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Editorial

Special Issue: Nonlinear Dynamics in Complex Systems via Fractals and Fractional Calculus

Viorel-Puiu Paun [1,2]

[1] Physics Department, Faculty of Applied Sciences, University Politehnica of Bucharest, 060042 Bucharest, Romania; viorel.paun@physics.pub.ro
[2] Academy of Romanian Scientists, 050094 Bucharest, Romania

Advances in our knowledge of nonlinear dynamical networks, systems and processes (as well as their unified repercussions) currently allow us to study many typical complex phenomena taking place in nature, from the nanoscale to the extra-galactic scale, in an comprehensive manner. Thus, systems generally deemed dynamical systems, chaotic systems or fractal systems clearly have something essential in common, and can be considered to belong to the same class of complex phenomena discussed herein. In other words, the physical, biological and financial data of complex systems, as well as the technological data (observed using mechanical or electronic devices), available today can be managed using same unique conceptual approach; this approach works both analytically and through computer simulations, using effective nonlinear dynamics procedures. The works presented in this technical publication are those that have appeared in the *Fractal and Fractional* journal in a Special Issue of the same name, which included the thirteen individually published papers plus an Editorial signed by the editor of this book.

In the first work introduced in this volume, the authors affirmed that the accurate determination of atmospheric temperature using telemetric platforms is an active issue, and one that can also be tackled with the aid of multifractal theory to observe the fundamental behaviors of the lower atmosphere. These observations can then be used to facilitate such determinations [1]. Thereby, within the framework of the scale relativity theory, PBL dynamics can be analyzed with the aid of a multifractal hydrodynamic scenario. Considering the PBL as a complex system that is assimilated into mathematical objects of a multifractal type, its various dynamics exert a multifractal tunnel effect. Such a treatment allows one to define both a multifractal atmospheric transparency coefficient and a multifractal atmospheric reflectance coefficient. These products are then used to create theoretical temperature profiles, which lead to correlations with real results obtained using radiometer data (RPG-HATPRO radiometer), with favorable results. Such methods could be further used and refined in future applications to efficiently produce theoretical atmospheric temperature profiles.

In the reference [2], the authors consider that the study of hidden attractors plays a very important role in the engineering applications of nonlinear dynamical systems. Thus, in this paper, a new three-dimensional (3D) chaotic system is proposed, in which hidden attractors and self-excited attractors appear as the parameters change. Meanwhile, asymmetric coexisting attractors are also found as a result of the system's symmetry. The complex dynamical behaviors of the proposed system were investigated using various tools, including time series diagrams, Poincaré first return maps, bifurcation diagrams, and basins of attraction. Moreover, unstable periodic orbits within a topological length of 3 in the hidden chaotic attractor were calculated systematically using the variational method, which required six letters to establish suitable symbolic dynamics [2]. Furthermore, the practicality of the hidden attractor chaotic system was verified using circuit simulations. Finally, offset boosting control and adaptive synchronization were used to investigate the utility of the proposed chaotic system in engineering applications.

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The third selected work refers to the fractal analysis of some nuclear ceramic materials. SEM micrographs of the fracture surface of UO2 ceramic materials have been analyzed. In this paper, some algorithms were introduced, and a computer application based on the non-linear time series method was developed. Utilizing the embedding technique of phase space, the attractor is reconstructed. In addition, the fractal dimension, lacunarity, and autocorrelation dimension average value have been calculated [3].

To further understand the dynamical characteristics of chaotic systems with a hidden attractor and coexisting attractors, we refer readers to the fourth work. Here, the fundamental dynamics of a novel three-dimensional (3D) chaotic system, derived by adding a simple constant term to the Yang–Chen system, were investigated under different parameters; these include the bifurcation diagram, Lyapunov exponents spectrum, and basin of attraction [4]. Additionally, an offset-boosting control method is presented to the state variable, and a numerical simulation of the system is also introduced. Furthermore, the unstable cycles embedded in the hidden chaotic attractors are extracted in detail, which shows the effectiveness of the variational method and the one-dimensional symbolic dynamics. Finally, the adaptive synchronization of the novel system is successfully designed, and a circuit simulation is implemented to illustrate the flexibility and validity of the numerical results. Theoretical analysis and simulation results indicate that the new system has complex dynamical properties, and can be used to facilitate engineering applications [4].

The fifth work is dedicated to fuzzy dispersion entropy (FuzzDE), a very recently proposed non-linear dynamical indicator which combines the advantages of both dispersion entropy (DE) and fuzzy entropy (FuzzEn) to detect dynamic changes in a time series. However, FuzzDE only reflects the information of the original signal, and is not very sensitive to dynamic changes. To address these drawbacks, a fractional order calculation on the basis of FuzzDE was proposed; it is referred to as FuzzDEα. The calculation may be used as a tool for signal analysis and the fault diagnosis of bearings [5]. In addition, other fractional order entropies were introduced, including fractional order DE (DEα), fractional order permutation entropy (PEα), and fractional order fluctuation-based DE (FDEα); a mixed-features extraction diagnosis method was also proposed. Both simulated as well as real-world experimental results demonstrated that the FuzzDEα, at different fractional orders, is more sensitive to changes in the dynamics of the time series. The proposed mixed-features bearing fault diagnosis method achieves a 100% recognition rate with only three features, among which the mixed-feature combinations with the highest recognition rates all include FuzzDEα. FuzzDEα also appears most frequently [5].

Investigating global bifurcation behaviors, the vibrating structures of micro-electro-mechanical systems (MEMS) received substantial attention in the sixth work of this collection. This paper considers the vibrating system of a typical bilateral MEMS resonator containing fractional functions and multiple potential wells. By introducing new variations, the Melnikov method is applied to derive the critical conditions for global bifurcations. By engaging in the fractal erosion of the safe basin to intuitively depict the phenomenon of pull-in instability, the point-mapping approach is used to present numerical simulations that are in close agreement with analytical predictions, showing the validity of the analysis. It is found that chaos and pull-in instability, two initially sensitive phenomena of MEMS resonators, may be due to homoclinic bifurcation and heteroclinic bifurcation, respectively [6]. On this basis, two types of delayed feedback are proposed to control the complex dynamics successively. Their control mechanisms and effects are then studied. It follows that under a positive gain coefficient, delayed position feedback and delayed velocity feedback can both reduce pull-in instability; nevertheless, in suppressing chaos, only the former is effective. The results may have some potential value in broadening the application fields of global bifurcation theory, and in improving the performance reliability of capacitive MEMS devices [6].

In the seventh paper presented in this book, SEM microfractographies of Zircaloy-4 are studied using fractal analysis and the time series method. First, a computer application that associates a fractal dimension and lacunarity with each SEM micrograph picture was developed; the application also produced a nonlinear analysis of the data acquired from the quantitatively evaluated time series. Utilizing the phase space-embedding technique to reconstruct the attractor and to compute the autocorrelation dimension, the fracture surface of the Zircaloy-4 samples was investigated. The fractal analysis method manages to highlight damage complications, and provides a description of the morphological parameters of various fractures by calculating the fractal dimension and lacunarity [7].

In the eighth study presented herein, the authors discuss the engineering and construction of a special sixth-generation (6G) antenna based on the fractal known as Minkowski's loop. The antenna has the shape of this known fractal, set at four iterations, to obtain maximum performance. The frequency bands for which this 6G fractal antenna was designed are 170 GHz to 260 GHz (WR-4) and 110 GHz to 170 GHz (WR-6), respectively. The three resonant frequencies, optimally used, are equal to 140 GHz (WR-6) for the first, 182 GHz (WR-4) for the second, and 191 GHz (WR-4) for the third. The electromagnetic behaviors of the fractal antennas and their graphical representations are highlighted at these frequencies [8].

In the next work, the ninth of the thirteen, it is established that slope entropy (SlEn) is a time series complexity indicator proposed in recent years that has shown excellent performance in the fields of medicine and hydroacoustics. In order to improve the ability of SlEn to distinguish between different types of signals and solve the problem of selecting two threshold parameters, a new time series complexity indicator is proposed on the basis of SlEn. This was achieved by introducing fractional calculus and combining particle swarm optimization (PSO) in an indicator named PSO fractional SlEn (PSO-FrSlEn). Then, PSO-FrSlEn is applied to the field of fault diagnosis, and a single-feature extraction method and a double-feature extraction method based on PSO-FrSlEn are proposed for rolling bearing faults [9]. The experimental results illustrated that only PSO-FrSlEn can classify ten kinds of bearing signals with 100% classification accuracy (by using double features), which is at least 4% higher than the classification accuracies of the other four fractional entropies [9].

In the tenth work, it is shown that the metric of H may be denoted by d when H is a compact metric space. Then, we can let $(H,f1,\infty)$ be a non-autonomous discrete system, where $f1,\infty = \{fn\}\infty n = 1$ is a mapping sequence. This paper discusses the infinite sensitivity, m-sensitivity, and m-cofinite sensitivity of $f1,\infty$. It proves that if $fn(n\in\mathrm{N})$ are feebly open and uniformly converge to $f{:}H\to H$, $fi\circ f = f\circ fi$ for any $i\in\{1,2,\ldots\}$, and $\sum\infty i = 1D(fi,f) < \infty$, then (H,f) has the above sensitive property if and only if $(H,f1,\infty)$ has the same property, where $D(\cdot,\cdot)$ is the supremum metric [10].

The investigation of chaotic systems containing hidden and coexisting attractors has attracted extensive attention. The eleventh paper presents a four-dimensional (4D) novel hyperchaotic system that is advanced by adding a linear state feedback controller to a three-dimensional chaotic system with two stable node focus points [11]. The proposed system has no equilibrium point, or two lines of equilibria, depending on the value of the constant term. Complex dynamical behaviors, such as hidden chaotic and hyperchaotic attractors and the five types of coexisting attractors within the simple four-dimensional autonomous system, are investigated and discussed, and they are numerically verified through the analysis of phase diagrams, Poincaré maps, the Lyapunov exponent spectrum, and its bifurcation diagram. The short unstable cycles in the hyperchaotic system are systematically explored using the variational method, and symbol codings of cycles with four letters are produced based on the topological properties of the trajectory's projection onto the two-dimensional phase space. The bifurcations of the cycles are explored through a homotopy evolution approach. Finally, the novel four-dimensional system is implemented using an analog electronic circuit, and is found to be consistent with the numerical simulation results [11].

In the twelfth manuscript, multifractal theories of motion based on scale relativity theory are considered in the description of atmospheric dynamics. It is shown that these theories have the potential to highlight the nondimensional mass conduction laws that describe the propagation of atmospheric entities [12]. Then, using special operational procedures and harmonic mappings, these equations may be rewritten and simplified so that their plotting and analysis may be performed. The inhomogeneity of these conduction phenomena was analyzed, and the study found that it can fluctuate and increase at certain fractal dimensions, leading to the conclusion that certain atmospheric structures and phenomena of either atmospheric transmission or stability can be explained by atmospheric fractal dimension inversions. Finally, this hypothesis is verified using the ceilometer data found throughout the atmospheric profiles [12].

This Special Issue, which is the subject of our editorial, also collates some new insights into the theory of hidden attractors and multistability phenomena, which have considerable application prospects in engineering [13]. Thus, in the final work, the thirteenth, by modifying a simple three-dimensional continuous quadratic dynamical system, a new autonomous chaotic system with two stable node foci that can generate double-wing hidden chaotic attractors is reported. The rich dynamics of the proposed system were discussed; said system has some interesting characteristics in terms of its different parameters and initial conditions, which were found through the use of dynamic analysis tools such as the phase portrait, the Lyapunov exponent spectrum, and bifurcation diagrams. The topological classification of the periodic orbits of the system was investigated using a recently devised variational method. The symbolic dynamics of four and six letters have been successfully established under two sets of system parameters, including hidden and self-excited chaotic attractors [13]. The system was implemented using a corresponding analog electronic circuit to verify its realizability.

This volume gathers together information on some important advances in the fields of fractal curves, fractal analysis and fractional calculus [14,15]. Thereby, the Special Issue which is the subject of our editorial also collates some novel insights into the theory of complex systems; it is a significant and relevant volume for our field of study, and will be appreciated as a useful reference within the specialized literature.

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