



Article

Application of Fractional Fourier Transform and BP Neural Network in Prediction of Tumor Benignity and Malignancy

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Abstract: To address the limitations of traditional tumor diagnostic methods in image feature extraction and model generalization, this study innovatively proposes a synergistic diagnostic model that integrates fractional Fourier transform (FrFT) and error back-propagation (BP) neural networks. The model leverages the time–frequency analysis capability of FrFT and incorporates the fractal characteristics observed during tumor proliferation, effectively enhancing multi-scale feature extraction and representation. Experimental results show that the proposed model achieves an accuracy of 93.177% in classifying benign and malignant tumors, outperforming the support vector machine (SVM) method. The integration of FrFT improves feature distinguishability and reduces dependence on manual extraction. This study not only represents a breakthrough in tumor diagnostic technology but also paves new avenues for the application of fractional calculus and fractal geometry in medical image analysis. The findings show great potential for clinical application and future development.

Keywords: tumor diagnosis; fractional calculus; fractal feature; fractional Fourier transform; BP neural network



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1. Introduction

1.1. Background

Tumors, especially malignant ones, remain among the major global health threats. According to the World Health Organization, in 2022, approximately 19.96 million new malignant tumor cases were reported worldwide, resulting in 9.73 million deaths [1]. Accurate prediction of tumor benignity or malignancy plays a crucial role in early detection and improving patient outcomes.

Conventional diagnostic methods such as medical imaging and biopsy are often invasive, inefficient, and prone to false positives, limiting their suitability for early screening. Computer-aided diagnosis (CAD) technologies have emerged as promising tools in medical image analysis, enhancing diagnostic efficiency and objectivity through machine learning. However, existing CAD systems still struggle to extract informative features and achieve robust classification, especially when dealing with complex, non-stationary medical signals.

Tumor images often exhibit fractal-like patterns and complex time–frequency structures that are challenging to capture using traditional Fourier transforms. The fractional Fourier transform (FrFT), by introducing a fractional-order parameter, extends classical Fourier analysis and enables more flexible and effective multi-scale feature extraction. Meanwhile, backpropagation (BP) neural networks are widely used for classification due to their powerful nonlinear modeling capability, but they are sensitive to input features and susceptible to overfitting.

To address these challenges, this study proposes an FrFT-BP hybrid model. FrFT is used to preprocess tumor images and extract fractional time–frequency features, which are then fed into a BP neural network for classification, aiming to improve both feature representation and classification performance.

1.2. Status of Research

FrFT breaks through the limitation of traditional Fourier transform in time–frequency resolution by introducing the fractional-order parameter, which is especially suitable for feature extraction of non-smooth signals. In 1980, Namias first proposed the mathematical theory of FrFT, which laid the theoretical foundation for its use in the field of signal processing [2]. In 1994, Almeida verified the advantages of FrFT in the time–frequency analysis of biological signals [3]; in 2019, Zhan Hongfeng et al. extended the feature domain with FrFT to extract as much valid information as possible from different dimensions, which significantly improved the correct recognition rate of classification results [4]. However, most of the existing studies focus on the processing of single-modal data and lack an efficient synergy mechanism with deep learning models.

BP neural networks are widely used in medical classification tasks due to their powerful nonlinear fitting ability. In 1986, the backpropagation algorithm proposed by Rumelhart et al. laid the foundation for the development of BP networks [5]. In 2023, Ge Mengfei et al. combined a neural network with the Adaboost method to increase the accuracy of predictive classification of breast cancer to 95.55% [6]. However, the high sensitivity of traditional BP neural networks to input features and their reliance on manually designed feature extraction methods limit their performance in complex medical imaging.

1.3. Research Objectives, Hypotheses, and Innovativeness

The rationale of this study lies in combining FrFT with BP neural networks for tumor classification. FrFT enables the extraction of fine-grained, non-stationary features from tumor images, improving the quality of inputs. The BP network enhances classification with its nonlinear learning capability. The synergy reduces reliance on manual feature design, enhances generalization, and improves accuracy [7].

The innovation of this study lies in the integration of the FrFT as a preprocessing tool for input features into the BP neural network, thereby constructing a novel framework of fractional neural networks. The FrFT possesses enhanced time–frequency resolution, enabling it to capture non-stationary and complex signal characteristics that traditional Fourier transforms struggle to extract. This addresses key limitations of existing approaches, such as low feature utilization and high dependency on manual feature engineering. In particular, FrFT effectively enhances the representation of tumor-specific fractal features, leading to higher-quality inputs for the neural network. When combined with the powerful nonlinear fitting capabilities of the BP neural network, the model achieves improved generalization and stability, especially in small-sample medical data scenarios where traditional networks are prone to overfitting. This synergy not only reduces reliance on handcrafted features but also paves the way for building accurate and low-invasive intelligent diagnostic tools for tumor prediction, demonstrating both structural innovation and practical relevance.

Based on this, the research objective of this paper is to construct and validate a benign and malignant tumor prediction model integrating FrFT and a BP neural network and to improve its classification performance and generalization ability when dealing with complex medical data. Considering that medical data are characterized by high dimensionality, nonlinearity, and noise sensitivity, this paper further proposes the hypothesis that if the input data have a high signal-to-noise ratio and feature integrity after FrFT preprocessing, the constructed FrFT-BP model will significantly outperform the traditional methods in terms of classification accuracy and robustness.

1.4. Structure of the Paper

This paper adopts the research path of “theory construction–method design–experimental verification” and is organized as follows:

Section 2 systematically elaborates on the theoretical foundation of FrFT and neural networks. It includes the mathematical definition, basic properties, and unique advantages of FrFT in signal processing and analyzes the core algorithm and architectural features of BP neural networks.

Section 3 constructs the FrFT-BP collaborative diagnostic model. The composition of the experimental dataset, the FrFT preprocessing process, and the network parameter optimization strategy are explained in detail, and a hybrid training method based on the L-M algorithm is proposed.

Section 4 conducts the analysis of benign and malignant tumor prediction results and carries out comparative experimental research. Through multi-dimensional visualization tools such as a performance chart, training status chart, and regression chart, the model’s prediction performance on clinical data is analyzed and compared with traditional SVM methods.

Section 5 summarizes the research results, argues for the theoretical contribution of FrFT in enhancing feature differentiability, and explores the prospect of the application of the fusion technique of fractional calculus and deep learning in the field of smart healthcare.

2. Theoretical Foundation

2.1. Fractional Fourier Transform

2.1.1. Definition of FrFT

The FrFT is a tool for transforming a signal from the time (or spatial) domain to the frequency domain. Unlike the classical Fourier transform, the FrFT not only involves the frequency domain, but also allows the “order” of the transform angle to be adjusted α in order to realize different levels of signal transformation. This gives the FrFT a unique advantage in signal processing applications, especially in non-local characterization, where the FrFT is able to retain more information about the signal [8].

For a one-dimensional signal $x(t)$, FrFT is defined as follows [9]:

$$X_p(\mu) = F_p[X(t)] = \int_{-\infty}^{+\infty} K_p(\mu, t)x(t) dt, \quad (1)$$

where $K_p(\mu, t) = A_p e^{i\mu^2 \cot \alpha - it^2 \csc^2 \alpha}$ is called the kernel function of the fractional Fourier. $A_p = \sqrt{1 - j \cot \alpha}$, α is the counterclockwise rotation angle of the fractional-order Fourier domain with respect to the time domain, and the corresponding fractional order is $P = 2\alpha/\pi$. When $p = 4n$, $K_p(\mu, t) = \delta(\mu - t)$; when $p = 4n + 2$, $K_p(\mu, t) = \delta(\mu + t)$. The fractional Fourier transform kernel is denoted as follows.

$$K_p(\mu, t) = \begin{cases} \sqrt{\frac{1-j \cot \alpha}{2\pi}} \exp\left(j\frac{\mu^2+t^2}{2} \cot \alpha - j\frac{\mu t}{\sin \alpha}\right), & \alpha \neq n\pi, \\ \delta(\mu - t), & \alpha = 2n\pi, \\ \delta(\mu + t), & \alpha = (2n + 1)\pi. \end{cases} \quad (2)$$

The inverse of the fractional order Fourier is

$$x(t) = \int_{-\infty}^{+\infty} K_p(\mu, t)X_p(\mu)d\mu. \quad (3)$$

When the FrFT of the transform of $p = 1$ and when $\alpha = \pi/2$, from Equation (1),

$$x_1(u) = \int_{-\infty}^{+\infty} e^{-j2\pi ut} x(t)dt. \quad (4)$$

From Equation (4), the FrFT is the ordinary Fourier transform when $p = 1$. Therefore, the FrFT can also be seen as an extension of the conventional Fourier transform [10].

2.1.2. Basic Properties

- **Linear properties:** FrFTs have the characteristics of linear transforms and can be combined—that is,

$$F^p \left[\sum_n c_n f_n(u) \right] = \sum_n c_n [F^p f_n(u)].$$

- **Order additivity (rotational additivity):**

$$F^{p_1} F^{p_2} = F^{p_1 + p_2}.$$

- **Reversible nature**

$$(F^p)^{-1} = F^{-p}.$$

According to the rotational additivity, it is known that after performing an FrFT with the transformed angle $\alpha = p\pi/2$, its inverse transformation is equivalent to performing an FrFT with the angle $-\alpha$. This property implies that if a signal's time–frequency domain has been rotated by a specific angle through the FrFT, the restoration of the signal's original state can be easily achieved by performing an FrFT with the corresponding negative angle on the signal [11].

- **Nature of exchange**

$$F^{p_1} F^{p_2} = F^{p_2} F^{p_1}.$$

- **Combining properties**

$$(F^{p_1} F^{p_2}) F^{p_3} = F^{p_1} (F^{p_2} F^{p_3}).$$

- **Time shift characteristic**

$$F^p [x(t - \nu)] = X_p(\mu - \nu \cos \alpha) \exp \left(j \frac{\nu^2}{2} \sin \alpha \cos \alpha - j \nu \mu \sin \alpha \right).$$

indicates the frequency shift of the signal.

- **Frequency shift characteristics**

$$F^p [x(t) e^{j\sigma t}] = X_p(\mu - \sigma \cos \alpha) \exp \left(-i \frac{\sigma^2}{2} \sin \alpha \cos \alpha - i \mu \sigma \sin \alpha \right),$$

σ indicates the frequency shift of the signal.

2.1.3. Application of FrFT in Data Processing

Fractional neural networks can be categorized into two types in terms of structural form: the first type combines FrFT with feedforward neural networks, where the activation function of the nodes in the hidden layer is replaced by the fractional-order kernel function, and the weight from the input layer to the hidden layer is replaced by the fractional-order parameter. This method searches for the optimal order parameter through network training and gradient correction iterations, and performs data prediction at that optimal order. The second category is the use of FrFT as a preprocessing tool for neural network inputs, which is suitable for data characterized by Chirp-like signals. The Chirp-like signals are analyzed by fractional Fourier Transform to extract their unique time–frequency features, and the neural network performs prediction, function approximation, or classification identification based on these feature parameters. The optimal fractional neural network proposed in this

paper belongs to this category. In this paper, FrFT is used as a data preprocessing tool, aiming to improve the non-local characteristics of the data and thus enhance the learning ability of the neural network on the data. By adjusting the value of the order parameter α of FrFT, the input features can be optimized so that the neural network can better extract the key information from the data, and thus improve the accuracy of benign and malignant tumor prediction [12].

2.2. Neural Network Theory

2.2.1. Overview of Neural Networks

Neural networks are a kind of machine learning model inspired by biological nervous systems, which consist of a large number of artificial neurons, and realize complex function approximation through multi-layer nonlinear transformations, which makes them show their unique value in medical pattern recognition tasks, especially in the problem of tumor benignity and malignancy discrimination, which is characterized by high dimensionality, nonlinearities, and small samples.

2.2.2. The BP Neural Network

The BP neural network is a typical multilayer feedforward neural network composed of input, hidden, and output layers, which is trained through four steps: forward propagation, computation of error, back-propagation, and iterative training. The BP neural network has a strong nonlinear fitting capability [13]. In this study, the BP neural network is selected as the baseline model due to its adaptability to medical prediction problems, particularly in capturing complex nonlinear relationships in structured, non-sequential data and in offering stable gradient-based optimization [14].

Compared with other neural network architectures such as Convolutional Neural Networks (CNNs) and Recurrent Neural Networks (RNNs), the BP network presents several advantages in the context of this research. CNNs are primarily designed for image-related tasks and rely on local spatial features for convolution operations. However, the input features in this study are extracted numerical descriptors of tumors with no inherent spatial correlation, making CNNs structurally mismatched. RNNs, on the other hand, are designed for sequential or temporal data modeling, such as speech or text. Since the input data in this study consist of independent samples without temporal dependency, RNNs are not suitable for this scenario either.

In contrast, BP neural networks impose minimal assumptions on input data structure and are capable of learning high-order nonlinear mappings through multilayer nonlinear transformations in the hidden layers. Their universal approximation capability has been theoretically validated by the Universal Approximation Theorem. Furthermore, BP networks leverage the chain rule for efficient gradient computation and can be optimized using second-order algorithms such as the Levenberg–Marquardt method, ensuring rapid convergence. In this study, although the FrFT-based preprocessing significantly increases the dimensionality of the input features, the BP network maintains efficient training and convergence performance, with computational complexity remaining within clinically acceptable limits. Therefore, considering structural compatibility, computational efficiency, and generalization performance, the BP neural network is a rational and effective model choice for this specific application.

3. Application of Fractional-Order Calculus Theory to Neural Networks

3.1. Experimental Data

The data for this study were obtained from a clinical dataset publicly released by the Northwest University Mathematical Modeling Competition Organizing Committee in

2023, which consisted of data with nine characteristics, with each cancer case assigned one tumor benign and malignant label provided by a hospital. The dataset contains a total of 608 case samples, including 389 benign and 219 malignant cases, and the image data are anonymized.

3.2. Experimental Methodology

In this paper, we propose a fusion prediction model based on FrFT and the BP neural network, aiming to enhance the neural network's ability to characterize the tumor data through the non-local characteristics of fractional-order calculus [15]. The method uses FrFT to preprocess the original tumor data with a time–frequency domain transform and realizes multi-dimensional signal analysis by adjusting the order in the FrFT, which effectively overcomes the problem of limited generalization performance of traditional neural networks due to local feature extraction [16].

Specifically, we first preprocess the tumor data using FrFT, which effectively enhances the non-local characteristics of the data and extracts unique time–frequency features by adjusting the transform order. The preprocessed data are converted into real-valued features, which are used as inputs to the BP neural network. The output layer of the BP neural network is responsible for predicting the benignness or malignancy of the tumor. During the training process, we use the Levenberg–Marquardt algorithm to optimize the weights and biases of the network to minimize the prediction error. The L-M algorithm is a second-order optimization method that improves the speed of convergence during the learning process by combining the advantages of the gradient descent method and the Newton method [17]. Its update formula is $\Delta\omega = (J^T J + \mu I)^{-1} J^T e$, where J is the Jacobi matrix of error pair weights, μ is the damping factor, and e is the error vector. The L-M algorithm balances convergence speed and stability by dynamically adjusting μ .

In order to validate the prediction performance of the model, we used a comprehensive assessment of the sum of squared errors (SSE), mean square error (MSE), and root mean square error (RMSE).

The Figure 1 shows the structure of a fractional BP neural network, which is detailed as follows:

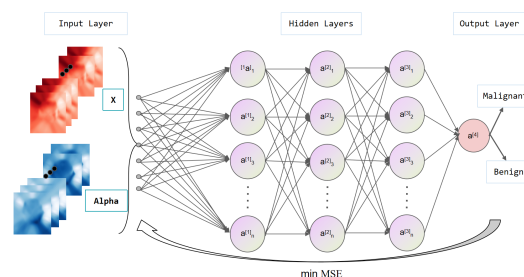


Figure 1. Structure of fractional BP neural network.

(A) Input Layer: The annotation X represents medical instance data input, and the annotation α represents the FrFT parameters.

(B) Hidden Layers: Multiple fully connected layers with node notation $a_i^{[l]}$ (l : layer index, i : node index).

(C) Output Layer: Binary classification (Malignant/Benign) with MSE optimization target.

3.3. Experimental Design

(1) Data processing and FrFT preprocessing

First, this paper uses a dataset in Excel table format containing nine features and a label for tumor benignity and malignancy. The data are transformed by FrFT and the output is a

complex-valued result, and we take its magnitude as the final feature data and keep it as a real value. With this treatment, the non-local characteristics of the signal are preserved [18].

(2) Data normalization and training set and test set division

In order to eliminate the scale differences between features and ensure that the training of the model is not affected by the data magnitude, the input data are normalized in [19].

In terms of dataset division, random assignment was used to divide the dataset into training and test sets, where the training set accounted for 80% of the total data and the test set accounted for 20%.

(3) Selection of optimal hidden layer nodes

Too few nodes in the hidden layer will limit the network's ability to learn complex nonlinear relationships, leading to underfitting, while too many nodes will easily trigger the phenomenon of overfitting, reducing the model's ability to generalize on new samples. In order to ensure the training effect of a BP neural network, this study adopts the method of cross-validation to select the optimal number of nodes in the hidden layer. The specific steps are as follows:

Step 1: Set different number of implicit layer nodes (10, 20, 30, 40) for experimentation.

Step 2: Train the network 10 times for each hidden layer node and record the test set mean square error (MSE), that is,

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2,$$

where y_i is the true value, $\hat{y}_i$ is the predicted value, and N is the sample size.

Step 3: Select the number of hidden layer nodes with minimum MSE as the final network structure.

(4) Network training and prediction

Network training is optimized using the Levenberg–Marquardt (L-M) algorithm. The specific process is as follows:

Step 1: Initialize the weights ω and bias b , and set the initial damping factor $\mu = 0.01$ and the maximum number of iterations $T = 2000$.

Step 2: Forward propagation calculates the output error e and constructs the Jacobian matrix J .

Step 3: Solve the linear equation, that is,

$$\Delta\omega = \left(J^T J + \mu I \right)^T J^T e,$$

and update the weights, that is,

$$\omega_{k+1} = \omega_k + \Delta\omega.$$

Step 4: If the error decreases after the update, accept the update and set $\mu = \mu/10$; otherwise, reject the update and order $\mu = \mu \times 10$.

Termination condition: The maximum number of iterations is reached or the error is less than the threshold $\varepsilon = 10^{-7}$.

After the training is completed, predictions are made using the test set data and compared with the actual labels to evaluate the performance of the model.

Remark 1. If μ is too small, the training MSE will converge fast but the test MSE will be elevated, causing the algorithm to fall into local minima. If μ is too large, the training convergence speed

will be significantly reduced and the test MSE will be elevated, leading to over-regularization and damaging the model capacity. Therefore, it is important to calculate the appropriate initial μ value, set the initial value of μ as [0.001, 0.01, 0.1, 1, 10], and record the training MSE convergence curve with the test MSE (as shown in Figure 2). Based on the convergence curve analysis, taking into account MSE and number of rounds, we chose $\mu = 0.01$ as it presents the optimal balance between training efficiency and stability, satisfying the dual constraints of medical AI models on training efficiency and computing resources.

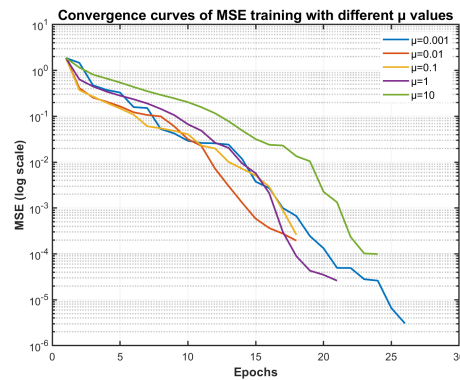


Figure 2. Convergence curves of MSE training with different μ values.

(5) Error analysis and performance evaluation

After the model prediction results are obtained, this paper provides a comprehensive assessment of the model's prediction performance using a variety of error metrics, including the sum of squared errors (SSE), mean absolute error (MAE), mean square error (MSE), root mean square error (RMSE), and mean absolute percentage error (MAPE).

SSE and MSE measure the sum of the squares of the deviations of the predicted values from the true values, which can reflect the overall error level; MAE provides an average assessment of the absolute values of the errors, which is a more intuitive description of the average prediction bias; RMSE places more emphasis on the large error values based on the MAE, which is more sensitive to the fluctuation of the errors; and MAPE is used to assess the relative percentage of the errors, which facilitates the comparison of the data with different scales.

Although the above metrics provide a multifaceted perspective for performance evaluation, there are some limitations in each metric: SSE and MSE are extremely sensitive to outliers, which may lead to underestimation of the overall performance due to a few extreme error values; MAPE produces abnormally large values when the target value is close to zero, which affects the interpretability; and RMSE magnifies the effect of large errors, which may hide the fact that the model performs well in most of the samples. Therefore, relying on a single metric may lead to a misjudgment of model performance.

To compensate for the limitations of these error metrics, this study further combines graphical analysis tools, such as a Regression Chart and a Comparison Chart of Predicted Results, to assess the model's fitting ability and prediction stability. In addition, group comparative analysis of the performance metrics of the training set, validation set, and test set are conducted in this study (i.e., Performance Chart), so as to verify the generalization ability and robustness of the model at different stages. The combination of multiple means makes the model assessment more comprehensive and reliable, ensuring the objectivity and credibility of the conclusions.

This flowchart (Figure 3) presents the key implementation steps of the FrFT-BP hybrid model:

- (1) Perform multi-scale time–frequency decomposition on the original tumor features using FrFT ($\alpha = 0.7$) (Step 1).
- (2) Using normalization to process the training set data (Step 2).
- (3) Determine the optimal number of hidden layer nodes through cross validation (Step 3).
- (4) Apply Levenberg–Marquardt second-order optimization algorithm for network training, including forward propagation $\rightarrow$ error backpropagation $\rightarrow$ Hessian matrix update (Step 4).
- (5) Evaluate the performance of the model based on seven indicators including SSE, MAE, MAPE, etc. (Step 5).

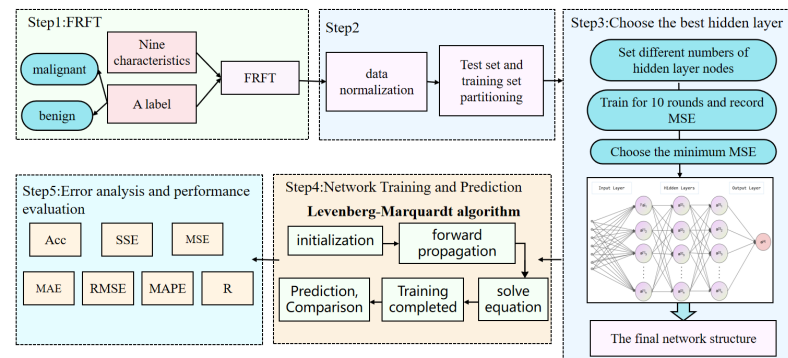


Figure 3. Flowchart of experimental design.

4. Analysis of Outcomes of Tumor Benignity and Malignancy Prediction

4.1. Implementation Details

The experiments were implemented in MATLAB R2022b using a customized function to perform the FrFT on the input data. The FrFT kernel was designed based on the time-domain definition with adjustable fractional order α . Each sample's real and imaginary parts were extracted and concatenated to form the new input features.

The BP neural network was constructed using MATLAB's `newff` function from the Neural Network Toolbox. The input data were normalized to $[0, 1]$, and the output labels were normalized using `mapminmax`. The training was conducted using the Levenberg–Marquardt algorithm (`trainlm`), with the following parameters:

- Learning rate: 0.001.
- Maximum epochs: 2000.
- Performance goal (MSE): 10^{-7} .
- Transfer functions: `tansig` for hidden layer and `purelin` for output layer.

The optimal number of hidden neurons was determined through empirical testing, starting from $\sqrt{n+m} + 1$ to $\sqrt{n+m} + 10$, where n and m denote the number of input and output neurons, respectively. The best architecture was selected based on the lowest mean squared error (MSE) on the training set.

All experiments were conducted on a personal computer with the following specifications:

- CPU: R7-5825U
- RAM: 16 GB
- Operating System: Windows 11

4.2. Evaluation of the Neural Network Training Outcomes

This paper uses three charts, the performance chart, the training state chart, and the regression chart, to demonstrate the neural network's training effect and prediction ability while training. These figures thoroughly highlight the key indicators used during the training process as well as the model's effect after training.

• Performance Chart

The performance chart (as shown in Figure 4) shows how the mean square error (MSE) varies with the number of iterations during the BP neural network's training process. The chart shows how well the network fits the training data in each cycle. The performance chart shows gradually decreasing inaccuracy, which often suggests that the network is constantly improving its fit to the training data. As the training develops, the error stabilizes, indicating that the network is close to optimal and the training process converges.

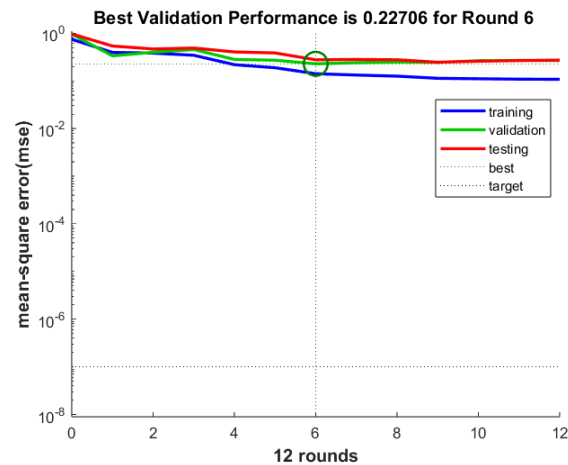


Figure 4. Performance chart.

The model's training quality can be visualized by observing the chart, where the error decreases faster and more smoothly, indicating better training results.

• Training Status Chart

The training status chart usually shows the network training state at various stages of the training process. The figure (as shown in Figure 5) shows the training dynamics of the neural network model in 10 rounds of iterations; the gradient continues to decrease, the μ exponential level decreases, and the validation check reaches the early stopping condition within a reasonable number of rounds, all of which indicate that the neural network has effectively completed the learning of the data features and successfully avoided the occurrence of the overfitting phenomenon.

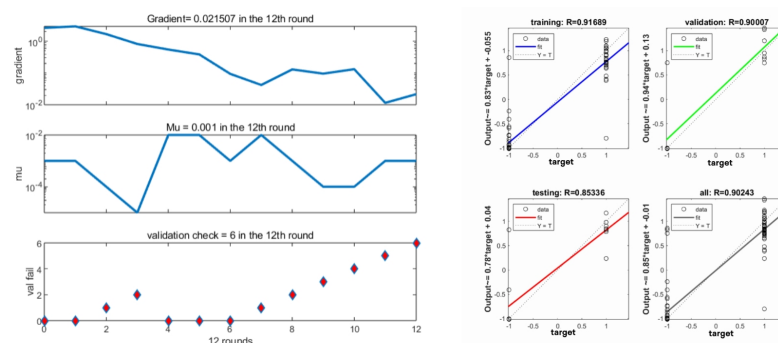


Figure 5. Training status chart (left) and regression chart (right).

• Regression Chart

The regression graph shows the relationship between the predicted values and the true values. The graph illustrates the relationship for the four processes: Training, Validation, Testing, and All.

- (1) Training set regression performance (Training: $R = 0.9806$)

The blue fitted line in the figure is very close to the ideal line ($Y = T$), with a correlation coefficient R of 0.9806. This indicates that the neural network has an extremely strong fit on the training data. The regression equation is as follows:

$$\text{Output} \approx 0.95 \times \text{target} - 0.018$$

The slope is close to 1 and the intercept is close to 0, further indicating that the deviation between the predicted values and the true values is small, and the fitting is excellent.

(2) Validation set regression performance (Validation: $R = 0.86481$)

The green fitted line deviates slightly from the ideal line, with an R value of 0.86481. Although slightly lower than the training set, it still shows a strong correlation. The regression equation is

$$\text{Output} \approx 0.8 \times \text{Target} - 0.07$$

This shows that the model still has some generalization ability on data not involved in training and does not show serious overfitting.

(3) Regression performance on the test set (Testing: $R = 0.91254$)

The fitting performance in the test set is good, with an R value of 0.91254, a slope of 0.83, and an intercept of -0.14 . This indicates that the model predicts unknown data more accurately and has good robustness.

(4) Overall regression performance (All: $R = 0.95612$)

The regression plot of the integrated dataset shows an overall correlation coefficient R of 0.95612, indicating strong overall prediction ability. The regression equation is

$$\text{Output} \approx 0.92 \times \text{Target} - 0.037$$

This is also close to the ideal line, further validating the model's good learning ability across the sample space.

All four regression plots demonstrate a high correlation between the predicted output and the actual target. The R -values of the training and overall datasets are particularly close to 1, indicating that the model accurately captures the nonlinear relationship between inputs and targets, showing strong potential for practical application.

Further comparison of the prediction results is shown in Figure 6. The left figure verifies the discriminative ability of the BP neural network in benign and malignant tumor prediction. The network output is highly consistent with the real labels on the whole, indicating that the model has good generalization ability and robustness in the binary classification task. The right panel reveals the predictive stability and robustness of the BP neural network in tumor benign and malignant classification. The model predicts the vast majority of test samples accurately with high error concentration and no systematic bias in the overall error.

With a prediction accuracy of 93.177%, the experimental results show that the model based on the combination of the FrFT and BP neural network makes more accurate predictions on the test set. The model accurately predicts whether a tumor is benign or malignant, as evidenced by the small gap between the predicted and true values in the chart of true and predicted values.

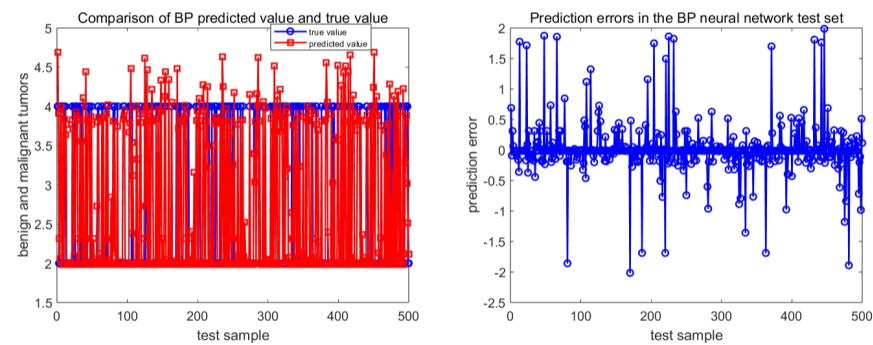


Figure 6. Comparison chart of predicted results.

4.3. Prediction Accuracy Under Different Orders

The figure (as shown in Figure 7) shows the variation in prediction accuracy of the neural network model under different orders α . The accuracy peaks at $\alpha = 0.7$ (93.177%), indicating the importance of selecting an appropriate order for optimal model performance.

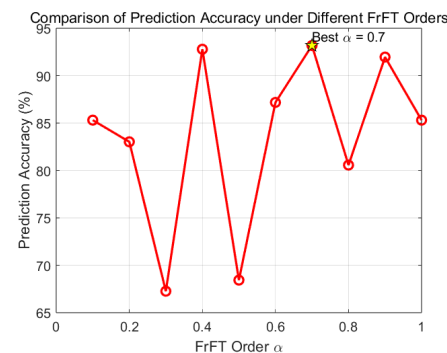


Figure 7. Comparison of prediction accuracy under different FrFT orders.

4.4. Comparative Experimental Design

Since the support vector machine (SVM) method is suitable for processing high-dimensional medical data and can classify nonlinear data through the kernel function, this paper uses it as a comparison model and conducts comparative experiments based on the same preprocessed dataset to evaluate the performance of the two methods under several evaluation metrics such as accuracy, sum of squares of errors (SSE), mean absolute error (MAE), mean square error (MSE), root mean square error (RMSE), mean percentage error (MAPE) and correlation coefficient (R) under several evaluation indexes.

The following (Table 1) are the results:

Table 1. Comparison of the results of the indicators of the two models.

INDEX	Acc	SSE	MAE	MSE	RMSE	MAPE	R
BP	93.177	83.789	0.193	0.178	0.421	6.823	0.901
SVM	64.130	132	0.717	1.435	1.198	17.935	−0.560

The experimental results show that the model based on FrFT combined with the BP neural network proposed in this paper outperforms the SVM model in all the metrics in the benign–malignant tumor prediction task.

Specifically, the model has smaller SSE and MSE, which indicates that it fits the sample data better during the training process; the MAE and RMSE are also lower, which show stronger predictive stability and robustness; the correlation coefficient R value is close to

1, which further verifies that the model prediction results are highly consistent with the real values.

In contrast, despite its good generalization ability, the performance of the SVM method is highly dependent on the selection of the kernel function and the accurate tuning of parameters when facing the complex nonlinear structure of tumor data. If the kernel function is not properly set, the model may not be able to fully explore the implicit associations between higher-order features. In contrast, the BP neural network used in this paper constructs the higher-order feature interaction structure through multi-layer nested nonlinear activation functions and combines with the frequency domain feature enhancement provided by FrFT, which enables the model to adaptively capture the multi-scale information, thus improving the ability to recognize the benignness and malignancy of tumors.

In addition, under the premise of moderate sample size, high feature dimensions, and complex nonlinear relationships among variables, BP neural networks have more modeling flexibility and expressive ability than SVMs. Therefore, the method in this paper shows better performance and adaptability in such medical prediction scenarios.

4.5. Discussion on Computational Complexity

In terms of computational complexity, the proposed FrFT-BP neural network includes a preprocessing step where the fractional Fourier transform is applied to each input sample. This step has a complexity of $O(N \cdot d)$ for N samples and d features. While this introduces additional computation compared to traditional BP or SVM models, it significantly enhances the feature representation, leading to improved learning efficiency and accuracy.

Compared with SVMs, especially those using nonlinear kernels, which typically require solving quadratic programming problems with complexity ranging from $O(N^2)$ to $O(N^3)$, our model is computationally more efficient and scalable. Additionally, SVMs often require parameter tuning and kernel selection, which further adds to the computation burden.

Overall, the FrFT-BP method achieves a favorable balance between accuracy and computational cost, making it a practical choice for large-scale classification problems.

4.6. Error Sources and Research Limitations

Although the FrFT-BP model proposed in this paper shows high classification accuracy on the current dataset, there are still some non-negligible sources of error and research limitations. First, the input data have certain fractal characteristics and non-stationarity, and although the feature extraction stage improves the signal characterization ability with the help of FrFT, the selection of fractional order is still dependent on experiments and lacks the theoretical optimality guarantee; second, there is a certain category imbalance and labeling subjectivity in the training data, which may lead to unstable prediction of the model on the boundary samples. In addition, although the overall accuracy of SVM is lower than that of the present method, it shows better generalization ability in the case of high-dimensional features, which suggests that we should pay attention to the adaptability of different models to the data structure in model selection.

From a system perspective, the model has not yet been validated cross-platform in different imaging devices and clinical scenarios, and its ability to process high-resolution images (e.g., 4K pathology slices) and computational efficiency still need to be further optimized. Meanwhile, due to the sensitivity of medical data, the regulatory constraints on data access and sharing are also a realistic bottleneck for future large-scale deployment. Therefore, despite the superior performance of the model in experiments, its generalized application still needs to be continuously improved in terms of model interpretability, real-time reasoning capability, and system integration.

5. Conclusions and Outlook

5.1. Conclusions

In this paper, a benign–malignant tumor prediction model combining FrFT and BP neural networks is proposed. The experimental results show that the FrFT can effectively enhance the non-local characteristics of the data and improve the prediction accuracy of the BP neural network. The model not only has high prediction accuracy, but also performs well in error analysis, proving its potential application in tumor prediction.

5.2. Impact on Field of Medical Image Analysis and Tumor Diagnosis

(1) Enhanced characterization of medical image features

The FrFT-BP neural network fusion model proposed in this study achieves the enhanced characterization of tumor morphological features through the time–frequency rotational property of FrFT. Experiments show that the model improves the prediction accuracy while improving its classification performance compared with the traditional integer-order Fourier transform. This method provides a new mathematical tool for mining the deep time–frequency features of tumors.

(2) Promoting real-time aided diagnosis

The FrFT-BP neural network fusion model proposed in this study effectively improves the model efficiency by reducing the number of training iterations through the second-order convergence property of the Levenberg–Marquardt algorithm. It provides a technical prototype for outpatient real-time screening and helps the intelligent transformation of domestic medical equipment.

(3) Optimization of clinical decision support system

The FrFT-BP neural network fusion model proposed in this study can be used to assist doctors in diagnosis, and at the same time, doctors can be reminded to make follow-up consultations in time for complicated cases, in order to prevent patients from missing the golden stage of early cancer treatment.

5.3. Future Prospects

- **Expand the application of medical data:** Apply existing mathematical frameworks to the analysis of pathological sections, genomic data, etc., and explore how to combine different types of medical data through fractional-order methods. At the same time, develop new mathematical tools to more accurately identify the features of tumor edges.
- **Interdisciplinary research:** Combining mathematics and artificial intelligence technology to predict the development trend of tumors is promising direction of research. By using fractional time series analysis and recurrent neural networks, a model can be constructed to predict the probability of tumor deterioration, which could help with early detection and treatment. Meanwhile, studying the efficacy of drugs in cancer treatment and exploring the relationship between tumor characteristics and drug metabolism are important directions of study.
- **Exploration of clinical application:** It is important to conduct actual testing on a certain type of tumor case to verify the stability of the model under different devices and scanning methods, ensuring its wide applicability in clinical practice. Simultaneously optimizing the algorithm to meet the fast processing requirements of 4K medical images should also be explored.

5.4. Potential Challenges in Clinical Application

While the proposed method shows promising results in classification accuracy and generalization, several challenges remain for its clinical deployment. These include the following:

- **Data Privacy:** Medical data are sensitive and subject to strict regulations, making data access and sharing a potential barrier.
- **Model Interpretability:** Deep learning models, including BP neural networks, are often criticized for their “black box” nature, which limits clinicians’ trust.
- **Scalability:** Real-time application in clinical workflows requires integration with hospital systems and optimization for inference speed.
- **Patient Rights:** The model is designed to assist physicians in decision-making or prompt follow-up examinations, rather than replacing clinical diagnoses, thereby potentially reducing the risk of misdiagnosis. Furthermore, the diagnostic report will clearly indicate that it is an “AI-assisted result”, and patients can access the model version and performance metrics through the hospital information system.

Future work will focus on addressing these issues through interpretable model designs, federated learning frameworks, and collaborations with clinical institutions.

5.5. Ethical Statement

This study is based on publicly available datasets that do not involve any personally identifiable patient information. No clinical trials or patient interventions were conducted. Any future clinical application of the proposed method will be subject to formal ethical review and institutional approval to ensure compliance with data protection and research ethics guidelines.

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Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request. The dataset utilized in this study originates from the Northwest University Mathematical Modeling Competition Organizing Committee, which has made the data publicly available through competition problems. Although the specific location of the database has not been disclosed, the relevant data can be obtained by contacting the relevant Committee. In this research, we have rigorously filtered the published dataset, excluding any unknown or ambiguous data points to ensure the accuracy and reliability of our study. This data processing procedure not only enhances the quality of the data but also provides a solid foundation for subsequent analysis.

Conflicts of Interest: The authors declare that they have no conflicts of interest.

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