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Pollution Information Acquisition and Discovering Pollution Sources †

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Abstract: This paper describes analytical methods used for the elaboration of the fast algorithm for detecting unknown sources of contamination release into water and/or air network systems. In addition, an analysis of existing theoretical approaches to computing pollutants released into water parameters is given and the necessity for a new methodology to solve multiple problems related to the real-time source identification is demonstrated.

Keywords: information; pollution; source; unknown; hypernumber; liquid; contaminant

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1. Introduction

The contaminant leakage into liquid or gas systems is a frequent event. Multiple publications disclose information related to the release of contaminants such as bacteria, heavy metals, PFAS (per- and polyfluoroalkyl substances) and many others from unknown sources. The bacterial contaminant release is a major cause of the beach closure. Per ACS publication, nearly all (93%) of the Los Angeles and Orange County advisories and closures were caused by unknown sources of fecal indicator bacteria (FIB). The PFAS contamination is another critical problem [1]. According to the Nordic Co-operation published socioeconomic analysis of environmental and human health impacts from exposure to PFAS estimates that EEA member countries incur EUR 52 to 84 billion in PFAS-related health costs annually. Similar to the bacterial contamination, there is a significant number of cases when the sources of PFAS contamination are unknown.

Finding a fast algorithm for detecting unknown sources of pollution is critical to solving many problems such as:

1. *Detecting the source of contamination in potable water systems.*
2. *Identifying the smoke source in ventilation systems.*
3. *Locating the source* of the explosive in air systems.
4. Detecting contamination in water streams while monitoring multiple streams from a remote location.

The approaches for detecting sources of pollution are covered in multiple publications [2–5].

One of these methods, which is discussed in publications [4,5], defines pollutant transport with a stochastic direct model and set S of pollutant source parameters by solving equations of the inverse engineering mode State of the Art Report on Mathematical Methods for Groundwater Pollution Source Identification. The set S of computed parameters for this ill-posted problem are provided below.

$$S = \{\text{Distance from sensor},\ \text{Rate of the discharge},\ \text{Coefficient of the dispersion}\}$$

The stochastic modeling allows us to extend the obtained solution for complex network systems. However, the numerical solution of the inverse model cannot be applied for real-time water security tasks and many other real-time processes.

Here, we describe a new direction for obtaining analytical solutions for the inverse model of the pollutant propagation using the theory of hypernumbers [6,7], which is a new field in functional analysis developed for rigorous and consistent operation with singular dynamics when integrals and series tend to infinity. Real hypernumbers form a natural extension of real numbers in the same way as real numbers extend rational numbers by topological constructions. In a similar way, complex hypernumbers are built from complex numbers.

In addition to the utilization in solving diverse differential, integral and functional equations, hypernumbers are efficiently used for the further development of the theory of the Feynman path integral and probability theory.

2. Defining Pollutant Source Location and Release Rate with the Theory of Hypernumbers

The stochastic approach leads to a simple integral formula for a pointed source. Furthermore, we can receive an analytical solution with this approach for any distributed source or for a set of pointed sources.

$$c = \int_0^t \frac{\mu e^{-\frac{(x-v\tau)^2}{2D\tau}}}{\sqrt{2\pi D\tau}} d\tau \tag{1}$$

where c—contaminant concentration; x—distance from the contamination source to the sensor; D—dispersion; v—contaminant transport velocity; $(\tau,\ t)$—time; and μ contaminant discharge rate.

$$U = \sum_{i=1}^{n} \left(c_i - \int_0^{t_i} \frac{\mu e^{-\frac{(x-v\tau)^2}{2D\tau}}}{\sqrt{2\pi D\tau}} d\tau \right)^2 \tag{2}$$

where U is the operator defining distance between monitoring and calculated contaminant's concentration and c_i is the concentration at time t_i.

The U minimization requirement must satisfy conditions (3)–(5).

$$F_1 = \frac{\partial U}{\partial x} = 2\sum_{i=1}^{n} \left(c_i - \int_0^{t_i} \frac{\mu e^{-\frac{(x-v\tau)^2}{2D\tau}}}{\sqrt{2\pi D\tau}} d\tau \right) \int_0^{t_i} \frac{\mu e^{-\frac{(x-v\tau)^2}{2D\tau}} (x - v\tau)}{\sqrt{2\pi D\tau}\ D\tau} d\tau = 0 \tag{3}$$

$$F_2 = \frac{\partial U}{\partial D} = 2\sum_{i=1}^{n} \left(c_i - \int_0^{t_i} \frac{\mu e^{-\frac{(x-v\tau)^2}{2D\tau}}}{\sqrt{2\pi D\tau}} d\tau \right) \int_0^{t_i} \frac{-\mu e^{-\frac{(x-v\tau)^2}{2D\tau}} (x - v\tau)^2}{2\sqrt{2\pi D\tau}(D\tau)^2} d\tau = 0 \tag{4}$$

This expression is simplified by the following replacement:

$$I_1 = \int_0^{t_i} \frac{\mu e^{-\frac{(x-v\tau)^2}{2D\tau}}}{\sqrt{2\pi D\tau}} d\tau,\ I_2 = \int_0^{t_i} \frac{\mu e^{-\frac{(x-v\tau)^2}{2D\tau}} (x - v\tau)}{\sqrt{2\pi D\tau}\ D},\ I_3 = \int_0^{t_i} \frac{-\mu e^{-\frac{(x-v\tau)^2}{2D\tau}} (x - v\tau)^2}{2\sqrt{2\pi D\tau}(D\tau)^2} d\tau \tag{5}$$

Replacing (3) and (4) with the expressions (3)–(5),we receive Equations (6), (8) and (9).

$$\frac{\partial U}{\partial \mu} = 2\ \sum_{i=1}^{n} (c_i - \mu I_1) I_1 = 2\sum_{i=1}^{n} c_i I_1 - \mu \sum_{i=1}^{n} I_1^2 = 0 \tag{6}$$

$$\mu = \frac{\sum_{i=1}^{n} c_i I_1}{\sum_{i=1}^{n} I_1^2} \tag{7}$$

$$\frac{\partial U}{\partial x} = 2\sum_{i=1}^{n}\left(c_i - \frac{\sum_{i=1}^{n} c_i I_1}{\sum_{i=1}^{n} I_1^2} I_1\right) I_2 \tag{8}$$

$$\frac{\partial U}{\partial D} = 2\sum_{i=1}^{n}\left(c_i - \frac{\sum_{i=1}^{n} c_i I_1}{\sum_{i=1}^{n} I_1^2} I_1\right) I_3 \tag{9}$$

Following the methods from the theory of hypernumbers for solving non-linear operator equations [8,9], we determine the solutions for the source distance and dispersion at (10)–(13).

$$x_m = H_n(x_m)_{m\in\omega} \tag{10}$$

$$D_m = H_n(D_m)_{m\in\omega} \tag{11}$$

$$x_{m+1} = x_m + \delta(x)_m \tag{12}$$

$$D_{m+1} = D_m + \delta(D)_m \tag{13}$$

The deviations of hypernumbers x_m and D_m are computed using Formulas (14) and (15).

$$\delta F_{1,m+1} = \delta(x)_m d_{11} + \delta(D)_m d_{12} = (1-\theta)F_{1,m} \tag{14}$$

$$\delta F_{2,m+1} = \delta(x)_m d_{21} + \delta(D)_m d_{22} = (1-\theta)F_{2,m} \tag{15}$$

The formula for defining d_{11} is given below.

$$d_{11} = \frac{\partial^2 U}{\partial x^2}\, 2\sum_{i=1}^{n}\left(c_i - \frac{\sum_{i=1}^{n} c_i I_1}{\sum_{i=1}^{n} I_1^2} I_1\right)\frac{dI_2}{dx} 2\sum_{i=1}^{n}\left(\frac{d\left(\frac{\sum_{i=1}^{n} c_i I_1}{\sum_{i=1}^{n} I_1^2}\right)}{dx} I_1 + \frac{\sum_{i=1}^{n} c_i I_1}{\sum_{i=1}^{n} I_1^2}\frac{dI_1}{dx}\right) I_2 \tag{16}$$

A similar computational approach is utilized to find the rest of the coefficients. Using Equations (1)–(16), we created a software for computing the pollutant discharge parameter.

3. Conclusions

The described algorithms are used for the development of the Pollution Positioning System (PPS), which detects location of pollution sources with the great precision, as well as for building the Contamination Positioning System (CPS), which efficiently and accurately finds critically contaminated places and regions.

Besides, the defined solution for locating the source of the pollution discharge into the liquid or gas network system will allow us to extend the applicability of the suggested approach for a variety of critical problems related to the security and technological process optimization. It is possible to use this approach to solve several other important problems such as locating the place of the liquid and gas leakage in underground pipe network systems or locating the source of the pollution with pollutant decay during its transport. The latter problem would require a new mathematical description of the stochastic contaminant transport model.

In addition, the suggested approach opens ways for utilization of real extrafunctions for solving even more serious ecological and technological problems.

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