



Proceedings Thermal Accretion Disk Spectra Based Tests of General Relativity[†]

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Abstract: The continuous X-ray flux of stellar-mass black holes provides an excellent source of data to learn about the astrophysics of accretion disks and about the spacetime itself. The extraction of information, however, depends heavily on our ability to correctly model the astrophysics and the theory of gravity, and the quality of the data. By combining a relativistic ray-tracing and Markov-Chain Monte-Carlo sampling technique, I show that the incorporation of the spin parameter through a slowly-rotating approximation, is not able to break the complex degeneracies of the model and therefore, when introducing modifications beyond general relativity it is very challenging to perform tests of general relativity with this type of observations. As a particular case, I show that it not possible to distinguish the small-coupling, slow-rotation black hole solution of dynamical Chern–Simons gravity from the Kerr solution with current instruments.

Keywords: black holes; accretion disk; thermal spectrum; dynamical Chern-Simons

1. Introduction

Before the detection of gravitational waves, the only source of information about black holes (BHs) was obtained purely through the electromagnetic spectrum [1]. In particular, for stellar-mass black holes the accretion disk spectrum [2], which peaks around $kT \sim 0.1-1$ keV, has shown to be a powerful tool to understand the astrophysics of the material surrounding them and about the spacetime itself. Most of the information of the spectrum produced by an accretion disk is obtained around its peak mainly because there are no special features at different wavelengths and the power drops by orders of magnitude away from the peak.

The electromagnetic spectrum of accreting black holes binaries can be described by at least three components: a thermal component in the soft X-ray band, a hard X-ray spectrum dominated by the direct radiation from the hot corona and a reflection component of the disk [3]. The resulting spectrum is extremely informative, given its many features, which is the heritage of the richness of the many physical processes occurring. However, when applying disk-reflection methods to estimate parameters, the results are strongly affected by systematic uncertainties related to both the spectral models as well as the calibration of the spectral data [4]. Nevertheless, despite these challenges, significant results have been delivered over the last years, and many sources have been studied, providing information about the astrophysics of black hole accretion [5]. These techniques continue to be refined as the quality of the data improves and soon will become a precision tool for astrophysics.

The vast majority of these models assume Einstein's general relativity (GR) as the underlying theory of gravity to describe the geometry of the black hole, which ultimately dictates the motion of the particles in the disk and the trajectory of the photons that arrive at our detectors. This assumption is to date well supported by a plethora of precision experiments and observations that GR has passed with flying colors in the *weak* field [6]. Black holes and the dynamics around them, however, belong to the *strong* gravity regime, i.e., where the curvature of the spacetime is large, the gravitational field is strong and dynamical, and the characteristic velocities are comparable to the speed of light. Because the light we observe from X-ray binaries comes from a region of strong gravity, these observations provide an ideal scenario to test our theory of gravity in the strong-field regime. But to guarantee that these tests of the theory are accurate, the accretion disk structure must be correctly modeled.

Ideally, one should study different models of accretion and include, consistently, modifications of GR at the same time, but in practice, this approach is not feasible. That is why over the past years different groups have studied different parts of the model, piece by piece, by changing a component, either in the astrophysics of the disk or in the theory of gravity, and keeping the rest unmodified (see, for instance, Refs. [4,7–13] for different types of modifications). If one keeps the astrophysical model of the disk fixed and only varies its parameters, there are four types of simulations, listed in Table 1 and explained in detail in Ref. [14], to study either the astrophysical model itself or the gravitational sector. These types of simulations are classified based on whether the synthetic injection is constructed within GR or outside GR and whether the model used to recover the injection is built within GR or outside GR. Thus, each case is composed of a *base* model and a deformation of the base model and the signal is constructed through a realization of a model, be it a pure GR model or a non-GR model.

Table 1. Studies considered in this work. In cases A and B, GR signals are injected and then attempted to be extracted with a GR and a non-GR model, respectively. Case A aims to estimate the accuracy to which accretion disk model parameters can be measured, while case B aims to determine how well a non-GR deviation can be constrained. Conversely, in cases C and D, non-GR signals are injected and then extracted with a GR model and a non-GR model, respectively. Case C aims to estimate the systematic uncertainties introduced in the extraction of accretion disk model parameters due to the a priori assumption that GR is correct, while D aims to determine whether GR deviations can be detected if they are present in the data.

	Signal	GR	non-GR
Model			
GR		А	С
non-GR		В	D

A detailed study of these four cases was carried out in Ref. [14] through a parametrization of potential deviations from the Schwarzschild metric in terms of a *bumpy* parameter. It was shown there that for small values of the injected deformation, the modified spectrum is not sufficiently different from those produced in GR to break the strong degeneracies with the disk model parameters. As the degeneracies in a simple model do not necessarily imply worse degeneracies in a more complex one, it is natural to ask if a more complicated model is capable of better-capturing features present in the signal. Furthermore, as black holes are expected to be spinning in nature, the correct base for the models should be a rotating BH solution, so the incorporation of spin in the model is a good candidate to increase the complexity of the model.

The chosen bumpy parameter used in Ref. [14] was shown to change the location of essential signatures of the strong gravity that are characteristic to BHs: the event horizon, the ergosphere, the innermost stable circular orbit (ISCO), and the photon sphere, in a similar way as the rotation does in

the Kerr case. Therefore, the bumpy parameter should be, in principle, degenerate with the spin if the spin were included in the model. These two parameters, the spin, and the bumpy parameter, however, cannot be 100% degenerate with each other, as the spin introduces frame-dragging effects and other modifications to the metric that are different from those introduced by the bumpy parameter.

Following closely Ref. [14], in this work I study the thermal component of the accretion disk and introduce the spin through a slow-rotation expansion of the Kerr solution, which assumes that the magnitude of the BH spin angular momentum *S* is much smaller than its mass, *M*, squared, i.e., $\chi = S/M^2 \ll 1$. At zeroth order in rotation, the slow-rotation expansion of the Kerr metric reduces precisely to the Schwarzschild metric, and at next order in rotation, it represents a deformation of Schwarzschild. Instead of considering a bumpy parameter as it was done in Ref. [14], I chose a well-motivated, parity-violating effective field theory; dynamical Chern-Simons (dCS), to be the underlying theory of gravity. The results presented here extend the findings of Ref. [14] to the rotation case, and agree with the ones presented in Refs. [9,13], showing that it not possible to distinguish the small-coupling, slow-rotation black hole solution of dynamical Chern–Simons gravity from the Kerr solution with current and next-generation instruments.

The remainder of this paper shows the details of the calculations that led to the above conclusions. Section 2 describes the dCS metric, Section 3 presents the results of the simulations performed and finally Section 4 concludes. When not mentioned, geometric units are used in which G = 1 = c.

2. Dynamical Chern-Simons

Rotating BH solutions in extended theories of gravity have mostly been found in the small-coupling and slow-rotation approximations, i.e., assuming that the solution is a small deformation from GR, and that the dimensionless spin parameter is much smaller than unity. This assumptions do not guarantee the existence of exact integrals of the motion [15], as in GR, making the study of geodesics only possible numerically. One well motivated example of this type of solutions is dynamical Chern–Simons gravity, a four-dimensional effective theory that derives from loop quantum gravity [16], string theory [17] and inflation [18]. This theory introduces parity-violating interactions and is defined through the action [19]

$$S \equiv \int d^4x \sqrt{-g} \left\{ \kappa_g R + \frac{\alpha}{4} \vartheta R_{\nu\mu\rho\sigma}^* R^{\mu\nu\rho\sigma} - \frac{\beta}{2} [\nabla_\mu \vartheta \nabla^\mu \vartheta] + \mathcal{L}_{\text{mat}} \right\},\tag{1}$$

where *g* is the determinant of the metric $g_{\mu\nu}$, $\kappa_g \equiv (16\pi)^{-1}$, α and β are coupling constants, $R_{\mu\nu\delta\sigma}$ is the Riemann tensor, \mathcal{L}_{mat} is the matter Lagrangian density, ϑ is a dimensionless pseudo-scalar field, and $*R^{\mu\nu\rho\sigma}$ is the dual of the Riemann tensor, defined by

$${}^{*}R^{\mu\nu\rho\sigma} \equiv \frac{1}{2}\epsilon^{\rho\sigma\alpha\beta}R^{\mu\nu}{}_{\alpha\beta}, \qquad (2)$$

with $\epsilon^{\mu\nu\alpha\beta}$ the Levi-Civita tensor. The coupling constants are taken such that β is dimensionless and α has dimensions of length squared, so that deformations from GR are proportional to following dimensionless parameter

$$\zeta \equiv \frac{\alpha^2}{\kappa_g \beta \mathcal{M}^4} \,, \tag{3}$$

where \mathcal{M} is a characteristic length of the system.

In this work, I consider the BH solutions derived using perturbation theory techniques presented in Ref. [19], which are valid to first order in the spin and coupling parameter. At this order the slow-rotation and small-coupling correction to the Kerr metric in dCS gravity is introduced only in the $g_{t\phi}$ component of the metric [19]

$$g_{t\phi}^{\text{dCS}} = \frac{5}{8} \zeta a \frac{M^4}{r^4} \left(1 + \frac{12}{7} \frac{M}{r} + \frac{27}{10} \frac{M^2}{r^2} \right) \sin^2 \theta \,, \tag{4}$$

where (t, r, θ, ϕ) are Boyer-Lindquist coordinates. Note that the dCS modification deforms the gravitational field of spinning BHs in GR, and when $\zeta = 0$ the solution reduces to the slow-rotation expansion of the Kerr background.

3. Simulations

The simulations presented in this work are set up following closely Ref. [14]. The time-averaged energy flux emitted from the disk [9]

$$\mathcal{F}(r) = \frac{\dot{M}}{4\pi} \frac{1}{\sqrt{-g_{rr}g_{tt}g_{\phi\phi}}} \frac{-\Omega_{,r}}{(E-\Omega L)^2} \int_{r_{\rm in}}^r (E-\Omega L) L_{,r'} dr'$$
(5)

is computed assuming a Novikov-Thorne model [20], i.e., a geometrically thin and optically thick accretion disk with the material in circular rotation. Here spherical polar coordinates are considered and *E*, *L* and Ω are the energy, (*z*-component of the) specific angular momentum and the orbital frequency, respectively. These quantities are found from the components of the metric. The inner edge of the disk, r_{in} , is assumed here to coincide with the innermost stable circular orbit (ISCO), and the disk extends up to r = 300M. GYOTO [21] is used to solve for the motion of photons from the observer, located a distance r_{obs} at an inclination angle *i*, to the accretion disk (integrating backwards in time), which produces the Planck function's B_{ν} on the screen, from which the computation of the observed flux is calculated [9]. Thus, the accretion disk entirely is determined by the parameters $\vec{\lambda} = (a, \zeta, \dot{M}, M, r_{obs}, i)$, i.e., the spin, the dimensionless coupling constant of dCS, the mass accretion rate, the mass, the location and the inclination angle of the black hole, respectively.

The analysis of the resulting spectra is carried out by a Markov-Chain Monte-Carlo (MCMC) exploration of the likelihood to find the marginalized posterior distributions for each parameter, through the affine invariant MCMC sampler emcee [22], as follows. Given a synthetic data injection L_{inj} characterized by \bar{N}^* injected parameters $\vec{\lambda}^* = (\lambda_1^*, \dots, \lambda_{\bar{N}^*}^*)$, the \bar{N} parameters $\vec{\lambda} = (\lambda_1, \dots, \lambda_{\bar{N}})$ are estimated in the model L_{mod} by minimizing the reduced χ^2_{red} , defined by

$$\chi^{2}_{\rm red}(\bar{N}) = \frac{\chi^{2}}{F} = \frac{1}{F} \sum_{i=1}^{F} \left[\frac{L_{\rm mod}(\nu_{i},\vec{\lambda}) - L_{\rm inj}(\nu_{i},\vec{\lambda}^{*})}{\sigma(\nu_{i})} \right]^{2}, \tag{6}$$

where the summation is over F = 50 sampling frequencies $v_i \in (10^{16.8}, 10^{18.3})$ [Hz] evenly spaced logarithmically. Here the standard deviation σ of the distribution is modeled via

$$\sigma(\nu_i) = \sum_{j}^{\bar{N}} \sigma_j(\nu_i), \tag{7}$$

where the sum is over the number of free parameters in the model, and

$$\sigma_j(\nu_i) = \frac{|L(\nu_i, \vec{\lambda}^*_{\notin n}, \lambda^*_n + \delta\lambda^*_n) - L(\nu_i, \vec{\lambda}^*_{\notin n}, \lambda^*_n - \delta\lambda^*_n)|}{2}.$$
(8)

The term $\delta\lambda_n^*$ is a measure of the observational error in the injected parameter $\lambda_n^* \in \vec{\lambda}^*$. For the GR cases the parameters are $\vec{\lambda} = (a, \log \dot{M}, M, r_{obs}, i)$, while in the non-GR cases the parameters are $\vec{\lambda} = (a, \zeta, \log \dot{M}, M, r_{obs}, i)$. The priors are assumed to be uninformative (flat) on the parameters a, ζ and \dot{M} , with ranges $-0.2 < a < 0.2, 0 \le \zeta \le 0.2$ and $17 < \log \dot{M} [g s^{-1}] < 19$, with $\delta a = 0.05, \delta\zeta = 0.05$ and $\delta \log \dot{M} = 0.2 g s^{-1}$, respectively. For the parameters r_{obs} , M and i, Gaussian priors were chosen with means $\mu_{r_{obs}} = r_{obs}^*, \mu_M = M^* \mu_i = i^*$ and standard deviations $\sigma_{r_{obs}} = 2 \text{ kpc}, \sigma_M = 1.5 M_{\odot}$ and $\sigma_i = 5^\circ$, as these parameters are typically known to some degree from independent observations [23–28].

Even though I will show results for mainly two representative examples, simulation of the four cases shown in Table 1 were performed, and the features are generic and based on an extensive numerical study in a very large region of parameter space. The corner plot shown in Figure 1 refers to the case when a GR injection is extracted with the same GR model, i.e., simulations of the type A. As expected, the posterior distribution of M, r_{obs} and i are dominated by the Gaussian priors. However, the spin and accretion rate show a complex degeneracy between them, which directly translates into a weak determination of these two parameters. In particular, these strong covariances make the spin parameter not measurable at all, while \dot{M} is measured to roughly one order of magnitude.



Figure 1. Results of the MCMC analysis for case A, where the vertical lines correspond to the injected values. The parameters M, r_{obs} and i are well constrained, mainly because of the Gaussian priors used. However, the parameters a and \dot{M} are not well estimated due to the large degeneracies between these two parameters.

The results of the simulation performed when a non-GR injection is extracted with a non-GR model, i.e., case D, are shown in Figure 2, where a non-GR signal was injected with $\zeta = 0.1$. The posterior distribution on the coupling constant is almost flat, resembling the prior distribution. This is also the case for the parameters *a* and \dot{M} , which is consistent with the results of Figure 1.



Figure 2. Results of the MCMC analysis for case D, where the vertical red lines correspond to the injected values. Similarly, as shown in Figure 1 the parameters M, r_{obs} and i are well constrained due to the Gaussian priors used, and a, ζ and \dot{M} are not well estimated due to the large degeneracies between them.

From the studied cases, one can focus on the posterior distribution of these parameter. In particular, the posterior distributions of the spin parameter *a* are shown in Figure 3 for all the simulations with injected values a = 0 and $\zeta = 0.1$. The recovered posterior is flat and consistent with the prior, indicating that one cannot constrain *a* at all in any of the four types of simulations. This behavior holds even for



Figure 3. Marginalized posterior distributions for the dimensionless spin parameter for case A (dashed black), case B (blue), case C (red) and case D (solid black). The dashed vertical line shows the injected value of the spin parameter. Note that this distributions are statistically indistinguishable.

The posterior distributions of the coupling constant ζ are shown in Figure 4 for simulations performed for cases B and D, with injected values a = 0.2, $\zeta = 0.1$. The results imply that the changes in the spectra are not significant enough to allow an observation of this type to distinguish between GR and non-GR, making the posteriors consistent with the priors. This implies that one could thus be in a situation in which a GR deviation is present in the BH background, yet the observed accretion disk spectra is not sensitive enough to detect it (or discern a GR model from a non-GR model), and consequently the amount of fundamental bias in the recovered parameters is negligible and well within the statistical uncertainties.



Figure 4. Marginalized posterior distributions for the dCS dimensionless coupling constant ζ for case B (blue) and case D (red), with an injected value of a = 0.2 for these simulations. The dashed vertical line shows the injected value for that parameter. Note that this distributions are statistically indistinguishable.

4. Conclusions

I have presented here a slight extension of the results presented in Ref. [14], allowing for a non-GR deformation of the Kerr metric, through a slowly-rotating approximation. I show that the use of the accretion disk spectrum to constrain or detect non-GR deformations from a GR background is extremely challenging due to the degeneracies between the accretion disk model parameters, even when considering a model that accounts for the rotation of the source.

Current accretion models are characterized by a large number of parameters that must be inferred from the fit of the spectrum. In particular, spin measurements of black holes are often obtained by imposing some strong assumptions on the value of other model parameters, and therefore they should thus be taken with some caution. Thus, the degeneracy of the spin and the parameter that controls the non-GR modification shown here limits the power of this type of measurements with current observations. The results presented here are in complete agreement with current spin measurements [5,28], i.e., large errors for slowly rotating sources.

Modifications of GR are expected to be in general small due to the plethora of other gravity tests already performed, and therefore, as shown in Ref. [14], if astrophysical BHs are not described by a GR solution, but instead there is a large deformation, then the estimation of the parameters could be systematically biased. By performing a detailed analysis of the likelihood surface to find the marginalized posterior distribution for the coupling constant of dynamical Chern–Simons gravity, I show that it is not possible to distinguish this solution from the Kerr solution with current instruments. This has been shown in previous studies using the information from the events detected so far by the LIGO-Virgo collaboration [29] and also through the thermal component of the spectrum where small portions of the parameter space were studied for a few cases [9] or on a grid [13].

The analysis presented here can be extended along several different directions, as the main focus was on a rather simple accretion disk model, which is suitable only for slowly rotating BHs. For instance, the accretion disk thickness and other accretion disk physics (beyond those approximated in a simple geometrically thin/optically thick model) could be included. One could thus repeat this type of analysis to include these effects and even quantify the systematic error of the models.

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Abbreviations

The following abbreviations are used in this manuscript:

BH	Black Hole
dCS	Dynamical Chern-Simons
ISCO	Innermost Stable Circular Orbit
MCMC	Markov Chain Monte Carlo

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