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Fixed-Time Adaptive Synchronization of Fractional-Order Memristive Fuzzy Neural Networks with Time-Varying Leakage and Transmission Delays

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Abstract: Finite-time synchronization depends on the initial conditions of the system in question. However, the initial conditions of an actual system are often difficult to estimate or even unknown. Therefore, a more valuable and pressing problem is fixed-time synchronization (FTS). This paper addresses the issue of FTS for a specific class of fractional-order memristive fuzzy neural networks (FOMFNNs) that include both leakage and transmission delays. We have designed two distinct discontinuous control methodologies that account for these delays: a state feedback control scheme and a fractional-order adaptive control strategy. Leveraging differential inclusion theory and fractional-order differential inequalities, we derive several novel algebraic conditions that are independent of delay. These conditions ensure the FTS of drive–response FOMFNNs in the presence of leakage and transmission delays. Additionally, we provide an estimate for the upper bound of the settling time required to achieve FTS. Finally, to validate the feasibility and applicability of our theoretical findings, we present two numerical examples which are accompanied by simulations.

Keywords: fractional-order memristive fuzzy neural networks; fixed-time synchronization; adaptive control; leakage delays



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1. Introduction

A memristor is a kind of nonlinear resistor with a memory function. The resistance of a memristor can change with the change in the current passing through it and displays corresponding resistance characteristics, which is very similar to the changing characteristic of the synaptic weight of a biological synapse under the stimulation of biological electrical signals [1]. So far, the memristor is the most apt component for simulating a biological synapse, and an artificial neural network using memristors to simulate neural synapses is called a memristive neural network. In addition, because the memristor is nano-sized, it has a higher level of integration and has incomparable advantages in the simulation of neuronal synapses; thus, memristive neural networks have rapidly become a research hotspot. Compared with traditional artificial neural networks, memristive neural networks can better simulate the structure and function of the human brain [2]. On the other hand, fractional-order calculus has “heredity” and “memory” characteristics, which makes fractional-order neural network more accurate than traditional integer-order neural networks in describing the complex relationship between a signal’s input and output

and the dynamic behaviors of a neural network, such as self-adaptation, cognition, and decision-making [3].

Due to their widespread applications, the synchronization of NNs has been intensively studied in areas such cryptography, image encryption, secure communication, etc. Synchronization time is an important evaluation index of the synchronization performance of a system, and most of the existing research focuses on asymptotic synchronization [4], exponential synchronization [5], projective synchronization [6], adaptive synchronization [7], etc., which require very long or even infinite synchronization times, limiting these synchronization schemes' practical application, so finding a fast synchronization method is a matter of great urgency.

Finite-time synchronization (FNS) ensures that the error between systems can converge in a finite time that can be calculated using an explicit expression of the synchronization time. Zhang et al. used the comparison principle to study the FNS of the switching hopping mismatched fractional memristive NN (FMNN) of BAM [8]. Du et al. presented an FNS control method for a type of time-delayed FMNN by using the fractional Gronwall inequality [9]. Anbalagand et al. investigated the FNS of a competitive FMNN [10]. Sun et al. studied the asymptotic synchronization and FNS of an inertial FMNN with a time-varying delay [11].

In FNS, the synchronization time cannot be estimated if the initial system is unknown. But, for FTS, one can calculate the upper bound of the synchronization time under the same conditions. Therefore, FTS is studied in this work. Reference [12] also considered adaptive control in the FTS of Cohen–Grossberg NNs with pulsed memristors. Reference [13] discussed the FTS control problem of random MNNs and presented two different controller design methods. The FTS of competing MNNs with different time scales has also been reported [14]. The FTS control of quaternion-valued MNNs with discontinuous activation functions was discussed in [15]. The FTS control of MNNs with discontinuous activation functions and mixed delays was presented in [16]. The FTS of fractional-order inertial fuzzy cellular NNs with mixed time-dependent delays was studied in [17].

On the other hand, the adaptive synchronization control strategy, which can modify the control intensity at any time, can reduce the cost of control more effectively. In addition, when the dynamic characteristics of the system fluctuate within a certain range, adaptive synchronization can still ensure a good control effect. The FTS of a fractional-order memristive fuzzy NN (FOMFNN) with leakage and transmission delays [18] and the adaptive FTS of fractional fuzzy cellular NNs with a time delay [19] have recently been studied.

Various types of time delays (e.g., time-dependent delays, proportional delays, and distributed delays) are often encountered in NNs [20]. The leakage delays in the negative feedback items related to the states of NNs can destabilize the dynamics of the system, making it more of a challenge to process the leakage item [21]. So, there is significance to considering the effect of leakage delays on NNs, and [22–25] are some recent reports on this topic.

In this paper, we discuss the FTS and AFTS of FOMFNNs with time-varying leakage and transmission delays. The main highlights of this paper are listed as follows:

(i) Memristors, leakage delays, transmission delays, fuzziness, and fractional derivatives are all considered in the FTS and AFTS analyses of the proposed NNs, which complement similar studies without fuzziness, without fuzziness or a leakage delay, and without memristors or fuzziness.

(ii) Two control schemes that consider leakage and transmission delays are proposed to achieve FTS and AFTS, which include the design of a state feedback controller and a fractional-order adaptive controller.

(iii) Based on the Lyapunov functional method and some fractional-order differential inequalities, some effective criteria, according to algebraic inequalities, are derived to accomplish the FTS of the considered NNs, which are easy to put into effect in reality to avert the complex computation of matrix inequalities.

The rest of this paper is organized as follows: Section 2 presents several definitions and lemmas. Section 3 analyzes the FTS and AFTS of FOMFNNs and derives some new criteria for achieving these synchronizations. Section 4 verifies the obtained results using two numerical examples. Section 5 concludes this article.

2. Preliminaries

Definition 1 ([11]). The Caputo fractional integral of order ω is defined as follows:

$$I^{-\omega}z(x) = \frac{1}{\Gamma(\omega)} \int_{x_0}^x (x-s)^{\omega-1} z(s) ds,$$

where $\omega \in \mathbb{R}^+$, $x > x_0$, and $\Gamma(\cdot)$ is the Gamma function.

Definition 2 ([11]). The Caputo fractional derivative of order ω is defined as follows:

$${}^C_{t_0}D^\omega z(x) = \frac{1}{\Gamma(m-\omega)} \int_{x_0}^x (x-s)^{m-\omega-1} z^{(m)}(s) ds,$$

where $m \in \mathbb{R}^+$ and $m-1 \leq \omega < m$.

In this paper, we investigate FMCNNs with the leakage delay that follows:

$$\begin{cases} {}^C_{t_0}D^\omega p_i(t) = -\alpha_i p_i(t - \tau_1(t)) + \sum_{r=1}^M \phi_{ir}(p_i(t)) f_r(p_r(t)) \\ \quad + \sum_{r=1}^M \psi_{ir}(p_i(t - \tau_2(t))) f_r(p_r(t - \tau_2(t))) + H_i(p(t)) + I_i \\ p_i(v) = \zeta_i(v), v \in [t_0 - \tau, t_0], i = 1, \dots, M, \end{cases} \quad (1)$$

where

$$\begin{aligned} H_i(p(t)) &= \sum_{r=1}^M g_{ir} v_r + \bigwedge_{r=1}^M T_{ir} v_r + \bigwedge_{r=1}^M a_{ir} f_r(p_r(t - \tau_2(t))) + \bigvee_{r=1}^M S_{ir} v_r \\ &\quad + \bigvee_{r=1}^M b_{ir} f_r(p_r(t - \tau_2(t))) \\ \phi_{ir}(p_i(t)) &= \begin{cases} \phi'_{ir}, |p_i(t)| \leq \Gamma_i \\ \phi''_{ir}, |p_i(t)| > \Gamma_i \end{cases}, \psi_{ir}(p_i(t - \tau_2(t))) = \begin{cases} \psi'_{ir}, |p_i(t - \tau_2(t))| \leq \Gamma_i \\ \psi''_{ir}, |p_i(t - \tau_2(t))| > \Gamma_i \end{cases} \end{aligned}$$

and ${}^C_{t_0}D^\omega$ represents a ω -order Caputo fractional derivation where $0 < \omega < 1$. $f_r(\cdot)$ denotes the activation function. $\alpha_i > 0$ denotes the weight of the self-feedback connection. $\tau(t)$ is the time-dependent delay, and $\tau_1(t), \tau_2(t) \in [0, \tau]$. $\phi_{ir}(p_i(t))$ and $\psi_{ir}(p_i(t))$ represent the memristive connection weights. g_{ir} denotes the feedforward template; a_{ir}, b_{ir} and T_{ir}, S_{ir} represent the elements of the fuzzy feedback and backward MIN and MAX template, respectively; $\bigvee$ and $\bigwedge$ are the fuzzy OR and AND operations, respectively; and I_i is the bias value. $\phi'_{ir}, \phi''_{ir}, \psi'_{ir}, \psi''_{ir}$ are known constants, and $\Gamma_i > 0$ denotes the switching jump value of the memristor. $\zeta_i(t)$ are the initial system values.

Remark 1. Compared to the systems in [18,26–28], System (1) is more general. It degenerates into Model [18] for constant $\tau_1(t)$ and $\tau_2(t)$. It degenerates into Model [26] if the memristive weights are constants. If $H_i(p(t)) = 0$, it degenerates into Model [27]. If $H_i(p(t)) = \tau_1(t) = \psi_{ir}(p_i(t - \tau_2(t))) = 0$, it degenerates into Model [28].

Let $\phi_{ir}^- = \min\{\phi'_{ir}, \phi''_{ir}\}$, $\phi_{ir}^+ = \max\{\phi'_{ir}, \phi''_{ir}\}$, $\psi_{ir}^- = \min\{\psi'_{ir}, \psi''_{ir}\}$, and $\psi_{ir}^+ = \max\{\psi'_{ir}, \psi''_{ir}\}$. Based on the differential inclusion theory and measurable selection theory [29], there exist $\phi_{ir} \in \text{co}(\phi_{ir}(q_i(t))) = [\phi_{ir}^-, \phi_{ir}^+]$, $\psi_{ir} \in \text{co}(\psi_{ir}(q_i(t - \tau_2(t)))) = [\psi_{ir}^-, \psi_{ir}^+]$ such that

$$\begin{cases} {}^C_{t_0} D^\omega p_i(t) = -\alpha_i p_i(t - \tau_1(t)) + \sum_{r=1}^M \psi_{ir} f_r(p_r(t - \tau_2(t))) \\ \quad + \sum_{r=1}^M \phi_{ir} f_r(p_r(t)) + H_i(p(t)) + I_i \\ p_i(v) = \zeta_i(v), v \in [t_0 - \tau, t_0], i = 1, \dots, M, \end{cases} \quad (2)$$

The response system is given by

$$\begin{cases} {}^C_{t_0} D^\omega q_i(t) = -\alpha_i q_i(t - \tau_1(t)) + \sum_{r=1}^M \phi_{ir} f_r(q_r(t)) \\ \quad + \sum_{r=1}^M \psi_{ir} f_r(q_r(t - \tau_2(t))) + H_i(q(t)) + u_i + I_i \\ q_i(v) = \xi_i(v), v \in [t_0 - \tau, t_0], i = 1, \dots, M, \end{cases} \quad (3)$$

where

$$\begin{aligned} H_i(q(t)) = & \sum_{r=1}^M g_{ir} v_r + \bigwedge_{r=1}^M T_{ir} v_r + \bigwedge_{r=1}^M a_{ir} f_r(q_r(t - \tau_2(t))) + \bigvee_{r=1}^M S_{ir} v_r \\ & + \bigvee_{r=1}^M b_{ir} f_r(q_r(t - \tau_2(t))) \end{aligned}$$

and $\xi_i(v)$ represents the initial values of System (3).

By defining the synchronization error as $e_i(t) = q_i(t) - p_i(t)$, we obtain the following error system:

$$\begin{cases} {}^C_{t_0} D^\omega e_i(t) = -\alpha_i e_i(t - \tau_1(t)) + \sum_{r=1}^M \phi_{ir} f_r(e_r(t)) + \sum_{r=1}^M \psi_{ir} f_r(e_r(t - \tau_2(t))) \\ \quad + \bigwedge_{r=1}^M a_{ir} f_r(e_r(t - \tau_2(t))) + \bigvee_{r=1}^M b_{ir} f_r(e_r(t - \tau_2(t))) + u_i(t) \\ e_i(v) = \xi_i(v) - \zeta_i(v), v \in [t_0 - \tau, t_0], i = 1, 2, \dots, M. \end{cases} \quad (4)$$

where

$$\begin{aligned} f_r(e_r(t)) &= f_r(q_r(t)) - f_r(p_r(t)) \\ f_r(e_r(t - \tau_2(t))) &= f_r(q_r(t - \tau_2(t))) - f_r(p_r(t - \tau_2(t))) \end{aligned}$$

Definition 3. FTS is said to be achieved for the above drive–response systems if there exists a settling time function $T(e(t_0))$ such that

$$\begin{cases} \lim_{t \rightarrow T(e(t_0))} \|e(t)\| = 0 \\ e(t) = 0, \forall t \geq T(e(t_0)) \\ T(e(t_0)) \leq T_{\max}, \forall e(t_0) \in C^n[t_0 - \tau, t_0] \end{cases}$$

where $\|\cdot\|$ represents the Euclidean norm and $T_{\max}$ is the settling time.

Assumption 1. For $\forall q_r(t), p_r(t) \in R$, there exists constants $L_r > 0$ such that

$$|f_r(q_r(t)) - f_r(p_r(t))| \leq L_r |q_r(t) - p_r(t)|, r = 1, 2, \dots, N.$$

Lemma 1 ([30]). Suppose that $x(t) \in C^1[0, T]$. Then,

$${}^C_{t_0} D^{\beta_1} {}^C_{t_0} D^{\beta_2} x(t) = {}^C_{t_0} D^{\beta_1 + \beta_2} x(t),$$

where $\beta_1, \beta_2 > 0$, $\beta_1 + \beta_2 \leq 1$, and T is a positive constant.

Lemma 2 ([31]). If $\chi(t) \in C^1[t_0, \infty]$, $0 < \omega \leq 1$, then

$${}^C_{t_0} D^\omega |\chi(t)| \leq \text{sign}(\chi(t)) {}^C_{t_0} D^\omega \chi(t)$$

holds.

Lemma 3 ([32]). Let $p_r(t), q_r(t), a_{ir}, b_{ir} \in R, f_r : R \rightarrow R$ be a continuous function, $i, r = 1, 2, \dots, N$. Then,

$$\begin{aligned} \left| \bigwedge_r a_{ir} f_r(p_r(t)) - \bigwedge_r a_{ir} f_r(q_r(t)) \right| &\leq \sum_r |a_{ir}| |f_r(p_r(t)) - f_r(q_r(t))| \\ \left| \bigvee_r b_{ir} f_r(p_r(t)) - \bigvee_r b_{ir} f_r(q_r(t)) \right| &\leq \sum_r |b_{ir}| |f_r(p_r(t)) - f_r(q_r(t))| \end{aligned}$$

Lemma 4 ([32]). If $a_1, a_2, \dots, a_M \geq 0, \mu > 1, 0 < \nu \leq 1$, then

$$\sum_i a_i^\mu \geq M^{1-\mu} \left(\sum_{i=1}^n a_i \right)^\mu, \quad \sum_i a_i^\nu \geq \left(\sum_i a_i \right)^\nu$$

Lemma 5 ([33]). If there exists a regular, positive, definite, and radially unbounded function $V(x) : R^n \rightarrow R$ and the constants $a, b, \delta, k > 0$ and $\delta k > 1$ meet,

$$\dot{V}(e(t)) \leq -(aV^\delta(e(t)) + b)^k, e(t) \in R^n \setminus \{0\}$$

then we obtain $V(e(t)) = 0, t \geq T(e(t_0))$, while the settling time is estimated by

$$T(e(t_0)) \leq T_{\max} = \frac{1}{b^k} \left(\frac{b}{a} \right)^{\frac{1}{\delta}} \left(1 + \frac{1}{\delta k - 1} \right)$$

3. Main Results

In this paper, two kinds of synchronization controllers are designed; one is feedback controller, and the other is adaptive controller. Both can make the considered neural network achieve fixed-time synchronization. The first controller is as follows:

$$u_i(t) = -\delta_i e_i(t) - \text{sign}(e_i(t)) \left(\theta_i |e_i(t - \tau_1(t))| + \eta_i |e_i(t - \tau_2(t))| + \gamma_i (\lambda_{i t_0}^C D^{\omega-1} |e_i(t)|)^l + \sigma_i \right) \quad (5)$$

The second controller is as follows:

$$\begin{cases} u_i(t) = -\delta_i(t)e_i(t) - \text{sign}(e_i(t))(\theta_i(t)|e_i(t - \tau_1(t))| + \eta_i(t)|e_i(t - \tau_2(t))| \\ \quad + \gamma_i(\lambda_{i t_0}^C D^{\omega-1}|e_i(t)|)^l + \sigma_i) \\ \dot{\delta}_i(t) = \lambda_i \kappa_i |e_i(t)| - \left(\frac{\epsilon_1}{2}\right)^l (\delta_i(t) - \hat{\delta}_i)^{2l-1} \\ \dot{\theta}_i(t) = \lambda_i \rho_i |e_i(t - \tau_1(t))| - \left(\frac{\epsilon_2}{2}\right)^l (\theta_i(t) - \hat{\theta}_i)^{2l-1} \\ \dot{\eta}_i(t) = \lambda_i \rho_i |e_i(t - \tau_2(t))| - \left(\frac{\epsilon_3}{2}\right)^l (\eta_i(t) - \hat{\eta}_i)^{2l-1} \end{cases} \quad (6)$$

where $\text{sign}(\cdot)$ is the symbolic function and $l > 1, \delta_i > 0, \sigma_i > 0, \theta_i > 0, \eta_i > 0, \gamma_i > 0$ are the constants to be ascertained. $\lambda_i, \kappa_i, \rho_i, \epsilon_1, \epsilon_2, \epsilon_3$ are appropriate constants, $\hat{\delta}_i, \hat{\theta}_i, \hat{\eta}_i$ are adaptive constants, and $\delta_i(t), \theta_i(t), \eta_i(t)$ denote adaptive control gains.

Remark 2. The feedback control scheme (5) and adaptive control scheme (6) are different from the existing state feedback schemes in [26] and adaptive control schemes in [27], respectively. Both Controllers (5) and (6) contain a leakage delay and transmission delay, and Controller (6) also allows for fractional derivative behavior, which can reduce the control cost by using state information. In (5) and (6), the terms with time delays are used to remove the time delays, and the term $\sigma_r \text{sign}(e_r(t))$ is a mixed term that can improve the speed of the response.

Theorem 1. Under Assumption 1, Systems (2) and (3) can achieve fixed-time synchronization using Controller (5), if there exist some positive numbers $\theta_i, \delta_i, \eta_i$ such that

$$\begin{cases} \theta_i \geq \alpha_i \\ \delta_i \geq \frac{L_i}{\lambda_i} \sum_{r=1}^M \lambda_r |\phi_{ri}| \\ \eta_i \geq \frac{L_i}{\lambda_i} \sum_{r=1}^M \lambda_r (|\psi_{ri}| + |a_{ri}| + |b_{ri}|) \end{cases} \quad (7)$$

for $i = 1, 2, \dots, M$. Furthermore, the settling time $T_{\max}$ can be calculated using the following equation:

$$T_{\max} = \frac{1}{\Theta^{(l-1)}} \left(\frac{\Theta}{\Lambda M^{1-l}} \right)^{\frac{1}{l}} \quad (8)$$

where $\Lambda = \min_{1 \leq i \leq M} \{\lambda_i \gamma_i\}$ and $\Theta = \sum_{i=1}^M \lambda_i \sigma_i$.

Proof. Given the Lyapunov function (9)

$$V(e(t)) = \sum_i \lambda_{i t_0}^C D^{\omega-1} |e_i(t)| \quad (9)$$

$V(e(t)) \geq 0$ and if and only if $e(t) = 0$, $V(e(t)) = 0$. Then, the derivative of () can be calculated by using Lemma 1.

$$\begin{aligned}\dot{V}(e(t)) &= {}^C D^\omega \left({}^C D^{1-\omega} V(e(t)) \right) \\ &= {}^C D^\omega \left({}^C D^{1-\omega} \sum_i \lambda_i {}^C D^{\omega-1} |e_i(t)| \right) \\ &= {}^C D^\omega \left({}^C D^{1-\omega} \left({}^C D^{\omega-1} \sum_i \lambda_i |e_i(t)| \right) \right) \\ &= \sum_i {}^C D^\omega |e_i(t)|\end{aligned}$$

Based on Lemma 2 and (4),

$$\begin{aligned}\dot{V}(e(t)) &\leq \sum_i \lambda_i \text{sign}(e_i(t)) {}^C D^\omega e_i(t) \\ &= \sum_i \lambda_i \text{sign}(e_i(t)) \left\{ -\alpha_i e_i(t - \tau_1(t)) + \sum_r \phi_{ir} f_r(e_r(t)) + \sum_r \psi_{ir} f_r(e_r(t - \tau_2(t))) \right. \\ &\quad \left. + \bigwedge_i a_{ir} f_r(e_r(t - \tau_2(t))) + \bigvee_r b_{ir} f_r(e_r(t - \tau_2(t))) + u_i(t) \right\} \\ &\leq \sum_i \lambda_i \alpha_i |e_i(t - \tau_1(t))| + \sum_i \sum_r \lambda_i |\phi_{ir} f_r(e_r(t))| + \sum_i \sum_r \lambda_i |\psi_{ir} f_r(e_r(t - \tau_2(t)))| \\ &\quad + \sum_i \lambda_i \bigwedge_r a_{ir} |f_r(e_r(t - \tau_2(t)))| + \sum_i \lambda_i \bigvee_r b_{ir} |f_r(e_r(t - \tau_2(t)))| \\ &\quad + \sum_i \lambda_i \text{sign}(e_i(t)) u_i(t).\end{aligned}\tag{10}$$

Using Assumption 1 and Lemma 3, we obtain

$$\sum_i \sum_r \lambda_i |\phi_{ir} f_r(e_r(t))| \leq \sum_i \sum_r \lambda_i L_r |\phi_{ir}| |e_r(t)|,\tag{11}$$

$$\sum_i \sum_r \lambda_i |\psi_{ir} f_r(e_r(t - \tau_2(t)))| \leq \sum_i \sum_r \lambda_i L_r |\psi_{ir}| |e_r(t - \tau_2(t))|,\tag{12}$$

$$\sum_i \lambda_i \bigwedge_r a_{ir} |f_r(e_r(t - \tau_2(t)))| \leq \sum_i \sum_r \lambda_i L_r |a_{ir}| |e_r(t - \tau_2(t))|,\tag{13}$$

$$\sum_i \lambda_i \bigvee_r b_{ir} |f_r(e_r(t - \tau_2(t)))| \leq \sum_i \sum_r \lambda_i L_r |b_{ir}| |e_r(t - \tau_2(t))|.\tag{14}$$

By substituting (11)–(14) into (10),

$$\begin{aligned}\dot{V}(e(t)) &\leq \sum_i \lambda_i \alpha_i |e_i(t - \tau_1(t))| + \sum_i \sum_r \lambda_i L_r |\phi_{ir}| |e_r(t)| \\ &\quad + \sum_i \sum_r \lambda_r L_i (|\psi_{ir}| + |a_{ir}| + |b_{ir}|) |e_i(t - \tau_2(t))| + \sum_i \lambda_i \text{sign}(e_i(t)) u_i(t)\end{aligned}\tag{15}$$

From Formula (5), we obtain

$$\begin{aligned}\sum_i \lambda_i \text{sign}(e_i(t)) u_i(t) &= - \sum_i \lambda_i (\theta_i |e_i(t - \tau_1(t))| + \eta_i |e_i(t - \tau_2(t))|) \\ &\quad - \sum_i \lambda_i \gamma_i (\lambda_i {}^C D^{\omega-1} |e_i(t)|)^l - \sum_i \lambda_i \sigma_i.\end{aligned}\tag{16}$$

Combining (15) and (16), one finds that

$$\begin{aligned} \dot{V}(e(t)) \leq & \sum_i \lambda_i (\alpha_i - \theta_i) |e_i(t - \tau_1(t))| - \sum_i \left(\delta_i - \sum_r \frac{\lambda_r L_i |\phi_{ri}|}{\lambda_i} \right) \lambda_i |e_i(t)| \\ & + \sum_i \left[\sum_r \lambda_r L_i (|\psi_{ri}| + |a_{ri}| + |b_{ri}|) - \eta_i \lambda_i \right] |e_i(t - \tau_2(t))| \\ & - \sum_i \lambda_i \gamma_i (\lambda_{i0}^C D^{\omega-1} |e_i(t)|)^l - \sum_i \lambda_i \sigma_i. \end{aligned} \quad (17)$$

From (7), we obtain

$$\dot{V}(e(t)) \leq - \sum_i \lambda_i \gamma_i \left(\lambda_{i0}^C D^{\omega-1} |e_i(t)| \right)^l - \sum_i \lambda_i \sigma_i \quad (18)$$

Thus,

$$\dot{V}(e(t)) \leq -\Lambda \sum_i \left(\lambda_{i0}^C D^{\omega-1} |e_i(t)| \right)^l - \Theta \quad (19)$$

where $\Lambda = \min_{1 \leq i \leq N} \{\lambda_i \gamma_i\}$ and $\Theta = \sum_i \lambda_i \sigma_i$. From Lemma 4,

$$\dot{V}(e(t)) \leq -\Lambda M^{1-l} (V(e(t)))^l - \Theta$$

Assume $k = 1$ in Lemma 5. By using the fixed-time stability of the origin, we can calculate $T_{\max}$ as follows:

$$T_{\max} = \frac{1}{\Theta(l-1)} \left(\frac{\Theta}{\Lambda M^{1-l}} \right)^{\frac{1}{l}}$$

□

Remark 3. Asymptotic stability has been widely studied in the dynamics of nonlinear systems. But in practical applications, people usually expect the system to be in a stable state for a certain duration, which makes asymptotic stability no longer applicable. From this, the concepts of time-limited stability and fixed-time stability emerged. Compared to finite-time stability, fixed-time stability has been improved based on its definition; that is, the upper bound of the system's stability time is not affected by the initial state of the system. This has significant practical significance, as the initial state of actual systems is often difficult or even impossible to obtain.

Remark 4. The adaptive controller in (6) can ascertain unknown model parameters from the input and output information and is extremely robust against external interference and uncertainties. It is not easy to choose the feedback control parameters δ_i , θ_i , and η_i for Controller (5), so the FTS of the driving response system will be achieved using the adaptive scheme.

Theorem 2. Under Assumption 1, Systems (2) and (3) can achieve fixed-time synchronization through Controller (6) if there exist some positive numbers λ_i , κ_i , q_i , ρ_i , σ_i , ϵ_1 , ϵ_2 , ϵ_3 , $\hat{\theta}_i$, $\hat{\delta}_i$, and $\hat{\eta}_i$ such that the following conditions hold:

$$\begin{cases} \hat{\theta}_i \geq \alpha_i \\ \hat{\delta}_i \geq \frac{L_i}{\lambda_i} \sum_{r=1}^M \lambda_r |\phi_{ri}| \\ \hat{\eta}_i \geq \frac{L_i}{\lambda_i} \sum_{r=1}^M \lambda_r (|\psi_{ri}| + |a_{ri}| + |b_{ri}|) \end{cases} \quad (20)$$

for $i = 1, 2, \dots, M$. Furthermore, the settling time $T_{\max}$ can be calculated using the following equation:

$$T_{\max} = \frac{1}{\Theta^{(l-1)}} \left(\frac{\Theta}{\Omega M^{1-l}} \right)^{\frac{1}{l}} \quad (21)$$

where $\Omega = \min_{1 \leq i \leq M} \left\{ \lambda_i \gamma_i, \kappa_i^{l-1} \epsilon_1^l, \varrho_i^{l-1} \epsilon_2^l, \rho_i^{l-1} \epsilon_3^l \right\}$ and $\Theta = \sum_{i=1}^M \lambda_i \sigma_i$.

Proof. Given Lyapunov Function (22),

$$\begin{aligned} V(e(t)) = & \sum_i \lambda_{it_0}^C D^{\omega-1} |e_i(t)| \\ & + \sum_i \frac{1}{2\kappa_i} (\delta_i(t) - \hat{\delta}_i)^2 + \sum_i \frac{1}{2\varrho_i} (\theta_i(t) - \hat{\theta}_i)^2 + \sum_i \frac{1}{2\rho_i} (\eta_i(t) - \hat{\eta}_i)^2 \end{aligned} \quad (22)$$

by deriving $V(e(t))$ along (4) and using Lemma 2, we obtain

$$\begin{aligned} \dot{V}(e(t)) \leq & \sum_i \lambda_i \text{sign}(e_i(t)) \lambda_{it_0}^C D^{\omega} e_i(t) \\ & + \sum_i \frac{1}{\kappa_i} (\delta_i(t) - \hat{\delta}_i) \dot{\delta}_i(t) + \sum_i \frac{1}{\varrho_i} (\theta_i(t) - \hat{\theta}_i) \dot{\theta}_i(t) + \sum_i \frac{1}{\rho_i} (\eta_i(t) - \hat{\eta}_i) \dot{\eta}_i(t) \end{aligned} \quad (23)$$

By combining Formulas (15), (23), and (6), one can obtain

$$\begin{aligned} \dot{V}(e(t)) \leq & \sum_i \lambda_i (\alpha_i - \hat{\theta}_i) |e_i(t - \tau_1(t))| - \sum_i \left(\hat{\delta}_i - \sum_r \frac{\lambda_r L_i |\phi_{ri}|}{\lambda_i} \right) \lambda_i |e_i(t)| \\ & + \sum_i \left[\sum_r \lambda_r L_i (|\psi_{ri}| + |a_{ri}| + |b_{ri}|) - \lambda_i \hat{\eta}_i \right] |e_i(t - \tau_2(t))| \\ & - \sum_i \lambda_i \gamma_i \left(\lambda_{it_0}^C D^{\omega-1} |e_i(t)| \right)^l - \sum_i \kappa_i^{l-1} \epsilon_1^l \left(\frac{1}{2\kappa_i} (\delta_i(t) - \hat{\delta}_i)^2 \right)^l \\ & - \sum_i \varrho_i^{l-1} \epsilon_2^l \left(\frac{1}{2\varrho_i} (\theta_i(t) - \hat{\theta}_i)^2 \right)^l - \sum_i \rho_i^{l-1} \epsilon_3^l \left(\frac{1}{2\rho_i} (\eta_i(t) - \hat{\eta}_i)^2 \right)^l - \sum_i \lambda_i \sigma_i \end{aligned} \quad (24)$$

From (20), we know that

$$\begin{aligned} \dot{V}(e(t)) \leq & - \sum_i \lambda_i \gamma_i \left(\lambda_{it_0}^C D^{\omega-1} |e_i(t)| \right)^l - \sum_i \kappa_i^{l-1} \epsilon_1^l \left(\frac{1}{2\kappa_i} (\delta_i(t) - \hat{\delta}_i)^2 \right)^l \\ & - \sum_i \varrho_i^{l-1} \epsilon_2^l \left(\frac{1}{2\varrho_i} (\theta_i(t) - \hat{\theta}_i)^2 \right)^l - \sum_i \rho_i^{l-1} \epsilon_3^l \left(\frac{1}{2\rho_i} (\eta_i(t) - \hat{\eta}_i)^2 \right)^l - \sum_i \lambda_i \sigma_i \end{aligned} \quad (25)$$

Let $\Omega = \min_{1 \leq i \leq M} \left\{ \lambda_i \gamma_i, \kappa_i^{l-1} \epsilon_1^l, \varrho_i^{l-1} \epsilon_2^l, \rho_i^{l-1} \epsilon_3^l \right\}$. Using Lemma 4 yields

$$\begin{aligned} \dot{V}(e(t)) \leq & -\Omega \left(\sum_i \left(\lambda_{it_0}^C D^{\omega-1} |e_i(t)| \right)^l + \sum_i \left(\frac{1}{2\kappa_i} (\delta_i(t) - \hat{\delta}_i)^2 \right)^l \right. \\ & \left. + \sum_i \left(\frac{1}{2\varrho_i} (\theta_i(t) - \hat{\theta}_i)^2 \right)^l + \sum_i \left(\frac{1}{2\rho_i} (\eta_i(t) - \hat{\eta}_i)^2 \right)^l \right) - \sum_i \lambda_i \sigma_i \\ & \leq -\Omega M^{1-l} (V(e(t)))^l - \Theta \end{aligned} \quad (26)$$

Assume $k = 1$ in Lemma 5. By using the fixed-time stability of the origin, we determine $T_{\max}$ as follows:

$$T_{\max} = \frac{1}{\Theta(l-1)} \left(\frac{\Theta}{\Omega M^{1-l}} \right)^{\frac{1}{l}}$$

□

Remark 5. The settling time in this paper is based on Lemma 5, and the algorithm of the settling time in Lemma 5 itself is an optimized result. In fact, in the proof of Lemma 1, an estimation of the settling time is given by

$$\begin{aligned} T_s &\leq \int_0^{+\infty} \frac{1}{(a\omega^\delta + b)^k} d\omega = \int_0^r \frac{1}{(a\omega^\delta + b)^k} d\omega \\ &\quad + \int_r^{+\infty} \frac{1}{(a\omega^\delta + b)^k} d\omega \\ &\leq \int_0^r \frac{1}{b^k} d\omega + \int_r^{+\infty} \frac{1}{a^k \omega^{\delta k}} d\omega \\ &= \frac{r}{b^k} + \frac{1}{a^k(\delta k - 1)} r^{1-\delta k} \end{aligned}$$

where r is an arbitrary positive number. To give a less conservative and more accurate estimation of T_s , let

$$\varphi(r) = \frac{r}{b^k} + \frac{1}{a^k(\delta k - 1)} r^{1-\delta k}$$

Then,

$$\dot{\varphi}(r) = \frac{1}{b^k} + \frac{1}{a^k(\delta k - 1)} r^{-\delta k}$$

which shows that $\varphi(r)$ reaches its minimum value $\hat{\varphi}$, given by

$$\hat{\varphi} = \frac{1}{b^k} \left(\frac{b}{a} \right)^{\frac{1}{\delta}} \left(1 + \frac{1}{\delta k - 1} \right)$$

On the other hand, we can optimize the estimation of the settling time by choosing suitable parameters a, b, k, δ for different applications. In fact, if $a > b$ and $\delta = \frac{\ln \frac{b}{a}}{\ln \frac{b}{a} + 1}$, a less conservative estimation of the settling time function T_s can be obtained by

$$T_s \leq T_{\max} = \frac{\ln \frac{b}{a}}{b} \left(\frac{b}{a} \right)^{1 + \frac{1}{\ln \frac{b}{a}}}.$$

4. Numerical Simulations

This section validates the effectiveness of Theorems 1 and 2 through two numerical examples.

Example 1. Take the drive-response systems

$$\begin{cases} {}^C_{t_0} D^\omega p_i(t) = -\alpha_i p_i(t - \tau_1(t)) + \sum_{r=1}^3 \phi_{ir}(p_i(t)) f_r(p_r(t)) \\ \quad + \sum_{r=1}^3 \psi_{ir}(p_i(t - \tau_2(t))) f_r(p_r(t - \tau_2(t))) + H_i(p(t)) + I_i \\ p_i(v) = \zeta_i(v), v \in [-\tau, t_0], i = 1, 2, 3, \end{cases} \quad (27)$$

where

$$H_i(p(t)) = \sum_{r=1}^3 g_{ir} v_r + \bigwedge_{r=1}^3 T_{ir} v_r + \bigwedge_{r=1}^3 a_{ir} f_r(p_r(t - \tau_2(t))) + \bigvee_{r=1}^3 S_{ir} v_r \\ + \bigvee_{r=1}^3 b_{ir} f_r(p_r(t - \tau_2(t)))$$

with following system parameters:

$$\left\{ \begin{array}{l} \Gamma_1 = \Gamma_2 = \Gamma_3 = 1, \alpha_1 = \alpha_2 = \alpha_3 = 1, l = 1.8, \omega = 0.95 \\ \phi'_{11} = 2.0, \phi'_{12} = -1, \phi'_{13} = 1.8, \phi'_{21} = 0.8, \phi'_{22} = 1.5, \phi'_{23} = -1.0, \\ \phi'_{31} = -1.1, \phi'_{32} = 2.0, \phi'_{33} = 1.5, \phi''_{11} = 2.2, \phi''_{12} = -1.2, \phi''_{13} = 2.0, \\ \phi''_{21} = 1.0, \phi''_{22} = 1.8, \phi''_{23} = -1.5, \phi''_{31} = -1.0, \phi''_{32} = 1.7, \phi''_{33} = 2.0 \\ \psi'_{11} = -2.0, \psi'_{12} = -0.5, \psi'_{13} = 1.5, \psi'_{21} = 2.5, \psi'_{22} = 5.0, \psi'_{23} = -2.5 \\ \psi'_{31} = 2.4, \psi'_{32} = -2.0, \psi'_{33} = 4.5, \psi''_{11} = -1.5, \psi''_{12} = -1.0, \psi''_{13} = 2.0 \\ \psi''_{21} = 2.2, \psi''_{22} = 4.5, \psi''_{23} = -3.0, \psi''_{31} = 2.0, \psi''_{32} = -1.8, \psi''_{33} = 5 \\ a_{11} = a_{13} = a_{22} = a_{23} = a_{31} = a_{32} = 0.1, a_{12} = a_{21} = -0.2, a_{33} = -0.1 \\ b_{11} = b_{13} = b_{22} = b_{31} = 0.2, b_{12} = b_{21} = -0.2, b_{23} = b_{32} = 0.1, b_{33} = -0.1 \\ S_{11} = S_{22} = S_{33} = -0.1, S_{12} = S_{13} = S_{21} = S_{23} = S_{31} = S_{32} = 0.1 \\ T_{11} = T_{22} = T_{33} = -0.1, T_{12} = T_{13} = T_{21} = T_{23} = T_{31} = T_{32} = 0.1 \\ g_{ir} = -1.0, v_r = -1, i, r = 1, 2, 3 \end{array} \right. \quad (28)$$

Let $f_i(p(t)) = \tanh(|p(t)|) - 1$, $I_i = 0.1$, $L_i = 1$, $i = 1, 2, 3$, $\tau_1(t) = \tanh(t)$, and $\tau_2(t) = \frac{e^t}{1+e^t}$. The initial values of System (27) are $\zeta_1(v) = 0.3$, $\zeta_2(v) = 0.6$, $\zeta_3(v) = -0.2$, $v \in [-\tau(t), 0]$.

The response system is

$$\left\{ \begin{array}{l} {}^C_{t_0} D^\omega q_i(t) = -\alpha_i q_i(t - \tau_1(t)) + \sum_{r=1}^3 \phi_{ir}(q_i(t)) f_r(q_r(t)) \\ \quad + \sum_{r=1}^3 \psi_{ir}(q_i(t - \tau_2(t))) f_r(q_r(t - \tau_2(t))) + H_i(q(t)) + u_i + I_i \\ q_i(v) = \zeta_i(v), v \in [-\tau, t_0], i = 1, 2, 3, \end{array} \right. \quad (29)$$

where

$$H_i(q(t)) = \sum_{r=1}^3 g_{ir} v_r + \bigwedge_{r=1}^3 T_{ir} v_r + \bigwedge_{r=1}^3 a_{ir} f_r(q_r(t - \tau_2(t))) + \bigvee_{r=1}^3 S_{ir} v_r \\ + \bigvee_{r=1}^3 b_{ir} f_r(q_r(t - \tau_2(t)))$$

The parameters are the same as those in System (27). The initial values of System (29) are $\zeta_1(v) = -0.3$, $\zeta_2(v) = -0.6$, $\zeta_3(v) = 0.2$, $v \in [-\tau(t), 0]$.

According to Theorem 1, the control parameters $\theta_i, \delta_i, \eta_i, i = 1, 2, 3$ should satisfy

$$\left\{ \begin{array}{l} \theta_i \geq \alpha_i \\ \delta_i \geq \frac{L_i}{\lambda_i} \sum_{r=1}^3 \lambda_r |\phi_{ri}| \\ \eta_i \geq \frac{L_i}{\lambda_i} \sum_{r=1}^3 \lambda_r (|\psi_{ri}| + |a_{ri}| + |b_{ri}|) \end{array} \right.$$

Here, we choose

$$\begin{cases} \eta_1 = 5.5, \eta_2 = 6.2, \eta_3 = 6.7, \\ \theta_1 = \theta_2 = \theta_3 = 1.0, \\ \delta_1 = \delta_2 = \delta_3 = 10.7 \\ \gamma_1 = \gamma_2 = \gamma_3 = 4 \\ \sigma_1 = \sigma_2 = \sigma_3 = 0.01 \\ \lambda_1 = \lambda_2 = \lambda_3 = 1.0 \end{cases} \quad (30)$$

Figures 1 and 2 present the state trajectories of Systems (27) and (29) with and without a controller. No synchronization can be achieved without a controller. But, synchronization is achieved within a finite time with a controller (5). In addition, according to Theorem 1, $T_{\max} = 1.6874$ can be computed using Formula (8). Therefore, Theorem 1 is effective.

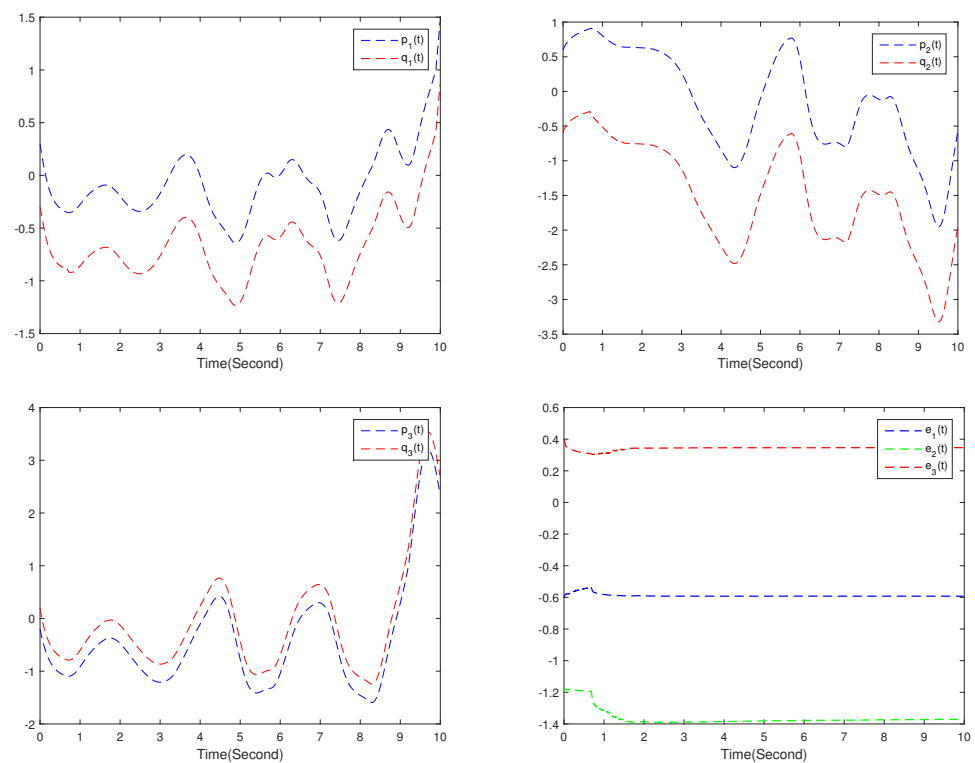


Figure 1. State trajectories of $p_i(t)$, $q_i(t)$, and $e_i(t)$ without controller.

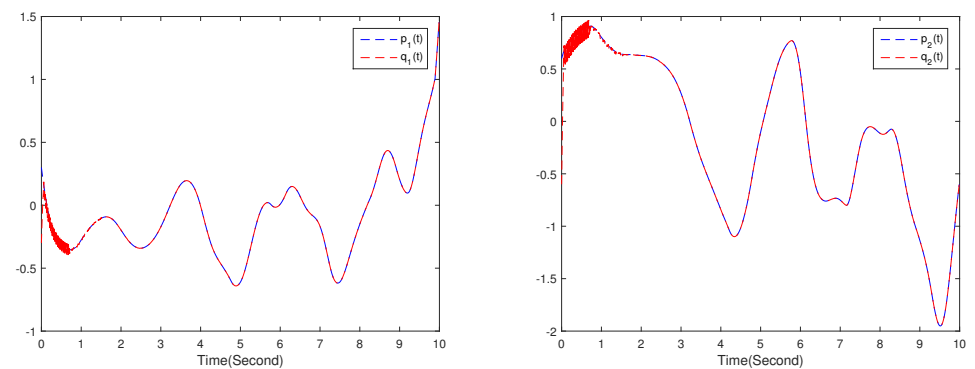


Figure 2. Cont.

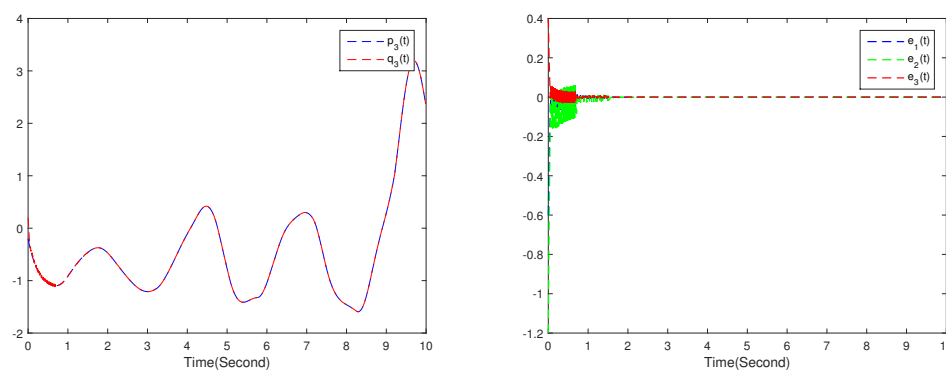


Figure 2. State trajectories of $p_i(t)$, $q_i(t)$, and $e_i(t)$ with controller.

Example 2. For System (27), the system's parameter selection is as follows:

$$\left\{ \begin{array}{l} \Gamma_1 = \Gamma_2 = \Gamma_3 = 1, \alpha_1 = \alpha_2 = \alpha_3 = 1, l = 1.8, \omega = 0.9 \\ \phi'_{11} = 2.0, \phi'_{12} = -1.1, \phi'_{13} = 2.1, \phi'_{21} = 1.1, \phi'_{22} = 1.8, \phi'_{23} = -1.3, \\ \phi'_{31} = -1.2, \phi'_{32} = 1.7, \phi'_{33} = 2.1, \phi''_{11} = 2.1, \phi''_{12} = -1.0, \phi''_{13} = 1.8, \\ \phi''_{21} = 0.8, \phi''_{22} = 1.5, \phi''_{23} = -1.0, \phi''_{31} = -1.1, \phi''_{32} = 2.1, \phi''_{33} = 1.5 \\ \psi'_{11} = -1.8, \psi'_{12} = -1.1, \psi'_{13} = 2.1, \psi'_{21} = 2.2, \psi'_{22} = 4.5, \psi'_{23} = -3.1 \\ \psi'_{31} = 2.1, \psi'_{32} = -1.8, \psi'_{33} = 5.1, \psi''_{11} = -2.1, \psi''_{12} = -0.5, \psi''_{13} = 1.8 \\ \psi''_{21} = 2.5, \psi''_{22} = 5.1, \psi''_{23} = -2.5, \psi''_{31} = 2.4, \psi''_{32} = -2.1, \psi''_{33} = 4.5 \\ a_{11} = a_{13} = a_{22} = a_{23} = a_{31} = a_{32} = 0.1, a_{12} = a_{21} = -0.2, a_{33} = -0.1 \\ b_{11} = b_{13} = b_{22} = b_{31} = 0.2, b_{12} = b_{21} = -0.2, b_{23} = b_{32} = 0.1, b_{33} = -0.1 \\ S_{ir} = 0.1, T_{ir} = 0.1, g_{ir} = 1.0, v_r = 1.0, i, r = 1, 2, 3 \end{array} \right. \quad (31)$$

Let $f_i(p(t)) = \tanh(|p(t)|) - 1$, $I_i = 0.1$, $L_i = 1$, $i = 1, 2, 3$. $\tau_1(t) = \tanh(t)$, $\tau_2(t) = \frac{e^t}{1+e^t}$. The initial values of System (27) are $\zeta_1(v) = 1$, $\zeta_2(v) = 0.6$, $\zeta_3(v) = -0.2$, $v \in [-\tau(t), 0]$.

The parameters of Response System (29) are the same as those in System (27). The initial values of System (29) are $\xi_1(v) = -1$, $\xi_2(v) = -0.6$, $\xi_3(v) = 0.8$, $v \in [-\tau(t), 0]$.

According to Theorem 2, the control gains $\hat{\theta}_i, \hat{\delta}_i, \hat{\eta}_i, i = 1, 2, 3$ should satisfy

$$\left\{ \begin{array}{l} \hat{\theta}_i \geq \alpha_i \\ \hat{\delta}_i \geq \frac{L_i}{\lambda_i} \sum_{r=1}^M \lambda_r |\phi_{ri}| \\ \hat{\eta}_i \geq \frac{L_i}{\lambda_i} \sum_{r=1}^M \lambda_r (|\psi_{ri}| + |a_{ri}| + |b_{ri}|) \end{array} \right.$$

Here, we choose

$$\left\{ \begin{array}{l} \hat{\theta}_1 = 1.0, \hat{\theta}_2 = 2.0, \hat{\theta}_3 = 3.0, \\ \hat{\delta}_1 = 4.4, \hat{\delta}_2 = 6.0, \hat{\delta}_3 = 6.5, \\ \hat{\eta}_1 = 8.0, \hat{\eta}_2 = 9.2, \hat{\eta}_3 = 11.0, \\ \gamma_1 = \gamma_2 = \gamma_3 = 2, \sigma_1 = \sigma_2 = \sigma_3 = 0.1 \\ \lambda_1 = \lambda_2 = \lambda_3 = 1.0, \kappa_1 = \kappa_2 = \kappa_3 = 1, \\ \rho_1 = \rho_2 = \rho_3 = 3, q_1 = q_2 = q_3 = 2, \\ \epsilon_1 = 3, \epsilon_2 = 4, \epsilon_3 = 5. \end{array} \right. \quad (32)$$

Let $\hat{\theta}_i(0) = 1, \hat{\delta}_i(0) = 1, \hat{\eta}_i(0) = 1$, and $i = 1, 2, 3$,

Figures 3 and 4 depict the state trajectories of Systems (27) and (29) with and without a controller. No synchronization can be achieved without a controller. But, synchronization is achieved within a finite time with a controller (6). In addition, according to Theorem 2, $T_{\max} = 0.8913$ when using Formula (21). As shown in Figures 5 and 6, the adaptive control gains $\kappa_i(t)$ and $\theta_{ir}(t)$, ($i, r = 1, 2, 3$) converge to certain values within a finite time. Therefore, Theorem 2 is effective.

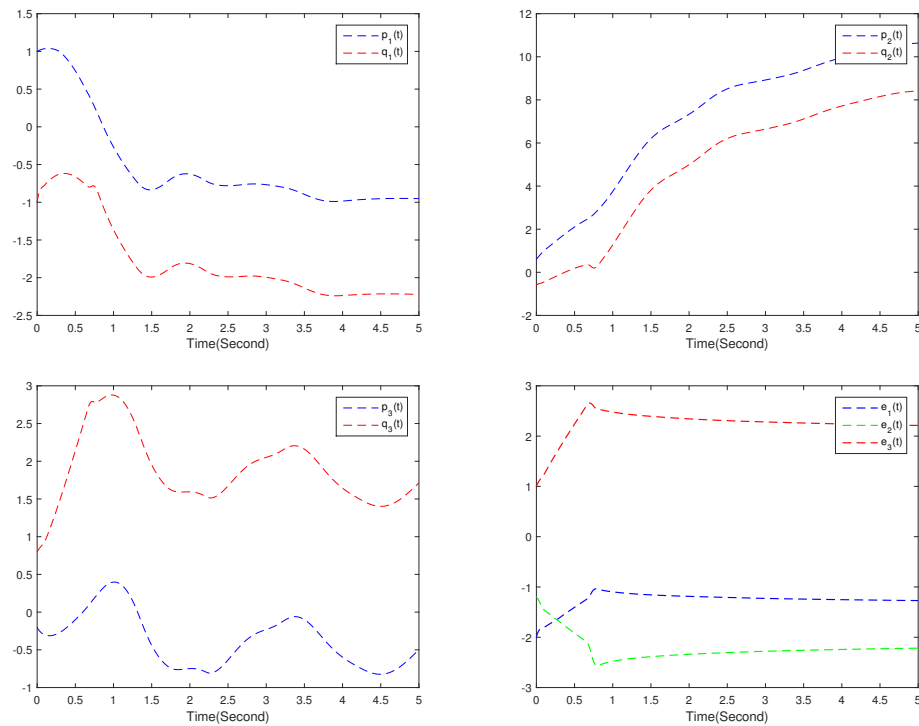


Figure 3. State trajectories of $p_i(t)$, $q_i(t)$, and $e_i(t)$ without controller.

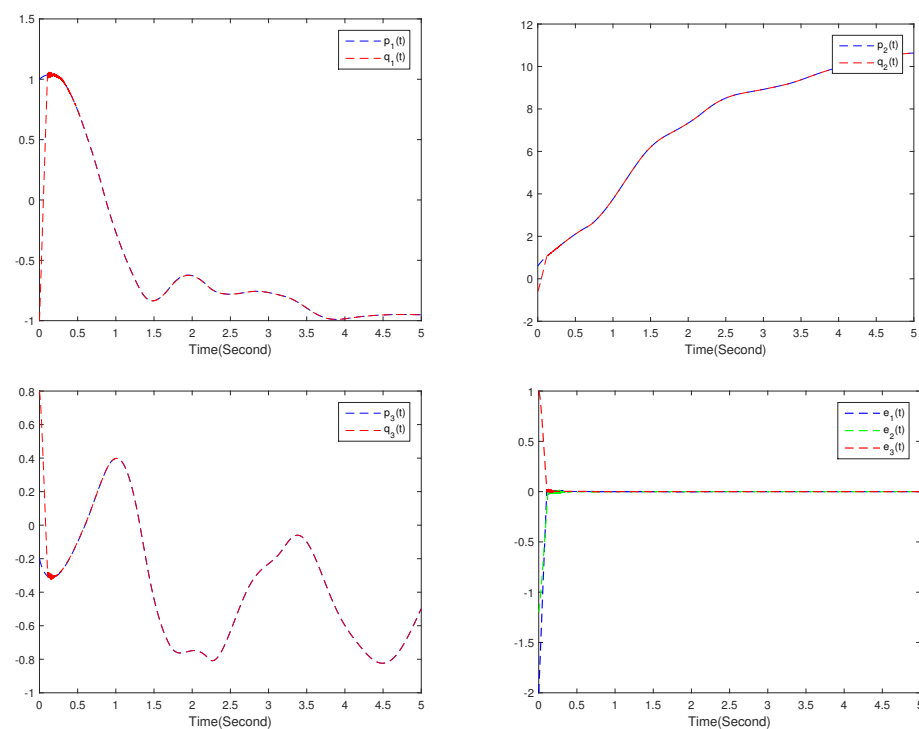


Figure 4. State trajectories of $p_i(t)$, $q_i(t)$, and $e_i(t)$ with controller.

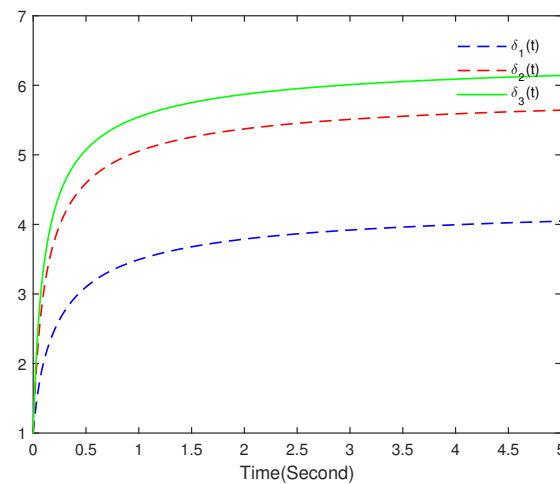


Figure 5. State trajectories of control gains $\delta_i(t)$, $i = 1, 2, 3$.

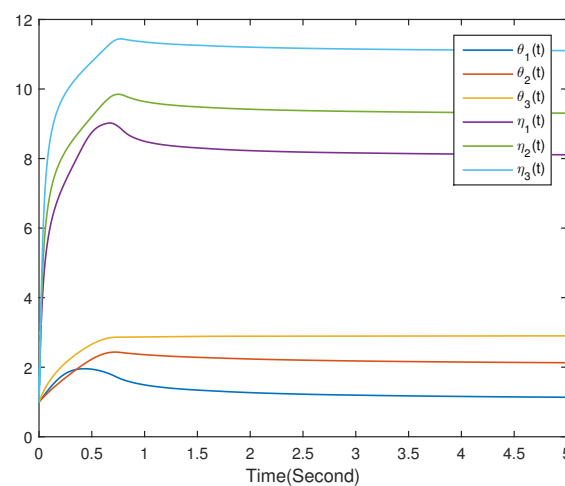


Figure 6. State trajectories of control gains $\theta_i(t)$ and $\eta_i(t)$, $i = 1, 2, 3$.

5. Conclusions

In this paper, the FTS and AFTS of FOMFNNs with leakage and transmission delays were investigated. Under a Filippov-like fractional-order differential inclusions theory and set-valued map, applying the Lyapunov functional method and the fractional-order differential inequalities allowed us to establish several new and useful criteria to achieve FTS and AFTS for FOMFNNs with leakage and transmission delays. In addition, the settling time is also presented. Finally, two numerical examples and their simulations are provided to demonstrate the validity of the obtained results. Future works will focus on the study of the described performance synchronization control for FOMFNNs with leakage and transmission delays.

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References

1. Struko, D.; Snider, G.; Stewart, D. The missing memristor found. *Nature* **2008**, *453*, 80–83. [\[CrossRef\]](#) [\[PubMed\]](#)
2. Yao, P.; Wu, H.; Gao, B.; Tang, J.; Zhang, Q.; Zhang, W.; Yang, J.J.; Qian, H. Fully hardware-implemented memristor convolutional neural network. *Nature* **2020**, *577*, 641–646. [\[CrossRef\]](#) [\[PubMed\]](#)
3. Chen, J.; Zeng, Z.; Jiang, P. Global Mittag-Leffler stability and synchronization of memristor-based fractional-order neural networks. *Neural Netw.* **2014**, *51*, 1–8. [\[CrossRef\]](#) [\[PubMed\]](#)
4. Zhang, J.; Lu, J.; Jin, X.; Yang, X.Y. Novel results on asymptotic stability and synchronization of fractional-order memristive neural networks with time delays: The $0 < \delta \leq 1$ case. *Neural Netw.* **2023**, *167*, 680–691.
5. Yao, X.; Liu, X.; Zhong, S. Exponential stability and synchronization of Memristor-based fractional-order fuzzy cellular neural networks with multiple delays. *Neurocomputing* **2021**, *419*, 239–250. [\[CrossRef\]](#)
6. Ding, Z.; Chen, C.; Wen, S.; Li, S.; Wang, L. Lag projective synchronization of nonidentical fractional delayed memristive neural networks. *Neurocomputing* **2022**, *469*, 138–150. [\[CrossRef\]](#)
7. Li, H.; Cao, J.; Hu, C.; Zhang, L.; Jiang, H. Adaptive control-based synchronization of discrete-time fractional-order fuzzy neural networks with time-varying delays. *Neural Netw.* **2023**, *168*, 59–73. [\[CrossRef\]](#)
8. Zhang, L.; Yang, Y. Finite time impulsive synchronization of fractional order memristive BAM neural networks. *Neurocomputing* **2020**, *384*, 213–224. [\[CrossRef\]](#)
9. Du, F.; Lu, J. New criterion for finite-time synchronization of fractional order memristor-based neural networks with time delay. *Appl. Math. Comput.* **2021**, *389*, 125616. [\[CrossRef\]](#)
10. Anbalagan, P.; Ramachandran, R.; Alzabut, J.; Hincal, E.; Niezabitowski, M. Improved results on finite-time passivity and synchronization problem for fractional-order memristor-based competitive neural networks: Interval matrix approach. *Fractal Fract.* **2022**, *6*, 36. [\[CrossRef\]](#)
11. Sun, Y.; Liu, Y.; Liu, L. Asymptotic and finite-time synchronization of fractional-order memristor-based inertial neural networks with time-varying delay. *Fractal Fract.* **2022**, *6*, 350. [\[CrossRef\]](#)
12. Wei, R.; Cao, J.; Alsaedi, A. Fixed-time synchronization of memristive Cohen-Grossberg neural networks with impulsive effects. *Int. J. Control Autom. Syst.* **2018**, *16*, 2214–2224. [\[CrossRef\]](#)
13. Ren, H.; Peng, Z.; Gu, Y. Fixed-time synchronization of stochastic memristor-based neural networks with adaptive control. *Neural Netw.* **2020**, *130*, 165–175. [\[CrossRef\]](#) [\[PubMed\]](#)
14. Zhao, Y.; Ren, S.; Kurths, J. Finite-time and fixed-time synchronization for a class of memristor-based competitive neural networks with different time scales. *Chaos Solitons Fractals* **2021**, *148*, 111033. [\[CrossRef\]](#)
15. Wei, R.; Cao, J.; Gorbachev, S. Fixed-time control for memristor-based quaternion-valued neural networks with discontinuous activation functions. *Cogn. Comput.* **2023**, *15*, 50–60. [\[CrossRef\]](#)
16. Jia, T.; Chen, X.; Yao, X.; Zhao, F.; Qiu, J. Adaptive fixed-time synchronization of delayed memristor-based neural networks with discontinuous activations. *CMES-Comput. Model. Eng. Sci.* **2023**, *134*, 221–239. [\[CrossRef\]](#)
17. Sun, Y.; Liu, Y.; Liu, L. Fixed-time synchronization for fractional-order cellular inertial fuzzy neural networks with mixed time-varying delays. *Fractal Fract.* **2024**, *8*, 97. [\[CrossRef\]](#)
18. Zhao, F.; Jian, J.; Wang, B. Finite-time synchronization of fractional-order delayed memristive fuzzy neural networks. *Fuzzy Sets Syst.* **2023**, *467*, 108578. [\[CrossRef\]](#)
19. Du, F.; Lu, J. Adaptive finite-time synchronization of fractional-order delayed fuzzy cellular neural networks. *Fuzzy Sets Syst.* **2023**, *466*, 108480. [\[CrossRef\]](#)
20. Jian, J.; Duan, L. Finite-time synchronization for fuzzy neutral-type inertial neural networks with time-varying coefficients and proportional delays. *Fuzzy Sets Syst.* **2020**, *381*, 51–67. [\[CrossRef\]](#)
21. Gopalsamy, K. Leakage delays in BAM. *J. Math. Anal. Appl.* **2007**, *325*, 1117–1132. [\[CrossRef\]](#)
22. Alia, M.; Narayanana, G.; Sarohab, S.; Priya, B.; Thakur, G.K. Finite-time stability analysis of fractional-order memristive fuzzy cellular neural networks with time delay and leakage term. *Math. Comput. Simul.* **2021**, *185*, 468–485.
23. Yan, H.; Qiao, Y.; Duan, L.; Miao, J. New results of quasi-projective synchronization for fractional-order complex-valued neural networks with leakage and discrete delays. *Chaos Solitons Fractals* **2022**, *159*, 112121. [\[CrossRef\]](#)
24. Zhu, M.; Wang, B.; Wu, Y. Stability and Hopf bifurcation for a quaternion-valued three-neuron neural network with leakage delay and communication delay. *J. Frankl. Inst.* **2023**, *360*, 12969–12989. [\[CrossRef\]](#)
25. Yuan, S.; Wang, Y.; Zhang, X. Global exponential synchronization of switching neural networks with leakage time-varying delays. *Commun. Nonlinear Sci. Numer. Simul.* **2024**, *133*, 107979. [\[CrossRef\]](#)
26. Zhang, H.; Cheng, J.; Zhang, H.; Zhang, W.; Cao, J. Quasi-uniform synchronization of Caputo type fractional neural networks with leakage and discrete delays. *Chaos Solitons Fractals* **2021**, *152*, 11432. [\[CrossRef\]](#)

27. Kao, Y.; Li, Y.; Park, J.; Chen, X. Mittag-Leffler synchronization of delayed fractional memristor neural networks via adaptive control. *IEEE Trans. Neural Netw. Learn. Syst.* **2021**, *32*, 2279–2284. [[CrossRef](#)] [[PubMed](#)]
28. Yang, S.; Yu, J.; Hu, C.; Jiang, H. Finite-time synchronization of memristive neural networks with fractional-order. *IEEE Trans. Syst. Man Cybern. Syst.* **2021**, *51*, 3739–3750. [[CrossRef](#)]
29. Filippov, A. *Differential Equations with Discontinuous Righthand Sides*; Kluwer Academic Publishers: Boston, MA, USA, 1988.
30. Li, C.; Deng, W. Remarks on fractional derivatives. *Aied Math. Comput.* **2007**, *187*, 777–784. [[CrossRef](#)]
31. Zhang, S.; Yu, Y.; Wang, H. Mittag-Leffler stability of fractional-order hopfield neural networks. *Nonlinear Anal. Hybrid Syst.* **2015**, *16*, 104–121. [[CrossRef](#)]
32. Zheng, M.; Li, L.; Peng, H.; Xiao, J.; Yang, Y.; Zhang, Y.; Zhao, H. Fixed-time synchronization of memristor-based fuzzy cellular neural network with time-varying delay. *J. Frankl. Inst.* **2018**, *355*, 6780–6809. [[CrossRef](#)]
33. Sun, Y.; Liu, Y. Fixed-time synchronization of delayed fractional-order memristor-based fuzzy cellular neural networks. *IEEE Access* **2020**, *8*, 165951–165962. [[CrossRef](#)]

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