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Generation of Photon Pairs in the Light-Matter Ultrastrong Coupling Regime: From Casimir Radiation to Stimulated Raman Adiabatic Passage †

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Abstract: The ultrastrong coupling regime of light-matter interaction is achieved when the coupling strength is a significant fraction of the natural frequencies of the noninteracting parts. Physics in this regime has recently attracted great interest, both theoretically and experimentally being a fruitful platform to test fundamental quantum mechanics in a new non-perturbative regime, and for applications to quantum technologies.Here we discuss the generation of photon-pair states, which is a distinctive feature of this new regime, and interesting new dynamicsl effects both in optomechanics and in circuit-QED architectures.

Keywords: ultrastrong light-matter coupling; STIRAP; superconducting qubits

1. Introduction

Light-matter interaction is a fundamental building block of Nature leading to countless applications, including the forefront demonstration of supremacy of quantum technologies [1,2]. Circuit-QED solid-state systems are one of the forefront platforms for quantum hardware [3] implementing paradigmatic models where fundamental physics from measurement theory [4] to quantum thermodynamics [5] and quantum communication [6] can be studied. In the last few years a new ultrastrong coupling (USC) regime, where light-matter coupling is large enough to break the rotating wave approximation (RWA), has been achieved in cavity and circuit QED, THz metamaterials, intersubband polaritons and other physical systems [7,8], and has been subject of extensive theoretical analysis [7–14] showing thatnovel and outstanding physics arises in this non-perturbative regime. In very recent work it has been shown that USC can be a powerful tool for the generation of photon pairs from entangled states in optomechanical systems [15] and circuit-QED architectures [16]. New results on the dynamics of photons are here reported.

2. Casimir Radiation from Optomechanical Systems

Dynamical Casimir effect (DCE) consists in generation of photons from the quantum vacuum due to rapid changes of the boundary conditions. Creation of photons triggered by moving mirrors, was first predicted in 1970 [17], and later it was shown that photons can be generated also by a single mirror [18]. Tho different strategies have been considered for the experimental demonstration of DCE, namely: (1) in a device with a movable mirror able to oscillate very fast [19], or (2) inducing modulated boundary conditions mimicking an effective motion [20,21]. According to the theoretical description [22] the experimental detection of the DCE is still challenging due to the fabrication of very fast movable mirrors. Indeed for the DCE to be observed the mechanical oscillation frequency ω_m has to be at least twice the frequency of the lowest cavity mode, ω_c. Nowadays, such a condition still represents the biggest issue for the demontration of the mechanical DCE. This is why a lot of interest was driven to the implementation of the analog of DCE by time dependent boundary conditions. In the last decade [21] this strategy was implemented by superconducting circuits coupled to a coplanar transmission line terminated with a SQUID, whose inductance modulated at high frequency results in a time dependent reflectivity. Recently, a very interesting way to overcome the experimental observation of a true mechanical DCE was proposed by Macrí et al. [15]. In this work a careful theoretical description of the optomechanical interaction was studied, showing that the required matching condition $2\omega_c = \omega_m$ can be softened. Indeed the full Hamiltonian, for a single light mode interacting with a mechanical moveable mirror is:

$$H_{sys} = \omega_c a^\dagger a + \omega_m b^\dagger b + \frac{G}{2}(a + a^\dagger)^2(b + b^\dagger) \tag{1}$$

where $a(a^\dagger)$ and $b(b^\dagger)$, represent respectively the annihilation (creation) operators of cavity mode and mechanical oscillator, G being the coupling strength. Notice that by writing in normal order the interaction term we can identify the standard optomechanical coupling term describing the energy induced by the radiation pressure, $H_{om} = Ga^\dagger a(b + b^\dagger)$, and the additional term $H_{DCE} = \frac{G}{2}(a^2 + a^{\dagger 2})(b + b^\dagger)$. This latter is routinely neglected owing to its perturbative action, since the mechanical frequency is usually considered much smaller than the optical frequency (as it happens in the standard experimental conditions) and because of the weak coupling regime, i.e., G is small compared to the bare energies. However this is not true in the non-perturbative USC regime where H_{DCE} cannot be neglected anymore.

Eigenvalues of H_{sys} as a function of ω_c for fixed G and ω_m are reported in Figure 1b, showing the appearance of a peculiar avoided level crossing between states with 2 photons and 4 phonos, and between 2 photons and 3 phonons (magnified in Figure 1c,d). In general H_{DCE} is able to induce resonant optomechanical scattering processes involving $|n, k_n\rangle \leftrightarrow |n+2, (k-q)_{n+2}\rangle$, where n (k) labels the photon (phonon) state, the subscript indicates which mechanical manifold is considered, and q is an integer number. Such avoided crossings occur when the energies of the final and initial states coincide, i.e., $2\omega_c \sim q\omega_m$, marking the coherent exchange of excitations between light and mechanical oscillator. This result shows that the main difficulty for the detection of mechanical DCE, namely very large mechanical frequency are not necessary, is overcome since ω_m can be also smaller than ω_c even for $q = 3$. Of course for larger q the induced level splitting decreases, and the dynamical detection may become fragile with respect to the decoherence and losses, but for $q = 4$, the splitting is still larger than the losses rate achievable with the best state-of-art optomechanical systems. The reversible dynamics subsumed by H_{DCE}, can be tested either by preparing the system in one of the mechanical Fock state and let it evolve, or by an external coherent drive (pulsed or continuous) exciting the oscillator and detecting the photon-conversion of the excitation. In both cases measurement of the generated Casimir photons requires that the optomechanical system is viewed as an open quantum system, whose dynamics is described by a Master Equation accounting for the generalized photodetection scheme appropriate for USC [15].

In Figure 2 the dynamics of an interacting optomechanical systems prepared in the two-photons Fock state of the mechanical oscillator $|0, 2\rangle$, exchanging coherently excitations with the electromagnetic mode $|2, 0\rangle$. Figure 2b shows the strong coupling regime, when the assocated level splitting $\Omega^{2,0}_{0,2}$ is much larger than the losses, where an almost complete energy conversion occurs. In Figure 2a larger losses are considered, showing that a non-negligible photon rate is still measurable out of the cavity.

During the last year, such a new theoretical proposal for mechanical DCE has driven particular attention. In particular, in Ref. [23] the authors showed how to exploit a generalized version of DCE, in an interacting tripartite system consisting of two mechanical mirrors coupled to the same optical mode, where an excitation of a mirror can be transferred to the other involving virtual photon processes.

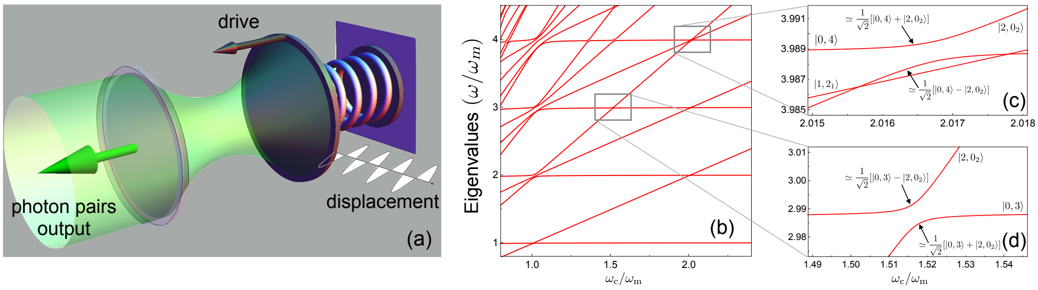

Figure 1. (**a**) Schematic picture of optomechanical system: the mechanical motion of the mirror induces the generation of Casimir photons that are detectable outside the optical cavity. (**b**) The lowest energy levels of H_{sys} as a function of ω_c/ω_m for fixed $G/\omega_m = 0.1$. (**c**,**d**) show the avoided level crossings due to the hybridization of zero- and two-photon states with mechanical states.

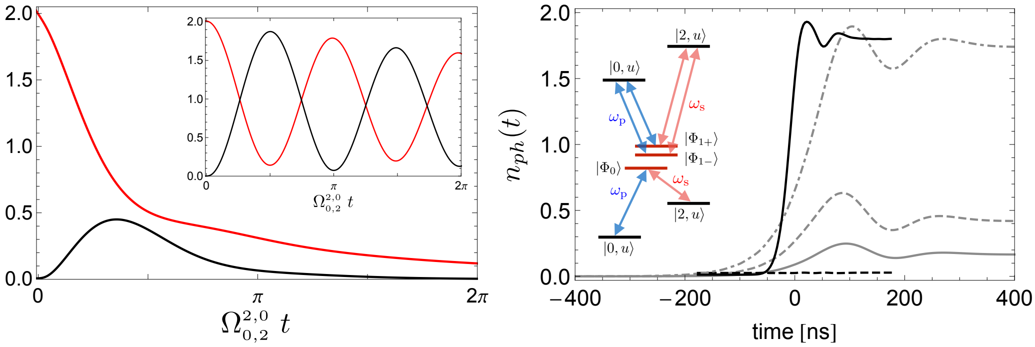

Figure 2. Left panel. Evolution of phonons (red curves) and photons (black curves) for an optomechanical system prepared in the mechanical Fock state $|0,2\rangle$, for matching energy splitting, $\omega_c \simeq \omega_m$, in the weak-coupling regime with an oscillator loss rate $\gamma = 0.2 \times \Omega_{0,2}^{2,0}$, and a cavity loss rate $\kappa = 2.5\gamma$. Here $\Omega_{0,2}^{2,0}$ the energy splitting of levels $|0,2\rangle$ and $|2,0\rangle$ calculated for $G/\omega_m = 0.1$. The inset shows the same dynamics for $\gamma = \kappa = \Omega_{0,2}^{2,0}/80$, while the coherent exchange of excitations is almost complete. Right panel. Average photon number $n_{ph}(t)$ of STIRAP protocol for Vee (black curves) and Λ (gray curves) scheme. Thick curve represents $n_{ph}(t)$ for the Rabi Hamiltonian including the stray couplings, while dashed lines refer to the case where only the stray RWA coupling is present. The dot dashed gray line refers to the Λ scheme in the absence of stray couplings. The level schemes for STIRAP are shown in the inset: the Stokes (frequency ω_s, red arrows) and the pump (ω_p, blue arrows) fields couple factorized eigenstates $|nu\rangle$ of H_R with selected dressed eigentates. In the Λ scheme they are coupled to the dressed ground state $|\Phi_0\rangle$ (lower energy levels) while in the Vee scheme with the excited states $|\Phi_{1\pm}\rangle$ (larger energy levels).

3. Generation of Two-Photon Pairs by STIRAP

Stimulated Raman Adiabatic Passage (STIRAP) is a coherent protocol that in its simplest version, transfers population between two quantum states along a dark state of a three-level atom, enforced by shining two light pulses in a *counterintuitive* sequence [26]. STIRAP is known to implement faithful complete and robust population transfer. STIRAP in superconducting nanocrcuits has been studied theoretically [27–29] and experimentally [30,31] since it can be used for new type of quantum gates [32] possibly resilient to solid state quantum noise [33–38]. In the USC scenario STIRAP can be used to detect virtual photons in the dressed eigenstates of the system, by coherently amplifying their conversion to real photons [16,24,25]. We consider a three-level atom (basis $\{|u\rangle, |g\rangle, |e\rangle\}$) ultrastrongly coupled to a single light mode of frequency ω_c, resonant with the eg transition energy ε, described by the Hamiltonian

$$H_R = \varepsilon\,|e\rangle\langle e| + \varepsilon_u\,|u\rangle\langle u| + \omega_c\, a^\dagger a + g(a + a^\dagger)(|e\rangle\langle g| + |g\rangle\langle e|) \tag{2}$$

where ε_u is the splitting between levels $u-g$ respectively, and g is the coupling between the mode and the $e-g$ transition. Eigenstates of H_R are partitioned in two sets: (1) factorized states $|nu\rangle$, where $|n\rangle$ is a Fock state, with energy $\varepsilon_u + n\omega_c$; (2) eigenstates of the two-level Rabi model $|\Phi_j\rangle$ with eigenvalue E_j. Due to USC these latter are strongly entangled atom-mode states, dressed by by virtual photon pairs. A unique signature of USC in H_R is the fact that the presence of virtual photon pairs allow in principle to transfer population from $|0u\rangle$ to $|2u\rangle$ [12]. Coherent amplification of this channel is obtained by using STIRAP. We first consider the Λ configuration, where $\varepsilon_u < 0$ (see the inset of the right panel of Figure 2). The atom is prepared in $|0u\rangle$ and driven by an electric field yieldind $H_c(t) = W(t)\,[|u\rangle\langle g| + |g\rangle\langle u| + (1/\eta)(|e\rangle\langle g| + |g\rangle\langle e|)]$, where η is the ratio between atomic dipole matrix elements and $W(t) = \mathscr{W}_s(t)\cos(\omega_s t) + \mathscr{W}_p(t)\cos(\omega_p t)$ is a two-tone signal. STIRAP is obtained by taking $\omega_p \approx E_0 - \varepsilon_u$, $\omega_s \approx \omega_p - 2\omega_c$ and the envelopes $\mathscr{W}_{s/p}(t) = \mathscr{W}_{s,p}\exp[-(t \pm \tau)^2/T^2]$ where $\mathscr{W}_{s/p}$ is the Stokes/pump field amplitude, τ is the delay, T is width of the pulses [16,25]. It may yield $\sim$ 100% coherent population transfer $|0u\rangle \to |2u\rangle$ iff $\langle\Phi_0|H_c|nu\rangle \neq 0$, i.e., the Rabi ground state contains pairs of virtual photons. The evolution of the average number of photons $n_{ph}(t)$ in this case is shown in Figure 2 (right panel, dot-dashed gray curve). Calculations are carried out for $g = 0.25\omega_c$, $\epsilon' = 4\epsilon$ and $\epsilon = \omega_c$. Field amplitudes $\mathscr{W}_s = 0.1\,\omega_c$ and $\mathscr{W}_p$ are chosen such that they yield the same Rabi frequencies $\Omega_p = \Omega_s$ for the relevant transitions, large enough to guarantee adiabaticity, $\mathscr{W}_s T = 900$, with delay $\tau = 0.6T$. Absolute times reported in the figure correspond to $\omega_c = 6\,\text{GHz}$.

Implementation of the protocol in artificial atoms faces the major problem that all the atomic transitions are coupled to the mode. We focus on superconducting artificial atoms where the stray coupling is accounted for by the extra term $H^{\Lambda}_{stray} = \eta g(a + a^\dagger)(|g\rangle\langle u| + |u\rangle\langle g|)$ [16]. The effect is significant since a second channel for population transfer $|0u\rangle \to |2u\rangle$ opens. The two channels interfere destructively, lowering the efficiency, as shown in Figure 2 (right panel, thick gray curve) for $\eta = 1$. Since the stray coupling alone in the RWA may yield the desired population transfer (Figure 2, right panel, dashed gray curve) measuring two photons in the cavity is anymore a smoking gun for USC, unless the stray channel is weak enough not to guarantee adiabaticity in STIRAP. This requires large anharmonicities, $|\varepsilon_u| \gg \varepsilon$, *and* small stray coupling, $\eta \ll 1$, conditions which are not met in available devices.

This problem is circumvented by using the Vee field configuration, where $\varepsilon_u > 0$ (see the inset of the right panel of Figure 2), with intermediate states $|\Phi_{1\pm}\rangle$ (the first excited doublet of the Rabi model). The atom is driven by a two-tone signal $W(t)$ with $\omega_p \approx \varepsilon_u - E_{1\pm}$ and $\omega_s \approx \omega_p + 2\omega_c$. Again STIRAP occurs only if $\langle\Phi_{1\pm}|H_c|2u\rangle$ is large enough, being nonzero only if the intermediate states contain virtual photons. Figure 2 (right panel, thick black curve) shows how the two-photon state populates under Vee STIRAP. Differently than before, the Vee configuration allows to cancel the effect of the stray coupling with available hardware, for instance flux qubits, as shown in Figure 2 (right panel, dashed black curve). Therefore Vee STIRAP guarantees that we finally observe in the cavity two virtual photons converted in real, i.e., a smoking gun of USC.

Curves for Vee STIRAP in Figure 2 are obtained for $g = 0.25$, $\varepsilon_u = 2.5\,\varepsilon$ and $\eta = 1$ showing that suppression of the stray channel occurs for much softer spectral constraints than for Λ STIRAP. This remarkable property is due to the fact that the stray pattern for population transfer does not contribute to $\langle\Phi_{1\pm}|H_c|2u\rangle$ since transfer via RWA couplings only may in principle occur via a different intermediate state which is largely detuned and so weak to spoil adiabaticity and yield practically zero final population of the target state. This has the important practical consequence that the protocol requires external fields with frequencies not larger than $\sim 10\,\text{GHz}$. Moreover since the two-photon components $\langle 2u|\Phi_{1\pm}\rangle$ of the dressed intermediate states are larger, the protocol in the Vee scheme is much faster. Indeed black curves in the right panel of Figure 2 are obtained for $\mathscr{W}_s T = 111$. Therefore the Vee protocol is less sensitive to decoherence and/or it may be operated with smaller field amplitudes.

4. Conclusions

In conclusion we have shown that Vee STIRAP yields the unambiguous signature of USC in available architectures of superconducting artificial atoms, via detection of two-photons by coherently amplified population transfer. On the contrary the usual Λ scheme for STIRAP is sensitive to the presence of stray coupling, which may allow population transfer also in RWA, unless stringent spectral contraints are met, which is not true for superconducting devices. Observing also that population transfer requires less stringent conditions on the spectrum and on the external fields, we conclude that demonstration of dynamical detection of USC by Vee STIRAP is well feasible in available superconducting hardware.

Conflicts of Interest: The authors declare no conflict of interest.

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