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Editorial

Complexity, Fractality and Fractional Dynamics Applied to Science and Engineering

Alexandra M. S. F. Galhano [1], Sergio Adriani David [2] and António M. Lopes [3,*]

[1] Faculdade de Ciências Naturais, Engenharias e Tecnologias, Universidade Lusófona do Porto, Rua Augusto Rosa 24, 4000-098 Porto, Portugal; alexandra.galhano@ulusofona.pt
[2] Faculdade de Zootecnia e Engenharia de Alimentos da USP, University of São Paulo, Av. Duque de Caxias-Norte, 225, Jardim Elite, Pirassununga 13635-900, SP, Brazil; sergiodavid@usp.br
[3] LAETA/INEGI, Faculty of Engineering, University of Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
* Correspondence: aml@fe.up.pt

Many problems in classical and quantum physics, statistical physics, engineering, biology, psychology, economics, and finance are inherently global rather than merely local, often exhibiting long-range correlations in time and space, memory effects, fractality, and power-law dynamics. Fractional calculus and fractional processes have been widely adopted across these disciplines, emerging as highly effective tools for capturing non-local behaviors and representing long-term memory effects in diverse applied sciences. The fractional paradigm extends beyond calculus to stochastic processes, providing a unified framework to model complex phenomena. Furthermore, big data analysis, organization, retrieval, and modeling have become essential computational approaches for addressing complex, fractal, and fractional dynamics.

This Special Issue, titled "Complexity, Fractality and Fractional Dynamics Applied to Science and Engineering" aims not only to present the state of the art in complex, fractal, and fractional dynamics and their applications but also to explore the potential and broader applicability of these tools in modeling real-world phenomena. It includes 11 manuscripts addressing novel issues and specific topics that illustrate the richness and applicability of fractional calculus. In the follow-up the selected manuscripts are presented in alphabetic order of their titles.

In the paper "A Fractional Time–Space Stochastic Advection–Diffusion Equation for Modeling Atmospheric Moisture Transport at Ocean–Atmosphere Interfaces" [1], the authors propose a one-dimensional fractional time–space stochastic advection–diffusion equation for modeling moisture transport in atmospheric boundary layers over ocean surfaces. Combining fractional calculus, advective transport, and pink-noise stochasticity, the model captures temporal memory, non-local turbulent diffusion, and correlated ocean–atmosphere fluctuations. Using Caputo derivatives, a fractional Laplacian, and a $1/f$ stochastic term, the framework is solved through Fourier–Laplace techniques, yielding closed-form Mittag–Leffler solutions. Applied to a coastal domain, results show that stronger advection expands regions exceeding fog-formation thresholds and that the model reproduces anomalous diffusion and realistic variability with robust numerical performance.

In "Approaching Multifractal Complexity in Decentralized Cryptocurrency Trading" [2], decentralized cryptocurrency trading is studied to uncover potential multifractal features known from mature financial markets. Using tick-by-tick transaction data from Uniswap's Universal Router (June 2023–June 2024) and applying multifractal detrended fluctuation analysis, the study finds signs of emerging multifractality despite lower liquidity relative to centralized exchanges. The multifractal spectra exhibit strong left-sided asymmetry, indicating that large fluctuations dominate the multifractal structure, while

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small fluctuations behave similar to uncorrelated noise. The results also show that multifractality is more pronounced in transaction volumes than in returns and that large events reveal multifractal cross-correlations between the two indices.

In "Dynamics of the Traveling Wave Solutions of Fractional Date–Jimbo–Kashiwara–Miwa Equation via Riccati–Bernoulli Sub-ODE Method through Bäcklund Transformation" [3], the wave solutions of the time–space fractional Date–Jimbo–Kashiwara–Miwa (DJKM) equation are obtained using Riccati–Bernoulli sub-ODE method through Bäcklund transformation. The regular dynamical wave solutions of the DJKM equation encompass trigonometric, hyperbolic, and rational functions. These solutions are graphically classified into compacton kink solitary wave solutions, kink soliton wave solutions, and anti-kink soliton wave solutions. To explore the impact of the fractional parameter on those solutions, 2D plots are adopted, while 3D plots are applied to present solutions involving integer-order derivatives.

In their work "Formation of Optical Fractals by Chaotic Solitons in Coupled Nonlinear Helmholtz Equations" [4], the authors construct and examine self-similarity of optical solitons by employing the Riccati modified extended simple equation method (RMESEM) within the framework of non-integrable coupled nonlinear Helmholtz equations (CNHEs). Initially, a complex transformation is used to convert the model into a single nonlinear ordinary differential equation from which hyperbolic, exponential, rational, trigonometric, and rational hyperbolic solutions are produced. Several 3D, contour, and 2D maps are provided, highlighting the behavior of some optical solitons and demonstrating that, under certain conditions, acquired optical solitons lead to the generation of optical fractals. The suggested RMESEM offers important insights into the dynamics of CNHEs and suggests possible applications in the management of nonlinear models.

The paper "Fractional Transfer Entropy Networks: Short-and Long-Memory Perspectives on Global Stock Market Interactions" [5], introduces fractional transfer entropy (FTE), a framework that incorporates fractional calculus into transfer entropy to capture both short-run volatility and long-run dependencies in global stock markets. By tuning memory parameters, FTE reveals how varying temporal emphases reshape directional information networks among major financial indices. Results show that short-memory settings produce fast-adapting but shock-sensitive networks, balanced memory yields more stable structures, and long-memory configurations highlight persistent historical ties. FTE is shown to offer a versatile tool for assessing stability, contagion risk, and the interplay between short-term signals and long-term market structure.

In the work "Information Properties of Consecutive Systems Using Fractional Generalized Cumulative Residual Entropy" [6], the authors investigate information properties of consecutive k-out-of-n:G systems using fractional generalized cumulative residual entropy. They derive a formula for computing this entropy for the system's lifetime and examine its preservation properties under established stochastic orders, also providing useful bounds. To support practical implementation, they propose two nonparametric estimators for the fractional generalized cumulative residual entropy, whose efficiency and performance are demonstrated through both simulated and real datasets.

In "Multifractal-Aware Convolutional Attention Synergistic Network for Carbon Market Price Forecasting" [7], the challenge of accurately predicting carbon market prices is tackled by developing a multifractal-aware deep learning model, MF-Transformer-DEC. Recognizing that traditional methods struggle to capture the multifractal nature of carbon price dynamics, the study introduces a multi-scale convolution module, a fractal attention mechanism, and a dynamic error correction component to better represent scale-invariant structures, nonlinear fluctuations, and uncertainty. Applied to carbon price data from

Shanghai and Guangdong, the model is shown to achieve high predictive accuracy, revealing the benefit of integrating fractal theory with AI.

The manuscript "Navigating Choppy Waters: Interplay between Financial Stress and Commodity Market Indices" [8], analyzes how financial stress interacts with major commodity markets by examining the nonlinear and multifractal cross-correlations between the financial stress index (FSI) and four key commodity indices. Using daily data from 2016 to 2023 and applying multifractal detrended cross-correlation analysis, the study identifies strong multifractal behavior and power-law cross-correlations, indicating that significant changes in financial stress tend to coincide with shifts in commodity prices. The findings offer valuable insights for investors and policymakers concerned with market stability and risk transmission.

In "New Class of Complex Models of Materials with Piezoelectric Properties with Differential Constitutive Relations of Fractional Order: An Overview" [9], a new class of fractional rheological models for elastoviscous and viscoelastic materials with piezoelectric properties is reviewed. By introducing two new fractional elements, namely a generalized Newton fluid element and a Faraday elastic element with polarization, the authors develop seven complex models governed by fractional-order constitutive relations. These models describe ideal materials with combined mechanical and piezoelectric behavior and provide a theoretical basis for future research, including applications in energy harvesting.

In their paper "Rheological Burgers–Faraday Models and Rheological Dynamical Systems with Fractional Derivatives and Their Application in Biomechanics" [10], the authors introduce two fractional Burgers–Faraday rheological models and their corresponding dynamical systems, capturing the coupled mechanical and piezoelectric behavior of materials via Kelvin–Voigt–Faraday and Maxwell–Faraday elements. The models feature fractional-order constitutive relations and two internal degrees of freedom, with different element configurations producing either fractional-type oscillations or creeping/pulsating responses under periodic forcing. These systems provide a basis for modeling biomaterials that combine viscoelastic or viscoplastic behavior with piezoelectric properties.

In the paper "The Multiscale Principle in Nature (Principium luxuriæ): Linking Multiscale Thermodynamics to Tiving and Non-living Complex Systems" [11], the authors explore a unifying physical principle behind the widespread occurrence of fractals, proposing a multiscale thermodynamic framework in which fractal and power-law patterns arise as mechanisms for dissipating excess energy across scales. In this view, the thermodynamic fractal dimension reflects how efficiently a system releases energy at microscopic versus macroscopic levels. The paper examines diverse physical, astrophysical, biological, and social systems to show that many can be reinterpreted through the "Principium luxuriæ", suggesting that it may underlie the emergence of fractals in complex systems.

The Guest Editors hope that the selected papers will help scholars and researchers to push forward the progress in complexity, fractality, and fractional dynamics applied to science and engineering.

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