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Editorial

Fractional Mathematical Modelling: Theory, Methods and Applications

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1. Introduction

Fractional calculus shares its historical roots with classical calculus and has lately become a powerful mathematical tool for modeling complex systems. The concept dates back to 1695, when L'Hôpital posed a question to Leibniz about the possibility of a derivative of non-integer order, later developed by Liouville, Riemann, Caputo, Grünwald, Letnikov, Hadamard, and others [1–3]. These contributions have led to the establishment of multiple definitions of fractional-order operators, each with distinct advantages and limitations [4]. By extending classical calculus through fractional operators, it captures memory, hereditary effects, and nonlocal interactions [5,6]. Unlike integer-order models, fractional formulations are uniquely capable of describing processes with long-term memory and anomalous diffusion. Its wide-ranging applications in physics, engineering, biology, medicine and health sciences, finance, and the social sciences [7–10] make it an indispensable tool for analyzing complex processes. In recent years, the field has gained remarkable momentum, supported by the introduction of new operators, improvements in numerical schemes, and diverse real-world applications [11,12]. These advances have strengthened its theoretical foundations while expanding its practical relevance, positioning fractional calculus as a unifying framework that bridges mathematics with diverse applications.

Building on this broad relevance, the aim of this Reprint is to showcase recent advances in fractional calculus across theory, methodology, and applications. Out of 51 submissions received, 12 high-quality papers were accepted for publication, giving an acceptance rate of 23.5%. The selected contributions highlight new operator formulations, analytical results, numerical techniques, and interdisciplinary applications ranging from control theory and fuzzy systems to engineering devices, materials science, and biomedical systems. The rapid pace of progress in the field, particularly with the emergence of novel operator definitions, advanced numerical techniques, and diverse real-world applications, motivated the launch of this Reprint. By bringing together contributions from different perspectives, it provides a concise snapshot of current developments in fractional calculus and serves as a resource to foster further collaboration across diverse disciplines.

2. Overview of the Contributions in the Reprint

The contributions in this Reprint are grouped into three main themes, in line with the Special Issue: theoretical developments, methodological advances, and applications.

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2.1. Theoretical Developments

Theoretical developments are illustrated in two contributions. Albidah (1.) investigated two forms of the Riemann–Liouville derivative for second-order fractional differential equations, showing that the choice of lower bound yields either implicit solutions in terms of Mittag-Leffler functions or explicit solutions involving trigonometric and hyperbolic functions. Alkandari et al. (2.) developed anomalous diffusion models using regularized general fractional derivatives with Sonin kernels, linking them to continuous-time random walks and deriving explicit expressions for waiting-time densities, mean squared displacement, and conditions ensuring non-negativity and maximum principles.

2.2. Methodological Advances

Methodological advances are demonstrated in several papers. Sengül et al. (3.) employed the optimal q-Homotopy Analysis Method to study Abel-type equations, demonstrating improved convergence and accuracy over classical approaches. AlBaidani (4.) compared the homotopy perturbation transform method with a new iterative method for the time-fractional Burger-Fisher equation, showing that both approaches yield reliable and computationally efficient solutions, further validated against techniques such as Haar wavelets, OHAM, and q-HATM. Abdelfattah et al. (5.) extended the fractional differential quadrature method (FDQM) to nonlinear Riccati and Lorenz systems using generalized Caputo derivatives, demonstrating superior accuracy and convergence compared with existing methods.

2.3. Applications

Applications are explored across fuzzy systems, engineering models, materials science, and biomedical systems. Muhammad et al. (6.) analyzed fuzzy fractional two-dimensional continuous-time linear systems based on Roesser and Fornasini-Marchesini models, using granular Laplace transforms to address parameter uncertainty and validating their approach with applications in signal processing and wireless sensor networks. Al-Dosari (7.) examined Hilfer fuzzy fractional inclusions with infinite delay, proving controllability of mild solutions through nonlinear functional analytic techniques and establishing new results supported by the properties of Mittag-Leffler functions.

Engineering and materials science applications include Yu et al. (8.), who proposed a Caputo-Fabrizio-based model of a fractional-order boost converter with inductive loads, constructing both large- and small-signal models and confirming their accuracy through simulations. Xu et al. (9.) introduced a fractional-order Zener model incorporating temperature-order equivalence for viscoelastic dampers, validated experimentally and optimized using a chaotic fractional-order particle swarm algorithm. García-de-los-Ríos et al. (10.) applied fractional models to ZnO micro- and nanostructures, explaining photoconduction and nonlinear optical effects relevant for optoelectronic devices. Abdelfattah et al. (11.) applied FDQM to simulate charge dynamics in polymer solar cells, achieving high accuracy and efficiency.

In the biomedical field, Mihai et al. (12.) proposed a personalized fractional-order autotuner for the maintenance phase of anaesthesia. Using small-amplitude sine tests to non-invasively estimate patient parameters, they designed a fractional-order PID controller to regulate the Bispectral Index during Propofol infusion. Closed-loop simulations confirmed the effectiveness of this approach, highlighting its potential for clinical practice.

3. Concluding Remarks

The contributions gathered in this Reprint reflect the richness and vitality of fractional calculus research today. They bring together rigorous theoretical work, efficient numerical methods, and impactful applications spanning engineering, materials science, physics, and medicine. As guest editors, we are grateful to the authors for their contributions, the reviewers for their careful evaluations, and the editorial team of *Fractal and Fractional* for their support. We hope this collection will serve as a valuable reference for the research community and stimulate further studies at the intersection of mathematics, engineering, and applied sciences, reinforcing the role of fractional calculus as a unifying tool across disciplines.

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List of Contributions

1. Albidah, A.B. Application of Riemann–Liouville Derivatives on Second-Order Fractional Differential Equations: The Exact Solution. *Fractal Fract.* **2023**, *7*, 843. https://doi.org/10.3390/fractalfract7120843.
2. Alkandari, M.; Loutchko, D.; Luchko, Y. Anomalous Diffusion Models Involving Regularized General Fractional Derivatives with Sonin Kernels. *Fractal Fract.* **2025**, *9*, 363. https://doi.org/10.3390/fractalfract9060363.
3. Şengül, S.; Bekiryazici, Z.; Merdan, M. Approximate Solutions of Fractional Differential Equations Using Optimal q-Homotopy Analysis Method: A Case Study of Abel Differential Equations. *Fractal Fract.* **2024**, *8*, 533. https://doi.org/10.3390/fractalfract8090533.
4. AlBaidani, M.M. Comparative Study of the Nonlinear Fractional Generalized Burger–Fisher Equations Using the Homotopy Perturbation Transform Method and New Iterative Transform Method. *Fractal Fract.* **2025**, *9*, 390. https://doi.org/10.3390/fractalfract9060390.
5. Abdelfattah, W.M.; Ragb, O.; Salah, M.; Mohamed, M. A Robust and Versatile Numerical Framework for Modeling Complex Fractional Phenomena: Applications to Riccati and Lorenz Systems. *Fractal Fract.* **2024**, *8*, 647. https://doi.org/10.3390/fractalfract8110647.
6. Muhammad, G.; Akram, M.; Alsulami, H.; Hussain, N. Granular Fuzzy Fractional Continuous-Time Linear Systems: Roesser and Fornasini–Marchesini Models. *Fractal Fract.* **2025**, *9*, 398. https://doi.org/10.3390/fractalfract9070398.
7. Al-Dosari, A.A.M. Controllability of Mild Solution to Hilfer Fuzzy Fractional Differential Inclusion with Infinite Continuous Delay. *Fractal Fract.* **2024**, *8*, 235. https://doi.org/10.3390/fractalfract8040235.
8. Yu, D.; Liao, X.; Wang, Y. Modeling and Analysis of Caputo–Fabrizio Definition-Based Fractional-Order Boost Converter with Inductive Loads. *Fractal Fract.* **2024**, *8*, 81. https://doi.org/10.3390/fractalfract8020081.
9. Xu, K.; Chen, L.; Lopes, A.M.; Wang, M.; Wu, R.; Zhu, M. Fractional-Order Zener Model with Temperature–Order Equivalence for Viscoelastic Dampers. *Fractal Fract.* **2023**, *7*, 714. https://doi.org/10.3390/fractalfract7100714.
10. Garcia-de-los-Rios, V.M.; Arano-Martínez, J.A.; Trejo-Valdez, M.; Hernández-Pichardo, M.L.; Vidales-Hurtado, M.A.; Torres-Torres, C. Fractional Photoconduction and Nonlinear Optical Behavior in ZnO Micro and Nanostructures. *Fractal Fract.* **2023**, *7*, 885. https://doi.org/10.3390/fractalfract7120885.
11. Abdelfattah, W.M.; Ragb, O.; Salah, M.; Matbuly, M.S.; Mohamed, M. Fractional Partial Differential Equation Modeling for Solar Cell Charge Dynamics. *Fractal Fract.* **2024**, *8*, 729. https://doi.org/10.3390/fractalfract8120729.
12. Mihai, M.D.; Birs, I.R.; Badau, N.E.; Hegedus, E.T.; Ynineb, A.; Muresan, C.I. Personalised Fractional-Order Autotuner for the Maintenance Phase of Anaesthesia Using Sine-Tests. *Fractal Fract.* **2025**, *9*, 317. https://doi.org/10.3390/fractalfract9050317.

References

1. Oliveira, E.C.D.; Machado, J.A.T. A Review of Definitions for Fractional Derivatives and Integral. *Math. Probl. Eng.* **2014**, *2014*, 238459 . [CrossRef]
2. Caputo, M.; Fabrizio, M. A new definition of fractional derivative without singular kernel. *Prog. Fract. Differ. Appl.* **2015**, *1*, 73–85.
3. Atangana, A.; Baleanu, D. New fractional derivatives with nonlocal and non-singular kernel: Theory and application to heat transfer model. *Therm. Sci.* **2016**, *20*, 763–769. [CrossRef]
4. Atangana, A. Fractional Operators and Their Applications. In *Fractional Operators with Constant and Variable Order with Application to Geo-Hydrology*; Academic Press: Amsterdam, The Netherlands , 2018; pp. 79–112. [CrossRef]
5. Podlubny, I. *Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications*; Elsevier: Amsterdam, The Netherlands, 1998; Volume 198.
6. Kilbas, A.A.; Srivastava, H.M.; Trujillo, J.J. *Theory and Applications of Fractional Differential Equations*; Elsevier: Amsterdam, The Netherlands, 2006; Volume 204.
7. Baleanu, D.; Agarwal, R.P. Fractional calculus in the sky. *Adv. Differ. Equ.* **2021**, *117*, 1–9 . [CrossRef]
8. Sun, H.; Zhang, Y.; Baleanu, D.; Chen, W.; Chen, Y.Q. A new collection of real world applications of fractional calculus in science and engineering. *Commun. Nonlinear Sci. Numer. Simul.* **2018**, *64*, 213–231. [CrossRef]
9. Madani, Y.A.; Ali, Z.; Rabih, M.; Alsulami, A.; Eljaneid, N.H.E.; Aldwoah, K.; Muflh, B. Discrete Fractional-Order Modeling of Recurrent Childhood Diseases Using the Caputo Difference Operator. *Fractal Fract.* **2025**, *9*, 55. [CrossRef]
10. Kumar, D.; Singh, J. (Eds.) *Fractional Calculus in Medical and Health Science*, 1st ed.; CRC Press: Boca Raton, FL, USA, 2020. [CrossRef]
11. Hafez, M.; Alshowaikh, F.; Voon, B.W.N.; Alkhazaleh, S.; Al-Faiz, H. Review on Recent Advances in Fractional Differentiation and its Applications. *Prog. Fract. Differ. Appl.* **2025**, *11*, 245–261. [CrossRef]
12. Volos, C. *Introductory Chapter: Fractional Calculus-From Theory to Applications*; IntechOpen: London, UK, 2025. [CrossRef]