

Dissipative Synthesis of Mechanical Fock-Like States [†]

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Abstract: The observation of genuine quantum features of nano-mechanical motion is a key goal for both fundamental and applied quantum science. To this end, a promising approach is the stabilization of nonclassical features in the presence of dissipation, by means of the tunable coupling with a photonic environment. Here we present a scheme that combines dissipative squeezing with a mechanical nonlinearity to stabilize arbitrary approximations of (displaced) mechanical Fock state of any number. We consider an optomechanical system driven by three control lasers—at the cavity resonance and at the two mechanical sidebands—that couple the amplitude of the cavity field to the resonator's position and position squared. When the amplitude of the resonant drive is tuned to some specific values, the mechanical steady state is found in a (displaced) superposition of a finite number of Fock states, which for large enough squeezing achieves near-unit fidelity with a (displaced) Fock state of any desired number.

Keywords: quantum optomechanics; reservoir engineering; dissipative state preparation; non-Gaussian states

1. Introduction

The motional state of atomic or mechanical degrees of freedom can be manipulated via the interaction with the electromagnetic field confined in a cavity. Such a possibility is best illustrated by cavity cooling, which has been successfully applied to single atoms [1], ions [2], and micro- and nano-mechanical resonators [3–5]. Recent breakthroughs in the dissipative preparation of mechanical squeezed states [6–9], where a cavity-assisted scheme is designed to *cool* the target system directly into a squeezed state of motion, can be thought of as a powerful development of this paradigm [10–14]. However, for many applications, ranging from fundamental tests of quantum mechanics to quantum information processing, the stabilization of highly pure states with non-Gaussian features is needed instead. In cavity optomechanics, the quadratic optomechanical coupling has been exploited for the dissipative preparation of Schrödinger cat states [15,16], but the existence of multiple steady states requires the unpractical initialization of the system in a state of definite parity. Recently we have shown that a tunable optomechanical coupling which has both a linear and quadratic component enables the stabilization of pure non-Gaussian states without requiring any initialization [17,18]. For specific values of the amplitude of the laser drives new families of nonclassical states can be stabilized, which correspond to (squeezed and displaced) superpositions of a finite number of Fock

states. Here we focus on a specific instance, namely on one such (displaced) finite superposition that approximates—in principle with arbitrary fidelity—any number state in the harmonic ladder (modulo a displacement).

2. Results

We consider an optomechanical system where the frequency of a cavity mode parametrically couples to the displacement and squared displacement of a mechanical resonator. The Hamiltonian is given by (we set $\hbar = 1$ throughout)

$$\hat{H} = \omega_c \hat{a}^\dagger \hat{a} + \omega_m \hat{b}^\dagger \hat{b} - g_0^{(1)} \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger) - g_0^{(2)} \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger)^2 + \hat{H}_{\text{drive}}, \quad (1)$$

where \hat{a} (\hat{b}) is the annihilation operator of the cavity (mechanical) mode of frequency ω_c (ω_m) and $g_0^{(1)}$, $g_0^{(2)}$ respectively quantifies the linear and quadratic single-photon coupling. Such linear-and-quadratic coupling can be realized in membrane-in-the-middle setups [19–21], cold atoms [22], microdisk resonators [23] and photonic crystal cavities [24,25]. The cavity has a decay rate κ and is driven with three tones

$$\hat{H}_{\text{drive}} = \hat{a}^\dagger (\varepsilon_- e^{-i\omega_- t} + \varepsilon_0 e^{-i\omega_0 t} + \varepsilon_+ e^{-i\omega_+ t}) + \text{H.c.}, \quad (2)$$

applied on the cavity resonance ($\omega_0 = \omega_c$), and on the lower and upper mechanical sideband ($\omega_\pm = \omega_c \pm \omega_m$). After standard linearization (we dub \hat{d} the fluctuation operator of the cavity field), moving into a frame rotating with the free cavity and mechanical Hamiltonian, and focusing on the good cavity limit ($\kappa \ll \omega_m$) we get

$$\hat{H}_{\text{RWA}} = -\hat{d}^\dagger (G_- \hat{b} + G_+ \hat{b}^\dagger + G_0 \{\hat{b}, \hat{b}^\dagger\}) + \text{H.c.}, \quad (3)$$

where we set $G_\pm = g_0^{(1)} \alpha_\pm$, $G_0 = g_0^{(2)} \alpha_0$, and $\alpha_{\pm,0}$ are the steady values of the cavity amplitude at each frequency component; we will assume these couplings to be real and positive without loss of generality. After a transient time the cavity field is found in the vacuum while the mechanical resonator in a pure state $|\varphi\rangle$ that satisfies the condition

$$(G_- \hat{b} + G_+ \hat{b}^\dagger + G_0 \{\hat{b}, \hat{b}^\dagger\}) |\varphi\rangle = 0. \quad (4)$$

Note that when the nonlinear term is absent, namely $G_0 \equiv 0$, we recover dissipative squeezing with a squeezing degree $r = \tanh^{-1}(G_+/G_-)$ [11].

In order to characterize the steady state $|\varphi\rangle$, let us first assume that the amplitudes at the two mechanical sidebands are equal, i.e., $G_\pm = G$. In this case it is enough to notice that for the following values of the resonant coupling

$$G_0 = \frac{G}{\sqrt{2(2n+1)}}, \quad (5)$$

the condition expressed in Equation (4) becomes

$$\hat{b}^\dagger \hat{b} \hat{D} \left(\sqrt{n + \frac{1}{2}} \right) |\varphi\rangle = n \hat{D} \left(\sqrt{n + \frac{1}{2}} \right) |\varphi\rangle, \quad (6)$$

where \hat{D} is the displacement operator and $n \in \mathbb{N}$ a non-negative integer (to stress this dependence we set $|\varphi_n\rangle \equiv |\varphi\rangle$ from now on). This is in turn equivalent to

$$|\varphi_n\rangle = \hat{D} \left(-\sqrt{n + \frac{1}{2}} \right) |n\rangle \quad (7)$$

and proves that the steady state is indeed a displaced Fock state. In particular, by tuning the amplitude of the resonant drive in Equation (5) any state in the Fock state ladder can be stabilized.

The class of steady states obtained in Equation (7) turns out to be unstable [18]. However, it can be seen as the limit $G_+ \rightarrow G_-$ of the more general case $G_+ \neq G_-$ with

$$G_0 = \sqrt{\frac{G_+ G_-}{2(2n+1)}} , \quad (8)$$

which is guaranteed to be stable as long as $G_+ < G_-$. In order to find the new steady state, we can project Equation (4) onto the position eigenstate $|q\rangle$ and obtain a differential equation for the associated wave function $\varphi_n(q)$. The solution of such equation reads

$$\varphi_n(q) \propto e^{-q\sqrt{\zeta(1+2n)}} e^{-\frac{q^2}{2}} H_n \left(q + \frac{(1+\zeta)\sqrt{\zeta(1+2n)}}{2\zeta} \right) , \quad (9)$$

where we set $\zeta = \tanh r \in [0, 1)$. Note that the integer order of the Hermite polynomial is determined by the resonant coupling in Equation (8). By completing the square in the exponent we get

$$\varphi_n(q) \propto e^{-\frac{1}{2}(q-\xi_n)^2} H_n \left(q - \xi_n + \frac{(1-\zeta)\sqrt{\zeta(1+2n)}}{2\zeta} \right) , \quad (10)$$

where $\xi_n = -\sqrt{\zeta(1+2n)}$. Note that for $\zeta \rightarrow 1$ we correctly recover the wave function of a displaced quantum harmonic oscillator. We now exploit the following property of the Hermite polynomials, $H_n(x+y) = \sum_{k=0}^n \binom{n}{k} H_k(x) (2y)^{n-k}$, which leads us to

$$\varphi_n(q) \propto \sum_{k=0}^n \binom{n}{k} c_n^{-k} e^{-\frac{1}{2}(q-\xi_n)^2} H_k(q - \xi_n) , \quad (11)$$

with $c_n = -\frac{(1-\zeta)}{4\zeta} \xi_n$. From the last line we can finally read the explicit expression of the state

$$|\varphi_n\rangle = \mathcal{N}_n \hat{D}(\xi_n/\sqrt{2}) \sum_{k=0}^n \binom{n}{k} c_n^{-k} |k\rangle , \quad (12)$$

where the normalization factor is given by $\mathcal{N}_n = [{}_2F_1(-n, -n; 1; c_n^{-2})]^{-1/2}$. The steady state is now given by the action of a n -dependent displacement on a superposition of a finite number $(n+1)$ of elements. It is easily checked that in the limit $\zeta \rightarrow 1$ the superposition collapses to the single element of Equation (7). On the other hand, for any non-zero value of the squeezing parameter the state $|\varphi_n\rangle$ displays negativity in the Wigner distribution and the larger the amount squeezing the closer the resemblance with a Fock state. This feature is clear from Figure 1, where we show the Wigner distribution for a given n ($n=5$) and different values of the squeezing parameter ζ . We clearly see that the distribution, which for lower values of ζ is skewed toward one side, progressively straightens to approach that of a Fock state. We can thus think of $|\varphi_n\rangle$ as a state that approximates any given displaced Fock state, to an extent that improves with the amount of squeezing available. Mechanical dissipation—not considered here—sets a limit on the precision of such approximation. Yet, one can show that it is still possible to approximate with near-unit fidelity any Fock state [18].

Coming back to Equation (4), we notice that $|\varphi_n\rangle$ is the state *uniquely* annihilated by the nonlinear operator

$$\hat{f} = \mathcal{G} \hat{\beta} + \sqrt{\frac{\cosh r \sinh r}{2(2n+1)}} \{\hat{b}^\dagger, \hat{b}\} , \quad (13)$$

where $\hat{\beta} = \cosh r \hat{b} + \sinh r \hat{b}^\dagger$ is a Bogoliubov mode and $\mathcal{G} = \sqrt{G_-^2 - G_+^2}$. The nonlinear contribution added to the Bogoliubov transformation makes the nature of \hat{f} non bosonic.

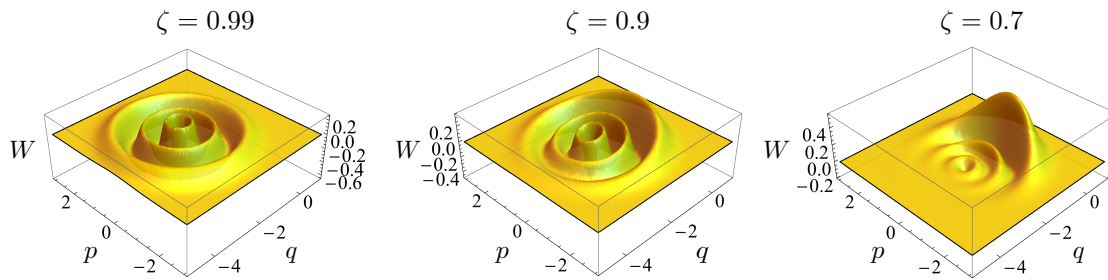


Figure 1. Wigner function $W(q, p) = \frac{1}{\pi} \int_{\mathbb{R}} dy e^{2ipy} \varphi_5(q+y)^* \varphi_5(q-y)$ of the state $|\varphi_5\rangle$ for different values $\zeta = 0.99$ (left), $\zeta = 0.9$ (centre), $\zeta = 0.7$ (right).

3. Discussion

We presented an exactly solvable model to augment dissipative squeezing by means of a quadratic nonlinearity. The model can be implemented in optomechanical cavity and the states stabilized by our protocol approximate displaced multi-phonon Fock state of any desired number.

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