical trajectory rather than the current target position. Therefore, wake-based localization usually requires wake-age estimation and environmental correction. Wake age can be inferred from diffusion, intensity decay, spatial spreading, thermal contrast, or bubble persistence, whereas the displacement of the wake should be corrected using background currents, wind forcing, stratification, and turbulence intensity. After the wake age and environmental advection are estimated, the historical track can be extrapolated to obtain a possible current target region rather than a precise point location. Thus, wake-based methods are more suitable for cueing, track reconstruction, and search-area reduction than for independent real-time localization.

False-alarm suppression is another essential step in wake-to-target interpretation. Natural internal waves, surface wakes, wind streaks, surface currents, temperature fronts,

biological activity, and illumination variations may produce wake-like signatures. Therefore, practical wake-based detection should combine geometric constraints, multi-temporal observation, environmental background data, and cross-signature consistency. For example, a target-induced wake should be consistent with possible target motion, should show reasonable temporal evolution, and should not be easily explained by wind direction, natural internal-wave propagation, or surface-vessel traffic. The main wake-to-target information links and false-alarm suppression strategies are summarized in Table 4.

Table 4. Wake-to-target information extraction and false-alarm suppression.

Wake Observation	Extracted Feature	Possible Target Information	Major False Alarms	Suppression Strategy
Kelvin or free-surface wake	Wake skeleton, wake angle, symmetry, surface elevation	Heading, speed, shallow-depth indication, approximate trajectory	Surface-vessel wakes, wind streaks, wave patterns	Geometric consistency, multi-temporal images, surface-traffic exclusion
Internal wake	Surface roughness modulation, long wake streaks, propagation direction	Historical trajectory, passage trace, search-area reduction	Natural internal waves, surface currents, frontal structures	Stratification data, propagation-direction analysis, background ocean correction
Turbulent or pressure wake	Pressure fluctuation, vortex pattern, wake velocity disturbance	Relative position, wake tracking, short-range confirmation	Platform flow noise, ambient turbulence, sensor vibration	Sensor-array correlation, platform-noise calibration, near-field consistency check
Bubble wake	Backscattering intensity, bubble cloud distribution, persistence	Recent activity indication, local confirmation	Breaking waves, suspended particles, biological particles	Optical-depth correction, temporal persistence, multi-band or multi-sensor confirmation
Thermal or biological wake	Temperature contrast, bioluminescence variation, optical anomaly	Passage indication under suitable conditions	Thermal fronts, solar heating, biological background noise	Background temperature correction, night/day condition screening, environmental reference data

3.5. Modality-Specific Limitations of Wake-Based Detection

Although wake-based detection has unique advantages, several modality-specific limitations still restrict its operational application to large underwater target detection and localization. First, wake signatures are strongly affected by environmental conditions. Sea state, wind speed, stratification, turbulence intensity, background currents, water clarity, thermal exchange, and biological activity can modify wake evolution and observability. Natural internal waves, surface-vessel wakes, wind streaks, surface currents, temperature fronts, biological luminescence, and illumination variations may also produce wake-like signatures. Therefore, reliable wake-based detection requires environmental correction, false-alarm suppression, and cross-signature consistency checks.

Second, wake-based localization is inherently indirect. Some wake signatures may persist after the target has left the area, which increases the probability of detection but weakens the relationship between the observed wake and the current target position. As a result, wake-based localization usually requires wake-age estimation, trajectory reconstruction, and environmental advection correction. Even after these corrections, the output is more likely to be a possible target region or trajectory constraint than a precise real-time position.

Third, the scaling relationship among laboratory experiments, numerical simulations, and real-ocean observations remains insufficiently established. Many existing wake studies are based on canonical bodies, model-scale experiments, or simplified CFD conditions,

whereas real underwater vehicles operate in complex stratified, turbulent, and nonuniform ocean environments. Future studies should strengthen similarity analysis among laboratory wakes, numerical wakes, and full-scale ocean wakes, especially with respect to Reynolds number, Froude number, internal Froude number, wake persistence, turbulent mixing, bubble transport, and surface signatures.

Finally, wake-based detection still lacks a unified wake-to-target inversion framework. Different wake signatures provide different types of information: Kelvin wakes may indicate velocity and heading, thermal wakes may indicate passage time, bubble wakes may indicate recent activity, and pressure wakes may support relative localization. Therefore, future wake-based studies should focus on multi-wake feature fusion, uncertainty estimation, wake-age estimation, and trajectory reconstruction. At the current stage, wake-based detection is better positioned as a cueing and trajectory-inference method rather than an independent precise localization solution.

4. Magnetic Anomaly Detection and Localization

4.1. Physical Origin and Sensing Principle

When the distance between the sensor and the magnetic target is greater than three times the characteristic size of the target, the target can be approximated as a single magnetic dipole. Under this far-field condition, the magnetic field generated by the magnetic dipole can be expressed as [124]

$$\mathbf{B}_a = \frac{\mu_0}{4\pi} \left(\frac{3(\mathbf{M} \cdot \mathbf{r})\mathbf{r}}{r^5} - \frac{\mathbf{M}}{r^3} \right) = \frac{\mu_0}{4\pi r^5} \begin{bmatrix} 3x^2 - r^2 & 3xy & 3xz \\ 3xy & 3y^2 - r^2 & 3yz \\ 3xz & 3yz & 3z^2 - r^2 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}, \quad (1)$$

where μ_0 denotes the magnetic permeability of vacuum, $\mathbf{M}$ denotes the magnetic dipole moment, and $\mathbf{r}$ and $r = |\mathbf{r}|$ denote the position vector and the distance from the magnetic dipole to the sensor, respectively.

The measured magnetic field $\mathbf{B}_m$ is the vector superposition of the geomagnetic field $\mathbf{B}_e$ and the target magnetic field $\mathbf{B}_a$:

$$\mathbf{B}_m = \mathbf{B}_e + \mathbf{B}_a. \quad (2)$$

Equation (2) can be rewritten in scalar magnitude form as

$$B_m = \sqrt{B_e^2 + B_a^2 + 2\mathbf{B}_e \cdot \mathbf{B}_a}. \quad (3)$$

Because $B_a \ll B_e$, Equation (3) can be approximated as

$$B_m \approx B_e + \mathbf{e} \cdot \mathbf{B}_a, \quad (4)$$

where $\mathbf{e}$ is the unit vector of the geomagnetic field. Let $S = \mathbf{e} \cdot \mathbf{B}_a$, which represents the projection of magnetic anomaly in the direction of the geomagnetic field.

In this review, conventional MAD mainly refers to the detection and localization of static or quasi-static magnetic anomalies produced by ferromagnetic structures and represented by equivalent magnetic dipole or multi-dipole models. The algorithms discussed in the following subsections, including OBF, MED, EMD, HOC, SR, AI-based detection, and MGT-based localization, are primarily developed within this conventional MAD framework.

In contrast, shaft-rate magnetic fields, active electromagnetic responses, extremely low-frequency electromagnetic fields, and wake-induced electromagnetic disturbances are not considered part of the core scope of conventional static MAD. Instead, they are discussed

as electromagnetic extensions or complementary signatures because they are more closely related to target operation, propulsion systems, induced currents, or interactions among target motion, conductive seawater, and the geomagnetic field.

4.2. Magnetic Anomaly Detection Algorithms

4.2.1. Target-Based MAD Approaches

The OBF method is one of the most representative target-based MAD algorithms. It was first introduced to underwater magnetic anomaly detection by Ginzburg [25]. The basic idea of OBF is that, under the magnetic dipole approximation and simple relative motion between the target and the sensor, the magnetic anomaly signal exhibits a characteristic waveform. This waveform can be represented by a set of predefined orthogonal basis functions. By projecting the measured magnetic signal onto these functions, the anomaly-related components can be extracted from the background magnetic field.

In practical implementation, OBF is usually combined with a sliding window strategy. Within each observation window, the measured magnetic signal is decomposed into several orthogonal components, and the corresponding projection coefficients are used to construct an energy-like detection statistic. When this statistic exceeds a specified threshold, the presence of a magnetic target is declared. Figure 6 shows the workflow of OBF-based MAD.

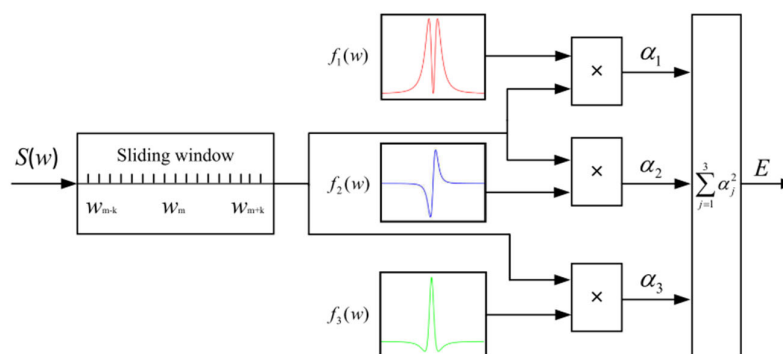


Figure 6. A flow chart of the OBF decomposition algorithm.

After the standard OBF method was proposed, a series of extensions and improvements were developed to enhance its applicability in realistic magnetic environments. Ginzburg further investigated the use of OBFs for magnetic anomaly data acquired from gradient magnetometers [125] and simplified the original OBF structure from five to four functions [126], thereby reducing computational complexity and improving practical robustness. Because the standard OBF detector performs best under Gaussian white noise, subsequent studies mainly focused on improving its robustness under colored noise and nonstationary magnetic background disturbances. Wavelet decomposition was introduced for magnetic signal denoising before OBF detection [127]. Sheinker used an autoregressive preprocessing procedure to whiten background noise before extracting target information through OBF [128]. Zalevsky combined wavelet transform with OBF-based detection and achieved a detection rate of more than 90% in field experiments [129]. Yang incorporated finite impulse response (FIR) low-pass filtering into the OBF framework to suppress high-frequency noise in the original magnetic signal [130]. Sheinker extended MAD to ferromagnetic pipeline detection by using principal component analysis and validated the detector with real data [131]. Wan proposed an OBF detector based on Karhunen–Loeve expansion, which improved detection performance under colored Gaussian noise [132]. Wang further combined wavelet packet denoising, multi-feature extraction, and support vector machine classification to improve magnetic anomaly recognition [133].

More recently, OBF has also been integrated with multi-signal magnetic detection frameworks. For example, Wang combined OBF-based magnetic anomaly detection with adaptive line spectrum enhancement for shaft-rate magnetic field detection, and introduced locally estimated scatterplot smoothing (LOESS) and constant false alarm rate (CFAR) decision-making to improve detection accuracy [134]. This type of work indicates that OBF is no longer used only as an isolated scalar magnetic anomaly detector, but is increasingly combined with preprocessing, adaptive thresholding, machine learning, and multi-feature magnetic signal fusion.

Overall, OBF-based methods offer clear physical interpretability, relatively low computational complexity, and good performance when the magnetic dipole assumption is valid. However, their performance depends on the target-motion model, the dipole approximation, and the background noise characteristics. In complex marine magnetic environments, especially in the presence of colored noise, platform magnetic interference, and nonstationary geomagnetic disturbances, OBF usually needs to be combined with denoising, whitening, adaptive thresholding, or data-driven classifiers.

4.2.2. Noise-Based MAD Approaches

Noise-based MAD approaches do not require an explicit target model. Instead, they detect magnetic anomalies by exploiting statistical or structural differences between background magnetic noise and signal-containing observations. The main noise-based detection techniques include MED, EMD, HOC, and SR. In general, these methods follow a common workflow: signal preprocessing, statistical or structural feature extraction, detection-statistic construction, and threshold-based decision-making.

MED is a typical noise-based MAD method. Unlike target-based methods, it does not require an explicit magnetic dipole model or prior information about the target trajectory. Instead, MED detects magnetic anomaly signals by analyzing the statistical difference between background magnetic noise and signal-containing observation windows.

The basic assumption of MED is that magnetic background noise usually exhibits relatively stable statistical characteristics within a short observation period, whereas the occurrence of a magnetic anomaly changes the probability distribution of the measured signal. This change can be reflected by information entropy. In general, a pure background window has a relatively stable entropy level, whereas the presence of a structured anomaly signal reduces the randomness of the window and causes a detectable entropy variation. Therefore, MED uses entropy-related statistics as detection features and applies a threshold to determine whether a magnetic anomaly is present.

Sheinker introduced MED for magnetic anomaly detection by exploiting the entropy difference between geomagnetic background signals and anomaly-containing signals [30]. Later studies improved this framework by incorporating differential measurements, kernel density estimation, EMD-assisted denoising, and coherent noise suppression. For example, adaptive coherent noise suppression was used to suppress background noise based on the spatial transition relationship between two magnetic sensors [135]. Kernel-smoothed probability-density estimation was introduced to improve entropy-based anomaly detection in differential magnetic signals [136]. Ensemble empirical mode decomposition and minimum entropy features were further combined to improve detection under low-SNR conditions [137], while coherent differential minimum entropy was proposed to suppress common-mode magnetic noise and improve SNR [138].

The main advantage of MED is that it requires less prior knowledge of the target and is suitable for weak-anomaly detection under uncertain target states. However, its detection performance is sensitive to window length, threshold selection, and the statistical stability of the magnetic background. In complex marine environments, MED usually

needs to be combined with differential measurement, adaptive thresholding, or signal-decomposition methods.

EMD is an adaptive time-frequency analysis method that is well-suited to nonlinear and nonstationary signals. In MAD applications, the measured magnetic signal often contains geomagnetic background variations, platform interference, environmental noise, and weak target anomalies. EMD decomposes the original signal into several intrinsic mode functions with different characteristic scales, making it possible to separate anomaly-related components from background trends and high-frequency noise.

Compared with fixed-basis methods such as OBF, EMD does not require predefined basis functions and is therefore more flexible for complex marine magnetic environments. Hasan introduced EMD into marine MAD applications [139], and Zhou proposed an energy detector based on EMD trend filtering for adaptive magnetic anomaly detection [140]. EMD has also been combined with magnetic gradient tensor analysis for pipeline defect localization [141], whereas complete ensemble EMD and statistical EMD variants have been used for magnetic anomaly localization and multi-scale anomaly identification [142,143]. In addition, EMD-based trend removal has been used in signal-modulation-based MAD algorithms [144].

However, EMD-based methods may suffer from mode mixing, endpoint effects, and parameter sensitivity. Therefore, improved variants such as ensemble empirical mode decomposition (EEMD), complementary ensemble empirical mode decomposition (CEEMD), complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN), and statistical EMD are often used to improve decomposition stability and detection robustness. These methods are especially suitable for nonstationary magnetic environments, but their performance still depends on the proper selection of intrinsic components and the reliability of subsequent detection criteria.

HOC is another noise-based technique for magnetic anomaly detection. It is based on the observation that background magnetic noise and anomaly-containing signals exhibit different oscillation patterns and zero-crossing behaviors. Instead of explicitly modeling the target signal, HOC characterizes a time series by counting the zero-crossing events of the original signal and its successively filtered or differenced sequences.

For stationary random signals, the distribution of higher-order crossings is closely related to the autocorrelation function and the power spectral characteristics of the signal. When a magnetic anomaly appears in the observation window, the local waveform structure changes, leading to a deviation in the higher-order crossing pattern from that of the background noise. By comparing the crossing-rate sequence of the current observation window with a background reference, a statistical detector can be constructed to identify magnetic anomaly signals.

The HOC method offers simple implementation, low computational cost, and no requirement for explicit target modeling [32]. These characteristics make it attractive for real-time or low-power MAD systems. However, its performance depends on the stability of the background reference and may be affected by nonstationary noise, abrupt environmental disturbances, and improper selection of the crossing order. Therefore, HOC is more suitable as a lightweight anomaly detector or as a complementary feature in multi-feature fusion frameworks.

SR provides a special weak-signal detection mechanism in which a suitable level of noise can enhance, rather than degrade, signal detectability. In MAD applications, a weak magnetic anomaly signal may be too small to be directly distinguished from the geomagnetic background. By passing the signal through a nonlinear resonance system with properly selected parameters, part of the noise energy can be transferred to the anomaly component, thereby increasing the output SNR.

The classical SR framework is usually implemented using a nonlinear bistable or monostable system. When the input magnetic anomaly signal, background noise, and nonlinear system parameters are properly matched, the system output changes significantly as the anomaly signal passes through the observation window. This change can then be used as a detection feature. Compared with conventional denoising methods, SR does not simply suppress noise; instead, it attempts to exploit noise to enhance weak anomaly signals.

SR-based MAD methods have shown potential under ultra-low-SNR conditions. Wan introduced SR into magnetic anomaly signal detection and achieved promising detection performance [33]. Liu proposed a MAD method based on a parallel monostable stochastic resonance system [145]. Qin developed a fourth-order non-periodic SR method for target recognition in complex interference environments [146]. Sun proposed an adaptive cascaded weak magnetic anomaly detection method based on SR optimized by the marine predator algorithm [147], and further investigated weak magnetic signal enhancement methods in subsequent studies [148,149]. Dai combined SR with orthogonal normalized basis functions and bistable SR systems for airborne MAD [150,151]. Zheng proposed an adaptive variable-structure SR system that can enhance and reconstruct weak magnetic signals [152].

The main advantage of SR lies in its ability to enhance weak anomaly signals under very-low-SNR conditions. However, its performance is highly dependent on system parameter tuning, noise intensity, and the frequency characteristics of the signal. In practical applications, adaptive parameter optimization is usually required to ensure robust detection.

4.2.3. AI-Based MAD Approaches

With rapid advances in machine learning and deep learning, AI-based methods have become an important research direction for MAD. Unlike target-based methods, which depend on magnetic dipole models or predefined anomaly waveforms, and noise-based methods, which rely on statistical variations in the background magnetic field, AI-based methods attempt to learn discriminative features directly from measured or simulated magnetic signals. According to the input representation and feature construction strategy, existing AI-based MAD methods can be broadly divided into methods based on time-domain signals, time-frequency representations, and multi-feature fusion.

Methods based on time-domain signals usually take raw or preprocessed magnetic signals as input and use neural networks to identify anomaly-related patterns. Liu proposed a MAD method based on a fully connected neural network (FCN), in which statistical characteristics of the signal and magnetic-moment-related features were used as inputs to the network [153]. Citak designed a feedforward artificial neural network (ANN) to detect buried magnetic materials at different heights [154]. Xu proposed DeepMAD, an end-to-end deep learning framework consisting of a binary classification network for magnetic anomaly detection and a regression network for geomagnetic noise suppression [155]. Miao combined OBF detection with a fast-convergence wavelet neural network to improve detection performance under low-SNR conditions [156].

Wang designed a residual convolutional network for magnetic anomaly feature extraction and recognition [157]. Chen introduced a Siamese neural network to improve the stability and generalization ability of deep-learning-based MAD [34]. Gao proposed a background-noise cancellation method based on a back-propagation (BP) neural network for a tunneling magnetoresistance (TMR) sensor array, which improved the detection of extremely weak magnetic anomaly signals [35]. Liu developed residual and end-to-end neural networks for submarine cable localization, demonstrating the potential of deep learning in magnetic-source localization tasks [36,37]. Wang proposed a self-supervised ocean-noise learning network based on a sparse autoencoder for MAD [158]. Yuan further

developed U-Net-based denoising networks and residual networks enhanced by a convolutional block attention module (CBAM) to improve weak magnetic anomaly detection under low-SNR conditions [159,160].

Methods based on time-frequency representations transform magnetic signals into time-frequency maps or multi-scale representations before classification. They are useful when anomaly features cannot be fully characterized in either the time domain or the frequency domain alone. Hu proposed two convolutional neural network (CNN) models for magnetic anomaly detection: a one-dimensional (1D) CNN using signal features and a two-dimensional (2D) CNN using time-frequency maps [161]. Wang proposed a network with adaptive time-frequency feature representation, which achieved better detection performance under low-SNR conditions than conventional time-frequency fusion methods [162]. These studies show that time-frequency representations can provide complementary information for weak magnetic anomaly recognition, especially when the measured signal contains nonstationary noise or complex background interference.

Support vector machine (SVM)-based methods have also been widely used for magnetic anomaly classification because of their robustness in small-sample scenarios. Zheng extracted local binary pattern (LBP) features from denoised magnetic anomaly signals and used SVM to classify magnetic objects with different orientations [163]. In a subsequent study, Zheng used singular value decomposition (SVD) to extract magnetic anomaly features and employed SVM for target classification [164]. Fan proposed an adaptive MAD method based on SVM, in which OBF energy and magnetic entropy were used as input features [165]. Du combined CEEMDAN with SVM by decomposing the measured magnetic signal into components with different time scales and extracting time-frequency features for classification [166]. Compared with deep neural networks, SVM-based methods usually require fewer training samples and have greater interpretability, but their performance depends strongly on feature selection and the quality of preprocessing.

Multi-feature fusion methods represent an important trend in AI-based MAD. Because a single feature type may be insufficient to characterize weak magnetic anomaly signals in complex marine environments, many studies have attempted to combine physically interpretable features with data-driven classifiers. Fan proposed a 1D CNN model with multi-feature fusion, in which the Hilbert-Huang transform and discrete wavelet transform were used to extract features from different feature dimensions before neural network classification [167]. Xu proposed a multi-feature-fusion neural network with three sub-classifiers, using time-frequency features, statistical features, and magnetic-moment features for multi-attribute decision-making [168]. Han developed a 2D CNN-based multi-feature fusion method by integrating time-domain and frequency-domain information with minimum entropy features. This method showed good detection performance for simulated underwater vehicle magnetic anomaly signals at different SNRs and under different noise types [169]. The main characteristics of representative AI-based MAD methods are summarized in Table 5.

Overall, AI-based MAD methods provide a flexible solution for weak magnetic anomaly detection in complex environments. Their main advantages lie in nonlinear feature extraction, multi-source feature fusion, and adaptability to complex noise backgrounds. However, several limitations remain. First, many existing models rely heavily on simulated datasets, whereas public benchmark datasets and large-scale sea-trial data remain insufficient. Second, trained models still have limited generalization ability across different platforms, sea areas, target types, and magnetic-background conditions. Third, deep-learning-based MAD methods often lack physical interpretability, making it difficult to assess their reliability in real applications. Therefore, future AI-based MAD research should focus on physics-informed learning, interpretable feature fusion, uncertainty estimation, domain adaptation, and benchmark dataset construction.

Table 5. Comparison of AI-based MAD methods.

AI-Based MAD Type	Typical Models	Input or Feature	Strength	Limitation
Time-domain methods	FCN, CNN, deep neural network (DNN), residual network	Raw or denoised magnetic signal, statistical features	Direct anomaly recognition with compact input	Sensitive to training data and background change
Time-frequency methods	2D CNN, adaptive time-frequency network	Spectrogram, wavelet map, multi-scale representation	Good for nonstationary magnetic signals	Higher preprocessing and parameter dependence
SVM-based methods	SVM, kernel SVM	LBP, SVD, CEEMDAN, OBF energy, entropy features	Useful for small-sample classification	Performance depends on hand-crafted features
Multi-feature fusion methods	Hybrid CNN, multi-branch network, U-Net variants	Time, frequency, entropy, magnetic-moment features	Stronger robustness in complex noise	Limited interpretability and cross-domain generalization

4.3. Magnetic Localization and Inversion Methods

Compared with scalar MAD methods, magnetic localization and inversion methods aim not only to determine whether a magnetic target exists, but also to estimate its position, magnetic moment, or motion trajectory. Because scalar magnetic-field measurements do not retain vector and spatial-gradient information, recent studies have increasingly focused on MGT measurements, tensor invariants, Euler deconvolution, and multi-point inversion.

Euler deconvolution methods provide a practical route for locating magnetic dipole sources from magnetic-field or magnetic-gradient measurements. Nara applied the Euler deconvolution formula to single-dipole localization [170]. This method is straightforward and computationally efficient, but it is sensitive to geomagnetic interference and unknown background fields. Clark analyzed the relationship between the eigenvalues and eigenvectors of the MGT and the position and magnetic moment of a dipole target [171]. Such eigenstructure-based methods provide useful geometric constraints, but they may produce multiple candidate solutions and often require auxiliary information for solution selection.

Tensor invariant methods attempt to improve localization robustness by exploiting the rotation-invariant properties of the MGT. Li proposed a magnetic dipole localization method based on tensor spatial invariant relations and used the dipole-field constraint to eliminate false solutions, as shown in Figure 7 [172]. Wiegert introduced the scalar triangulation and ranging (STAR) method based on the rotational invariance of MGT [24,173]. The STAR method can estimate the target position in real time and is suitable for moving platforms. However, non-sphericity errors may introduce systematic deviations in the estimated range and direction. Therefore, many studies have focused on compensating for these errors and improving the robustness of tensor-based localization [174–180].

Another important trend is the combination of MGT and signal-detection methods. Jin proposed an OBF detection and positioning method using magnetic tensor contraction C_T [181]. Chen combined minimum entropy and gradient OBF for MAD [182]. Yan proposed OBF methods based on MGT invariants [183]. Jin further developed a correlation detection and localization method based on normalized magnetic moment, which is suitable for magnetic exploration using dynamic platforms [184]. These studies indicate that magnetic localization is increasingly evolving from isolated inversion toward integrated detection-and-localization frameworks.

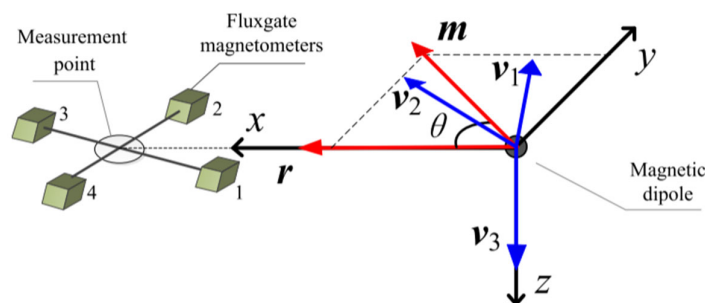


Figure 7. Magnetic dipole localization based on tensor spatial invariant relations.

Recent studies have further extended tensor-based localization to multi-point, two-point, and simplified-gradient measurement schemes. For example, Liu proposed a two-point localization method based on tensor invariants to reduce the influence of geomagnetism, environmental noise, and measurement-point spacing [185]. Wang optimized a multi-point MGT positioning algorithm by considering seawater-induced magnetic-field noise and the spacing between positioning points [186]. In addition, uniaxial MGT-based localization provides a simplified measurement scheme that reduces the requirement for full-tensor arrays [187].

4.4. Practical Implementation of MAD on Unmanned Platforms

For unmanned platforms, the practical performance of MAD is determined not only by the magnetic detector itself but also by platform magnetic cleanliness, sensor layout, attitude correction, navigation accuracy, and background-field compensation. UAVs, USVs, AUVs, and UUVs have different payload capacities, motion characteristics, and self-magnetic interference sources. Therefore, the same MAD algorithm may perform very differently when deployed on different platforms.

Platform magnetic interference is one of the most important factors in moving-platform MAD. Motors, batteries, power cables, ferromagnetic structures, payload electronics, propulsion systems, and attitude changes may generate self-magnetic fields or attitude-dependent magnetic disturbances. UAVs are often affected by motors, batteries, and frame materials; USVs may be affected by engines, electrical equipment, and wave-induced attitude variations; AUVs and UUVs may suffer from propulsion interference, battery-pack magnetization, internal cables, and hydrodynamic vibration. Therefore, platform magnetic compensation should be regarded as a prerequisite rather than a post-processing detail.

Practical compensation usually includes magnetic cleanliness design, hard-iron and soft-iron calibration, three-axis magnetometer calibration, attitude correction, and optimized sensor placement. In airborne or surface applications, boom-mounted or towed magnetometers can increase the distance between the sensor and the host platform. In underwater applications, nose-mounted, tail-mounted, towed, or array-based configurations may be used according to platform size and mission requirements. Differential or gradiometric measurements can suppress common-mode background fields and improve the robustness of local anomaly detection, but they also require careful baseline design and synchronization.

Background-field removal and detection decisions are also critical for operational MAD. The measured magnetic signal may contain geomagnetic diurnal variations, regional magnetic gradients, platform maneuvering effects, ocean-related electromagnetic noise, and target-induced anomalies. Practical systems usually need trend removal, reference-field correction, adaptive filtering, magnetic-gradient measurement, or constant false alarm rate (CFAR) decision-making. Detection thresholds should not be determined solely from idealized simulated noise, but should also account for platform self-noise, environmental

background, mission altitude or depth, sensor-target distance, and the required false-alarm probability.

For magnetic localization, the achievable accuracy is affected by target magnetic moment, sensor-target range, sensor sensitivity, platform navigation error, attitude errors, array-baseline design, model mismatch, and background magnetic noise. Tensor-based or multi-point localization methods can provide richer spatial information than scalar magnetic measurements, but their performance depends strongly on the accuracy of magnetic-gradient estimation and the validity of equivalent-source models. In near-field or multi-source scenarios, single-dipole inversion can introduce systematic errors, and multi-dipole or distributed-source models may be needed.

AI-based MAD provides useful tools for weak-anomaly recognition, denoising, and classification, but its practical deployment should be treated carefully. Many existing studies rely on simulated datasets or controlled experimental data, whereas large-scale sea-trial datasets covering different platforms, sea areas, target types, and background magnetic conditions are still limited. Therefore, future operational AI-assisted MAD systems should emphasize domain adaptation, physics-informed learning, uncertainty estimation, and benchmark dataset construction. The main practical factors affecting MAD on unmanned platforms are summarized in Table 6.

Table 6. Practical factors affecting MAD on unmanned platforms.

Factor	UAV	USV	AUV/UUV	Mitigation
Platform magnetic interference	Motors, batteries, frame materials, payload electronics	Engines, electrical equipment, wave-induced attitude changes	Propulsion system, batteries, cables, internal magnetic components	Magnetic cleanliness design, hard/soft iron calibration, sensor separation
Attitude and navigation error	Strong attitude variation and positioning uncertainty at low altitude	Moderate attitude variation under sea state	INS/DVL drift, attitude error, hydrodynamic disturbance	Attitude correction, navigation fusion, motion compensation
Background magnetic field	Regional geomagnetic gradient, temporal variation	Regional and temporal background variation	Local magnetic background and platform-induced disturbance	Reference-field correction, trend removal, differential or gradiometric measurement
Sensor layout	Boom-mounted or suspended magnetometer	Mast-mounted, boom-mounted, or towed magnetometer	Nose-mounted, tail-mounted, towed, or array-based sensors	Baseline optimization, synchronization, vibration isolation
Detection decision	Threshold affected by flight altitude and platform motion	Threshold affected by sea state and platform electronics	Threshold affected by self-noise and near-field uncertainty	Adaptive thresholding, CFAR, multi-feature decision
Localization accuracy	Limited by altitude, navigation error, and platform motion	Limited by range, baseline, and background noise	Limited by sensor-target distance, attitude error, and model mismatch	MGT, multi-point inversion, sensor fusion, uncertainty estimation
AI model generalization	Limited transfer from simulated to flight data	Limited cross-platform robustness	Limited sea-trial and multi-target datasets	Domain adaptation, physics-informed learning, benchmark datasets

4.5. Modality-Specific Limitations and Electromagnetic Extensions of MAD

Although magnetic anomaly detection and localization have developed rapidly, robust application in complex marine environments remains constrained by several modality-specific limitations. The first limitation is the extraction of weak anomalies under low-SNR conditions. A distant or magnetically compensated target may generate a magnetic anomaly much weaker than geomagnetic fluctuations, platform interference, and environmental magnetic noise. Therefore, practical MAD requires stable background suppression, adaptive decision thresholds, and reliable feature extraction rather than a single detector optimized only for ideal noise.

For moving platforms, MAD performance is strongly affected by platform magnetic interference. As discussed in Section 4.4, airborne, surface, and underwater platforms can introduce self-magnetic fields and attitude-dependent disturbances. For AUV- or UUV-mounted magnetic sensors, magnetic cleanliness design, platform compensation, attitude correction, and sensor-baseline optimization are prerequisites for stable detection and localization. These factors should be considered together with the detection algorithm, because an advanced detector cannot compensate for poorly controlled platform interference.

Another limitation is the validity of the equivalent magnetic source model. The magnetic dipole approximation is effective in far-field conditions, but it becomes less reliable in near-field, multi-source, or structurally complex target scenarios. Large underwater targets may contain distributed ferromagnetic components, rotating machinery, electric currents, and nonuniform magnetization. In such cases, multi-dipole models, distributed magnetic-source models, tensor-invariant constraints, and multi-point inversion are needed to reduce model mismatch and improve localization accuracy.

Conventional static MAD may also be weakened by magnetic stealth technologies, including degaussing, magnetic compensation, and the use of non-ferromagnetic materials. This does not mean that magnetic or electromagnetic detection becomes ineffective. Rather, it indicates that the detectable signature may shift from static ferromagnetic anomalies to operation-induced or motion-induced electromagnetic features. Shaft-rate magnetic fields, active electromagnetic responses, extremely low-frequency electromagnetic fields, and wake-induced electromagnetic disturbances should therefore be regarded as complementary electromagnetic extensions beyond conventional static MAD, rather than direct replacements for the magnetic dipole anomaly model.

Future MAD studies should therefore focus on three closely related issues: robust weak-signal extraction under realistic magnetic backgrounds, platform-level magnetic compensation for unmanned platforms, and detection-localization integration under model uncertainty. Data-driven methods can support denoising, classification, and feature fusion, but they should be combined with physical constraints, benchmark datasets, and uncertainty estimation. From this perspective, the future development of MAD should not rely only on increasingly complex signal-processing or AI models; it should also emphasize magnetic cleanliness, sensor-array design, real-time compensation, and multi-signature interpretation within a clearly defined MAD framework.

5. Gravity and Gravity-Gradient Anomaly Detection and Localization

In this paper, GAD is used as a broad term covering both gravity anomaly detection and gravity-gradient anomaly detection, unless otherwise specified. GAD is a passive non-acoustic detection technique based on the gravitational disturbances caused by the mass-density distribution of a target. Unlike acoustic, optical, and electromagnetic methods, gravity-related signals are difficult to shield in the conventional sense and do not require active emission. Therefore, gravity and gravity-gradient anomaly detection have potential value for passive, non-emitting detection of large underwater targets.

However, compared with wake-based detection and MAD, GAD remains at a feasibility-oriented stage and should not yet be regarded as an operational, standalone detection method for unmanned platforms. This is because the gravity and gravity-gradient signals generated by large underwater targets are extremely weak and are easily affected by platform motion, background gravity variations, and instrument noise and sensitivity. In addition, real-sea validation of GAD for this application remains very limited, and most existing studies are still based on theoretical modeling, numerical simulation, or sensor-requirement analysis.

5.1. Physical Basis and Feasibility of GAD

Large underwater targets may generate gravity and gravity-gradient anomalies because of their nonuniform mass-density distribution, internal equipment layout, hull geometry, and possible offset between the center of gravity and the center of buoyancy. Even when an underwater vehicle is approximately neutrally buoyant, neutral buoyancy only constrains its total mass relative to the displaced water and does not imply a uniform internal density distribution. Therefore, the surrounding gravitational field may still contain weak spatial perturbations related to the target structure.

In theory, the gravitational potential generated by a mass distribution can be differentiated to obtain both the gravity vector and the gravity-gradient tensor (GGT). The gravity vector reflects the first spatial derivative of the gravitational potential, whereas the GGT describes the second spatial derivative and is more sensitive to local mass distribution. For large underwater target detection, gravity-gradient measurements are generally considered more promising than scalar gravity measurements because gradient components can suppress common-mode or slowly varying background components and provide richer spatial information for target characterization.

In the Cartesian coordinate system, the gravitational potential Φ at the observation point $P(a, b, c)$ generated by a point mass M located at $O(x, y, z)$ can be expressed as

$$\Phi = \frac{GM}{\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}}. \quad (5)$$

The gravitational field and GGT can then be obtained from the first- and second-order spatial derivatives of Φ , respectively, and written as

$$\mathbf{g} = \begin{bmatrix} g_x & g_y & g_z \end{bmatrix} = \frac{GM}{R^3} \begin{bmatrix} \Delta x & \Delta y & \Delta z \end{bmatrix} \quad (6)$$

$$\mathbf{\Gamma} = \frac{GM}{R^5} \begin{bmatrix} 3(\Delta x)^2 - R^2 & 3\Delta x\Delta y & 3\Delta x\Delta z \\ 3\Delta x\Delta y & 3(\Delta y)^2 - R^2 & 3\Delta y\Delta z \\ 3\Delta x\Delta z & 3\Delta y\Delta z & 3(\Delta z)^2 - R^2 \end{bmatrix} \quad (7)$$

where $\Delta x = x - a$, $\Delta y = y - b$, $\Delta z = z - c$, and $R = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$ denotes the distance between the point mass and the observation point. These equations provide the basic physical relationship among gravitational potential, gravity anomaly, and gravity-gradient anomaly. They also show that gravity-gradient measurements contain richer spatial information than scalar gravity measurements, which explains why GGT has been widely used in theoretical studies of underwater target localization.

Nevertheless, the feasibility of GAD depends strongly on the target model, sensor accuracy, platform stability, measurement geometry, and environmental background. Existing studies can be broadly divided into two groups. The first group focuses on localization and inversion algorithms based on simplified point-mass models, whereas the second group

focuses on the forward modeling and feasibility analysis of gravity or gravity-gradient signals generated by large underwater targets with specific geometric structures.

5.2. Target Modeling for Gravity Gradient Detection

Target modeling is a key issue in gravity-gradient-based detection of large underwater targets. A simplified target model directly determines the predicted signal magnitude, spatial distribution, inversion feasibility, and engineering applicability. Existing target models can be broadly divided into point-mass models and geometric models.

Point-mass models are mathematically simple and have been widely used in early gravity-gradient inversion studies. Wu proposed automated gravity gradient tensor inversion for underwater object detection [39,40]. This type of method provides an important theoretical basis for gravity-gradient localization, but it usually requires the target to remain stationary for a relatively long time and requires extensive measurements around the target to estimate its mass distribution. These conditions are difficult to satisfy for moving underwater vehicles. Moreover, point-mass-based methods usually assume that the object has a large density contrast or net-mass anomaly relative to the surrounding medium. This assumption may be reasonable for seabed objects, reefs, or geological bodies, but it is questionable for neutrally buoyant underwater vehicles, whose total mass is approximately balanced by the displaced seawater.

To better describe large underwater targets, geometric models have been introduced to analyze the spatial distribution of gravity-gradient anomalies. Sun used a rotating ellipsoid model to simulate an underwater vehicle and calculated the corresponding gravity-gradient distribution [38]. This model provided an early feasibility analysis of underwater vehicle detection using gravity gradients, but it assumed that the mass was distributed on the outer shell. Zhang modeled an underwater vehicle as a three-section structure and analyzed the feasibility of airborne gravity-gradient detection by considering the hull geometry and the relative positions of the center of gravity and the center of buoyancy [19]. Wang introduced a gravitational dipole model by considering that the center of buoyancy is slightly higher than the center of gravity in underwater vehicles [44]. In this model, the upper and lower parts of the vehicle are represented by negative and positive equivalent mass perturbations relative to the displaced seawater, respectively. The results indicated that gravity-gradient anomaly detection may impose less stringent platform-stability requirements than scalar gravity anomaly detection.

More recently, the homogeneous ellipsoidal hull model has provided another way to evaluate the feasibility of large underwater target detection using gravity gradients [51]. Compared with point-mass models, this model is more consistent with the overall geometric characteristics of underwater vehicles. Compared with shell-based ellipsoid models, it provides a more direct description of the gravity-gradient response caused by a homogeneous hull structure. This type of model is valuable for estimating the signal magnitude, spatial distribution, detectable range, and sensor-sensitivity requirements of gravity-gradient-based detection of large underwater targets.

Figure 8 summarizes representative gravity-gradient target models for large underwater targets, including the rotating ellipsoid model, the three-section underwater vehicle model, the gravitational dipole model, and the homogeneous ellipsoidal hull model.

The main characteristics of representative gravity-gradient target models are summarized in Table 7. As shown in Table 7, the modeling of large underwater targets has gradually shifted from point-mass or excess-mass models to geometry-consistent models. This shift is important because the detectability of a neutrally buoyant underwater vehicle depends more on spatial mass distribution and gravity-gradient perturbations than on a simple net-mass anomaly.

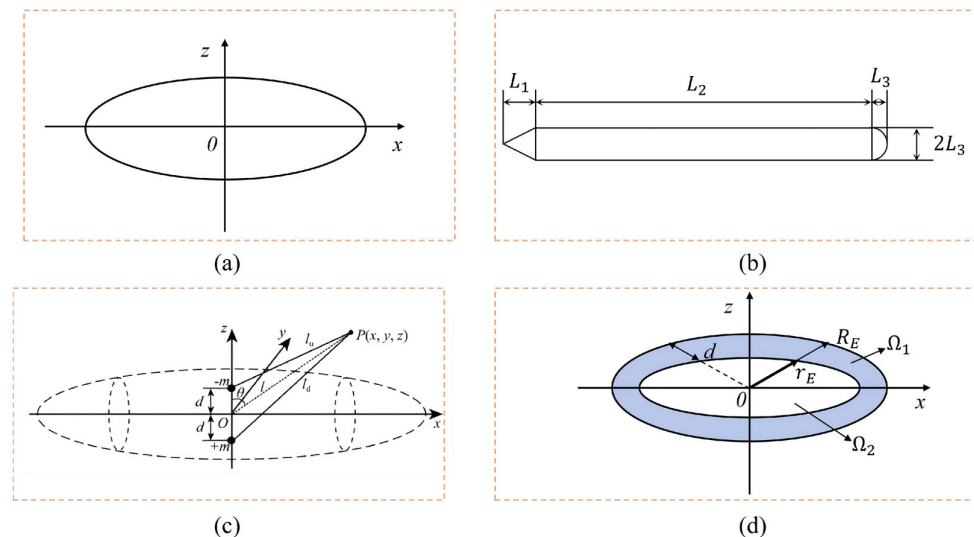


Figure 8. Representative gravity-gradient target models for underwater vehicles: (a) rotating ellipsoid model; (b) three-subsection model; (c) gravitational dipole model; (d) homogeneous ellipsoidal hull model.

Table 7. Comparison of gravity-gradient target models for large underwater targets.

Model	Representation	Main Use	Advantage	Limitation
Point or excess-mass model	Target simplified as a compact mass anomaly	Analytical inversion and initial localization	Simple and convenient for formula derivation	Weak suitability for neutrally buoyant vehicles
Rotating ellipsoid model	Underwater vehicle approximated by an ellipsoidal body	Gravity-gradient forward simulation	Reflects basic hull geometry	Mass distribution is still highly simplified
Three-section model	Hull divided into bow, middle, and stern sections	Feasibility analysis of underwater-vehicle-like targets	Closer to real vehicle shape	Requires assumed geometry and density parameters
Gravitational dipole model	Equivalent positive and negative mass perturbations	Modeling gravity-buoyancy offset	Captures vertical mass-buoyancy imbalance	Equivalent parameters are difficult to validate
Homogeneous ellipsoidal hull model	Continuous ellipsoidal volume or hull distribution	Signal magnitude and spatial-pattern estimation	More consistent than point-mass models	Still mainly used for forward prediction

This transition also reveals a key methodological gap in current GAD research. Point-mass and excess-mass models are mathematically convenient and are therefore widely used in localization and inversion algorithms. However, geometry-consistent models, such as rotating ellipsoid models, three-section models, gravitational dipole models, and homogeneous ellipsoidal hull models, are physically more suitable for describing large neutrally buoyant underwater vehicles. In other words, models that are easier to invert do not necessarily best represent real targets, whereas models that better represent target geometry are still mainly used for forward modeling and feasibility analysis. Bridging this gap between point-mass-model-based inversion and geometry-model-based forward prediction is therefore a central issue for future gravity-gradient-based localization.

5.3. Localization and Inversion Methods Based on Gravity and Gravity Gradient Anomalies

Localization and inversion methods aim to estimate the position, equivalent mass parameters, or motion-related state of a target from gravity or gravity-gradient observations. Existing methods have achieved important theoretical progress, but most of them are still

derived from simplified point-mass or excess-mass models. In these methods, the target is usually represented as a point-mass source, and analytical relationships are established between the target position and the measured gravity or gravity-gradient components. This treatment simplifies the inverse problem and makes analytical or semi-analytical localization possible. However, it also introduces a model mismatch when applied to large underwater targets. Real underwater vehicles are not compact point-mass objects with strong net-mass anomalies. Instead, their detectable gravity-gradient signatures are more likely to originate from hull geometry, nonuniform internal mass distribution, and the possible offset between the center of gravity and the center of buoyancy.

Wu first calculated the mass of an underwater object from gravity-gradient tensor measurements and developed an automated gravity gradient tensor inversion method [39,40]. This method has important theoretical significance, but it requires the target to remain stationary and relies on large-scale measurements around the object. To reduce the dependence on full-field measurements, Wu further studied joint gravity and gravity-gradient inversion for subsurface object detection [41]. Yan proposed a gravity-gradient differential ratio method, in which an AUV-mounted gravity gradiometer measures the gravity-gradient field at multiple positions and then uses the gravity-gradient difference and differential ratio to locate the object [188]. Yan and Tang later proposed analytical inversion formulas based on gravity and gravity-gradient anomalies, which improved the efficiency and accuracy of object localization [42,43].

These localization methods demonstrate that gravity and gravity-gradient observations can, in principle, support target positioning. Nevertheless, their assumptions should be carefully reconsidered when they are applied to large underwater targets. Most existing inversion formulas are more suitable for compact objects with clear excess-mass anomalies, such as seabed objects, dense blocks, or geological anomalies. For a neutrally buoyant underwater vehicle, the net-mass anomaly may be weak or partly compensated by the displaced seawater. In this case, useful localization information is not limited to the amplitude of an equivalent point-mass anomaly, but also includes the spatial gravity-gradient pattern generated by hull geometry, nonuniform internal mass distribution, and the possible offset between the center of gravity and the center of buoyancy.

Future GAD localization should therefore move from standalone point-mass inversion toward hybrid inversion frameworks. Point-mass-model-based methods can provide analytical constraints or initial position estimates, whereas geometry-consistent forward models can provide physically realistic gravity-gradient patterns for optimization, Bayesian estimation, data assimilation, or learning-based inversion. Such integration is essential for transforming GAD from feasibility analysis toward practical localization of large underwater targets.

5.4. Modality-Specific Limitations and Practical Feasibility of GAD

Although GAD offers passivity, low observability, and difficulty of shielding, it remains far from routine operational use for large underwater target detection. Its primary constraint is signal weakness. The gravity and gravity-gradient anomalies of a neutrally buoyant or deeply submerged target are extremely small and depend on target size, density contrast, internal mass distribution, diving depth, sensor-target range, measurement geometry, and gradiometer sensitivity. Reliable assessment, therefore, requires geometry-consistent models and sensor-requirement analysis, rather than only idealized point-mass assumptions.

Practical deployment is further limited by platform and environmental disturbances. Acceleration noise, attitude variations, vibration, navigation error, instrument drift, seafloor topography, geological density variations, tides, and surrounding objects may all produce

gravity-gradient variations comparable to or larger than the target signal. Thus, high-stability platforms, motion compensation, background correction, common-mode noise suppression, and optimized measurement geometry are prerequisites for any moving-platform GAD system.

A central methodological limitation is the weak connection between realistic target models and localization algorithms. Existing inversion methods are mostly based on point-mass or excess-mass assumptions, whereas geometry-consistent models are still mainly used for forward simulation and feasibility evaluation. Future work should develop hybrid inversion frameworks that incorporate hull geometry, internal mass distribution, gravity-buoyancy offset, and equivalent-source constraints into practical localization algorithms.

Finally, GAD research is still dominated by theory and numerical simulation, with limited sea-trial data, benchmark models, and sensor-system demonstrations. Table 8 summarizes key requirements for sensor performance, platform stability, background correction, validation, and operational use. At the current stage, GAD is better regarded as a long-term exploratory and complementary technique than as an independent operational localization method for unmanned platforms. Its value lies in providing a passive, difficult-to-shield physical signature, but practical application will require substantial progress in sensors, platforms, modeling, inversion, and validation.

Table 8. Practical requirements and current status of GAD for unmanned-platform detection.

Requirement	Practical Implication	Current Status
Signal magnitude	Gravity and gravity-gradient anomalies generated by underwater vehicles are extremely weak and decay rapidly with measurement distance.	Mainly evaluated by theoretical models and numerical simulations.
Sensor sensitivity	Practical detection requires high-sensitivity gravity or gravity-gradient instruments with sufficient resolution for weak local anomalies.	Still difficult for compact unmanned platforms.
Platform stability	Platform acceleration, vibration, attitude variation, and navigation errors can easily mask the target-induced signal.	One of the major bottlenecks for moving-platform GAD.
Background correction	Ocean tides, seafloor topography, geological density variations, nearby masses, and platform motion may introduce strong background disturbances.	Robust real-time correction remains challenging.
Validation condition	Practical validation requires controlled target geometry, calibrated sensor performance, stable carrier platforms, and known environmental backgrounds.	Real-sea verification for underwater target detection remains limited.
Operational role	GAD is unlikely to serve as a near-term standalone localization method.	More suitable for feasibility assessment, sensor-requirement analysis, and fusion-aided confirmation.

6. Discussion and Prospects for Unmanned Platform-Based Non-Acoustic UODL

To provide a concise comparison of the reviewed methods, Table 9 summarizes the main observables, functional roles, advantages, and limitations of wake-based detection, MAD, and GAD. This comparison shows that the three methods differ not only in sensing mechanisms but also in localization capability, environmental sensitivity, and technical maturity. For unmanned-platform-based UODL, however, practical value also depends on platform compatibility. Therefore, Table 10 further provides a platform-oriented as-

assessment from the perspectives of platform compatibility, real-time potential, localization ability, and technical maturity. The qualitative ratings in Table 10 are synthesized from the reviewed literature according to sensor maturity, platform integration feasibility, real-time detection/localization capability, and field-validation level. They should be interpreted as qualitative assessments rather than direct numerical comparisons under identical experimental conditions.

Table 9. Classification and comparison of representative non-acoustic UODL methods.

Method	Main Observable	Main Role	Advantage	Limitation
Wake-based detection	Hydrodynamic, bubble, thermal, and biological wakes	Wide-area search, trajectory inference, approximate localization	Large spatial footprint and persistent trace	Environmental sensitivity and limited real-time localization
MAD	Magnetic anomaly and magnetic gradient	Short-range confirmation, local positioning, source inversion	Relatively mature models and algorithms	Weak anomaly, platform interference, and magnetic stealth
GAD	Gravity or gravity-gradient anomaly	Passive and non-emitting detection and feasibility-oriented localization	Passive and difficult to shield	Extremely weak signal and strict platform stability requirement

Table 10. Platform-oriented assessment of representative non-acoustic UODL methods.

Method	Platform Compatibility	Real-Time Potential	Localization Capability	Technical Maturity
Wake-based detection	Satellite/UAV: high; AUV/UUV: medium	Low–medium	Approximate trajectory/location	Medium
MAD	UAV/USV/AUV/UUV: high	Medium–high	Relative/local positioning	High
GAD	Large UAV/USV: low–medium; AUV/UUV: low	Low	Theoretical/local inversion	Low

From this platform-oriented perspective, wake-based detection is suitable for wide-area cueing, remote sensing observation, and trajectory inference, especially when wake signatures can be observed by satellite or UAV payloads. However, its real-time localization capability is limited by wake-age uncertainty, environmental sensitivity, and the indirect relationship between the observed wake and the current target position. MAD shows the highest near-term deployability because compact magnetometers, magnetic gradiometers, and mature signal-processing algorithms can be integrated with UAVs, USVs, AUVs, and UUVs. Its main practical limitations are platform magnetic interference, background compensation, and target magnetic-signature reduction. GAD provides a passive and difficult-to-shield physical signature, but its near-term applicability is limited by extremely weak gravity-gradient signals, strict platform-stability requirements, and insufficient field validation. Thus, wake-based detection is more suitable for search-area reduction and trajectory inference, MAD is more suitable for near-term confirmation and local positioning, and GAD is better regarded as a long-term complementary or fusion-aided sensing route.

Although the reviewed studies differ in target scale, sensing distance, platform type, sea condition, and evaluation dataset, their reported performance can still be organized using several common quantitative indicators. Wake-based studies commonly report wake amplitude, wake persistence, signal-to-background ratio, feature capture rate, or velocity/heading inversion error. MAD studies usually report SNR improvement, detection

probability, false-alarm rate, localization error, or classification accuracy. GAD studies mainly report gravity or gravity-gradient anomaly magnitude, detectable range, sensor-sensitivity requirement, and platform-stability requirement. Because these indicators are obtained under different experimental or simulation conditions, they should not be directly compared as universal performance rankings. Future studies should report not only algorithmic accuracy, but also platform-related indicators such as payload compatibility, power consumption, real-time processing capability, navigation-error tolerance, and motion-compensation requirement. To make these indicators more explicit for future practical studies, Table 11 summarizes suggested quantitative evaluation metrics for wake-based detection, MAD, and GAD.

Table 11. Suggested quantitative evaluation metrics for practical non-acoustic UODL.

Metric	Wake-Based Detection	MAD	GAD
Detection probability	Wake detection rate	Magnetic anomaly detection probability	Gravity-gradient anomaly detection probability
False-alarm rate	Confusion with natural wakes, wind streaks, or surface-vessel wakes	False alarms caused by geomagnetic or platform noise	False alarms caused by geological or background gravity anomalies
SNR or signal-to-background ratio	Wake contrast, thermal contrast, or scattering contrast	Magnetic anomaly SNR	Gravity-gradient anomaly/background ratio
Localization error	Trajectory or current-position error	Position or range error	Inversion or location error
Detection range	Imaging swath and wake visibility range	Sensor-target range	Theoretical detectable range
Speed/heading error	Wake-geometry inversion error	Trajectory fitting or motion-estimation error	Usually limited or indirectly estimated
Processing time	Image processing and wake extraction time	Real-time detector cost	Inversion cost
Robustness	Sea state, stratification, wind, and current	Geomagnetic noise and platform motion	Platform stability and gravity background

Overall, wake-based detection, MAD, and GAD should be viewed as complementary rather than mutually exclusive routes for non-acoustic UODL. Future systems should therefore move beyond single-signature detection and develop toward physically realistic modeling, robust weak-signal extraction, high-performance sensors, stable unmanned platforms, multi-source fusion, and multi-platform collaborative operation. The following issues are particularly important.

6.1. Practical Detection and Localization Workflow for Unmanned Platforms

For practical unmanned-platform-based non-acoustic UODL, wake-based detection, MAD, and GAD should not be treated as isolated or interchangeable alternatives. Instead, they should be organized into a mission-oriented workflow according to observation range, platform capability, sensor payload, communication conditions, and localization requirements. A typical workflow may include wide-area cueing, regional confirmation, close-range localization, and multi-source fusion.

In the wide-area cueing stage, satellites or UAVs can use SAR, optical, infrared, LiDAR, or laser backscattering payloads to search for suspicious wake-related signatures. The main output is not a precise target position, but a possible wake region, passage trace, or approximate trajectory. Such information can reduce the search area and constrain target heading, speed, or historical path. Because wake signatures may persist after the target has

passed, wake-age estimation and environmental correction are needed before wake-derived cues are used for follow-up search planning.

In the regional confirmation stage, mobile unmanned platforms can approach the suspected area and collect higher-resolution measurements using magnetic, optical, pressure, or hydrodynamic sensors. MAD is particularly useful at this stage because compact magnetometers and magnetic gradiometers can be integrated with UAVs, USVs, AUVs, and UUVs. When platform magnetic interference is properly compensated, magnetic anomalies can support both target confirmation and local positioning.

In the close-range localization stage, AUVs or UUVs can further verify the suspected signal and refine the target estimate. Pressure sensors, bio-inspired flow sensors, magnetometers, or magnetic-gradient arrays may be used to examine whether the local disturbance is consistent with a target-induced wake or magnetic anomaly. The main outputs at this stage include relative position, local track, confirmation confidence, and uncertainty range. Because underwater communication is limited, onboard processing, autonomous decision-making, and cooperative path planning are essential.

In the fusion stage, wake-derived trajectory constraints, MAD-derived position or range estimates, and, when available, GAD-derived gravity-gradient constraints should be combined. The fusion result should be expressed not only as a target location, but also as an uncertainty region or probability distribution. Within this workflow, wake-based detection mainly supports cueing and trajectory inference, MAD supports near-term confirmation and local positioning, and GAD serves as a long-term complementary constraint or fusion-aided confirmation route. The platform–sensor–method suitability for practical non-acoustic UODL is summarized in Table 12.

Table 12. Platform–sensor–method suitability for practical non-acoustic UODL.

Platform	Typical Payload or Sensor	Suitable Method	Main Role in Workflow	Main Output or Limitation
Satellite	SAR, optical, infrared	Wake-based detection	Wide-area cueing and historical trace observation	Large coverage but limited revisit rate and real-time localization
UAV	Optical, infrared, LiDAR, magnetometer	Wake-based detection and MAD	Rapid regional survey and cue confirmation	Flexible deployment but affected by wind, altitude, payload, and platform magnetic interference
USV	Magnetometer, optical sensor, hydrodynamic sensor	MAD and local wake sensing	Regional confirmation and continuous monitoring	Good communication and payload capacity, but affected by sea state and platform self-noise
AUV	Magnetometer, pressure sensor, flow sensor	MAD and hydrodynamic wake sensing	Close-range confirmation and relative localization	Good underwater proximity, but limited communication and endurance
UUV	Magnetometer, magnetic-gradient sensor, pressure or bio-inspired sensor	MAD, pressure wake sensing, and local tracking	Short-range tracking and local positioning	Useful for covert confirmation, but limited by self-noise, navigation drift, and communication constraints
Future stable carrier platform	High-sensitivity gravity gradiometer	GAD	Fusion-aided confirmation and feasibility assessment	Potentially unshieldable signature, but limited by sensor sensitivity and platform stability

6.2. Physically Realistic and Computationally Efficient Target Modeling

Accurate target modeling is the basis of non-acoustic detection and localization. Existing models have reduced computational cost by simplifying the complex physical characteristics of large underwater targets. However, overly simplified models may not provide reliable predictions under realistic ocean conditions. The key issue is therefore not to maximize model complexity, but to balance physical fidelity, computational efficiency, and inversion usability. Future studies should develop task-oriented model hierarchies that can support signal prediction, sensor design, uncertainty estimation, and localization.

For wake-based detection, models should link target geometry, propulsion, navigation state, stratification, turbulence, and environmental forcing to observable wake signatures. Many existing studies are still based on canonical bodies, model-scale tests, or simplified CFD conditions. Future work should strengthen similarity analysis among laboratory experiments, numerical simulations, and full-scale ocean observations, especially with respect to Reynolds number, Froude number, internal Froude number, wake persistence, turbulent mixing, bubble transport, and surface expression. Such models should not only reproduce wake physics but also provide detection-oriented outputs such as wake age, trajectory constraints, and uncertainty bounds.

For MAD, the magnetic dipole model remains useful because of its simplicity and interpretability. However, large underwater targets may contain distributed ferromagnetic components, rotating machinery, electric currents, and nonuniform magnetization. Future MAD modeling should therefore combine single-dipole, multi-dipole, distributed-source, MGT-based, and platform-interference models according to sensing range and localization requirements. These models should support not only anomaly-field prediction, but also detector design, magnetic compensation, sensor-array layout, and detection-localization integration.

For GAD, modeling is closely tied to feasibility. Point-mass or excess-mass models provide analytical convenience, whereas geometry-consistent models better represent large neutrally buoyant targets. Future GAD studies should bridge this gap through hybrid frameworks that combine equivalent-source constraints with hull-geometry, internal-mass-distribution, and gravity-buoyancy-offset information. In this way, geometry-consistent forward models can be connected to optimization, Bayesian estimation, data assimilation, or learning-based inversion. This connection is essential if GAD is to move beyond signal-magnitude estimation toward practical localization.

6.3. Weak Signal Extraction and Background Field Suppression

Target-induced non-acoustic signals are usually weak and are often embedded in strong environmental backgrounds. Therefore, weak-signal extraction and background-field suppression remain central challenges for wake-based detection, MAD, and GAD. However, the dominant background sources differ across the three methods, and the suppression strategy should be designed according to the physical origin of each signal.

For wake-based detection, the main difficulty is to distinguish target-induced wake signatures from natural ocean phenomena and other wake-like disturbances. Internal waves, surface currents, surface-vessel wakes, wind streaks, biological luminescence, temperature fronts, and turbulence may all generate confusing signatures. Future studies should therefore combine physical wake models, remote sensing interpretation, ocean-background information, and uncertainty estimation. Wake detection should move beyond simple feature extraction toward wake-age estimation, trajectory reconstruction, false-alarm suppression, and environmental correction.

For MAD, weak magnetic anomalies may be masked by geomagnetic variations, platform magnetic interference, and environmental noise. The reviewed target-based, noise-based, and AI-based methods provide useful tools, but their performance still depends

on assumptions about target motion, noise characteristics, threshold selection, and data representativeness. Future MAD studies should combine physics-based magnetic models with adaptive denoising, platform magnetic compensation, tensor-based measurements, self-supervised learning, domain adaptation, and interpretable feature fusion. Benchmark datasets and sea-trial data are especially important for evaluating generalization across platforms, sea areas, and target types.

For GAD, background suppression is particularly difficult because the target-induced gravity-gradient signal is extremely weak. Seafloor topography, geological density variations, tides, platform acceleration, vibration, attitude variations, and nearby objects may all produce comparable disturbances. Future GAD algorithms should incorporate platform-noise models, motion compensation, common-mode suppression, regional gravity-background correction, multi-scale anomaly separation, and uncertainty-aware inversion. For this method, weak-signal extraction cannot be separated from sensor noise modeling, platform dynamics, and prior environmental information.

6.4. High-Precision Sensors and Stable Detection Platforms

Sensors and platforms jointly determine the practical measurement floor of non-acoustic UODL. Even advanced algorithms cannot compensate for insufficient sensor sensitivity, severe platform self-noise, or poor motion stability. Therefore, high-precision, energy-efficient, and platform-compatible sensors, together with stable unmanned platforms, are prerequisites for practical deployment.

For wake-based detection, future sensors should improve the observation of weak hydrodynamic, optical, thermal, and biological wake signatures. Satellite and UAV remote-sensing payloads can support wide-area wake screening, whereas underwater pressure sensors, velocity sensors, and bio-inspired flow sensors are more suitable for short-range confirmation and relative localization. Bio-inspired sensors inspired by fish lateral lines, seal whiskers, and cilia are particularly promising for detecting weak flow disturbances and vortex-induced signals, but their calibration, deployment strategy, environmental robustness, and long-term stability still require further investigation.

For MAD, sensor development should focus on compact high-sensitivity magnetometers, magnetic gradiometers, low-noise arrays, and platform magnetic compensation. Superconducting quantum interference devices (SQUIDs), optically pumped magnetometers, fluxgate magnetometers, and magnetoresistive sensors may be selected according to sensitivity, size, cost, power consumption, bandwidth, and platform compatibility. For moving-platform MAD, sensor performance must be evaluated together with magnetic cleanliness, attitude measurement, baseline design, vibration isolation, and real-time compensation.

For GAD, sensor sensitivity and platform stability are even more decisive. The gravity and gravity-gradient anomalies generated by large underwater targets are extremely weak, so practical detection requires high-sensitivity gravity gradiometers, stable carriers, accurate inertial navigation, and effective motion compensation. Cold-atom interferometers and superconducting gravity gradiometers may provide high-sensitivity measurement capabilities, whereas MEMS-based inertial sensors can support miniaturized navigation and motion correction. However, marine deployment, environmental robustness, miniaturization, and real-time operation remain major challenges.

Across all three methods, platform design should be treated as part of the detection system rather than a separate engineering detail. Airborne, surface, and underwater platforms may introduce acceleration noise, attitude variations, vibration, magnetic interference, hydrodynamic disturbances, navigation errors, and communication constraints. Future unmanned-platform systems should therefore integrate sensor layout, mechanical stabiliza-

tion, electromagnetic compatibility design, motion compensation, onboard processing, and cooperative operation into a unified detection architecture.

6.5. Multi-Source Fusion and Multi-Platform Collaboration

A single non-acoustic signature is usually insufficient for reliable detection and localization under complex ocean conditions. Wake, magnetic, electromagnetic, and gravity-gradient signatures differ in physical origin, observation range, temporal persistence, and environmental sensitivity. Therefore, future UODL systems should move from single-signature detection toward multi-source fusion and multi-platform collaboration.

The value of fusion lies in complementarity. Wake-based observations can provide wide-area cues and trajectory-related information, MAD can support passive confirmation and local positioning, and GAD may provide an additional mass-distribution-related constraint when sensor and platform conditions allow. Complementary electromagnetic signatures, such as operation-induced magnetic fields and motion-induced electromagnetic disturbances, may further provide information related to propulsion, operating state, and seawater-electromagnetic coupling. Combining these heterogeneous signatures can improve detection probability, reduce false alarms, and increase localization robustness.

However, multi-source fusion should not be limited to simple decision-level voting. Future studies should develop hierarchical fusion frameworks that include data-level, feature-level, model-level, and decision-level fusion. Physical consistency among different signatures should be explicitly considered. For example, wake geometry may constrain target heading and historical trajectory, magnetic anomalies may constrain relative position and equivalent magnetic-source distribution, and gravity-gradient patterns may constrain target geometry or mass distribution. These constraints should be integrated in a unified probabilistic framework so that the final output includes not only a target position, but also uncertainty, confidence level, and possible trajectory evolution.

Multi-platform collaboration is equally important. Satellite and UAV remote sensing can provide wide-area screening; USVs, buoys, and shore-based systems can support regional monitoring; and AUVs or UUVs can perform close-range confirmation, tracking, and local sensing. Such collaboration can improve spatial coverage, increase observation diversity, and reduce blind zones associated with single-platform detection. Future research should therefore focus on cross-platform time synchronization, coordinate registration, cooperative path planning, adaptive formation control, distributed data fusion, underwater communication constraints, and real-time task allocation.

6.6. Target Simulation, Multi-Target Localization, and Emerging Detection Mechanisms

Reliable evaluation and validation require a realistic target simulation. Large underwater targets are expensive and difficult to use frequently in controlled experiments, and many existing substitutes can reproduce only limited aspects of real targets. Decommissioned or specialized underwater vehicles, towed bodies, scaled models, and self-propelled underwater targets may provide useful acoustic, hydrodynamic, or maneuvering characteristics, but they often cannot simultaneously reproduce the scale, geometry, magnetic signature, wake signature, and gravity-related signature of a real large underwater target. A promising direction is to develop modular or cooperative target simulators. For example, multiple small UUVs or distributed artificial sources may be used to emulate selected multi-physical-field signatures, thereby balancing simulation fidelity, operational flexibility, and cost.

Multi-target localization is also becoming increasingly important. Many existing algorithms are developed under single-target assumptions, whereas realistic detection scenarios may include multiple targets, decoys, environmental anomalies, and interfering

platforms. Such scenarios introduce data association, signal overlap, false-target discrimination, and cooperative tracking problems. Future algorithms should distinguish multiple targets with similar signatures, estimate their trajectories, and quantify localization uncertainty. Multi-sensor and multi-platform observations will be essential for addressing these challenges.

Emerging signal sources and detection mechanisms should also be explored. As stealth and detection technologies co-evolve, conventional signatures may be weakened or masked. Greater diving depth can reduce surface wake observability, magnetic compensation can suppress static magnetic anomalies, and non-ferromagnetic materials can weaken conventional MAD performance. Future studies should therefore investigate operation-induced, motion-induced, and environment-coupled signatures beyond traditional mechanisms. Possible directions include wake-induced electromagnetic fields, shaft-rate magnetic signatures, extremely low-frequency electromagnetic fields, electrokinetic or ion-relaxation-related disturbances, micro-eddy signatures, bio-optical responses, and coupled hydrodynamic–electromagnetic–gravity effects. These mechanisms should be evaluated through physical modeling, controlled experiments, sensor feasibility analysis, and field validation.

Overall, future non-acoustic UODL should move from single-signature detection toward multi-physical-field perception, from simplified target assumptions toward realistic and inversion-compatible models, and from isolated sensors toward collaborative unmanned sensing networks. This transition will require high-precision sensors, stable platforms, robust background suppression, realistic target simulation, and uncertainty-aware fusion algorithms.

7. Conclusions

The detection and localization of large underwater targets are important for maritime security, marine resource exploration, underwater situational awareness, and marine scientific research. As acoustic stealth and noise-reduction technologies continue to develop, conventional acoustic detection faces increasing challenges. Non-acoustic detection and localization, therefore, provide an important complementary direction. This review systematically examined non-acoustic UODL technologies, with emphasis on wake-based detection, MAD, and GAD. These three methods represent different physical mechanisms, sensing platforms, modeling assumptions, localization capabilities, and maturity levels.

Wake-based detection exploits environmental disturbances generated by moving underwater targets, including hydrodynamic, bubble, thermal, and biological wakes. Its main advantage is that wake signatures may occupy a much larger spatial region than the target itself and can be observed by wide-area remote sensing or local flow sensors. Therefore, wake-based methods are suitable for wide-area cueing, trajectory inference, velocity estimation, and approximate localization. However, their performance is strongly affected by sea state, stratification, turbulence, background currents, optical conditions, thermal exchange, and biological activity. In addition, wake persistence increases the possibility of detection but weakens the real-time relationship between the observed wake and the current target position. Future wake-based studies should therefore emphasize wake-age estimation, trajectory reconstruction, environmental correction, false-alarm suppression, and the integration of remote sensing with underwater pressure or bio-inspired flow sensing.

MAD is currently the most mature non-acoustic route among the three reviewed methods. Based on magnetic dipole models, magnetic-gradient-tensor measurements, and weak magnetic anomaly extraction algorithms, MAD can support both detection and localization. Target-based, noise-based, and AI-based MAD methods have improved

weak-anomaly detection under different background conditions, while tensor-invariant, Euler deconvolution, STAR, and multi-point inversion methods provide additional spatial information for localization. Nevertheless, MAD remains constrained by low-SNR anomaly extraction, colored geomagnetic noise, platform magnetic interference, near-field model mismatch, limited training datasets, and target magnetic-signature reduction. Future MAD systems should combine physical magnetic models, adaptive denoising, platform magnetic compensation, magnetic-gradient measurements, interpretable AI, uncertainty estimation, and multi-sensor localization. Operation-induced and motion-induced electromagnetic signatures should be regarded as complementary extensions beyond conventional static MAD, rather than replacements for the basic magnetic dipole framework.

GAD provides a promising but less mature route for non-acoustic underwater target detection. Gravity-related signals are passive, non-emitting, and difficult to shield in the conventional sense, giving GAD unique theoretical value. However, the gravity and gravity-gradient anomalies generated by large underwater targets are extremely weak and are easily affected by platform acceleration, vibration, attitude variations, seafloor topography, geological density variations, ocean tides, and instrument noise. Existing GAD studies are still dominated by theoretical modeling and numerical simulation. A key limitation is the gap between point-mass-model-based inversion and geometry-model-based forward prediction. Point-mass or excess-mass models are mathematically convenient for analytical localization, but they are not fully suitable for neutrally buoyant targets. Geometry-consistent models can better describe hull geometry, internal mass distribution, and gravity-buoyancy offset, but they are still mainly used for feasibility analysis. Bridging this gap is essential for transforming GAD from theoretical feasibility toward practical localization.

Overall, wake-based detection, MAD, and GAD should not be treated as competing technologies with identical application scenarios. Instead, they should be organized according to platform capability, sensing range, and mission requirements. Wake-based detection is more suitable for wide-area cueing and trajectory inference, MAD is more suitable for near-term confirmation and local positioning, and GAD should be regarded as a long-term complementary or fusion-aided sensing route. Future UODL systems should move from single-signature detection toward multi-physical-field perception, from simplified target assumptions toward physically realistic and inversion-compatible models, from isolated sensors toward collaborative unmanned sensing networks, and from simulation-dominated studies toward experimental and sea-trial validation. By integrating wake, magnetic, electromagnetic, and gravity-gradient signatures with high-precision sensors, stable platforms, robust background suppression, and uncertainty-aware fusion algorithms, future UODL systems are expected to achieve higher detection reliability, stronger localization robustness, and better adaptability to complex marine environments.

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