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Non-Ideal X-Gate and Z-Gate in Semiconducting Spin Qubit Implementations †

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† Presented at the 11th Italian Quantum Information Science Conference (IQIS2018), Catania, Italy, 17–20 September 2018.

Published: 19 November 2019

Abstract: Several spin qubit architectures have been proposed, theoretically investigated and realized at least on the scale of single devices in view of quantum computation and simulation applications. We focus our study on five qubit types: quantum dot spin qubit, double quantum dot singlet-triplet qubit, double quantum dot hybrid qubit, donor qubit, quantum dot spin-donor qubit and for each one we derived a compact effective Hamiltonian. Single qubit gate fidelities when time interval error is included are compared. A realistic set of values for the error parameters of amplitude controls linked to the z and x contribution appearing in the Hamiltonian models has been used. This study provides a ranking of the gate fidelities for the different qubit architectures highlighting which one is the most robust with respect to the considered control noises.

Keywords: spin qubit; noise; quantum computation

1. Introduction

Semiconductor-based electron spin qubits are an interesting platform for universal quantum computation [1–4]. These types of qubits are realized confining electron spins in host semiconducting materials through electrostatic gates or self-assembled quantum dots (QDs) [5,6] or by donor nuclear spins in solid matrices [7,8]. They are an attractive scenario thanks to the electrons spin long coherence times, the fast gate operations and potential for scaling due to the integrability with the already existing CMOS infrastructure of the microelectronic industry. The five qubit types that we studied are: the quantum dot spin qubit (SQ) [1], the double quantum dot singlet-triplet qubit (STQ) [4], the double quantum dot hybrid qubit (HQ) [6,9–11], the donor qubit (DQ) [7] and the quantum dot spin-donor qubit (SDQ) [8]. We demonstrated that they have in common a compact effective Hamiltonian derived when each qubit is expressed in its proper logical basis [12].

The greatest challenge for semiconductor qubits is improving gate operation fidelity. The ideal realization of quantum gates is indeed deeply influenced by the unavoidable environmental noise that cause decoherence. We studied this problem from a theoretical point of view adopting the entanglement fidelity to test the resilience of the quantum gates with respect to disturbance sources. We consider two different sources of noise directly linked to the z and x contribution appearing in the effective Hamiltonian models. Such abstract controls become physical entities giving a real mean to the type of disturbance and a measure of it once that the qubit type is specified.

The paper is organized as follows. Section 2 defines the global scenario for all the qubit types under investigation providing a compact effective Hamiltonian model. It also contains the main results about the single qubit gate fidelities (X-gate ans Z-gate) and gives a comparison among all the five qubit types when they are subject to time interval error (TIE). In Section 3, concluding remarks are summarized.

2. Results

The effective Hamiltonian models are here presented. Then non idealities are included in the model to perform a good performance analysis in real systems. We account for error sources that is modeled as random variables with Gaussian distributions featuring zero mean and standard deviation σ that add up to the ideal values. The figure of merit used to estimate the disturbance effects is the entanglement fidelity F

$$F = tr[\rho^{RS}\mathbf{1}_R \otimes (U_i^{-1}U_d)_S \rho^{RS}\mathbf{1}_R \otimes (U_d^{-1}U_i)_S], \tag{1}$$

where $U_i(U_d)$ is the ideal (disturbed) time evolution and $\rho^{RS} = |\psi\rangle\langle\psi|$ with $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ a maximally entangled state in a double state space generated by two identical Hilbert spaces R and S. X-gate and Z-gate, that are rotations of π along x and z axis of the Bloch sphere, are chosen as reference gates for each qubit type. The sequences that realize such rotations are derived analytically for each qubit type [12].

2.1. Quantum Dot and Donor Spin Qubits Effective Hamiltonian Models

The five qubit types under study have in common a compact effective Hamiltonian when expressed each in its proper logical basis $\{|0\rangle, |1\rangle\}$ [12]. The effective Hamiltonian models in terms of 2×2 Pauli matrices σ_z and σ_x and the identity operator I_2 are expressed by

$$H = \alpha_z\sigma_z + \alpha_x\sigma_x + \alpha_0 I_2, \tag{2}$$

where α_z, α_x and α_0 are given in Table 1.

Table 1. Coefficients of the effective Hamiltonian models and logical bases.

Qubit	α_z	α_x	α_0	$\{\|0\rangle, \|1\rangle\}$
SQ (rot. frame)	$\frac{\hbar}{2}(\omega_z - \omega)$	$\frac{\hbar}{2}\Omega_x$	0	$\{\|\uparrow\rangle, \|\downarrow\rangle\}$
STQ	$-\frac{1}{2}J$	ΔE_z	$-\frac{1}{4}J$	$\{\|S\rangle, \|T_0\rangle\}$
HQ	$-\frac{1}{2}J' + \frac{1}{4}(J_1 + J_2)$	$-\frac{\sqrt{3}}{4}(J_1 - J_2)$	$-\frac{E_z}{2} - \frac{1}{4}(J' + J_1 + J_2)$	$\{\|S\rangle\|\uparrow\rangle, \sqrt{\frac{1}{3}}\|T_0\rangle\|\uparrow\rangle - \sqrt{\frac{2}{3}}\|T_+\rangle\|\downarrow\rangle\}$
DQ (rot. frame)	$\frac{\hbar}{2}(\omega_{12} - \omega)$	$\frac{\hbar}{2}\Omega_x$	0	$\{\|\uparrow\Downarrow\rangle, \|\downarrow\Downarrow\rangle\}$
SDQ	$-\frac{1}{8}J$	$\frac{A}{16}$	$\frac{1}{4}\gamma_n B_0 - \frac{1}{16}J$	$\{\|S\Downarrow\rangle, \|T_0\Downarrow\rangle\}$

The state $|\uparrow(\downarrow)\rangle$ denotes the single spin with up (down) projection; $|S(T)\rangle$ is the singlet (triplet) state of the pair of electronic spins and $|\Downarrow\rangle$ is the angular momentum state of the donor nulear spin. The parameters appearing in Table 1 are here defined for each qubit type:

- SQ: ω_z is the Zeeman frequency associated to the DC applied magnetic field, ω is the angular frequency of the AC local magnetic field and Ω_x is the angular frequency that depends on the amplitude of that field.
- STQ: ΔE_z is the magnetic field gradient between the QDs and J is the exchange coupling between the two spins.
- HQ: E_z is the Zeeman energy associated to the constant applied magnetic field and J', J_1, J_2 are the exchange couplings among couple of spins.
- DQ: the same as for the SQ, where ω_{12} is the analogous of ω_z and is equal to $\omega_{12} = \Delta_- + \sqrt{\Delta_+^2 + 4a^2} - 2a$, with $\Delta_\pm = 1/2(\gamma_e \pm \gamma_n)B_0$ and $a = A/4$. The parameters γ_e and γ_n are respectively the electron and nuclear gyromagnetic ratio, B_0 is the external DC magnetic field and A is the hyperfine coupling. The donor nuclear spin is supposed equal to $I = 1/2$.

- SDQ: J is the exchange coupling between the electron spins of the donor and of the dot, A is the hyperfine coupling between the electron spin and the nuclear spin of the donor and B_0 is the applied DC magnetic field. The donor nuclear spin is supposed equal to $I = 1/2$.

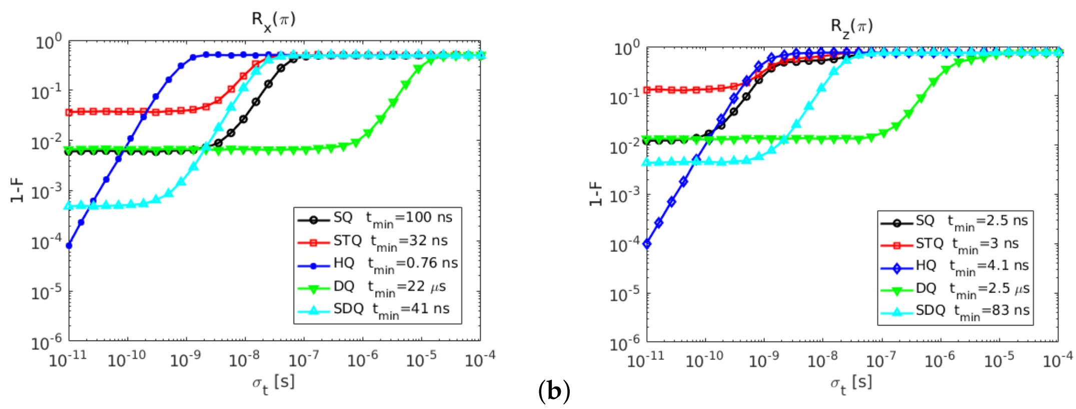
2.2. Comparison of Gate Fidelities among Qubit Types

A comparison of gate fidelities among all the qubit types due to TIE is presented. To this purpose, control error standard deviations on the amplitudes of the control parameters are set to the values taken from the literature reported in Table 2. As it is evident comparing Tables 1 and 2 the control parameters are directly linked to the x and z contributions of the effective Hamiltonian models.

Table 2. Control error standard deviations for the five qubit types.

Qubit	Error on Control Variables	Semiconductor
SQ	$\sigma_{\Delta\omega_z}$ = 20 Hz [13]; σ_{Ω_x}=0.25 MHz [1]	Si/SiGe
STQ	$\sigma_{\Delta E_z}$=4 neV [5]; σ_J=1 neV [5]	Si/SiGe
HQ	σ_J=1 neV [6]	Si/SiGe
DQ	$\sigma_{\Delta\omega_{12}}$=100 Hz [7]; σ_{Ω_x}=1.2 kHz	^{31}P in Si
SDQ	σ_J=4 neV [8]; σ_A=2.5 neV [8]	^{31}P in Si

Figure 1 shows a gate infidelities comparison for $R_x(\pi)$ and $R_z(\pi)$ as a function of the standard deviation σ_t.



(**a**) (**b**)

Figure 1. (**a**) Comparison of $R_x(\pi)$ gate infidelities among all qubit types as a function of the standard deviation σ_t. In the legend the smallest time of sequence step t_{min} for each qubit type is also reported. (**b**) Same as a) but for $R_z(\pi)$ gate.

For both the gates all the qubits show decreasing infidelities when σ_t is reduced. The roll off of each curve is roughly observed for σ_t close to the shortest step time t_{min} of the gate sequence for the corresponding qubit type. SQ, STQ, DQ and SDQ present a saturated behavior when σ_t is reduced, meaning that the TIE is no more the fidelity limiter in that range. Such graphs point out that HQ is the most sensitive one to TIE whereas DQ is the most robust to such kind of error for both operations. But such robustness of the DQ is achieved by imposing slower gates than those of other qubits. Given a qubit type, there is a trade-off between the speed of the gate operation and the robustness of the gate fidelity to TIE.

3. Discussion

We reported a comparative study of five spin qubit types realized through electron spin in electrostatically defined quantum dots and the electron spin of impurity atoms in semiconducting host (donors). For each qubit type, we presented a compact effective Hamiltonian model and starting from analytical time sequences that realize X-gate and Z-gate we estimated the effects on the gate fidelity of the disturbances by using a Gaussian noise model. A comparison of the gate fidelities of all the qubit implementations due to the TIE is presented using a realistic set of values for the error parameters of amplitude controls taken from the literature. We conclude that the HQ is very sensitive to TIE while the infidelity of the DQ due to TIE is not dominant till very large time errors, at a cost of very slow gate operations. This study offers a general platform for different spin qubit implementations providing an important instrument for future works.

Author Contributions: E.F. and M.D.M. conceive the study and perform the calculations. E.F., M.F. and M.D.M. wrote the paper.

Acknowledgments: This work was supported by the European Union's Horizon 2020 research and innovation program under grant agreement No 688539 MOS-QUITO.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Kawakami, E.; Scarlino, P.; Ward, D.R.; Braakman, F.R.; Savage, D.E.; Lagally, M.G.; Friesen, M.; Coppersmith, S.N.; Eriksson, M.A.; Vandersypen, L.M.K. Electrical control of a long-lived spin qubit in a Si/SiGe quantum dot. *Nat. Nanotchnol.* **2014**, *9*, 666–670, doi:10.1038/nnano.2014.153.
2. Pla, J.J.; Tan, K.Y.; Dehollain, J.P.; Lim, W.H.; Morton, J.J.; Jamieson, D.N.; Dzurak, A.S.; Morello, A. A single-atom electron spin qubit in silicon. *Nature* **2012**, *489*, 541–545, doi:10.1038/nature11449.
3. Rui, L.; Xuedong, H.; You, J.Q. Controllable exchange coupling between two singlet-triplet qubits. *Phys. Rev. B* **2012**, *86*, 205306, doi:10.1103/PhysRevB.86.205306.
4. Coish, W.A.; Loss, D. Singlet-triplet decoherence due to nuclear spins in a double quantum dot. *Phys. Rev. B* **2005**, *72*, 125337, doi:10.1103/PhysRevB.72.125337.
5. Wu, X.; Ward, D.R.; Prance, J.R.; Kim, D.; Gamble, J.K.; Mohr, R.T.; Shi, Z.; Savage, D.E.; Lagally, M.G.; Friesen, M.; et al. Two-axis control of a singlet–triplet qubit with an integrated micromagnet. *Proc. Natl. Acad. Sci. USA* **2014**, *111*, 11938–11942, doi:10.1073/pnas.1412230111.
6. Thorgrimsson, B.; Kim, D.; Yang, Y.C.; Smith, L.W.; Simmons, C.B.; Ward, D.R.; Foote, R.H.; Corrigan, J.; Savage, D.E.; Lagally, M.G.; et al. Extending the coherence of a quantum dot hybrid qubit. *NPJ Quantum Inf.* **2017**, *3*, 32, doi:10.1038/s41534-017-0034-2.
7. Laucht, A.; Muhonen, J.T.; Mohiyaddin, F.A.; Kalra, R.; Dehollain, J.P.; Freer, S.; Hudson, F.E.; Veldhorst, M.; Rahman, R.; Klimeck, G.; et al. Electrically controlling single-spin qubits in a continuous microwave field. *Sci. Adv.* **2015**, *1*, e1500022, doi:10.1126/sciadv.1500022.
8. Harvey-Collard, P.; Jacobson, N.T.; Rudolph, M.; Dominguez, J.; Ten Eyck, G.A.; Wendt, J.R.; Pluym, T.; Gamble, J.K.; Lilly, M.P.; Pioro-Ladrière, M.; et al. Coherent coupling between a quantum dot and a donor in silicon. *Nat. Commun.* **2017**, *8*, 1029, doi:10.1038/s41467-017-01113-2.
9. Ferraro, E.; De Michielis, M.; Mazzeo, G.; Fanciulli, M.; Prati, E. Effective Hamiltonian for the hybrid double quantum dot qubit. *Quantum Inf. Process.* **2014**, *13*, 1155–1173, doi:10.1007/s11128-013-0718-2.
10. Ferraro, E.; De Michielis, M.; Fanciulli, M.; Prati, E. Effective Hamiltonian for two interacting double-dot exchange-only qubits and their controlled-NOT operations. *Quantum Inf. Process.* **2015**, *14*, 47–65, doi:10.1007/s11128-014-0864-1.
11. De Michielis, M.; Ferraro, E.; Fanciulli, M.; Prati, E. Universal set of quantum gates for double-dot exchange-only spin qubits with intradot coupling. *J. Phys. A Math. Theor.* **2015**, *48*, 065304, doi:10.1088/1751-8113/48/6/065304.
12. Ferraro, E.; Fanciulli, M.; De Michielis, M. Gate fidelity comparison in semiconducting spin qubit implementations affected by control noises. *J. Phys. Commun.* **2018**, *2*, 115022, doi:10.1088/2399-6528/aaf088.
13. Technologies Keysight. *Keysight Technologies E8257D PSG Microwave Analog Signal Generator*; Technologies Keysight: Santa Rosa, CA, USA, 2017.