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Abstract

The Symmetric and Antisymmetric Eigenvalue Problem for Electromagnetic Equilateral Triangular Waveguides via Plane Wave Reconstruction †

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A plane wave reconstruction technique is presented for the solution of Helmholtz's equation governing wave propagation in equilateral triangular electromagnetic waveguides in order to aid in the classification of symmetric and antisymmetric modes as well as to resolve the problem of excitation of triangular patch antennas. The case of transverse magnetic (TM) modes of a patch antenna with magnetic wall boundary conditions is discussed as representative of problems with Neumann boundary conditions. The technique sheds light on how these antennas may be excited and is applicable to problems with Dirichlet boundary conditions as well as to other triangular shapes including the right isosceles triangle, the 30°–90°–60°, the 30°–120°–30°, and the 15°–90°–75° triangles. The solutions are classified into two major categories: triaxially symmetric with eigenvalues that are harmonic multiples of 4 mπ/($\sqrt{3}a$), and solutions that are symmetric or antisymmetric with respect to only one median of the equilateral triangle. The triaxially symmetric solution is a result of sources at the centroid sending out waves perpendicular to the sides of the triangle in a **Y** configuration, while the latter are a result of two sets of waves traveling in opposite directions (clockwise and anticlockwise) parallel to the sides of the triangle in a **Δ** configuration. The solutions symmetric/antisymmetric with respect to only one median divide into two groups: one group with eigenvalues that are harmonic multiples of 4 mπ/(3a) and another group whose eigenvalues are not harmonic multiples of one another (solutions become triaxially symmetric/antisymmetric if m is a multiple of 3). These cases will be discussed in light of Lamé's solution [1–3] as detailed by McCartin in a series of papers [4,5].

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