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The Stochastic Structural Modulations in Collapsing Maccari's Model Solitons

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Abstract: The two-dimensional Maccari nonlinear system performs the energy and wave dynamical features in fiber communications and modern physical science as hydrodynamic and space plasma. Several new forms of solutions for the Maccari's model are constructed by a unified solver method that mainly depends on He's variations method. The obtained solutions identify new wave stochastic structures with important features in energy physics such as rational explosive, breather, dispersive, explosive dissipated, dark solitons and blow-up (shock structure). It was elucidated that the random effects amend the energy wave strength or the collapsing due to model medium turbulence. Finally, the produced stochastic structures may be vital in some of these relationships between dispersions, nonlinearity and dissipative effects. The predominant energy waves that are collapsing or being forced may be applied to electrostatic auroral Langmuir structures and energy-generating ocean waves.

Keywords: Maccari system; nonlinearity structures; hyperbolic functions; physical parameters

MSC: 34A05; 34A34; 35Q35; 35Q62

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1. Introduction

Nonlinear partial differential equations (NPDEs) are frequently utilized to characterize different complex phenomena of applied sciences, such as plasma physics, optical fiber communications, fluid mechanics, biochemistry, networked systems, etc. [1–5]. Solving these equations results in a relatively correct recognition of the researchers from the outlined process, allowing them to learn about some truths that cannot be grasped through normal observation. In the contemporary scientific and technological period, numerous researchers have been hired to develop numerous analytical procedures to obtain solitary wave solutions for NPDEs [6–10].

A stochastic process represents an observation at a specific time where the result is a random variable. The Brownian motion process, commonly known as the Wiener process, is a frequent stochastic process that is both a martingale and a Markov process [11]. As the cornerstone of stochastic calculus, the Brownian process is crucial for modeling stochastic processes. The Brownian process is a frequently utilized stochastic process in dispersive situations [12,13]. We believe that recent advances in stochastic calculus via stochastic partial differential equations (SPDEs), will provide a foundation for modeling real life systems in a comprehensive way. The phrase "stochastic dynamics" refers to the temporal dynamics of random variables, which comprises the body of knowledge that includes stochastic processes, SPDEs, and applications of such knowledge to real-world systems [12]. The SPDEs and stochastic processes are still a domain where mathematicians more than anybody else are comfortable in applying to natural models.

Maccari's system (MS) denotes a type of NPDEs that is frequently used to depict the dynamics of isolated waves in a smallness area of space in various domains of superfluid, physics of plasmas, nonlinear optics, quantum mechanics, and so on [14–19]. This system was obtained from the Kadomtsev–Petviashvili equation by Maccari using a reduction strategy depending on spatiotemporal rescaling [20]. Some recent studies offered new original conceptions in wave dynamics in the survey of nonlinear unanticipated crucial behaviors; such acoustics waves in fluid and electrostatic noise in auroras [21–24]. Maccari [25] showed how the MS accurately characterized the very important characteristics of rogue waves and how they might be used to study different nonlinear forms in the form of standing waves, nonlinear optical fibers, and fluid mechanics. It was reported that the (2+1)-dimensions of rogue waves in the laboratory enabled controlled experimentation in optics and water waves. Moreover, it was suggested that the MS system may be used for more complex systems to study the dynamics of water waves and the production of energy waves. Additionally, the nonlinear auroral Langmuir electrostatic waveforms have been observed and discussed for collapsing energy and electrostatic waves [26,27]. Furthermore, all of these previous studies were taken from a deterministic point of view.

We consider the coupled MS [15,28,29] via Brownian motion process, given as follows:

$$\begin{aligned} &i\,Q_t + Q_{xx} + \Psi Q - i\,\delta\,Q\,\Xi_t = 0\,,\\ &\Psi_t + \Psi_y + (\mid Q \mid^2)_x = 0, \end{aligned} \tag{1}$$

where $Q = Q(x, y, t)$ represents the complex scalar field and $\Psi = \Psi(x, y, t)$ represents the real scalar field. The noise Ξ_t is a Brownian times derivative of $\Xi(t)$ and δ denotes noise amplitude [30]. Zhao [31] introduced some general solitary wave solutions for model (1). Moreover, a number of periodic and solitons of the aforementioned system have lately been reported [15,28,29,32]. All these papers are conducted for the coupled MS without the stochastic influence. It is appropriate to present a formulation of the Brownian motion process $\{\Xi(t)\}_{t\geq 0}$ that satisfies the following requirements:

(i) $\Xi(t)$; $t \geq 0$ is a continuous function of t,
(ii) $\Xi(r) - \Xi(t)$ is independent of increments, for $r < t$,
(iii) $\Xi(t) - \Xi(r)$ follows a normal distribution with mean 0; variance $t - r$.

To ensure that the investigation of Brownian motion is thorough, $Xi(t)$. The distributional derivative of Brownian motion $\dot{\Xi} = \Xi_t = \frac{d\Xi}{dt}$ is the white noise in time. It is a delta correlated in the sense that

$$\mathbb{E}(\dot{\Xi}(t)\dot{\Xi}(r)) = \delta_{t-r},$$

δ is the Dirac mass. It is common to view white noise as a mathematical idealization of events such as abrupt and enormous fluctuations. Additionally, there have been numerous numerical and analytical studies on NPDEs through the Brownian motion process [33,34].

In the ongoing work, we extract some new stochastic solutions for MS constrained by multiplicative noises in Itô sense via a unified technique. The suggested technique has a number of advantages over the majority of existing methods, including the avoidance of laborious and time-consuming computations and the production of accurate results. It is straightforward, reliable, and effective. This approach presents some types of solitary waves based on the physical parameters. These solutions allow for crucial applications in hydrodynamic, optical fiber communications, and plasma physics [35,36]. The presented method can be implemented as a box-solver for several systems in natural science. Furthermore, we introduce the potential form related to the energy equation of Equation (1). To the best of our knowledge, the proposed technique for solving the MS has never been used before.

In Section 3, we introduce new stochastic solutions for MS in Itô sense. Section 2 briefly describes the method that is used to find the solitary wave solutions to the two-dimensional nonlinear Maccari's model. In Section 4, we present the potential model that corresponds to the energy equation of model (1). We also illustrate the physical interpretation of the presented solutions and the influence of the noise term on the behavior of these solutions. Additionally, some graphs of some acquired answers are shown. The concluding remarks and future directions are provided in Section 5.

2. Description of the Method

Consider the NLPDEs in the form

$$\mathbb{G}(\Phi, \Phi_x, \Phi_t, \Phi_y, \Phi_{xx}, \Phi_{tt}, \Phi_{yy}, \Phi_{xt}, \dots) = 0. \tag{2}$$

Using the wave transformation:

$$\Phi(x, y, t) = \Phi(\xi), \qquad \xi = x + y - 2ct, \tag{3}$$

c is the velocity of the wave, Equation (2) converted to the following ODE:

$$\mathbb{H}(\Phi, \Phi', \Phi'', \Phi''', \dots) = 0. \tag{4}$$

Several applied science models of the form (2) were translated to the following ODE:

$$A\Phi'' + B\Phi^3 + C\Phi = 0. \tag{5}$$

Based on the main model's constants and the wave speed, A, B, and C are specific constants.

The closed form solutions of Equation (5) are [19]:

2.1. Family 1

The first family of solutions is

$$\Phi(\xi) = \pm\sqrt{\frac{-2C}{B}}\,\mathrm{sech}\left(\pm\sqrt{-\frac{C}{A}}\xi\right). \tag{6}$$

2.2. Family 2

The second family of solutions is

$$\Phi(\xi) = \pm\sqrt{\frac{-35\,C}{18\,B}}\mathrm{sech}^2\left(\pm\sqrt{-\frac{5\,C}{12\,A}}\,\xi\right). \tag{7}$$

2.3. Family 3

The third family of solutions is

$$\Phi(\xi) = \pm\sqrt{\frac{-C}{B}}\,\tanh\left(\pm\sqrt{\frac{C}{A}}\xi\right). \tag{8}$$

3. Solutions of MS

Using the transformation:

$$Q(x, y, t) = q(\xi)\, e^{i(cx+\alpha\, y+\gamma\, t)+\delta\, \Xi(t)-\delta^2\, t}, \quad \Psi(x, y, t) = \Psi(\xi), \; \xi = x + \beta y - 2ct, \tag{9}$$

c, α, γ and β are constants, the Equation (1) becomes

$$
\begin{aligned}
&q''(\xi) + \Psi(\xi)q(\xi) - (\gamma + c^2)q(\xi) = 0, \\
&2e^{2\delta(\Xi(t)-\delta t)}q(\xi)q'(\xi) - (2c - \beta)\Psi'(\xi) = 0\,.
\end{aligned}
\tag{10}
$$

Taking expectations on both sides for the second equation of (10) yields

$$
2e^{-2\delta^2 t}\, E(e^{2\delta\Xi(t)})q(\xi)q'(\xi) - (2c - \beta)\Psi'(\xi) = 0\,, \tag{11}
$$

since $E(e^{2\delta\Xi(t)}) = e^{2\delta^2 t}$, Equation (11) is reduced to

$$
2q(\xi)q'(\xi) - (2c - \beta)\Psi'(\xi) = 0.
$$

Integrating the last equation with an integration constant equals zero and solving the result gives

$$
\Psi(\xi) = \left(\frac{1}{2c - \beta}\right)q^2\,(\xi). \tag{12}
$$

Substituting Equation (12) into the first equation of system (10) yields:

$$
K_1 q''(\xi) + K_2 q^3(\xi) + K_3 q(\xi) = 0\,, \tag{13}
$$

where

$$
K_1 = 1, K_2 = \frac{1}{2c - \beta}, K_3 = -(\gamma + c^2). \tag{14}
$$

In view of solver method [19], Equation (1) stochastic forms of are:

The First Family of Solutions

$$
q_{1,2}(\xi) = \pm\sqrt{2(2c - \beta)(\gamma + c^2)}\,\mathrm{sech}\left(\pm\sqrt{\gamma + c^2}\,\xi\right). \tag{15}
$$

Thus:

$$
Q_{1,2}(x, y, t) = \pm\, e^{i(cx+\alpha\, y+\gamma\, t)+\delta\Xi(t)-\delta^2 t}\sqrt{2(2c - \beta)(\gamma + c^2)}\,\mathrm{sech}\left(\pm\sqrt{\gamma + c^2}\,(x + \beta y - 2ct)\right). \tag{16}
$$

$$
\Psi_{1,2}(x, y, t) = 2(\gamma + c^2)\,\mathrm{sech}^2\left(\pm\sqrt{\gamma + c^2}\,(x + \beta y - 2ct)\right). \tag{17}
$$

The Second Family of Solutions

$$
q_{3,4}(\xi) = \pm\sqrt{\frac{35(\gamma + c^2)(2c - \beta)}{18}}\,\mathrm{sech}^2\left(\pm\sqrt{\frac{5(\gamma + c^2)}{12}}\,\xi\right). \tag{18}
$$

Thus:

$$
Q_{3,4}(x, y, t) = \pm e^{i(cx+\alpha\, y+\gamma\, t)+\delta\Xi(t)-\delta^2 t}\sqrt{\frac{35(\gamma + c^2)(2c - \beta)}{18}}\,\mathrm{sech}^2\left(\pm\sqrt{\frac{5(\gamma + C^2)}{12}}\,(x + \beta y - 2ct)\right). \tag{19}
$$

$$
\Psi_{3,4}(x, y, t) = \frac{35(\gamma + c^2)}{18}\,\mathrm{sech}^4\left(\pm\sqrt{\frac{5(\gamma + c^2)}{12}}\,(x + \beta y - 2ct)\right). \tag{20}
$$

The Third Family of Solutions

$$q_{5,6}(\xi) = \pm\sqrt{(\gamma + c^2)(2c - \beta)}\tanh\left(\pm i\sqrt{\frac{(\gamma + c^2)}{2}}\xi\right). \tag{21}$$

Thus:

$$Q_{5,6}(x, y, t) = \pm\, e^{i(cx+\alpha\, y+\gamma\, t)+\delta\Xi(t)-\delta^2 t}\sqrt{(\gamma + c^2)(2c - \beta)}\tanh\left(\pm i\sqrt{\frac{\gamma + c^2}{2}}\,(x + \beta y - 2ct)\right). \tag{22}$$

$$\Psi_{5,6}(x, y, t) = (\gamma + c^2)\tanh^2\left(\pm i\sqrt{\frac{\gamma + c^2}{2}}\,(x + \beta y - 2ct)\right). \tag{23}$$

4. Results and Discussion

We have implemented the unified technique for finding new considerable stochastic forms for the coupled nonlinear Maccari's equations with multiplicative noises in Itô sense. This system, which can arise in domains such as plasma physics, superfluids, and optical fiber communications, is a complicated nonlinearly mode that characterizes the dynamics of isolated waves, confined in a smallness of parts of space [15,37].

This random Maccari model is converted to nonlinear ordinary differential equations via $\Xi(t)$ function. The expectations of this model with $E(e^{2\delta\Xi(t)}) = e^{2\delta^2 t}$ converts the model to Equation (13) that symbolizes the model of motion for potential form

$$V = -\frac{1}{2}\left(\gamma + c^2\right)q(\xi)^2 - \frac{1}{4(2c - \beta)}q(\xi)^4. \tag{24}$$

The dynamical potential equation has an exact solution for a nonlinear Maccari system in the form

$$q(x, y, t) = \frac{2\sqrt{2}c^2\sqrt{\frac{2c-\beta}{c^2+\gamma}}e^{\sqrt{c^2+\gamma}(x+\beta y-2ct)}}{e^{2\sqrt{c^2+\gamma}(x+\beta y-2ct)} + 1} + \frac{2\sqrt{2}\gamma e^{\sqrt{\gamma+c^2}(x+\beta y-2ct)}\sqrt{\frac{2c-\beta}{\gamma+c^2}}}{e^{2(x+\beta y-2ct)\sqrt{c^2+\gamma}} + 1}, \tag{25}$$

with complex scalar field

$$\begin{aligned} Q(x, y, t) \quad = \quad & \pm\, e^{i(cx+\alpha\, y+\gamma\, t)+\delta\Xi(t)-\delta^2 t}\Big(\frac{2\sqrt{2}c^2\sqrt{\frac{2c-\beta}{c^2+\gamma}}e^{\sqrt{c^2+\gamma}(x+\beta y-2ct)}}{e^{2\sqrt{c^2+\gamma}(x+\beta y-2ct)} + 1} \\ & + \frac{2\sqrt{2}\gamma e^{\sqrt{\gamma+c^2}(x+\beta y-2ct)}\sqrt{\frac{2c-\beta}{\gamma+c^2}}}{e^{2(x+\beta y-2ct)\sqrt{c^2+\gamma}} + 1}\Big), \end{aligned} \tag{26}$$

and real scalar field $\Psi(x, y, t)$ in the form

$$\begin{aligned} \Psi(x, y, t) \quad = \quad & \left(\frac{1}{2c - \beta}\right)\Big(\frac{2\sqrt{2}c^2\sqrt{\frac{2c-\beta}{c^2+\gamma}}e^{\sqrt{c^2+\gamma}(x+\beta y-2ct)}}{e^{2\sqrt{c^2+\gamma}(x+\beta y-2ct)} + 1} \\ & + \frac{2\sqrt{2}\gamma e^{\sqrt{\gamma+c^2}(x+\beta y-2ct)}\sqrt{\frac{2c-\beta}{\gamma+c^2}}}{e^{2(x+\beta y-2ct)\sqrt{c^2+\gamma}} + 1}\Big)^2. \end{aligned} \tag{27}$$

The proposed Maccari model was considered in most standard articles in the deterministic situation. In contrast to our method, we study this model in the stochastic situation, that is, one that is forced by multiplicative noise in the Itô sense. A unified theoretical solver approach has been used to identify some novel random solutions for the Maccari model in the Itô sense, which produced a variety of dissipative and dispersive structures in the form solutions of Equation (13). The discovered solutions are hyperbolic function results that accounted for a number of intriguing physical phenomena in diverse fields of engineering and new applied theoretical physics, e.g., the hyperbolic secant appears in laminar jet profiles, while the hyperbolic tangent appears in magnetic moment studies. Furthermore, the acquired solutions are not only bright, rational explosive, breather, dispersive, explosive dissipated and dark solitons, but also blow-up and shock structure solutions of the proposed model.

The solution (26) expresses a random structural representation as shown in Figure 1. This figure shows the symmetric random variations in the phase trajectory. The intense randomness coefficient's influence on structure, amplitude, band width, and energy is shown in Figure 2. It was noted that the increasing δ causes an increasing ability of randomness effectiveness to produce rabid wave collapsing. The solution (19) makes distinct the envelope periodic wave in time t. The random noise effect δ on the wave picture is introduced in Figures 3 and 4. As δ increased, the rate of fluctuations increased and the wave tends to behave as a dissipative wave as in Figure 4. In the same manner, the variations of dissipative solution (22) with space x, time t and the noise effect δ are shown in Figures 5–7. By increasing δ, the rate forcing wave increases with production of the shock random wave with high amplitude as in Figure 7.

In the absence of a noise term, the solution (26) describes both breather envelope and stationary soliton waves as in Figures 8 and 9. Additionally, the breather envelopes of solution (15) are plotted in Figure 10. The solution (19) may be formed by two kinds of important waves, the first is periodic envelope solitons and the other is the bell soliton shape as in Figures 11 and 12. On the other hand, The solution (22) is regarded as one of the related to planning applications in the investigation of dissipative and explosive waves in physics. The creation of the dissipated oscillatory waves is depicted with x and y axes in Figure 13. The oscillating form becomes a high-energy, rational explosive structure by changing the x axis. Finally, Figure 14 depicts a dissipative blow-up wave.

In summary, the important features of random structures of Maccari's system with noise effect produced new structures with different energy properties of the obtained envelopes, solitary and explosive waves.

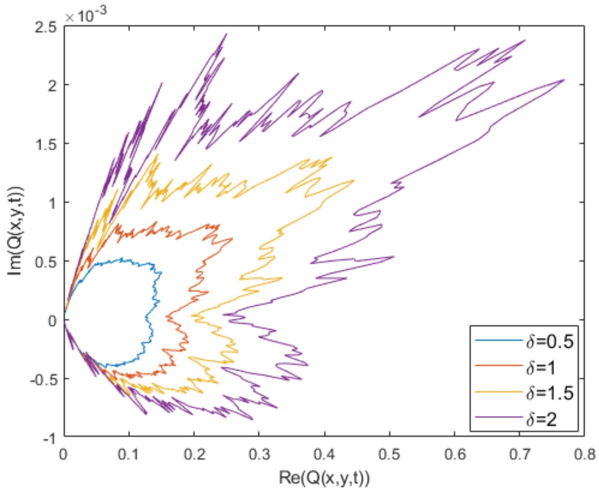

Figure 1. Trajectory of exact solution (26).

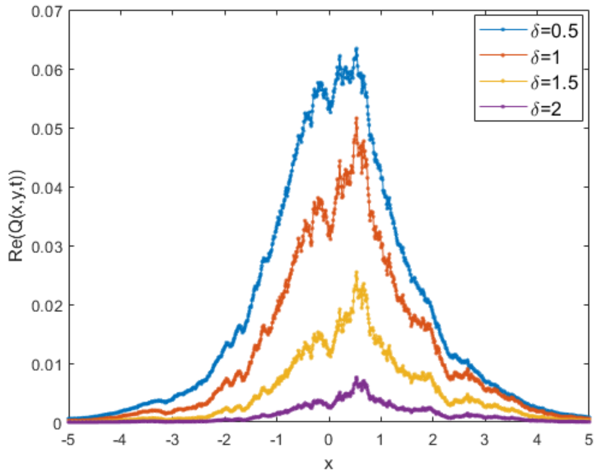

Figure 2. Change of solution (26) with x, δ.

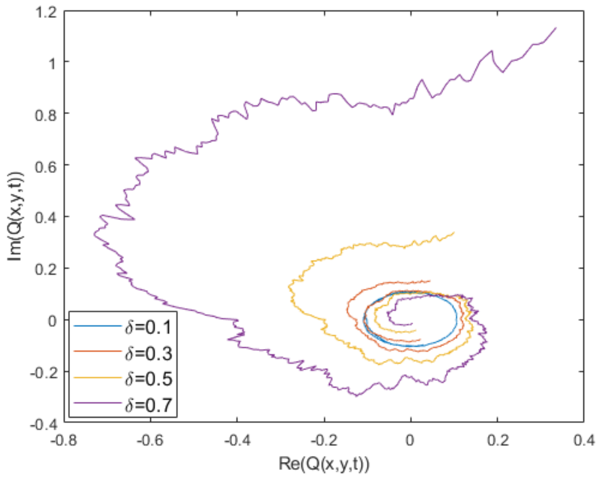

Figure 3. Trajectory of solution (19).

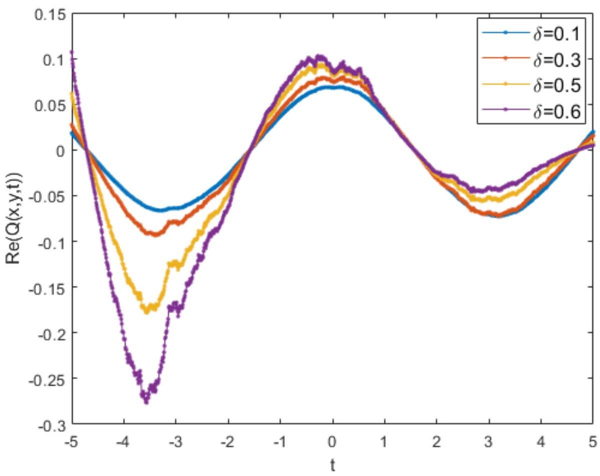

Figure 4. Change of solution (26) with t, δ.

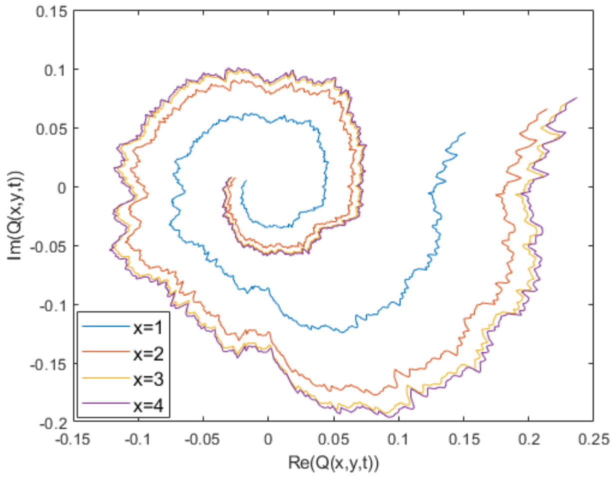

Figure 5. Trajectory of solution (22) with x.

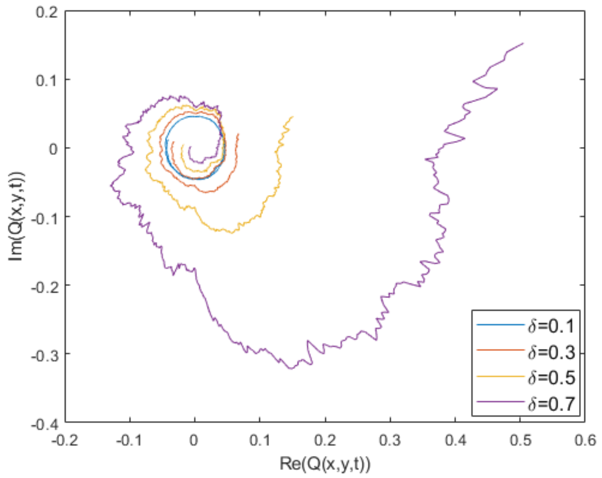

Figure 6. Trajectory of solution (22) with δ.

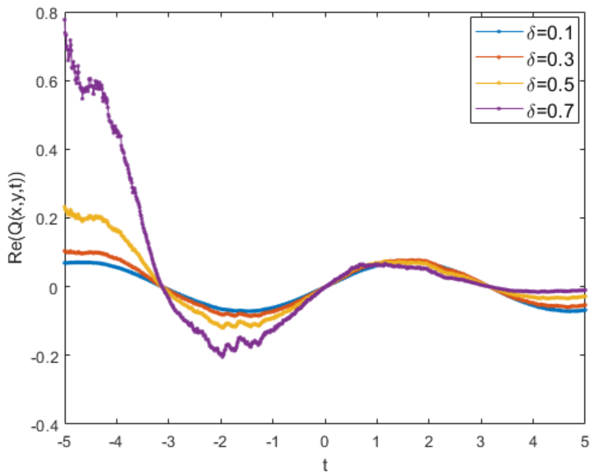

Figure 7. Change of solution (22) with t, δ.

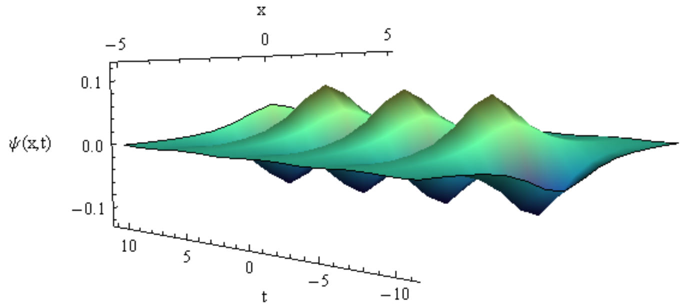

Figure 8. Change of solution (26) with x, t.

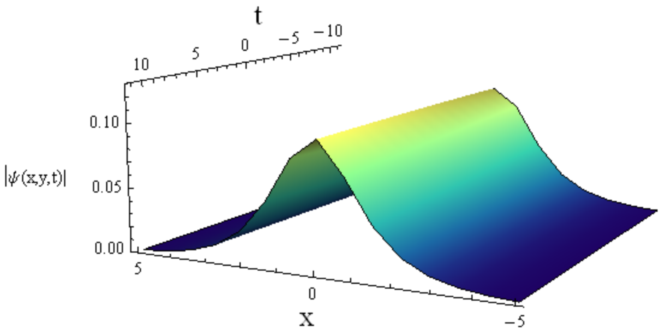

Figure 9. Change of $|Q(x, y, t)|$ with x, t.

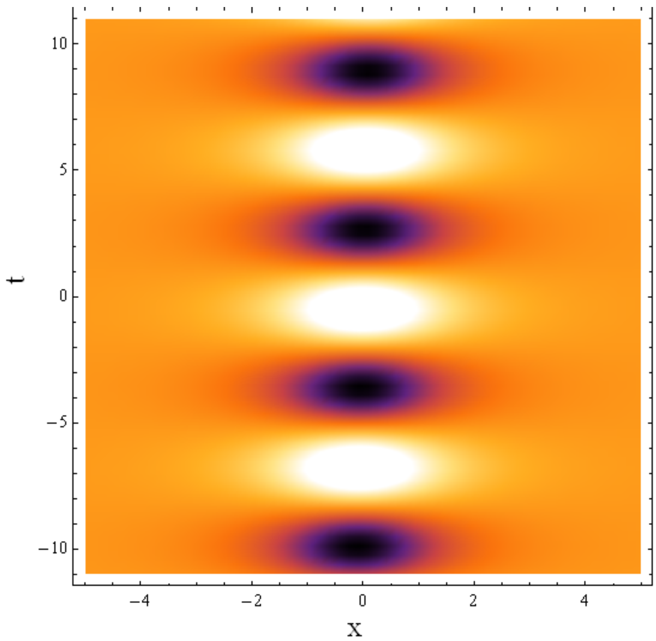

Figure 10. Change of $ReQ_1(x, y, t)$ with x, t.

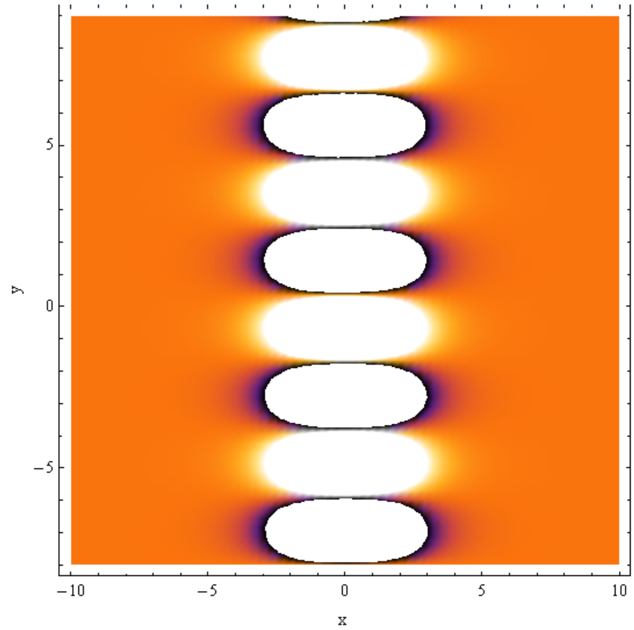

Figure 11. Change of $ReQ_3(x, y, t)$ with x, y at $t = 1$.

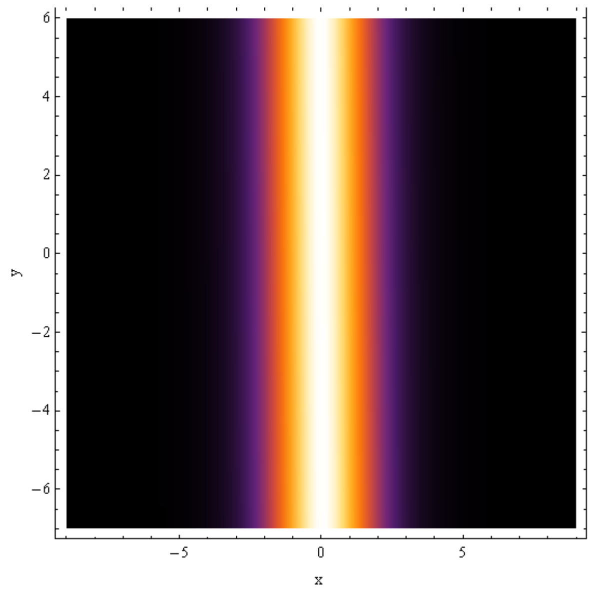

Figure 12. Change of $|Q_3(x, y, t)|$ with x, y at $t = 1$.

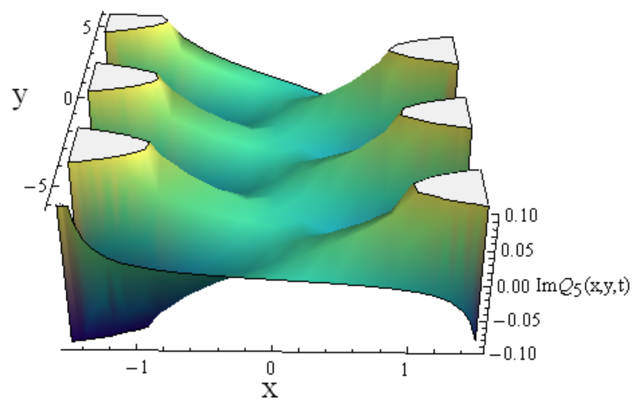

Figure 13. Change of $ImQ_5(x, y, t)$ with x, y.

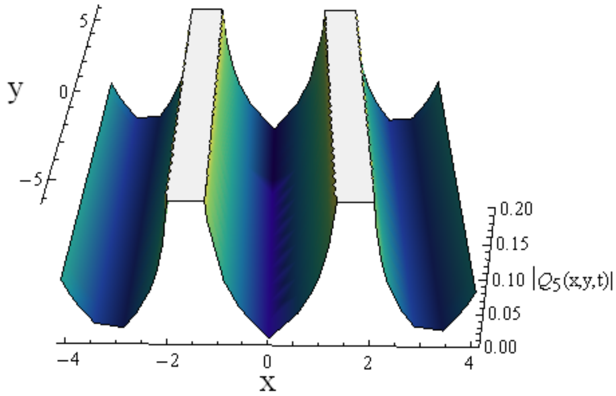

Figure 14. Change of $|Q_5(x, y, t)|$ with x, y.

5. Conclusions

In this work, the two-dimensional nonlinear Maccari's model was studied via a unified approach. Some important new solitary waves are produced. These waves admit vital physical aspects in several branches of science such in the form of bright periodic, explosive rational, breathers, dispersive, blow-up and shock structure solutions. The randomness parameters influence the envelope and solitonic structures and energy properties. It was reported that the increase of random parameters produced both rabid solitonic collapsing or forcing shock wave amplitudes. Additionally, the method used here can be applied in various nonlinear systems for new energy trends in natural science. Finally, the derived stochastic solutions may be crucial in limited relationships between nonlinearity, dispersions, and dissipative effects for predominant energy waves from collapsing or being forced, which applies to auroral electrostatic waves, Langmuir solar winds, and energy-generating water waves. In future work, we will use other analytical methods to obtain other forms of solutions. Additionally, we can analyze the bifurcation and chaotic patterns for the Maccari model.

Author Contributions: H.G.A.: conceptualization, software, formal analysis, writing—original draft. A.F.A.: conceptualization, data curation, writing—original draft. E.K.E.-S.: conceptualization, software, formal analysis, writing—original draft. M.A.E.A.: conceptualization, software, formal analysis, writing—review editing. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare that they have no competing interests.

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