



Article

Optimal Coordination of Directional Overcurrent Relays Using an Innovative Fractional-Order Derivative War Algorithm

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Abstract: Efficient coordination of directional overcurrent relays (DOCRs) is vital for maintaining the stability and reliability of electrical power systems (EPSs). The task of optimizing DOCR coordination in complex power networks is modeled as an optimization problem. This study aims to enhance the performance of protection systems by minimizing the cumulative operating time of DOCRs. This is achieved by effectively synchronizing primary and backup relays while ensuring that coordination time intervals (CTIs) remain within predefined limits (0.2 to 0.5 s). A novel optimization strategy, the fractional-order derivative war optimizer (FODWO), is proposed to address this challenge. This innovative approach integrates the principles of fractional calculus (FC) into the conventional war optimization (WO) algorithm, significantly improving its optimization properties. The incorporation of fractional-order derivatives (FODs) enhances the algorithm's ability to navigate complex optimization landscapes, avoiding local minima and achieving globally optimal solutions more efficiently. This leads to the reduced cumulative operating time of DOCRs and improved reliability of the protection system. The FODWO method was rigorously tested on standard EPSs, including IEEE three, eight, and fifteen bus systems, as well as on eleven benchmark optimization functions, encompassing unimodal and multimodal problems. The comparative analysis demonstrates that incorporating fractional-order derivatives (FODs) into the WO enhances its efficiency, enabling it to achieve globally optimal solutions and reduce the cumulative operating time of DOCRs by 3%, 6%, and 3% in the case of a three, eight, and fifteen bus system, respectively, compared to the traditional WO algorithm. To validate the effectiveness of FODWO, comprehensive statistical analyses were conducted, including box plots, quantile–quantile (QQ) plots, the empirical cumulative distribution function (ECDF), and minimal fitness evolution across simulations. These analyses confirm the robustness, reliability, and consistency of the FODWO approach. Comparative evaluations reveal that FODWO outperforms other state-of-the-art nature-inspired algorithms and traditional optimization methods, making it a highly effective tool for DOCR coordination in EPSs.



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1. Introduction

1.1. Conceptual Foundations and Motivational Frameworks

An effective protection plan is crucial for the reliable and consistent operation of a power network. This strategy would enable the swift detection and isolation of issues to maintain power supply to unaffected areas. DOCRs are critical elements of the protection strategy in multi-loop power systems, improving the system's efficiency and dependability. The setup and functioning of DOCRs are regulated by two essential parameters: the time dial setting (TDS) and the plug setting (PS). These parameters are essential for optimizing and coordinating the operation of the DOCRs. The TDS governs the relay response time, signifying a compromise between swift fault clearing and selectivity. Lowering the TDS will quickly reveal the fault but may lead to undesirable activities during transitional phases. An increased TDS enhances selectivity but may extend the fault clearance duration, potentially damaging equipment. The PS, on the other hand, specifies the present value that will activate the relay. Lowering the PS increases the sensitivity of DOCRs in detecting low-magnitude faults; nevertheless, a diminished PS may result in the incorrect tripping of DOCRs during non-fault events, such as load surges. Therefore, the sensitivity and selectivity parameters must be precisely calibrated to ensure the safety and reliability of the EPS. Effective DOCR coordination requires precise calculation of TDS and PS settings. With proper coordination, primary protection operates promptly and dependably with minimal impact on the fault. Additionally, DOCR settings must be synchronized to avert interference with other distance protection devices, hence preventing potential harm to adjacent equipment, which exacerbates the coordination difficulty. Traditional optimization methods often struggle with the non-linear, multimodal nature of the DOCR coordination problem. They can easily get trapped in local minima, leading to suboptimal solutions. These methods may also suffer from slow convergence rates and high computational costs, making them inefficient for large-scale power systems.

To address these challenges, a novel optimization strategy, the fractional-order derivative war optimizer (FODWO), is proposed. FODWO integrates the principles of fractional calculus (FC) into the conventional war optimization (WO) algorithm, significantly improving its optimization properties. The incorporation of fractional-order derivatives (FODs) enhances the algorithm's ability to navigate complex optimization landscapes, avoiding local minima and achieving globally optimal solutions more efficiently. This leads to the reduced cumulative operating time of DOCRs and improved reliability of the protection system.

1.2. Literature Review

To address the intricate challenges of DOCR coordination, numerous methodologies have been proposed in the literature. For instance, a linear programming approach was adopted in [1] to tackle overcurrent relay coordination, while [2] employed graphical sequential programming for relay configuration. The linear programming approach is straightforward and computationally efficient but may not handle non-linearities and complexities of real-world systems effectively. Graphical sequential programming offers a visual understanding but may lack scalability for large power systems. In [3], an expert system based on the minimum breakpoint set was utilized. A hybrid method combining interval linear programming and differential evolution was introduced in [4] to optimize DOCR coordination in microgrids. Meanwhile, ref. [5] presented an enhanced Elite Marine Predator Algorithm (EMPA), which demonstrated superior performance in DOCR coordination compared to conventional techniques. The EMPA is effective in exploring the solution space but suffers from slow convergence in complex scenarios. In distribution systems, ref. [6] proposed a method to optimize DOCR coordination by incorporating

distributed generation and fault current limiters, with a focus on the voltage sag energy index. While this method addresses specific issues in distribution systems, it may not be directly applicable to other network configurations. Other approaches include the curve intersection method [7] and graphical selection techniques [8], which provide simple and intuitive solutions but lack robustness and adaptability to varying network conditions. Researchers in [9–12] applied nature-inspired algorithms, such as whale optimization and JAYA, to resolve DOCR coordination challenges. These algorithms are robust and adaptable but require extensive parameter tuning and computational resources. The studies in [13,14] focused on optimizing system designs through physical and logical modifications. These approaches are effective in improving system reliability but involve high implementation costs and complexity. Hybrid genetic algorithms were explored in [15,16] to determine optimal TDS and PS values, considering scenarios like line or generator outages. Genetic algorithms are versatile and effective at exploring large solution spaces but converge slowly and require careful parameter tuning. The impact of single-line outage contingencies on DOCR coordination in multi-loop transmission systems was addressed in [17,18]. The importance of protection coordination is further emphasized by the risk of relay malfunctions due to changes in network configurations. In [19], fault current limiters were used to address coordination issues without considering line or distributed generation (DG) failures. New coordination parameters for microgrids were developed in [20] to handle line, substation, and DG failures. Optimal relay settings to minimize power outages were proposed in [21–24], taking into account primary system design. The DOCR coordination problem was formulated as a mixed-integer nonlinear programming (MINLP) problem and solved using population-based optimization techniques in [25,26]. Linear formulations combined with bio-inspired algorithms were developed in [27,28] to address DOCR coordination. Additionally, ref. [29] introduced a novel approach using non-standard multi-characteristics to enhance transient instability protection in power systems. This method is innovative but requires extensive validation for practical implementation. In [30], a new method for coordinating DOCRs and distance relays was proposed, leveraging various optimization algorithms to achieve significant improvements in performance and efficiency. Variants of particle swarm optimization (PSO) were explored in [31–34], to determine optimal DOCR settings. PSO variants are effective at exploring solution spaces but suffer from premature convergence and require careful parameter tuning. Modified differential evolution algorithms, which demonstrated superior performance, were reported in [35]. Eco-centric algorithms, such as the improved firefly algorithm (IFA), modified electromagnetic field optimization (MEFO), grey wolf optimizer (GWO), teaching–learning-based optimization (TLBO), biology-based optimization (BBO), and backtracking algorithms, were applied in [36–42] for DOCR coordination. A modified teaching-based optimization technique was used in [43], and an analytical approach was investigated in [44] to solve the DOCR problem. Enhanced group search algorithms were employed in [45], and an alternative firefly algorithm was applied in [46] for different IEEE test systems.

Current contributions indicate that the incorporation of fractional calculus (FC) and fractional derivatives in system models enhances system behavior, yielding substantial performance improvements in scientific and industrial applications. They have been effectively employed in domains such as feature selection, image processing, hyperspectral photography, robotic trajectory prediction, Kalman filtering, and fractional-order (FO) filtering. It is proposed that the integration of FC techniques with evolutionary algorithms is an emerging trend in optimization challenges within the energy sector. FOPSO has been effectively utilized in several applications, including Otsu image segmentation via a FO moth flame optimizer and FO Darwinian PSO for Kalman filtering [47–53]. These applications demonstrate the versatility and potential of FC techniques in improving algo-

rhythmic performance. However, the sensitivity of FO metaheuristic approaches to parameter configurations, including FO settings, inertial weights, and acceleration coefficients, is a significant challenge. The design of high-performance multiloop EPS stabilizers utilizes a hybrid dynamic genetic algorithm–PSO with lead-lag compensators [54], nonlinear system detection [55], the economic load dispatch problem addressed through FO swarm optimizers [56], a hybrid fractional computing framework employing a gravitational search algorithm for distribution-overcurrent relays [57], and optimal reactive power planning utilizing hybrid FO evolutionary strategies [58]. This study enhances research domains by advocating for the advancement of cutting-edge metaheuristic algorithms utilized in the energy and power sector through the integration of FC tools. FO metaheuristic approaches exhibit sensitivity to parameter configurations, including FO settings, inertial weights, and acceleration coefficients. The inappropriate selection of specific parameters may result in the algorithm failing to converge (premature convergence), making no progress (stagnation), or exhibiting oscillation, necessitating initial adjustment and optimization. In the context of EPSs, the DOCR coordination issue in multi-source networks is considered an optimization challenge. Conventional optimization techniques and metaheuristic algorithms often provide solutions that are suboptimal compared to those at the local optimum. This study evaluates the FODWO algorithm as an accurate and efficient way for coordinating DOCRs, showing its superiority over several sophisticated techniques.

1.3. Key Contributions and Research Framework

In EPSs, achieving perfect coordination of DOCRs demands meticulous adjustment of settings such as the PS and TDS. These settings are critical to ensuring the effectiveness of primary protection while meeting essential standards like sensitivity, reliability, selectivity, and speed. However, determining optimal settings for all operational scenarios is challenging due to uncertainties such as load fluctuations—including unexpected events or current surges—and variations in system configurations, such as radial, parallel, ring, or interconnected topologies.

Conventional computational approaches often struggle to identify global solutions, frequently getting trapped in local optima because of the presence of multiple optimal points, continuous variables like the PS and TDS, nonconvex characteristics, and nonlinear coordination time equations. To deal with these limitations, this study introduces a novel optimization technique called the fractional-order derivative war optimization (FODWO). This method enhances the traditional WO by integrating FC into its mathematical framework, improving its convergence rate and overall optimization performance.

The FODWO approach is assessed by minimizing the cumulative operating time for DOCR coordination, which enables faster fault detection and isolation, reducing system downtime and enhancing operational efficiency. This improvement is vital for maintaining system stability and ensuring uninterrupted operation, particularly in complex EPSs. The optimization is accomplished by fine-tuning control settings such as the PS and TDS. To validate the effectiveness of FODWO, three case studies based on IEEE standard systems are analyzed.

This research highlights the following key contributions:

- Enhancing the mathematical model of the WO algorithm by incorporating FC and fractional derivatives (FDs) to improve its optimization capabilities, particularly its convergence speed.
- Validating the performance of FODWO by solving eleven benchmark functions, incorporating both unimodal and multimodal problems, and evaluating the mean fitness value over a hundred autonomous runs.
- Applying a novel FODWO algorithm to minimize and improve the cumulative time of operation of DOCRs in standard test systems by altering TDS and PS values.

- Designing the FODWO scheme to reduce the cumulative time of operation of DOCRs in orthodox networks by constraining the PS and TDS within tolerable limits, accounting for various topological and working conditions.
- Developing statistical study plots, such as QQ, ECDF, stairs, and box plots, to assess the accuracy, robustness, and stability of the FODWO across autonomous runs.

2. Problem Formulation for DOCRs

The core objective of DOCR coordination is to quickly sense faults and separate the affected areas. To achieve this, it is essential to determine the optimal TDS and PS for each DOCR. The objective is to reduce the cumulative operating time (T_{op}) of all primary DOCRs while satisfying the constraints outlined by the objective function.

The mathematical representation of this objective is given by

$$\min f = \sum_{i=1}^n T_{i,K}, \quad (1)$$

where $T_{i,K}$ represents the operating time of the primary relay when a fault occurs in zone K. The operating curve for relay R_i is selected based on a predefined set of decisions, following the International Electrotechnical Commission (IEC) standards, as described by the following equation:

$$T_{op} = TDS_i \left[\frac{\alpha}{\left(\frac{IF_i}{PS \times CTR} \right)^k - 1} \right], \quad (2)$$

In this equation, T_{op} , IF , PS , TDS , and CTR represent the cumulative operating time, fault current, plug setting, time dial setting, and current transformer ratio, respectively, for typical inverted-type relays. The constants (α , k) are set to 0.14 and 0.02, respectively. Figure 1 provides a graphic representation that demonstrates the coordination of DOCRs within an EPS.

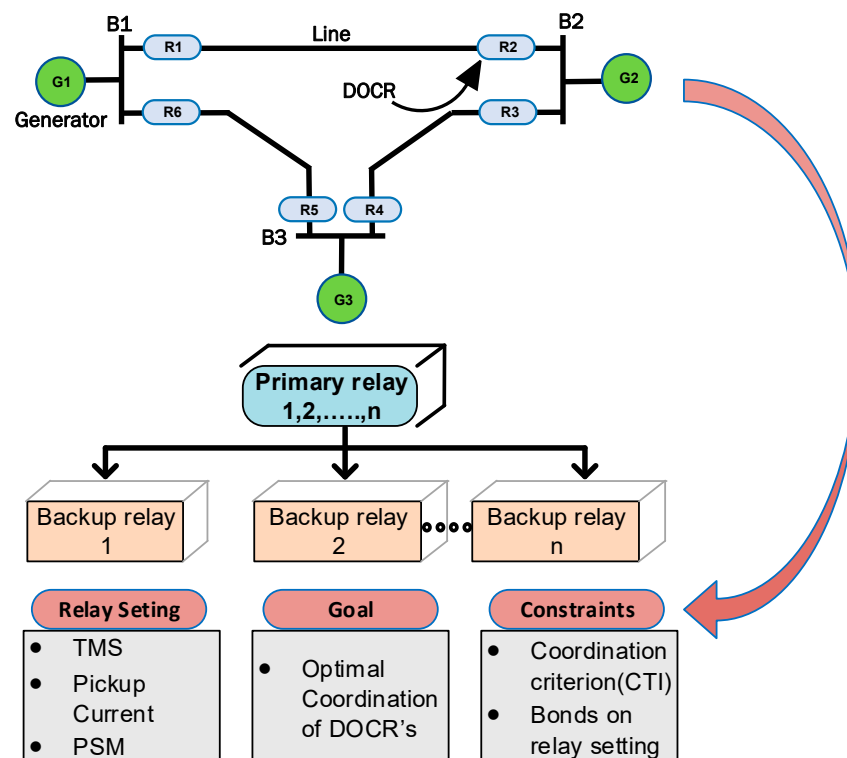


Figure 1. IEEE 3 bus electrical power network with DOCR coordination.

2.1. Synchronization Criterion

The Coordination Time Interval (CTI) ensures synchronization between primary and backup protection schemes in an EPS. Depending on various events and factors, the CTI limit is from 0.2 to 0.5 s. This section can be described as follows:

$$T_b \geq T_p + CTI, \quad (3)$$

where

T_p : operating time for the primary relays.

T_b : operating time for the backup relays.

2.2. Bounds for Relay Settings

The T_{op} can be minimized by considering two key factors: relay-specific constraints and coordination limits. The primary constraints establish the acceptable ranges for the TDS and PS, while secondary constraints ensure proper coordination between the primary and backup relays. The permissible values for relay configuration parameters are determined by these constraints and the relay's design specifications. These values fall within the following defined ranges:

$$TDS_i^{min} \leq TDS_i^{max} \quad (4)$$

$$PS_i^{min} \leq PS_i \leq PS_i^{max}, \quad (5)$$

3. Design Framework

We integrated the concepts of fractional mathematics into the structure of the conventional WO to solve the controlled optimization challenge. To assess the performance and efficiency of this enhanced optimization algorithm, three test bus systems (IEEE three, eight, and fifteen) were taken. The workflow of the proposed methodology is illustrated in Figure 2.

3.1. War Optimization (WO) Algorithm

Ancient kingdoms maintained armies to defend against attacks from rival dynasties. These armies consisted of various units, including infantry, chariots, and elephants. Each kingdom devised a "Vyuha", a specific formation or arrangement of these units, to launch assaults against opposing forces [59,60]. The emperor and commander-in-chief of each army would direct their troops in strategic patterns to achieve their objectives. War plans were developed based on the goals, risks, challenges, and opportunities presented in each conflict. Military coordination and engagement with the enemy were dynamic and continuous processes, adapting to the timing of events and the evolving battlefield conditions. The roles of the king and commanders influenced the status and movements of soldiers over time. Flags prominently displayed the positions of army commanders and the king, ensuring visibility to all troops. Warriors were trained to follow their strategies based on the sound of drums or other musical instruments. In the event of a general's death, a predetermined plan guided other generals on how to reorganize and maintain the war strategy. While individual soldiers aimed to attack the enemy and gain rank, the king's primary goal was to defeat the opposing leader. The following section outlines the various steps involved in military strategy.

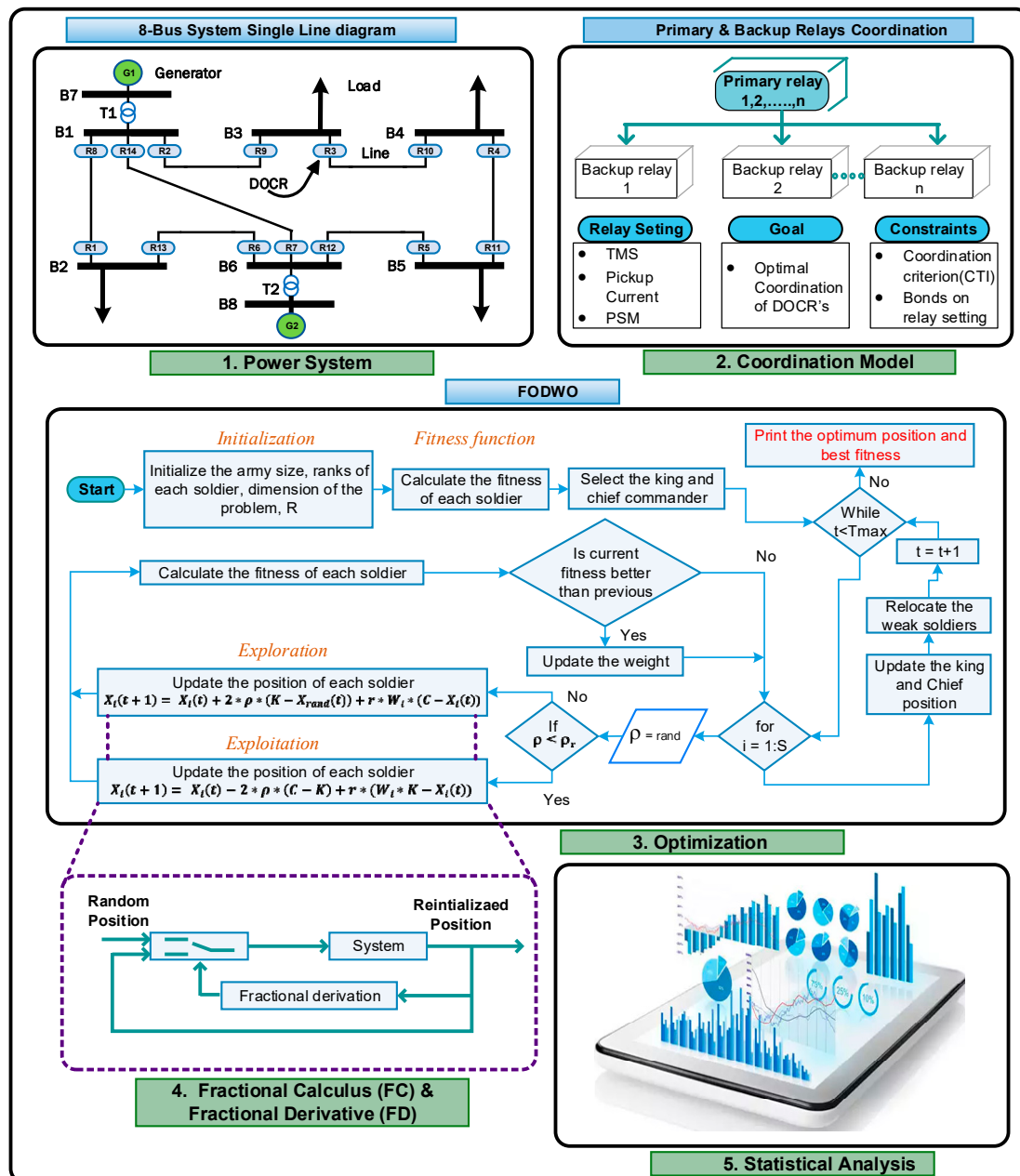


Figure 2. Proposed methodology (FODWO) workflow.

3.1.1. Random Attack

To launch an attack on the enemy forces, the army is evenly and randomly distributed. The army head possesses significant assault capability, making him the most powerful member of the military. The king oversees the various army heads.

3.1.2. Attack Strategy

The primary objective of this tactic is to attack the enemy forces. The king takes command, issuing directives to the army personnel. The soldiers conducting the assault identify the vulnerable points in the opposition's defenses (promising search areas). The king and the commander each drive distinct chariots with flags atop them. The soldiers' movements are influenced by the positions of the king and the commander. If a soldier successfully enhances his assault capabilities, his rank will increase. Conversely, if the new position is not suitable for combat, the soldier returns to his original location. At the

onset of the conflict, army units disperse in all directions, making significant movements to change their positions.

3.1.3. Signaling by Drums

Based on the evolving situation, the king issues commands to adjust the strategy. Consequently, a squad of troops rhythmically beats the drums. Following these rhythmic signals, the soldiers modify their tactics and reposition themselves accordingly.

3.1.4. Defense Strategy

The primary objective of this tactic is to ensure the king's safety. The commander or army chief, with the assistance of the troops, forms a protective chain around the king. Each soldier adjusts their position based on the surrounding soldiers and the king's stance. Throughout the battle, the army personnel endeavor to cover a significant portion of the battlefield (search space). The army dynamically modifies its strategy periodically to confuse and outmaneuver the opposing forces.

3.2. Mathematical Framework of the War Strategy

Two distinct military tactics have been developed. In the first approach, the position of each soldier is dynamically updated based on the locations of the king and the commander. This offensive strategy is visually represented and explained in Figure 3. The king positions himself strategically to deliver a decisive strike against the enemy. The soldier demonstrating the highest combat effectiveness is crowned as the king. Initially, all soldiers share the same rank and weight. Successful implementation of the strategy leads to a promotion in rank for the soldier. As the battle unfolds, the ranks and weights of all soldiers are recalibrated based on the success of the tactics employed. Toward the conclusion of the battle, the king, army chief, and soldiers converge closely, ensuring coordinated action.

$$X_i(t+1) = X_i(t) + 2 \times \rho \times (C - K) + rand \times (W_i \times K - X_i(t)) \quad (6)$$

In this scenario, $X_i(t+1)$ represents the updated position, X_i indicates the prior position, C corresponds to the commander's location, K denotes the king's position, and W_i symbolizes the weight. If the attack effectiveness (fitness) at the new position, F_n , is less than that at the previous position, F_p , the soldier returns to their original position.

$$X_i(t+1) = (X_i(t+1)) \times (F_n \geq F_p) + (X_i(t)) \times (F_n < F_p) \quad (7)$$

The soldier rank R_i will become high if the position is updated successfully.

$$R_i = (R_i + 1) \times (F_n \geq F_p) + (R_i) \times (F_n < F_p) \quad (8)$$

Based on the new rank, the weight will be updated as follows:

$$W_i = W_i \times \left(1 - \frac{R_i}{Max_iter}\right)^\alpha \quad (9)$$

Alpha (α) is a tunable parameter used to determine the second strategic position update, depending on the location of a randomly selected soldier, the commander, and the king. However, the process for rank and weight updating remains unchanged.

$$X_i(t+1) = X_i(t) + 2 \times \rho \times (K - X_{rand}(t)) + rand \times W_i \times (c - X_i(t)) \quad (10)$$

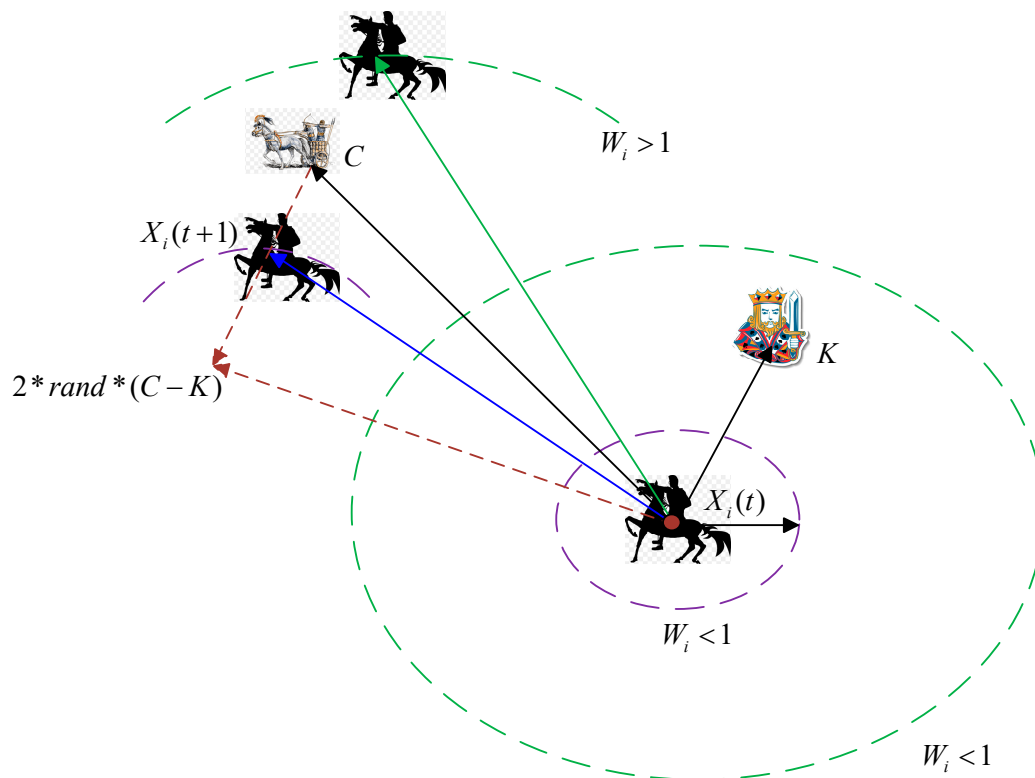


Figure 3. Attack strategy in WSO.

3.2.1. Fractional Calculus

Fractional calculus is a branch of mathematical analysis that extends the concepts of differentiation and integration to non-integer orders [61,62]. While classical calculus deals with derivatives and integrals of integer orders (e.g., first derivative, second derivative, etc.), fractional calculus generalizes these operations to real or complex numbers. This allows for the modeling of systems with memory effects, long-range dependencies, and fractal structures, which cannot be adequately described by classical calculus. The application of FC has gained considerable attention across diverse scientific disciplines, including engineering, computational mathematics, and physics, due to its ability to extend the principles of integer-order calculus and overcome its limitations.

Fractional calculus is primarily characterized by three fundamental concepts: the fractional derivative (differentiation of non-integer order), the fractional integral (integration of non-integer order), and non-locality. The non-locality property signifies that fractional derivatives and integrals are non-local operators, meaning their evaluation depends on the entire history of the function rather than its local behavior.

Several methods exist for defining and computing fractional-order derivatives, including the Riemann–Liouville fractional derivative, Caputo fractional derivative, Grünwald–Letnikov fractional derivative, Weyl fractional derivative, Hadamard fractional derivative, and Riesz fractional derivative. Among these, the Grünwald–Letnikov fractional derivative is particularly well suited for numerical implementation and computational simulations due to its discrete nature. It offers an intuitive interpretation of fractional derivatives as a weighted sum of past function values, highlighting the memory effect inherent in fractional calculus. Additionally, this approach approximates classical deriva-

tives when the order parameter δ approaches an integer value. The Grünwald–Letnikov fractional derivative of order δ (where $\delta > 0$) for a function $s(t)$ is defined as follows:

$$D^\delta[s(t)] = \lim_{h \rightarrow 0} \left[\frac{1}{h^\delta} \sum_{k=0}^{\infty} (-1)^k \binom{\delta}{k} f(t - kh) \right] \quad (11)$$

Let h represent the step size; $\tau(\cdot)$ is the gamma function and $\binom{\delta}{k}$ denotes the generalized binomial coefficient, defined as follows:

$$\binom{\delta}{k} = \frac{\tau(\delta + 1)}{\tau(\delta - k + 1)\tau(k + 1)} \quad (12)$$

The Grünwald–Letnikov formulation generalizes the first-order derivative approximation using finite differences to fractional orders by considering an infinite series of weighted function values at previous points $(t - kh)$. The weights associated with the function values $f(t - kh)$ depend on the fractional order δ and the index k . The step size h represents the interval between successive points $(t - kh)$, and as h approaches zero, the discrete summation converges to the fractional derivative.

3.2.2. Enhanced War Optimization Based on Fractional Calculus

The Grünwald–Letnikov theory offers a mathematical foundation for deriving fractional-calculus-based equations for war optimization using FODs. The proposed methodology integrates fractional calculus (FC) with the foundational war optimization (WO) algorithm to develop an FO variant of WO. This fusion is designed to tackle the challenge of rapid convergence while ensuring high-quality results. Although the WO algorithm excels in global exploration, its limited local search capability often leads to suboptimal convergence rates. To address this drawback, the proposed fractional-order derivative war optimization (FODWO) algorithm embeds FC into the core WO framework. During the exploitation phase, this enhancement leverages FC's inherent memory retention property, enabling the exchange of facts between solutions. As a result, both the convergence speed and precision of the solutions are significantly improved.

Justification for Fractional Calculus in FODWO

Memory Effects: The non-local property of fractional calculus allows the FODWO algorithm to consider the entire history of the optimization process, rather than just local changes. This memory effect helps in better navigating the optimization landscape, avoiding local minima, and accelerating convergence.

Complex Phenomena Modeling: FC is particularly adept at modeling complex phenomena such as irreversibility and disorder, thanks to its memory-dependent nature and unique characteristics. This makes FC an ideal tool for analyzing the dynamic and evolving trajectories generated by the WO algorithm.

Enhanced Search Capability: By incorporating fractional-order derivatives, the FODWO algorithm enhances the local search capabilities of the conventional WO algorithm. The location of each soldier is updated based on its position, utilizing the concept of fractional calculus previously defined (Equation (13)):

$$x_i(t + 1) = x_i(t) + \frac{1}{h^\delta} \sum_{k=0}^{\infty} (-1)^k \tau(\delta + 1) \times \frac{[r_1 \cdot (x_{\text{best}} - x_i(t - kh)) + r_2 \cdot (x_i(t - kh) - x_i(t - (k + 1)h))]}{\tau(\delta - k + 1)\tau(k + 1)} \quad (13)$$

where $x_i(t)$ is the position of the i – the soldier at time t , x_{best} is the best position, and δ is the fractional order of the derivative. Its range is from 0 to 1. Notably, when the fractional order δ is set to 1, the equation simplifies to a classical integer-order first derivative, aligning with conventional calculus. This reduction can be expressed as follows:

$$D^1[s(t)] = s(t+1) - s(t) \quad (14)$$

where h is the step size. r_1 and r_2 are random coefficients. τ is the gamma function. Memory Effects: the term $\frac{1}{h^\delta} \sum_{k=0}^{\infty} \frac{(-1)^k \tau(\delta+1)}{\tau(\delta-k+1)\tau(k+1)}$ captures the memory effects, allowing the current position to be influenced by previous positions.

Random Coefficients: The random coefficients r_1 and r_2 introduce stochasticity, making the algorithm adaptable to dynamic environments. **Position Components:** $r_1 \cdot (x_{best} - x_i(t - kh))$ represents the attraction towards the best position found so far. $r_2 \cdot (x_i(t - kh) - x_i(t - (k+1)h))$ accounts for the inertia or momentum of the soldier.

The weight updating mechanism is crucial for adjusting the influence of each soldier (or agent) based on their historical performance and current positions given in Equation (15). By incorporating fractional calculus, we can capture the memory effects and hereditary properties of the system, leading to a more nuanced and effective weight adjustment.

$$w_i(t+1) = w_i(t) + \frac{1}{h^\delta} \sum_{k=0}^{\infty} \frac{(-1)^k \tau(\delta+1) [r_3 \cdot (w_{best} - w_i(t - kh))]}{\tau(\delta - k + 1) \tau(k + 1)} \quad (15)$$

$w_i(t)$ is the weight of the i – the soldier at time t . w_{best} is the best weight. δ is the fractional order of the derivative. h is the step size. r_3 is a random coefficient. τ is the gamma function. Memory Effects: The term $\frac{1}{h^\delta} \sum_{k=0}^{\infty} \frac{(-1)^k \tau(\delta+1)}{\tau(\delta - k + 1) \tau(k + 1)}$ captures the memory effects, allowing the current weight to be influenced by previous weights. Random Coefficient: The random coefficient r_3 introduces stochasticity, helping different weight adjustments. Weight Components: $r_3 \cdot (w_{best} - w_i(t - kh))$ represents the attraction towards the best weight found so far, ensuring that weights are updated in a direction that has previously led to optimal performance. The pseudocode of this algorithm, specifically for the FODWO, is detailed as follows:

- Step 1: **Population Initialization.** Generate an initial population of search agents (soldiers) by randomly creating n agents. Each agent's dimensionality aligns with the number of controllable variables in the system. This population size is collectively represented by N and is taken as 30 for the algorithm.
- Step 2: **Fitness Calculation.** Assess the performance of each soldier by calculating its fitness value. This is achieved by inputting the agent's parameters into an objective function designed to quantify total operational time, thereby determining its effectiveness.
- Step 3: **Attack Strategy.** Soldiers move towards the best solution found so far (attack the enemy). This step involves updating the positions of soldiers based on their fitness values.
- Step 4: **Defense Strategy.** Soldiers also consider the worst solution found so far (defend against the weakest enemy). This step helps in maintaining diversity in the search space and prevents premature convergence.
- Step 5: **Update Positions.** Update the positions of soldiers using the attack and defense strategies. This involves calculating new positions based on the current best and worst solutions.

- Step 6: **Implement a fractional-order positioning strategy.** Refine each soldier's location by applying an FO update mechanism (refer to Equation (13)), which adjusts their current position relative to their prior coordinates.
- Step 7: **Establish termination conditions.** FODWO algorithm halts execution once a predefined maximum number of iterations is reached. The maximum number of iterations is taken as 200 for the algorithm.
- Step 8: **Archive optimal outcomes.** Determine the control variables for the DOCR problem by selecting the solution with the lowest active total operational time, corresponding to the optimal result identified by the moths or search agents.
- Step 9: **Comparative analysis.** A fair comparison with the state-of-the-art algorithms will be made, based upon the same parameters such as the population size and number of iterations.
- Step 10: **Perform statistical evaluations.** Analyze results from fifty independent runs using ECDF, stairs, QQ, and box plots, to assess performance consistency.

4. Results and Discussion

This segment analyzes the efficacy of the FODWO framework through rigorous testing on single-peak (unimodal) and multi-peak (multimodal) benchmark functions, supplemented by validation on an IEEE-standard test infrastructure. Benchmarking optimization algorithms against standardized unimodal and multimodal functions is a widely accepted validation methodology [59,63], where minimized error rates signify algorithmic superiority. To ensure robustness, eleven distinct benchmark functions are employed to scrutinize optimization performance across diverse problem landscapes. Functions F1–F7 are unimodal (single-solution search spaces), while F8–F11 represent multimodal challenges (complex, multi-solution environments). Empirical results demonstrate that FODWO consistently achieves superior performance across all experimental scenarios. Quantitative outcomes, summarized in Tables 1 and 2, highlight FODWO's stability and precision by reporting mean fitness values derived from 100 randomized trials. Comparative analysis reveals FODWO's dominance, surpassing all competing algorithms in accuracy and reliability during these assessments.

To demonstrate the efficacy of the FODWO algorithm, its performance in resolving DOCR coordination challenges—specifically, minimizing cumulative operational time—is rigorously assessed across three IEEE-standardized bus configurations: the three-bus, eight-bus, and fifteen-bus systems. These outcomes are compared against results from both established and emerging optimization techniques. The default settings for all of these algorithms used for comparative analysis are listed in Table 3. Each algorithm's robustness is verified through its application to continuous and discrete (mixed-integer) PTS-based frameworks, ensuring global applicability.

Table 1. Comparative performance analysis using benchmark functions (unimodal, multimodal).

F(x)	Dim	Moth Flame Opt [63]		Particle Swarm Opt [63]		Gravitational Search Opt [63]		Bat opt [63]		FODWO	
		Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
$F_1(x) = \sum_{i=1}^n x_i^2$	100	0.000117	0.00015	1.32115	1.15388	608.232	464.654	20,792.4	5892.40	4.23×10^{-34}	4.20×10^{-34}
$F_2(x) = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $	100	0.000639	0.000877	7.71556	4.13212	22.7526	3.36513	89.785	41.9577	4.02×10^{-17}	1.80×10^{-17}
$F_3(x) = \sum_{i=1}^n \left(\sum_{j=1}^i x_j \right)^2$	100	696.730	188.527	736.393	361.781	135,760	48,652.6	62,481.3	29,769.1	3.64×10^{-33}	4.64×10^{-33}
$F_4(x) = \max_i \{ x_i , 1 \leq i \leq n\}$	100	70.6864	5.27505	12.9728	2.63443	78.7819	2.81410	49.7432	10.1436	1.77×10^{-17}	6.40×10^{-18}
$F_5(x) = \sum_{i=1}^{n-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right]$	100	139.148	120.260	77,360.8	51,156.15	741.003	781.239	199,512	125,238	7.3340	0.1542
$F_6(x) = \sum_{i=1}^n ([x_i + 0.5])^2$	100	0.00011	9.87×10^{-5}	286.651	107.079	3080.96	898.635	17,053.4	4917.56	0.1210	0.0821
$F_7(x) = \sum_{i=1}^n ix_i^4 + \text{random}[0, 1)$	100	0.09115	0.04642	1.037316	0.310315	0.112975	0.037607	6.045055	3.045277	2.34×10^{-4}	1.73×10^{-4}
$F_8(x) = \sum_{i=1}^n -x_i \sin(\sqrt{ x_i })$	100	8496.78	725.873	3571	430.7989	2352.32	382.167	65535	0	2.51×10^3	317.3344
$F_9(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	100	84.600	16.1665	124.29	14.2509	31.0001	13.6605	96.2152	19.5875	0.2530	1.9868
$F_{10}(x) = -20 \exp \left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2} \right) - \exp \left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i) \right) + 20 + e$	100	1.2603	0.72956	9.1679	1.56898	3.74098	0.17126	15.9460	0.77495	3.98×10^{-15}	1.20×10^{-15}
$F_{11}(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos \left(\frac{x_i}{\sqrt{i}} \right) + 1$	100	0.0190	0.02173	12.418	4.16583	0.04978	0.04978	220.281	54.7066	0.0055	0.0291

Table 2. Comparative performance analysis using benchmark functions (unimodal, multimodal).

Functions	Dim	FPA [63]		SMS [63]		FA [63]		GA [63]		FODWO	
		Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
$F_1(x) = \sum_{i=1}^n x_i^2$	100	203.638	78.3984	120	0	7480.74	894.849	21,886.0	2879.58	4.23×10^{-34}	4.20×10^{-34}
$F_2(x) = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $	100	11.1687	2.91959	0.0205	0.00471	39.3253	2.46586	56.5175	5.66085	4.02×10^{-17}	1.80×10^{-17}
$F_3(x) = \sum_{i=1}^n \left(\sum_{j=1}^i x_j \right)^2$	100	237.56	136.6463	37820	0	17357.3	1740.11	37010.2	5572.21	3.64×10^{-33}	4.64×10^{-33}
$F_4(x) = \max_i \{ x_i , 1 \leq i \leq n\}$	100	12.5728	4 2.29	69.1700	3.87666	33.9535	1.86966	59.1433	4.64852	1.77×10^{-17}	6.40×10^{-18}
$F_5(x) = \sum_{i=1}^{n-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right]$	100	10,974.	12,057.2	638,224	729,967	3,795,009	75,9030.	3,132,141	5,264,496	7.3340	0.1542
$F_6(x) = \sum_{i=1}^n ([x_i + 0.5])^2$	100	175.38	63.4525	41,439.	3295.23	7828.72	975.210	20,964.8	3868.10	0.1210	0.0821
$F_7(x) = \sum_{i=1}^n ix_i^4 + \text{random}[0, 1)$	100	0.13594	0.061212	0.04952	0.024015	1.906313	0.460056	13.37504	3.08149	2.34×10^{-4}	1.73×10^{-4}
$F_8(x) = \sum_{i=1}^n -x_i \sin(\sqrt{ x_i })$	100	−8086.74	155.346	−3942.82	404.160	−3662.05	214.163	−6331.19	332.566	$−2.51 \times 10^3$	317.3
$F_9(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	100	92.6917	14.2239	152.844	18.5535	214.895	17.2191	236.82	19.0335	0.2530	1.9868
$F_{10}(x) = -20 \exp \left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2} \right) - \exp \left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i) \right) + 20 + e$	100	6.84483	1.24998	19.1325	0.23852	14.5676	0.46751	17.8461	0.53114	3.98×10^{-15}	1.20×10^{-15}
$F_{11}(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos \left(\frac{x_i}{\sqrt{i}} \right) + 1$	100	2.7160	0.72771	420.525	25.2561	69.6575	12.11393	179.904	32.4395	0.0055	0.0291

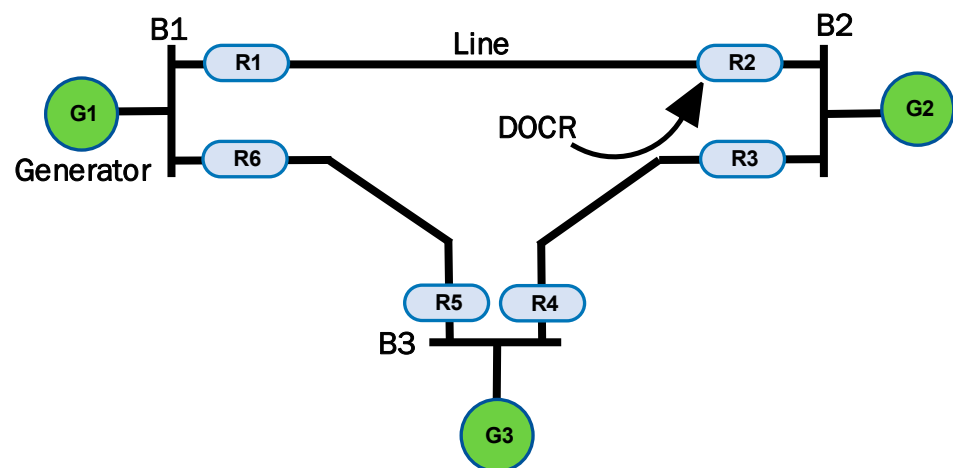
Table 3. Default settings for all of the algorithms used for comparative analysis.

Common Setting			Default Settings
Algorithm	Population Size	Number of Iteration	
JAYA	50	100	Distance-adaptive coefficient: dynamically adjusted during iterations.
OJAYA	50	100	
DJAYA	50	100	
LM	20	100	Constraints: CTI (0.2–0.3 s)
GA	50	100	Crossover probability (Pc): 0.8 Mutation probability (Pm): 0.01
GA-LP	50	100	Inertia weight = 0.7
PSO	30	1000	Perception value (β): 1.5 to 2 Learning coefficient (α): 0.5 to 1
SA	50	500	Perturbation factor: 0.01 to 0.1
EFO	50	200	Teaching factor (TF): 1 to 2
TLBO	20	500	Mutation or perturbation operators: sometimes added to avoid local minima
MTLBO	50	200	
GA	20	200	Crossover rate (Pc): 0.6 to 0.9 Mutation rate (Pm): 0.01 to 0.1
FAGA	50	100	Adaptive crossover rate (Pc): 0.9 to 1.0 Adaptive mutation rate (Pm): 0.1 to 0.3
BBO	50	200	Migration rate (λ, μ): 0.1 to 1.0
BBO-LP	50	200	Adaptive migration rate: adjusted dynamically

While the study focuses on phase relay coordination, the methodology remains extensible to ground relays due to their analogous operational principles. Simulations are solely focused on phase relays, as their behavior mirrors that of Earth relays in terms of inverse-time characteristics, fitness function linearity/nonlinearity, and constraint structures.

4.1. Test System 1: IEEE Three-Bus Configuration

The three-bus test system features three interconnected buses, three generation units, three transmission lines, and six protective relays (visualized in Figure 4). For evaluation, a symmetrical three-phase fault is simulated at the midpoint of each line. Critical parameters, including current transformer (CT) ratios, primary/backup relay pairings, and fault current magnitudes for each line, are adopted from prior research [25]. All relays in this configuration operate under the Inverse Definite Minimum Time (IDMT) tripping characteristic. To ensure compatibility with existing studies, the system is analyzed using mixed-integer nonlinear programming (MINLP) models, enabling direct comparisons with conventional optimization approaches.

**Figure 4.** Single-line diagram of IEEE three bus test system.

In this scenario, the optimization parameters are the TDS and PTS, with the TDS bounded between 0.1 and 1.1 and the PTS constrained within 1.5 to 5.0. The coordination challenge of DOCRs is formulated as an MINLP problem, treating both of the two variables as continuous.

4.1.1. Results and Discussions for Test System 1 (IEEE Three-Bus System)

The optimized TDS and PTS values derived from the WO and FODWO algorithms are detailed in Table 4. Table 5 further compares the net T_{op} achieved by FODWO and other advanced algorithms. The results confirm that the FODWO algorithm surpasses competing methods in minimizing DOCR operating times for the IEEE three-bus system. Table 4 also verifies that both WO and FODWO satisfy all constraints, with FODWO yielding superior DOCR configurations; i.e., in the IEEE three-bus system, FODWO reduces T_{op} by 3.91 s compared to TLBO, 3.36 s over MDE, 0.50 s over PSO, 0.18 s over SA and BBO-LP, 0.50 s over ABC, and 0.36 s over MFO, GA, and FA. The improvement over WO is 0.04 s. The percentage enhancement relative to TLBO is 73.34%, MDE is 70.24%, PSO is 26.13%, SA is 11.04%, BBO-LP is 11.02%, ABC is 26.13%, MFO is 20.10%, GA is 20.11%, FA is 20.10%, and WO is 2.83%. The computational complexity of the algorithm was evaluated based on the execution time of FODWO in comparison to WO. The WO algorithm required 2.8603 s, whereas FODWO completed the computation in 2.7861 s. These results indicate that FODWO exhibits lower computational complexity than WO.

Table 4. Optimized values of PTS and TDS for test system 1.

Relay #	WO		Relay #	FODWO	
	TDS	PTS		TDS	PTS
R_1	0.1000	2.2766	R_1	0.1129	1.6628
R_2	0.1000	1.8337	R_2	0.1001	1.5221
R_3	0.1001	2.0500	R_3	0.1087	1.6128
R_4	0.1107	1.5108	R_4	0.1001	2.0973
R_5	0.1001	2.1028	R_5	0.1000	1.6880
R_6	0.1001	1.8141	R_6	0.1001	1.5219
Objective function (s)	1.4640			1.4225	

Table 5. Comparative analysis using existing research (test system 1).

Algorithm	Objective Function (s)
SA [25]	1.599
PSO [34]	1.9258
MDE [35]	4.7806
BBO-LP [40]	1.59871
TLBO [41]	5.3349
ABC [46]	1.9258
FAGA [46]	1.78039
GA [46]	1.78047
MFA [46]	1.78039
WO	1.4640
FODWO	1.4225

Figure 5 highlights the convergence behavior of WO and FODWO during simulations, illustrating FODWO's faster convergence. Figure 6 quantifies the net improvement in T_{op} in seconds and Figure 7 in terms of percentage, emphasizing FODWO's dominance. Figure 8 compares the optimized operating times from the proposed algorithms against benchmarks in the literature, demonstrating their enhanced efficacy over existing state-of-the-art approaches.

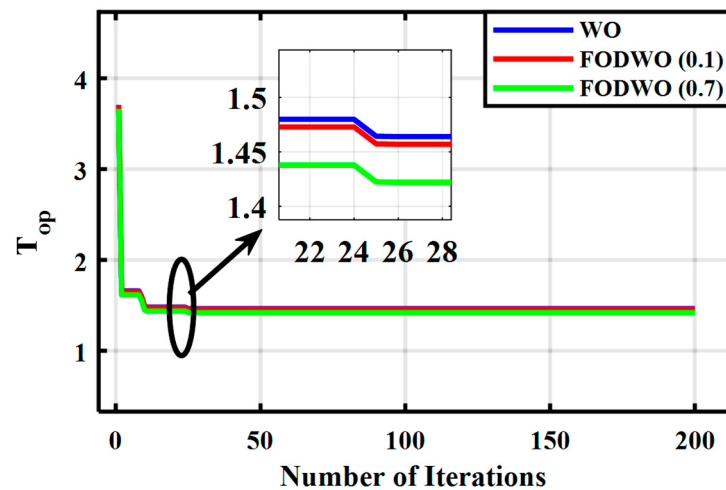


Figure 5. Convergence graph (test system 1).

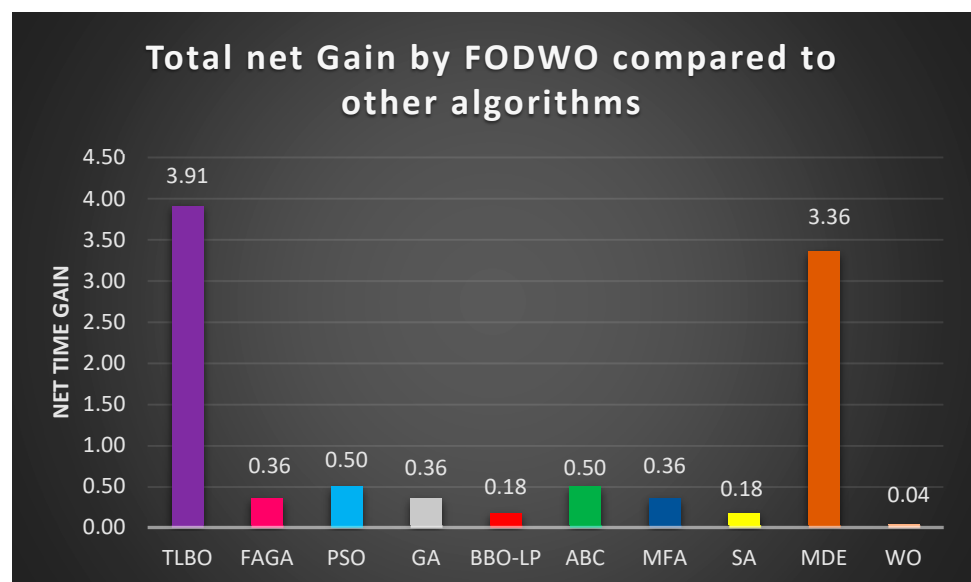


Figure 6. Total net gain by FODWO compared to other algorithms (test system 1).

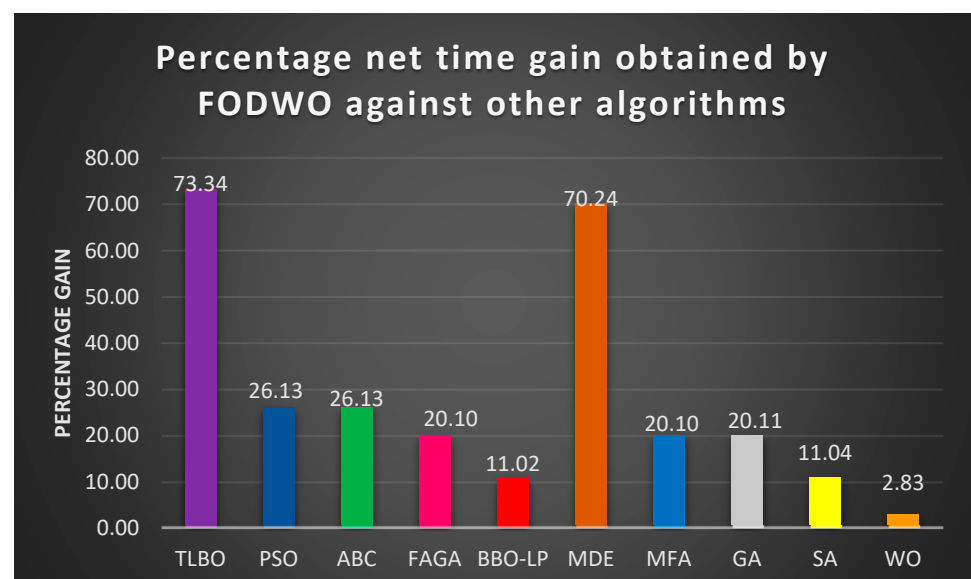


Figure 7. Percentage net time gain obtained by FODWO against other algorithms (test system 1).

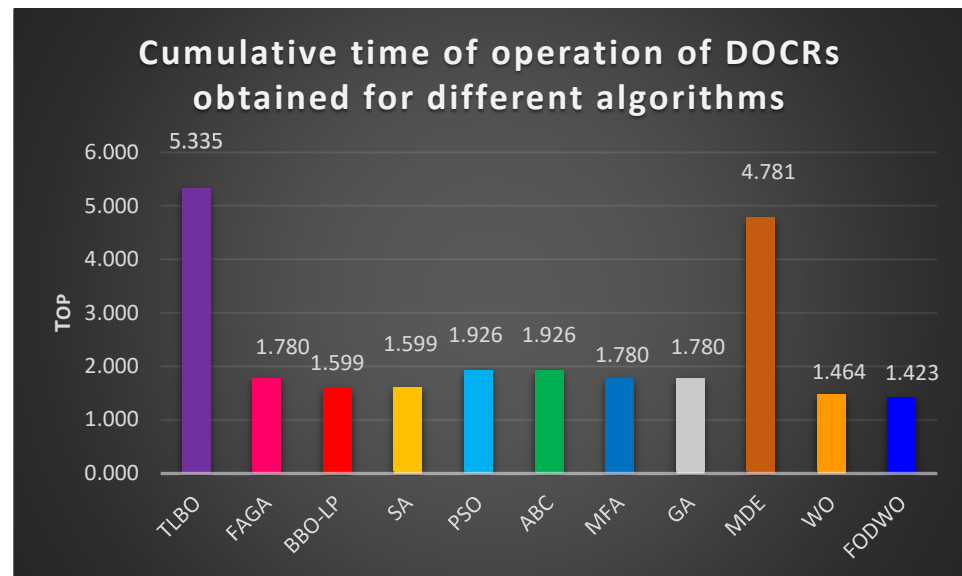


Figure 8. T_{op} for DOCRs obtained for different algorithms (test system 1).

4.1.2. Statistical Evaluation for IEEE Three-Bus Configuration (Test System 1)

To evaluate the dependability, robustness, and consistency of the proposed FODWO algorithm, an extensive statistical evaluation was carried out on the IEEE three-bus system to identify the optimal fractional order. For precision, 50 independent trials were executed, with the median of the outcomes serving as the benchmark for selecting the ideal fractional order. The statistical assessment utilized multiple analytical tools, such as ECDF, box, QQ, and stair plots across trials, as shown in Figure 9a–d. Figure 9a highlights that the likelihood of attaining an optimum solution using FODWO is markedly better than with traditional WO. Figure 9b reveals that the median solution across 50 trials is consistently reduced for FODWO relative to WO. Figure 9c underscores that FODWO attains favorable minimal fitness values compared to the normal distribution's quantiles. Figure 9d demonstrates FODWO's persistent superiority, maintaining lower fitness levels than WO over multiple independent runs. From these graphical analyses, it is evident that fractional calculus serves as an innovative mathematical framework for refining conventional optimization algorithms, significantly boosting their efficacy and reliability.

4.2. Test System 2: IEEE Eight-Bus Configuration

The IEEE eight-bus configuration is modeled as a mixed-integer nonlinear programming (MINLP) optimization framework. The network consists of eight buses, two generators, two transformers, seven transmission lines, and fourteen protective relays, as illustrated in Figure 10. The study examines a three-phase fault scenario located close to the power source, with a relay CTI fixed at 0.2 s. Values for the CT ratios and three-phase short-circuit currents corresponding to each primary-to-backup (P/B) relay pair are adapted from ref. [25].

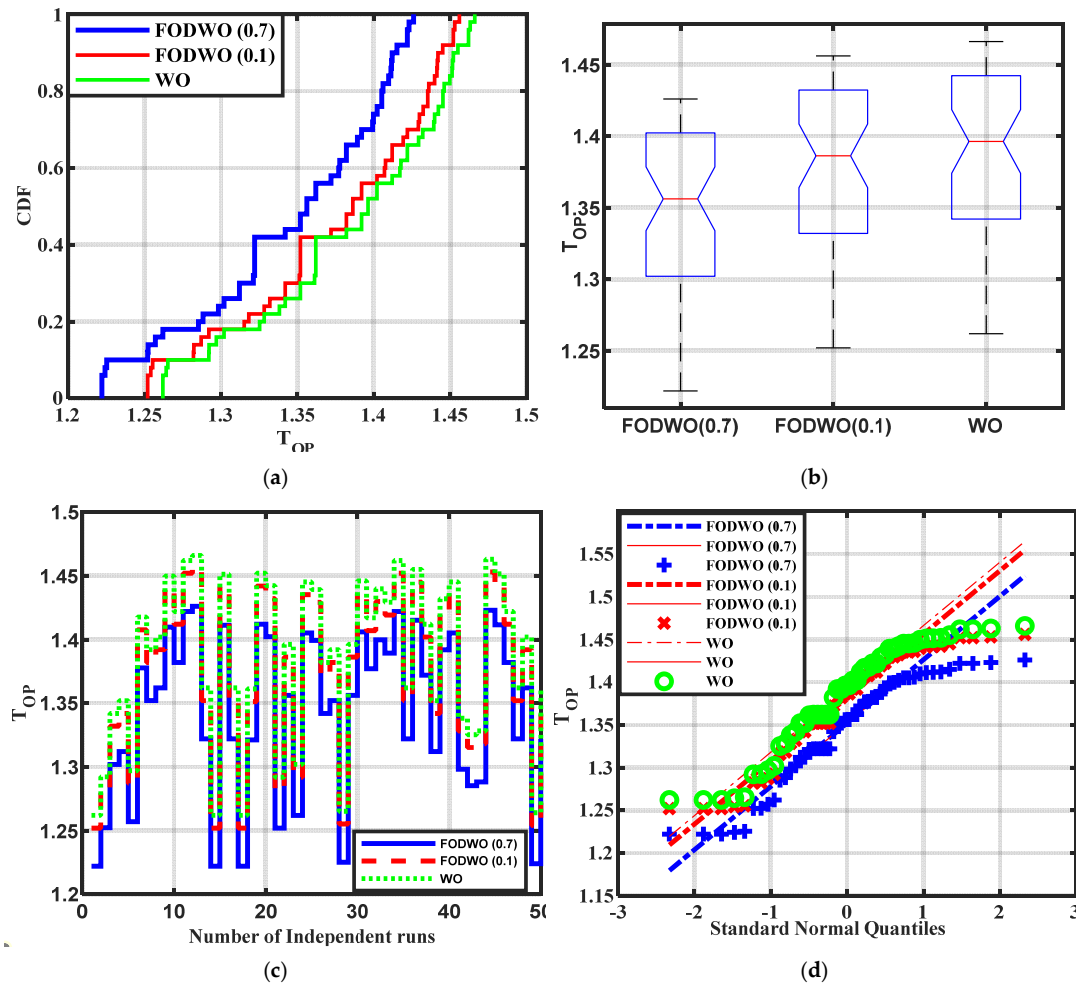


Figure 9. Statistical evaluation for IEEE three-bus system (test system 1): (a) CDF, (b) boxplot, (c) minimum fitness, and (d) quantile-quantile plot.

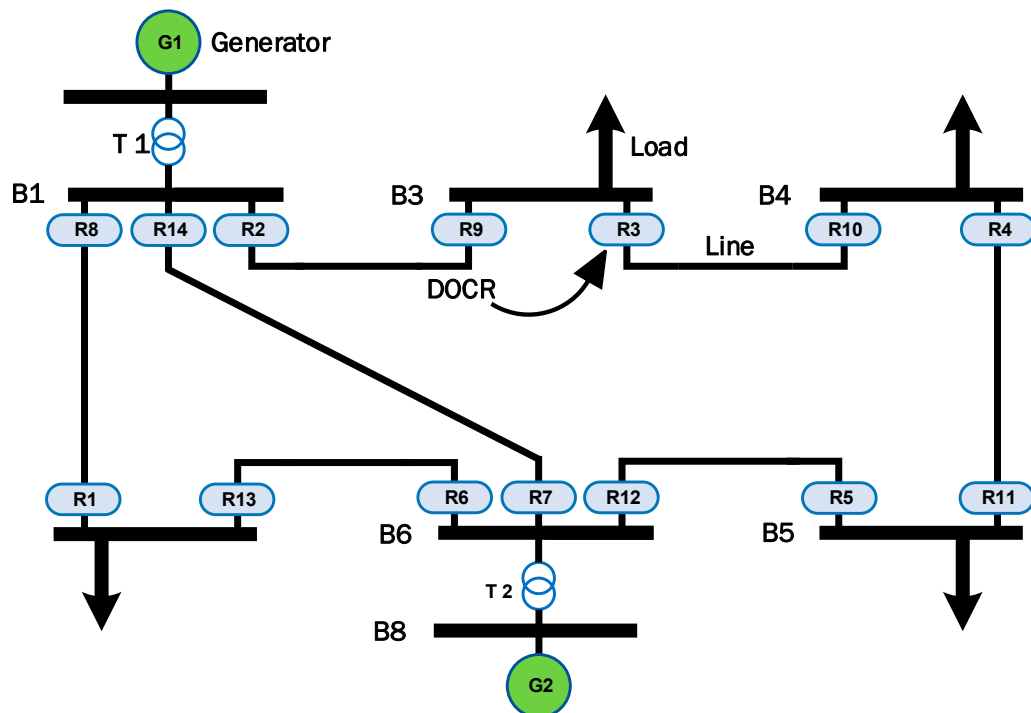


Figure 10. Single-line diagram of IEEE eight-bus configuration.

4.2.1. Results and Discussions for Test System 2 (IEEE Eight-Bus System)

The optimized results derived from the proposed FODWO and WO algorithms are presented in Table 6. These findings demonstrate that FODWO successfully reduced the T_{op} and delivered superior optimization outcomes. A comparative evaluation of WO, FODWO, and other methods addressing the DOCR coordination problem is provided in Table 7. The analysis reveals that FODWO surpasses conventional algorithms by achieving a substantial reduction in T_{op} , along with faster convergence, as visualized in Figure 11. For the IEEE eight-bus configuration, FODWO achieved performance improvements of 4.74 s over the genetic algorithm (GA), 4.68 s over GA-LP, 4.79 s over the LM method, 5.13 s over the BH algorithm, 5.49 s over Harmony Search (HS), 4.28 s over Biogeography-Based Optimization (BBO), and 3.96 s over JAYA. Additional enhancements include 3.70 s over DJAYA, 3.58 s over OJAYA, and 0.35 s over WO. When evaluated against the genetic algorithm, BH, HS, BBO, JAYA, DJAYA, OJAYA, GA-LP, LM, and WO, FODWO exhibited percentage improvements of 43.04%, 45.001%, 46.67%, 40.56%, 38.71%, 37.08%, 36.35%, 42.73%, 43.32%, and 5.25%, respectively. The computational complexity of the algorithm was evaluated based on the execution time of FODWO in comparison to WO. The WO algorithm required 6.4263 s, whereas FODWO completed the computation in 6.3560 s. These results indicate that FODWO exhibits lower computational complexity than WO. Furthermore, FODWO attained the optimal objective function value in fewer iterations. The convergence behavior of WO and FODWO during simulations is illustrated in Figure 10, emphasizing FODWO's ability to reach optimal solutions rapidly, even with limited iterations. Figure 12 highlights the algorithm's superiority in reducing T_{op} , while Figure 13 visualizes the percentage gains achieved. Figure 14 compares the optimized total operating time of FODWO with existing benchmarks, confirming its exceptional performance. The study concludes that the proposed FODWO algorithm delivers significant advancements in efficiency and convergence speed for the IEEE eight-bus system, outperforming established methodologies in both operating time reduction and computational effectiveness.

Table 6. Optimum values of PTS and TDS obtained for test system 2.

Relay #	WO		Relay #	FODWO	
	TDS	PTS		TDS	PTS
R_1	0.1000	2.0434	R_1	0.1000	2.4087
R_2	0.2422	1.8816	R_2	0.1834	2.3610
R_3	0.2326	1.6063	R_3	0.1781	2.0070
R_4	0.1829	1.5224	R_4	0.1328	2.0695
R_5	0.1000	2.2871	R_5	0.1103	1.5083
R_6	0.1732	1.6493	R_6	0.1540	2.3034
R_7	0.2062	2.1902	R_7	0.1941	1.8277
R_8	0.1525	2.1339	R_8	0.1731	1.6451
R_9	0.1098	2.1426	R_9	0.1020	2.3769
R_{10}	0.1156	2.4385	R_{10}	0.1246	2.2134
R_{11}	0.1476	1.7523	R_{11}	0.1473	1.8677
R_{12}	0.2110	1.5966	R_{12}	0.2050	1.8461
R_{13}	0.1000	2.1756	R_{13}	0.1000	2.0883
R_{14}	0.1933	1.7436	R_{14}	0.1790	2.1321
Objective function (s)	6.6187			6.2711	

Table 7. Comparative analysis using existing research (test system 2).

Method	Objective Function (s)
LM [8]	11.0645
JAYA [9]	10.2325
DJAYA [9]	9.9661

Table 7. Cont.

Method	Objective Function (s)
OJAYA [9]	9.8520
GA [16]	11.001
GA-LP [16]	10.9499
BH [23]	11.401
HS [23]	11.760
BBO [40]	10.5495
WO	6.6187
FODWO	6.2711

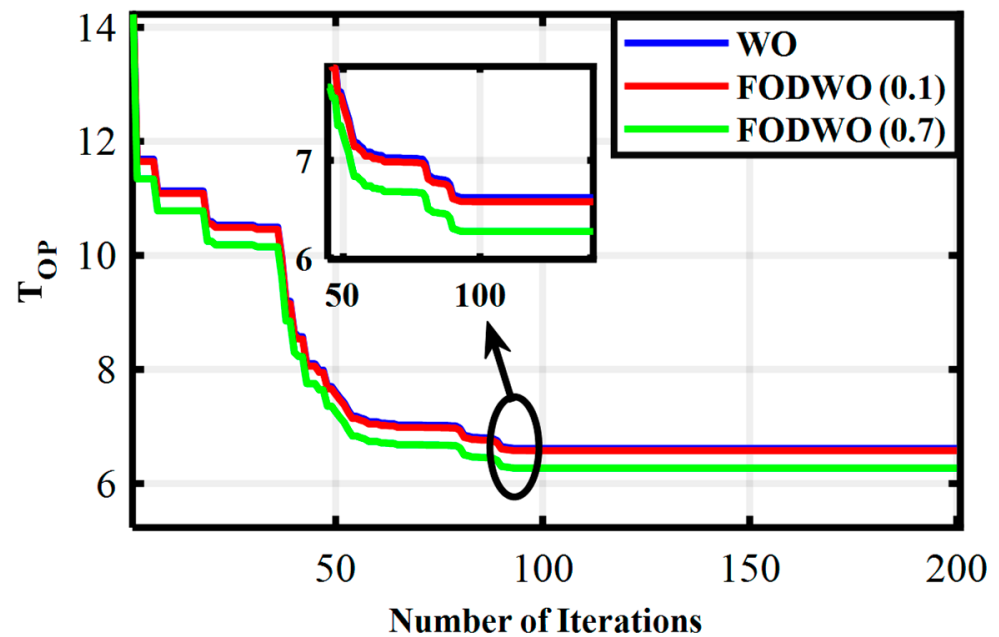


Figure 11. Convergence characteristic for WO and FODWO for IEEE eight-bus system.

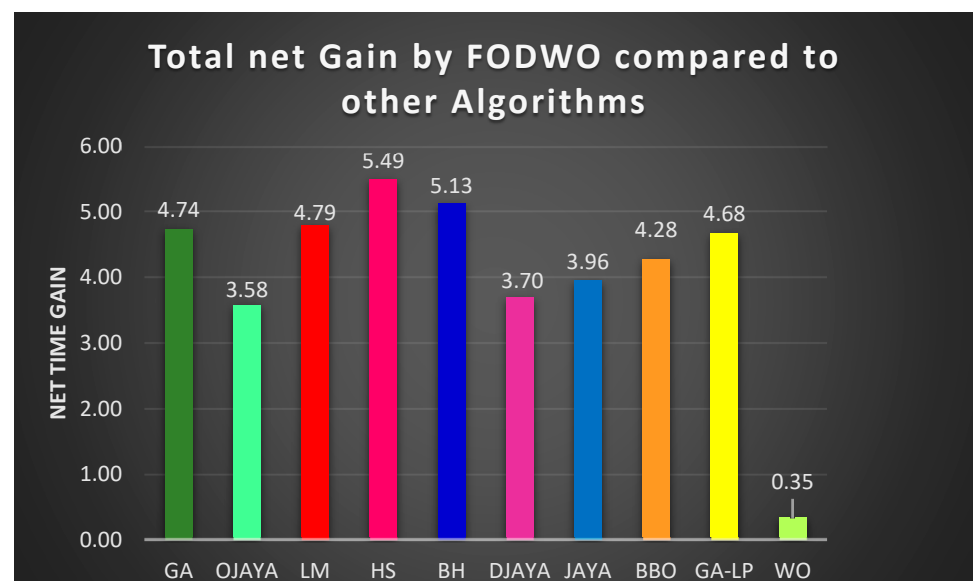


Figure 12. Total net gain by FODWO compared to other algorithms (test system 2).

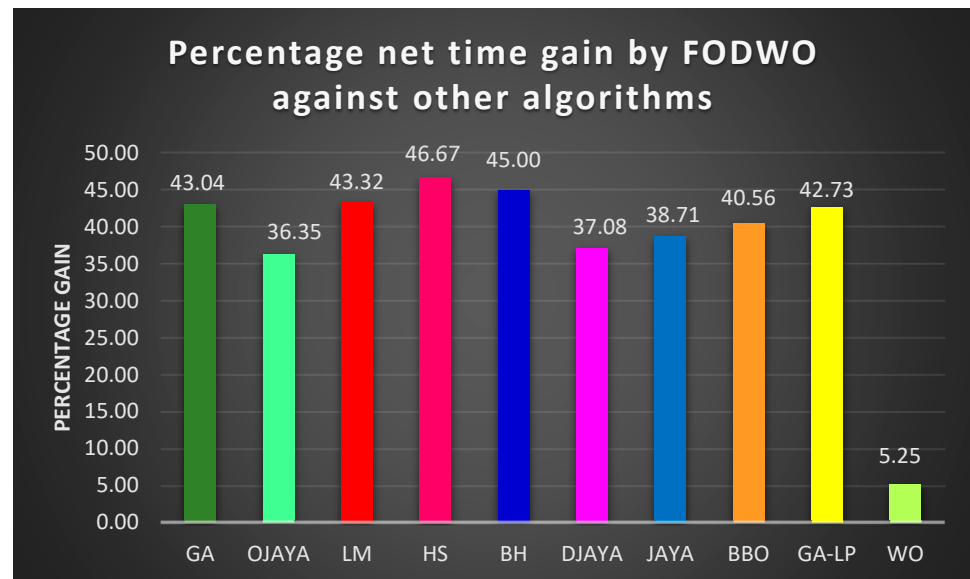


Figure 13. Percentage net time gain obtained by FODWO against other algorithms (test system 2).

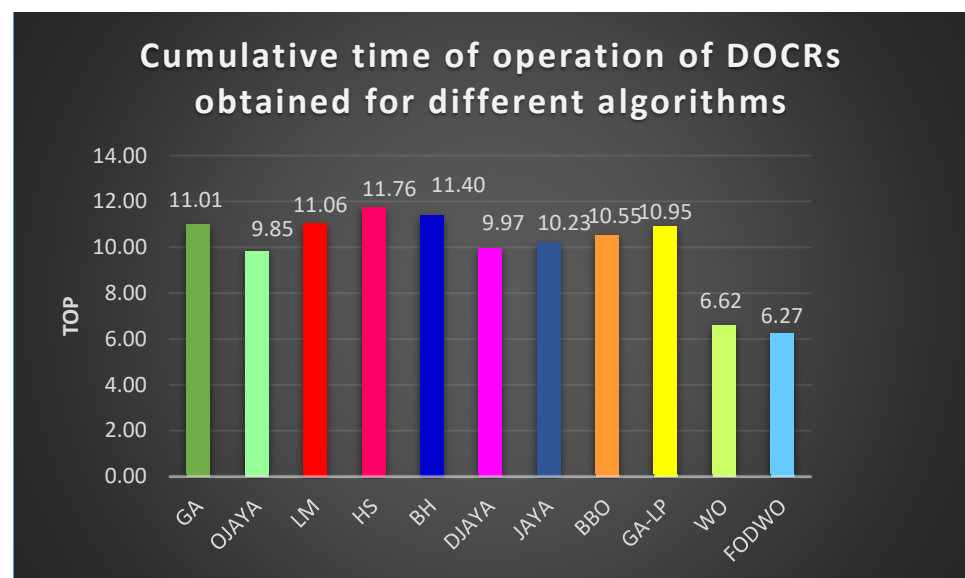


Figure 14. T_{op} of DOCRs obtained for different algorithms (test system 2).

4.2.2. Statistical Evaluation for IEEE Eight-Bus Configuration (Test System 2)

To evaluate the reliability, stability, and robustness of the proposed FODWO algorithm, a detailed statistical assessment was performed on the IEEE eight-bus system, focusing on identifying the optimal fractional order. Fifty independent simulations were conducted to ensure data accuracy, with the median of the final responses used as a baseline to determine the best-performing fractional order. The analysis employed multiple statistical tools, such as ECDF, box, QQ, and stair plots across trials, as shown in Figure 15a–d. Figure 15a reveals that FODWO achieves optimal solutions with significantly higher probability compared to the standard WO method. Figure 15b indicates that FODWO consistently yields lower median solution values across 50 trials than WO. Figure 15c demonstrates that FODWO's minimal fitness values align closely with normal distribution quantiles, confirming its statistical robustness. Additionally, Figure 15d illustrates FODWO's consistently lower fitness levels across multiple independent runs. These findings underscore that fractional calculus serves as an innovative mathematical framework for enhancing traditional optimization

algorithms, enabling improved performance and stability in complex systems like the IEEE eight-bus network.

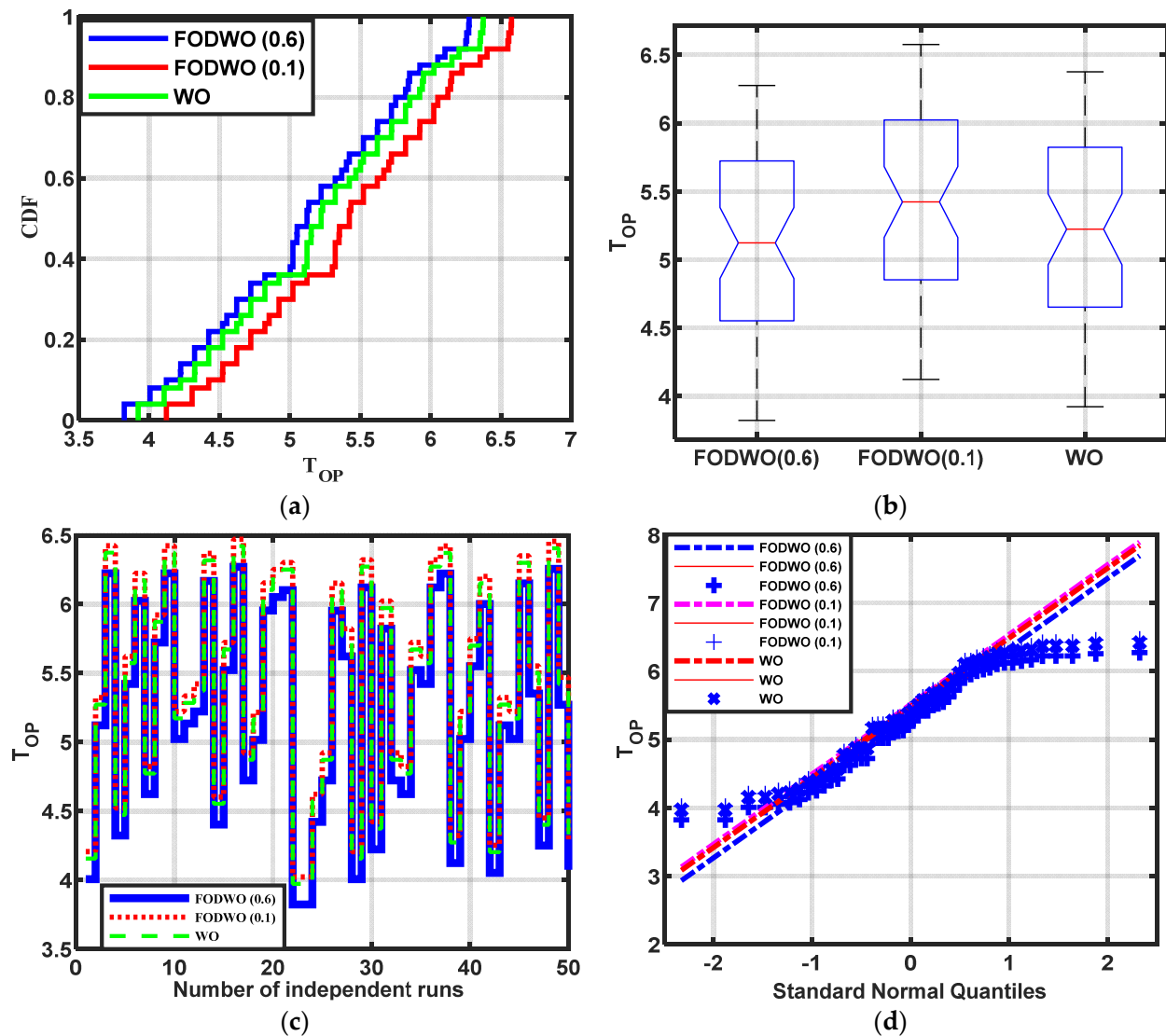


Figure 15. Statistical evaluation for IEEE eight-bus system (test system 2): (a) CDF, (b) boxplot, (c) minimum fitness, and (d) quantile-quantile plot.

4.3. Test System 3: IEEE Fifteen-Bus Configuration

The fifteen-bus power system is modeled as a mixed-integer nonlinear programming (MINLP) optimization problem, comprising fifteen buses, twenty-one transmission branches, forty-two directional overcurrent relays (DOCRs), and eighty-two primary/backup (P/B) relay pairs, as shown in Figure 16. The study examines a three-phase (3 ϕ) fault scenario occurring near the source across all transmission lines, accounting for the high penetration of distributed generation (DG) in the distribution network. A CTI of 0.2 s is applied, with the TDS constrained between 0.1 and 1.1 and the PS ranging from 0.5 to 2.5. Parameters such as CT ratios, P/B relay configurations, and three-phase fault current values are adopted from ref. [25].

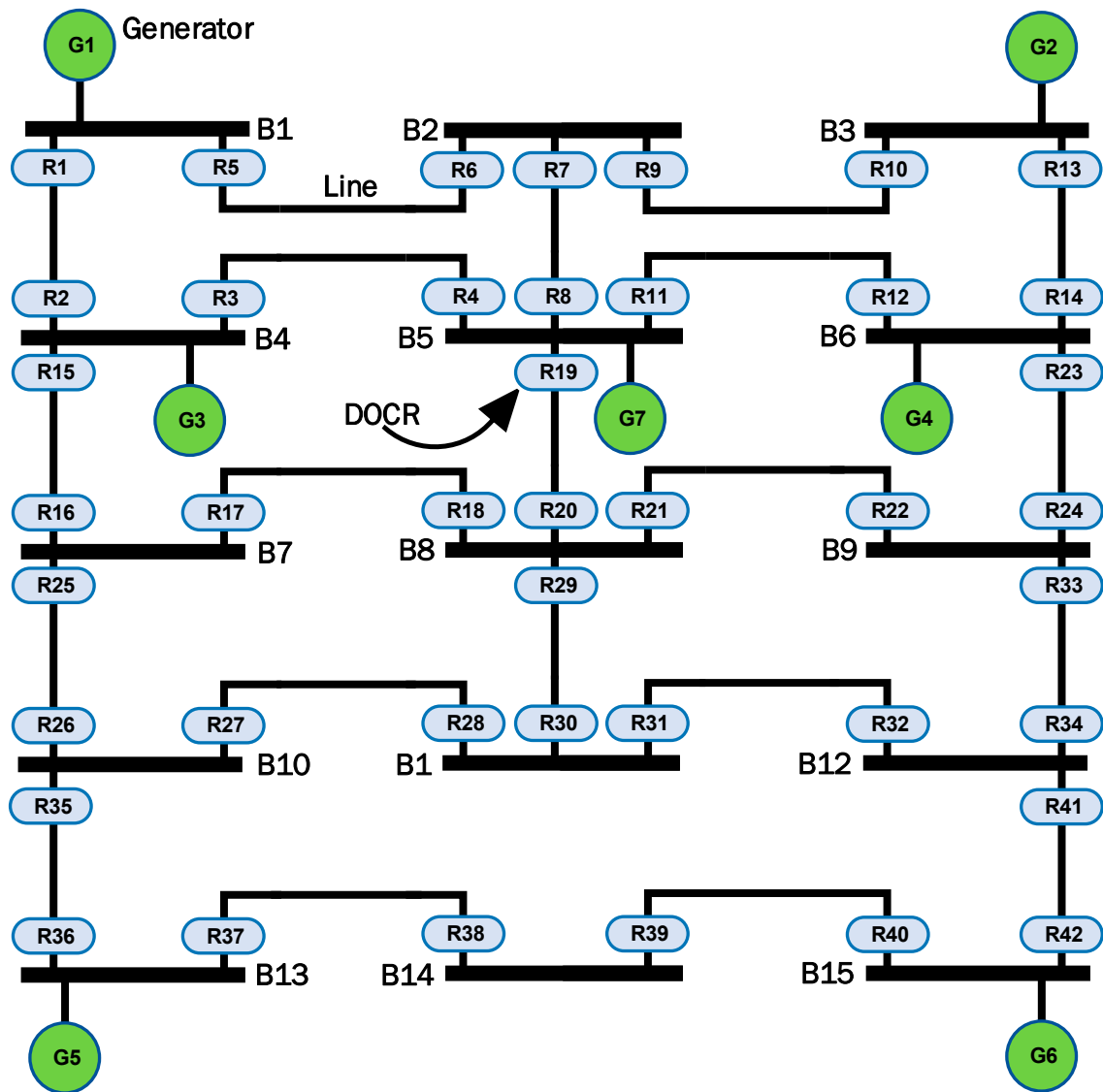


Figure 16. Single-line diagram of IEEE 15-bus configuration.

4.3.1. Results and Discussions for Test System 3 (IEEE Fifteen-Bus System)

The optimized results obtained through the proposed FODWO and WO algorithms are detailed in Table 8. These outcomes highlight FODWO's effectiveness in minimizing T_{op} while achieving superior optimization performance. A comparative analysis in Table 9 demonstrates FODWO's advantage over existing methods for resolving DOCR coordination challenges, showcasing its ability to reduce operating time significantly and converge rapidly, as evidenced in Figure 17. Notably, FODWO attains optimal objective function values with fewer computational iterations. For the IEEE fifteen-bus system, FODWO outperformed MINLP by 2.11 s, BSA by 3.06 s, MTLBO by 12.59 s, GSO by 0.42 s, GWO by 2.05 s, EFO by 4.68 s, ER-WCA by 2.68 s, DJAYA by 5.61 s, OJAYA by 2.29 s, and WO by 0.31 s. In relative terms, FODWO achieved improvements of 13.73% over MINLP, 18.80% over BSA, 48.75% over MTLBO, 3.11% over GSO, 13.40% over GWO, 26.12% over EFO, 16.85% over ER-WCA, 29.78% over DJAYA, 14.78% over OJAYA, and 2.29% over WO. The computational complexity of the algorithm was evaluated based on the execution time of FODWO in comparison to WO. The WO algorithm required 20.6892 s, whereas FODWO completed the computation in 19.9529 s. These results indicate that FODWO exhibits lower computational complexity than WO.

Table 8. Optimum values of PTS and TDS obtained for test system 3.

Relay #	WO		Relay #	FODWO	
	TDS	PTS		TDS	PTS
R ₁	0.1000	1.9687	R ₁	0.1000	1.8511
R ₂	0.1000	2.4115	R ₂	0.1000	1.8188
R ₃	0.1000	2.4554	R ₃	0.1001	2.2238
R ₄	0.1000	1.7577	R ₄	0.1000	2.2376
R ₅	0.1127	2.1764	R ₅	0.1185	1.9390
R ₆	0.1018	2.3793	R ₆	0.1023	2.3136
R ₇	0.1326	1.6711	R ₇	0.1160	2.0299
R ₈	0.1000	2.0865	R ₈	0.1000	1.7412
R ₉	0.1000	2.3439	R ₉	0.1091	1.8556
R ₁₀	0.1225	1.5184	R ₁₀	0.1000	2.1254
R ₁₁	0.1000	2.2989	R ₁₁	0.1000	1.8158
R ₁₂	0.1000	1.7872	R ₁₂	0.1067	1.5192
R ₁₃	0.1072	1.5771	R ₁₃	0.1000	2.1449
R ₁₄	0.1000	2.3759	R ₁₄	0.1000	1.8201
R ₁₅	0.1000	1.6649	R ₁₅	0.1000	2.1367
R ₁₆	0.1000	2.1795	R ₁₆	0.1000	2.1672
R ₁₇	0.1057	1.6571	R ₁₇	0.1000	2.4099
R ₁₈	0.1000	1.8209	R ₁₈	0.1000	2.1158
R ₁₉	0.1081	1.8204	R ₁₉	0.1000	2.1009
R ₂₀	0.1000	2.1216	R ₂₀	0.1000	2.0098
R ₂₁	0.1000	2.2860	R ₂₁	0.1000	1.7188
R ₂₂	0.1000	2.2645	R ₂₂	0.1000	1.9796
R ₂₃	0.1000	2.3577	R ₂₃	0.1000	2.4046
R ₂₄	0.1000	2.4526	R ₂₄	0.1000	1.8402
R ₂₅	0.1137	1.8584	R ₂₅	0.1000	2.3607
R ₂₆	0.1220	1.4983	R ₂₆	0.1049	1.8724
R ₂₇	0.1259	1.5443	R ₂₇	0.1213	1.6949
R ₂₈	0.1529	1.5313	R ₂₈	0.1266	2.1286
R ₂₉	0.1000	1.9518	R ₂₉	0.1000	1.9104
R ₃₀	0.1002	2.0262	R ₃₀	0.1010	1.9678
R ₃₁	0.1000	2.4828	R ₃₁	0.1265	1.5080
R ₃₂	0.1000	2.0277	R ₃₂	0.1101	1.5663
R ₃₃	0.1346	1.7904	R ₃₃	0.1204	1.9749
R ₃₄	0.1622	1.5098	R ₃₄	0.1442	1.6224
R ₃₅	0.1147	1.9840	R ₃₅	0.1220	1.7793
R ₃₆	0.1000	2.4676	R ₃₆	0.1000	2.0501
R ₃₇	0.14863	1.5420	R ₃₇	0.1238	2.2221
R ₃₈	0.1289	2.0262	R ₃₈	0.1171	2.1745
R ₃₉	0.1236	1.9334	R ₃₉	0.1443	1.6403
R ₄₀	0.1162	2.3507	R ₄₀	0.1096	2.4088
R ₄₁	0.1269	2.0351	R ₄₁	0.1090	2.4190
R ₄₂	0.1003	1.8421	R ₄₂	0.1000	2.3580
Objective function (s) 13.5394				13.2292	

Table 9. Comparative analysis using existing research (test system 3).

Algorithm	Objective Function
OJAYA [9]	15.523
DJAYA [9]	18.840
EFO [23]	17.906
ER-WCA [23]	15.910
GWO [23]	15.277
MINLP [25]	15.334
BSA [42]	16.293
MTLBO [43]	25.815
GSO [45]	13.654
WO	13.5394
FODWO	13.2292

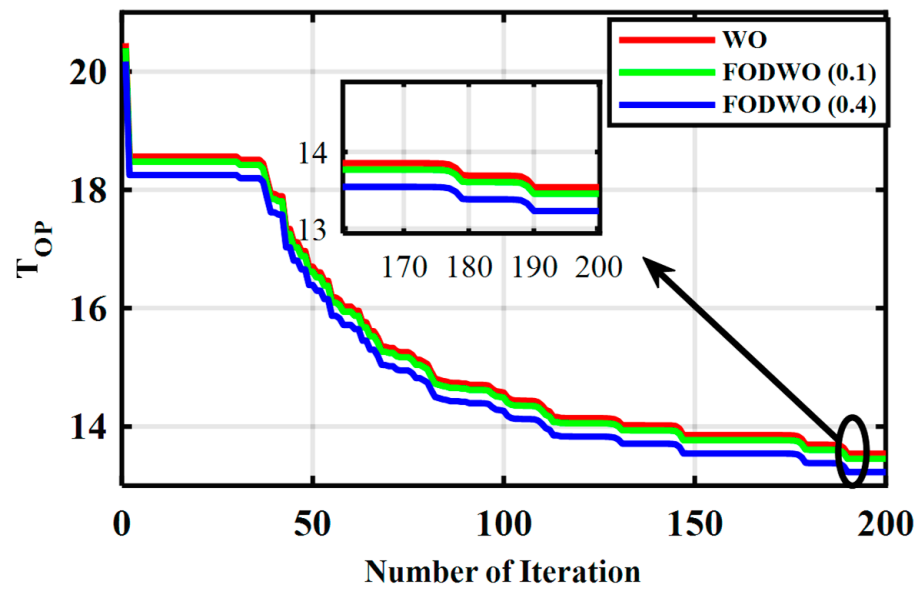


Figure 17. Convergence characteristic for WO and FODWO for IEEE 15-bus system.

Figure 18 illustrates FODWO's net gain advantage in T_{op} reduction, while Figure 19 quantifies its percentage improvements. Figure 20 further validates FODWO's superiority by comparing its optimized operating time against established benchmarks in the literature. The case study outcomes confirm that the proposed algorithm delivers marked enhancements in efficiency, outperforming existing methodologies in both absolute time reduction and computational effectiveness.

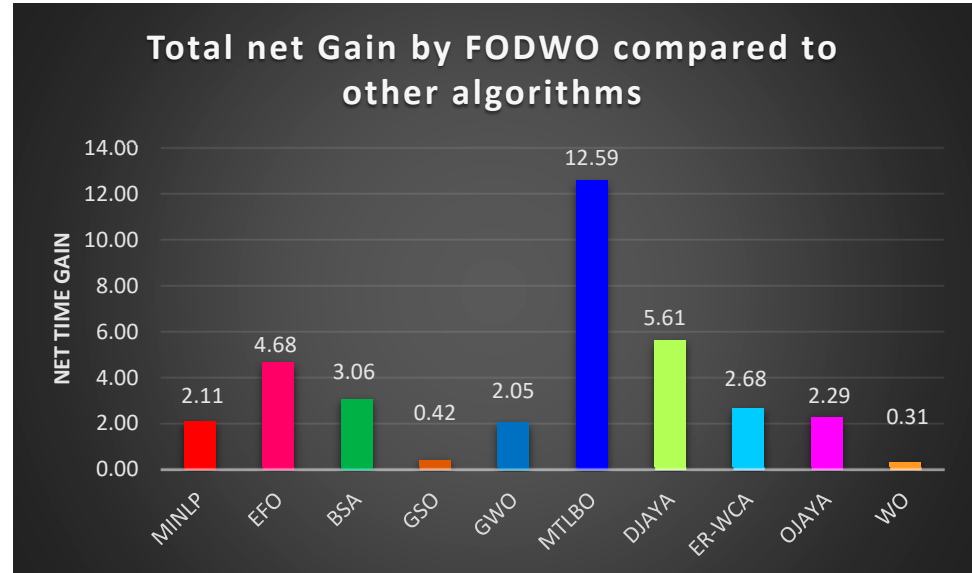


Figure 18. Total net gain by FODWO compared to other algorithms (test system 3).

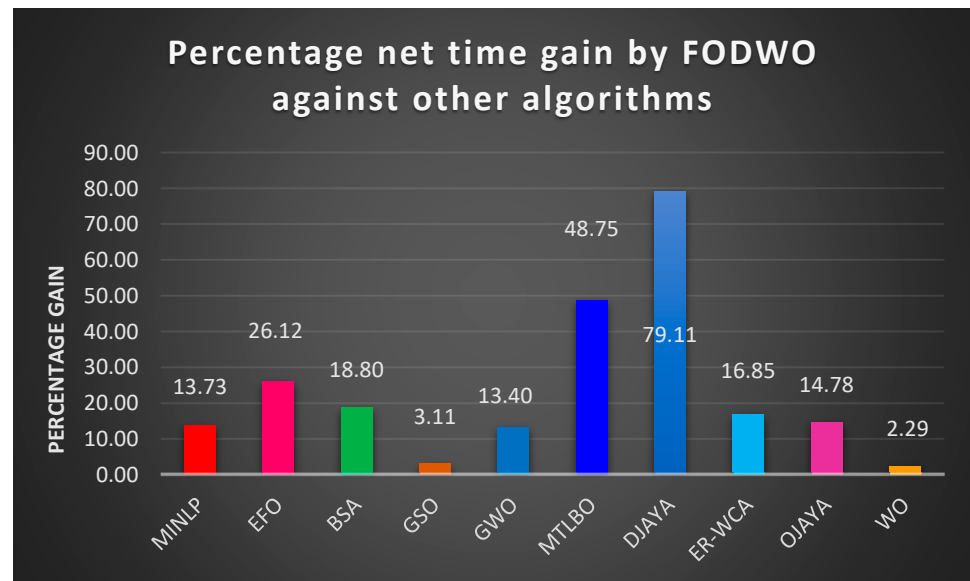


Figure 19. Percentage net time gain by FODWO against other algorithms (test system 3).

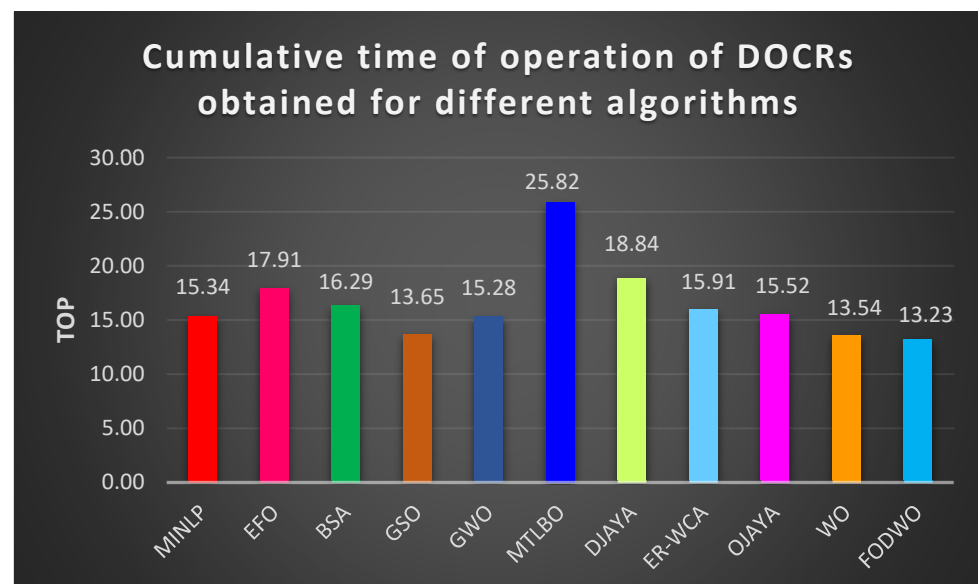


Figure 20. T_{op} of DOCRs for different algorithms (test system 3).

4.3.2. Statistical Evaluation for IEEE Fifteen-Bus Configuration (Test System 3)

A comprehensive statistical analysis of the IEEE fifteen-bus system was performed to evaluate the stability and reliability of the proposed FODWO algorithm, with an emphasis on determining the optimal fractional order parameter. Fifty independent simulation runs were conducted to ensure data robustness, using the median of the final outcomes as the reference benchmark for identifying the most effective fractional order. The study leveraged diverse statistical tools, including ECDF, box, QQ, and stair plots across trials, as visualized in Figure 21a–d. Figure 21a underscores FODWO's markedly higher success rate in attaining optimal solutions relative to conventional WO. Figure 21b reveals that FODWO achieves consistently lower median solution values over 50 trials compared to WO. Figure 21c validates FODWO's statistical soundness, as its minimal fitness values align closely with theoretical normal distribution quantiles. Figure 21d further reinforces FODWO's superiority, demonstrating sustained lower fitness levels throughout all independent trials. These findings collectively establish fractional calculus as an innovative

mathematical paradigm for augmenting classical optimization techniques, offering heightened precision and reliability in managing complex power system challenges.

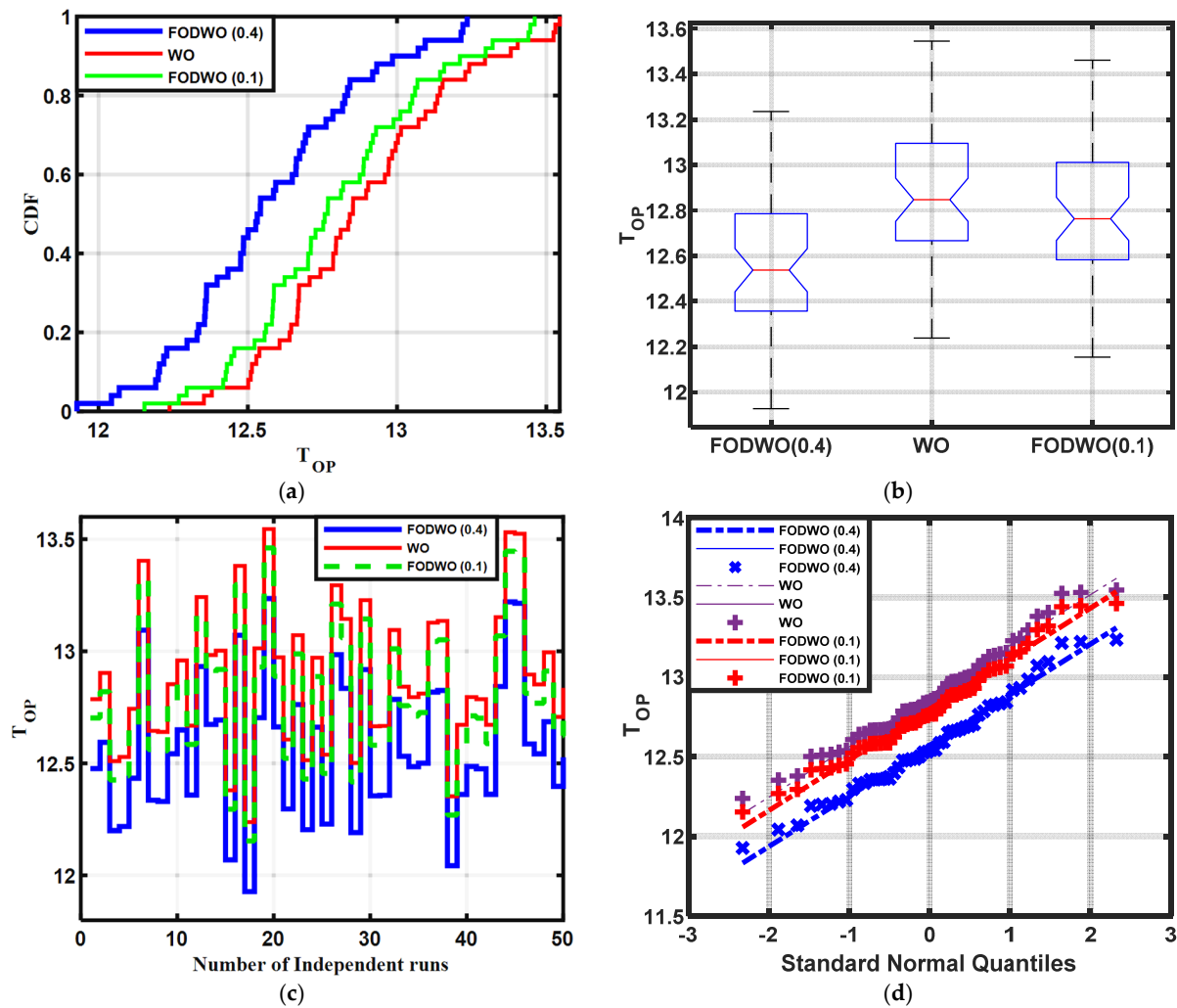


Figure 21. Statistical evaluation for IEEE fifteen-bus system (test system 3): (a) CDF, (b) boxplot, (c) minimum fitness, and (d) quantile-quantile plot.

5. Conclusions

The FODWO algorithm represents a groundbreaking approach to resolving coordination challenges in DOCRs within IEEE benchmark systems. By integrating fractional calculus into the traditional WO framework, FODWO introduces a novel, efficient, and highly accurate optimization technique. The algorithm enhances the conventional WO by embedding FOD into its mathematical model, explicitly refining the position update process during iterations. This modification improves the algorithm's convergence speed and mitigates the risk of premature convergence, thereby boosting its overall optimization capabilities. FODWO has been rigorously tested on three IEEE standard bus (three, eight, fifteen) systems, demonstrating its ability to minimize the T_{op} of DOCRs. Through the optimization of control parameters like PS and TDS for both primary and backup relays, FODWO achieves near-optimal configurations. According to comparative analyses with other advanced algorithms, FODWO achieved improvements of 13.73% over MINLP, 18.80% over BSA, 48.75% over MTLBO, 3.11% over GSO, 13.40% over GWO, 26.12% over EFO, 16.85% over ER-WCA, 29.78% over DJAYA, 14.78% over OJAYA, and 2.29% over WO in the case of the IEEE three-bus system. In the case of the eight-bus system, when evaluated against the genetic algorithm, BH, HS, BBO, JAYA, DJAYA, OJAYA, GA-LP, LM, and WO,

FODWO exhibited percentage improvements of 43.04%, 45.001%, 46.67%, 40.56%, 38.71%, 37.08%, 36.35%, 42.73%, 43.32%, and 5.25%, respectively, while in the case of the fifteen-bus system, FODWO achieved improvements of 13.73% over MINLP, 18.80% over BSA, 48.75% over MTLBO, 3.11% over GSO, 13.40% over GWO, 26.12% over EFO, 16.85% over ER-WCA, 29.78% over DJAYA, 14.78% over OJAYA, and 2.29% over WO. This statistic consistently highlights FODWO's superior performance in reducing DOCR operational times within MINLP models. The algorithm's robustness, reliability, and stability are further corroborated through extensive statistical evaluations, including ECDF, box, QQ, and stair plots. The integration of fractional calculus with WO has proven to significantly enhance the optimizer's efficiency by accelerating convergence rates. These outcomes underscore FODWO's capability to deliver optimal solutions for DOCR coordination, solidifying its role as a powerful tool for relay optimization. Looking ahead, FODWO will be used in the future to tackle protection coordination challenges in microgrids, encompassing both grid-connected and islanded operational modes on higher bus systems, such as IEEE 30, 57, or even 118-bus systems to further substantiate the approach's performance in large-scale power grids. This includes addressing scenarios involving line outages, substation failures, distributed generation disruptions, and diverse microgrid operating conditions.

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