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Performance of Portfolios Based on the Expected Utility-Entropy Fund Rating Approach †

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† Presented at the 5th International Electronic Conference on Entropy and Its Applications, 18–30 November 2019; Available online: https://ecea-5.sciforum.net/.

Published: 17 November 2019

Abstract: Yang and Qiu proposed and reframed an expected utility-entropy (EU-E) based decision model; later on, similar numerical representation for a risky choice was axiomatically developed by Luce et al. under the condition of segregation. Recently, we established a fund rating approach based on the EU-E decision model and Morningstar ratings. In this paper, we apply the approach to US mutual funds and construct portfolios using the best rated funds. Furthermore, we evaluate the performance of the fund ratings based on EU-E decision model against Morningstar ratings by examining the performance of the three models in portfolio selection. The conclusions show that portfolios constructed using the ratings based on the EU-E models with moderate tradeoff coefficients perform better than those constructed using Morningstar. The conclusion is robust to different rebalancing intervals.

Keywords: expected utility-entropy; fund rating; risk; portfolio; performance

1. Introduction

Mutual funds have become an increasingly dominant choice for retail investors in recent years, underlined by the large number of investors who attempt to beat the market and those who seek to diversify away unsystematic risk from their portfolio [1,2]. In selecting funds to comprise their portfolios, investors would seek to invest in the best performing funds. As a result, investors would explore the best rating category to guide their investment decisions. At present, the most prominent fund rating approaches have been developed by agencies including Morningstar and Lippers. These approaches rank funds using a star rating system on a scale of 5 to 1, where those with a 5-star rating are deemed to be the 'best' performing funds, based on a fund's calculated risk-adjusted return. Studies have shown Morningstar ratings play a powerful role in the mutual fund industry and are seen as a crucial metric for investors and fund managers [3,4]. However, Sharpe [5] and Lisi and Caporin [6] showed that risk adjustment made in calculating fund rating in Morningstar may not account for the risk faced by a fund appropriately. This inefficient adjustment for risk may be attributed, in part, to Morningstar's reliance on the expected utility theory, which is proven to draw conclusions which deviate from individuals' behavior under risk [7–9]. Specifically, Kahneman and Tversky [8] pointed out that people deviate from the description of expected utility theory in actual decision-making. Since its descriptive power for risky choices has been challenged and discussed by some famous paradoxes and researchers [7,10,11], many alternatives models have been developed to provide additional insights about decision-making under risk [8,12–17], especially the decision-making models to expand expected utility involving Shannon entropy [18–23]. Shannon entropy [24] has been applied to a wide body of financial literature to guide investors' investment decisions for its ability to describe risks. Recent studies evaluating entropy relative to standard deviation and beta in measuring financial risk have been supportive of entropy, primarily for its distribution-free nature and ability to incorporate more information of uncertainty than the latter two measures [19,25]. The conclusions of Bentes and Menezes [26] show that entropy can more clearly indicate market volatility. Caraiani [27] found that entropy can predict dynamic changes in the market, showing that entropy has important characteristics in predicting fund performance. Furthermore, there are some recent studies about entropy to measure uncertainty applied to decision-making. Wei et al. [28] proposed a general form of entropy measures for hesitant fuzzy sets. Wei et al. [29] further investigated how to measure uncertainty of hesitant fuzzy linguistic term sets and extended hesitant fuzzy linguistic term sets and then applied the measures to compute the weights in decision-making problems.

As expected utility cannot be a descriptive decision-making model and entropy alone is unable to consider the outcome of the risky action, Yang and Qiu [22] proposed the expected utility-entropy (EU-E) decision model. The EU-E model brings together the notions of expected utility and entropy to create a decision model which effectively considers the decision-maker's subjective preference and objective uncertainty at each state of nature. Additionally, this model is proven to solve typical decision problems such as the Allais paradox reasonably which the expected utility theory is known to be incapable of. As it has previously been argued that the behaviors of investors differ to the constraints set by the axioms of the expected utility theory, the finding that the EU-E model can solve these decision problems indicates the consideration of a behavioral component inherent in the model [30].

The EU-E decision model is not established on an axiomatic basis. Luce et al. [21] derived the similar numerical representations for risky actions under behavioral axioms, which can be taken as an axiomatic development of the EU-E decision model in Yang and Qiu [22]. Furthermore, Yang and Qiu [23] improved the model to a normalized EU-E decision model, allowing for comparison of risky choices where the number of states are widely dispersed. Dong et al. [31], Xia et al. [32], and Xia et al. [33] presented several discussions on the EU-E decision model, emphasizing the role of Shannon entropy in the field of decision-making under risk. Casquilho and Rego [34] used decision models with different utility values combined with weighted entropies, incorporating rarity factors associated to Gini-Simpson and Shannon measures. In their paper, they provided an example of this decision framework for landscape compositional scenarios in Portugal. Their results indicate that the likely best combination is achieved by the criterion using the Shannon weighted entropy and a square root utility function. Allahverdyan et al. [35] derived a measure of risk similar to the EU-E measure of risk and concluded that their risk measure has normative features. The EU-E model has also been applied in other fields, such as a decision-making model for large consumers on a smart grid [36] and in rainfall threshold analysis [37].

In addition, Yang et al. [38] applied the EU-E decision model to stock selection using different tradeoff coefficients within certain intervals to derive efficient portfolios with respect to the traditional mean-variance framework. They found that the efficient portfolios from stocks selected using the EU-E model with intermediate values of tradeoff coefficients are more efficient than that of the set of stocks selected using the expected utility criterion. The conclusions in their paper demonstrate the necessity of incorporating both the expected utility and Shannon entropy together for risky choices.

Recently, we, Daniel et al. [39], applied the EU-E decision model to fund ratings and proposed an alternative fund rating approach based on EU-E decision model. We applied this approach to mutual fund ratings in the US market and examined the predictive ability of this approach for its ability to potentially mitigate the drawbacks of the risk measure used in Morningstar ratings. In the paper, the ratings based on the EU-E model have been compared with longstanding fund rating measure, Morningstar ratings, across a 13 year in-sample period. We found that ratings based on the EU-E model where tradeoff coefficient takes values of 0.25 and 0.75 outperform Morningstar in predicting the best, but not the worst performing funds. This result is robust to varying market climates.

As we demonstrated that the fund rating approach based on EU-E model can predict the best performing funds [39], it raised the question of whether investors are able to utilize this fund-rating approach to guide their investment decisions and then to achieve excess returns. This motived us to investigate whether the established EU-E fund ratings approach is helpful in guiding investors to make investment decisions.

2. Fund Ratings in US Mutual Funds Based on EU-E Decision Model

We apply the fund rating approach based both on the EU-E decision model [39] and Morningstar ratings [40] to constructing portfolios from the best performing funds and then investigate the performance of these portfolios.

We apply the fund rating approach in US mutual funds. We use the same dataset as in Daniel et al. [39]. First, we retrieve monthly return and overall rating data for all US mutual funds over the period of August 1992 to July 2015 from the Morningstar Direct database. Then, we exclude funds which are not assigned an overall rating or have missing data points over the 23-year period. We also exclude funds without 10 years of data prior to August 2002 to calculate the 10-year EU-E ratings. This results in a final sample of 2159 US mutual funds. Furthermore, to conduct the portfolio performance analysis, we collect monthly returns on the S&P 500 index over the period August 2002 to July 2015 as proxy for our benchmark from Bloomberg.

The descriptive statistics of the monthly returns for US mutual funds included from August 2002 to July 2015 is shown in Table 1. The total number of observations, N, is defined as the total number of funds at each period in the sample period multiplied by the number of months in the sample period.

Table 1. Descriptive statistics of the monthly returns of US mutual funds.

N	Mean	S.D.	Skewness	Kurtosis	Min.	Med.	Max.	Jacque-Bera
336,804	0.59	3.61	−0.67	11.15	−46.20	0.58	34.09	956,805.55

To avoid the improper influence of extreme values among all the returns on the distribution of return series, we winsorize the data at the 1% level and perform the following investigation using the winsorized data.

3. Performance of Portfolios Constructed Using EU-E Model and Morningstar Ratings

3.1. Portfolio Rebalancing Periods

We assume that investors utilize fund ratings to guide their investment decisions and they only take into account the best performing funds to construct their portfolios. We evaluate the performance of portfolios constructed using Morningstar ratings, and ratings based on the EU-E (λ = 0.25) and EU-E (λ = 0.75) models, respectively.

Alexeev and Dungey [41] demonstrate that an investor is able to diversify away the majority of unsystematic risk by investing in 6–15 stocks. Thus, we use 15 funds in each portfolio. We construct 100 equally weighted portfolios by randomly selecting 15 funds from the 5-star category ranked by Morningstar. Similarly, another 100 equally weighted portfolios are constructed based on the 5-star funds ranked by each of the EU-E (λ = 0.25) and EU-E (λ = 0.75) models.

Figure 1 provides an illustration of the rebalancing interval regimes considered in the analysis of this study (e.g., if a fund has lost its previous 5-star rating at the time of rebalancing, it will be replaced by a randomly selected fund from the 5-star category).We adopt four different rebalancing intervals as follows: 12-, 18-, 36-, and 60-month rebalancing intervals throughout the overall sample period to examine the performance of portfolios formed using ratings based on either Morningstar or the EU-E models.

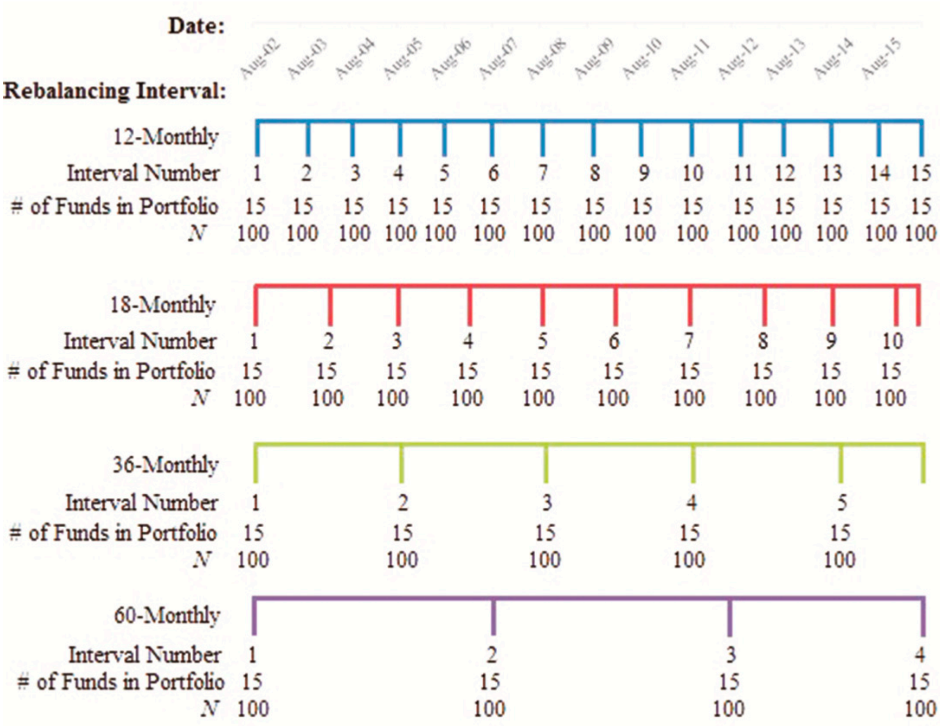

Figure 1. Summary of portfolio rebalancing periods. Note: Each portfolio is rebalanced and the performance of each portfolio is recorded at each interval as illustrated by the interval number. *N* refers to the total number of portfolios constructed using each of the ratings based on Morningstar and the EU-E model where λ takes a value of 0.25 and 0.75.

3.2. Portfolio Performance Evaluation

We present the methods used to evaluate the performance of portfolios following Loviscek and Jordan [42]. Firstly, we calculate the geometric mean of the excess returns relative to the benchmark, for each portfolio at every rebalancing interval. Next, we conduct the Wilcoxon signed-ranks test for differences between the performance of the Morningstar and EU-E model based portfolios relative to the benchmark. As shown by Loviscek and Jordan [42], the Wilcoxon signed-ranks test is used as opposed to a parametric test due to a potential bias associated with a low number of observations and a non-normal distribution.

In addition, we also assess the ability for an investor to construct trading strategies using the ratings based on Morningstar and the EU-E model to generate positive abnormal returns.

The abnormal return of portfolio *i* for a particular rebalancing interval is defined as follows:

$$AR_{it} = R_{it} - R_{mt} \tag{1}$$

where R_{it} is the return of portfolio *i* at rebalancing interval *t* and R_{mt} is the return of the benchmark at rebalancing interval *t*.

Therefore, we can define the average abnormal returns (AAR) for the 100 portfolios at each rebalancing interval *t* as:

$$AAR_t = \frac{1}{100}\sum_{i=1}^{100} AR_{it} \tag{2}$$

3.3. Abnormal Returns of the Portfolios Based on EU-E Decision Model and Morningstar

Table 2 provides a comparative summary of the performance of 100 randomly selected, equally weighted portfolios constructed using the 5-star categories of the ratings based on EU-E (λ = 0.25), EU-E (λ = 0.75), and Morningstar. The total number of portfolios constructed for each rating measure within each testing period is 100. The mean, minimum, maximum, and standard deviations are reported in terms of the return of the rating measure relative to the S&P 500. The number of significant outperformances and underperformances relative to the S&P 500 are reported as a percentage of the total number of periods per portfolio (*N*). Significance of the portfolios is tested at the 10% level.

Table 2. Summary of AAR of portfolio performance [1].

	Mean	Std. Dev.	Min.	Max.	Significant Outperformance (%)	Significant Underperformance (%)	*N*
Panel A: 12-Month Rebalancing Interval							
EU-E (λ = 0.25)	0.54	0.55	−2.18	2.44	68.31	7.00	1300
EU-E (λ = 0.75)	0.21	0.48	−1.79	2.31	29.77	8.77	1300
Morningstar	0.06	0.62	−1.55	2.32	17.62	21.08	1300
Panel B: 18-Month Rebalancing Interval							
EU-E (λ = 0.25)	0.46	0.54	−1.19	2.42	56.89	5.44	900
EU-E (λ = 0.75)	0.18	0.45	−1.44	1.96	32.11	10.00	900
Morningstar	−0.01	0.59	−1.15	1.79	10.67	21.44	900
Panel C: 36-Month Rebalancing Interval							
EU-E (λ = 0.25)	0.52	0.54	−1.22	2.29	56.40	4.40	500
EU-E (λ = 0.75)	0.26	0.45	−0.81	1.66	34.80	7.00	500
Morningstar	−0.06	0.41	−1.21	1.12	4.40	17.60	500
Panel D: 60-Month Rebalancing Interval							
EU-E (λ = 0.25)	0.48	0.47	−1.14	1.65	71.00	3.67	300
EU-E (λ = 0.75)	0.20	0.37	−0.75	1.31	25.33	8.33	300
Morningstar	−0.09	0.46	−0.98	1.25	5.67	30.33	300

[1] Table 2 summarizes the average abnormal returns (AAR) of portfolios constructed using the rating a based on the EU-E model and Morningstar across 12-, 18-, 36- and 60-month rebalancing intervals.

In general, the ratings based on EU-E (λ = 0.25) provide the best performing portfolios. In each panel, the number of times a portfolio constructed using the ratings based on EU-E (λ = 0.25) significantly outperforms the benchmark is above 56%. The outperformance statistic is greatest in the 12- and 60-month rebalancing intervals where the proportions are 68.31% and 71%, respectively. Additionally, the number of significant underperformances of the portfolios constructed using the ratings based on EU-E (λ = 0.25) relative to the benchmark is at a maximum in the 12-month rebalancing interval portfolios at 7% and declines as the rebalancing period increases.

The results are much weaker for the portfolios constructed using the ratings based on EU-E (λ = 0.75) and Morningstar. The highest proportions in which the portfolios constructed using the 5-star funds of EU-E (λ = 0.75) and Morningstar significantly outperform the benchmark are 34.8% and 17.62%, respectively. Furthermore, the result supports our expectation that portfolios constructed using the ratings based on EU-E (λ = 0.75) outperform the Morningstar ratings portfolios. The number of times the portfolios constructed using the ratings based on EU-E (λ = 0.75) outperform the benchmark is proportionally larger than that of Morningstar ratings, while the underperformance statistic of the portfolios based on Morningstar ratings is greater than the number of outperformances at each rebalancing interval. For instance, for the 12-month rebalancing interval, the proportion of Morningstar-rating based portfolios which significantly underperform the benchmark is 21.08%, whereas the number of significant outperformances is only 17.62%. For the ratings based on EU-E (λ = 0.75), the proportion of portfolios which significantly outperform the benchmark is 29.77%, whereas the proportion of significant underperformances is only 8.77%.

The statistics presented on the overall excess returns provides further support for our above-mentioned findings. In each rebalancing interval, the mean excess return on the portfolios using the ratings based on EU-E (λ = 0.25) is the highest among the three rating models with an excess return of 0.50%, on average across the four rebalancing intervals. This is followed by the portfolios constructed using the 5-star funds of EU-E (λ = 0.75) and Morningstar which yield average excess returns of 0.21% and −0.03%, respectively, across the four rebalancing intervals. For the portfolios based on Morningstar ratings, the mean excess return is only positive for the 12-month rebalancing interval, returning 0.06%. The standard deviation of excess returns is consistent across the rebalancing periods for each model.

In general, we find that the performance of portfolios based on the ratings of both EU-E ($\lambda = 0.25$) and EU-E ($\lambda = 0.75$) models tend to improve over longer rebalancing intervals. In contrast, performance of portfolios constructed using Morningstar ratings decreases as the length of the rebalancing interval increases.

4. Conclusions

We recently established the EU-E fund rating approach and showed that the fund rating approach can predict the best performing funds compared to Morningstar ratings. We applied the approach to the mutual fund ratings in the US market. In this paper, we investigate the practical applicability of investors utilizing this fund ratings approach to guide their investment decisions. The results in this paper indicate that the EU-E model as a fund rating measure can assist investors in constructing portfolios of mutual funds and that a portfolio constructed using best ranked funds from the EU-E model outperforms that of Morningstar. This finding is significant as it proves that the EU-E model is relevant in the decision-making process for investors.

Specifically, we construct portfolios based on the best rated funds to examine the ability for investors to use the ratings in fund selection. We construct 100 equally weighted portfolios by randomly selecting 15 funds from the 5-star category ranked both by fund rating based on EU-E models ($\lambda = 0.25$, $\lambda = 0.75$) and Morningstar. Overall, we find that the ratings based on the EU-E ($\lambda = 0.25$) model is the best performing measure followed by portfolios using the ratings based on the EU-E ($\lambda = 0.75$) model and Morningstar. This finding is proven by its superior near- to long-term predictive power which holds across volatile and stable markets. We adopt four different rebalancing intervals, namely 12-, 18-, 36-, and 60-month intervals. These findings are robust to these rebalancing intervals.

Author Contributions: D.C., J.Q., S.T. and J.Y. conceived and designed the experiments; D.C., J.Q. and S.T. performed the experiments; D.C., J.Q., S.T., J.Y. and C.S. analyzed the data; D.C., J.Q., S.T. and J.Y. contributed reagents/materials/analysis tools; D.C., J.Q., S.T., J.Y. and C.S. wrote the paper. All authors have read and agreed to the published version of the manuscript.

Acknowledgments: This research was funded by the National Natural Science Foundation of China, grant number 71571009.

Conflicts of Interest: The authors declare no conflict of interest.

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