fractal and fractional

Article

Analytical Solutions of the Predator–Prey Model with Fractional Derivative Order via Applications of Three Modified Mathematical Methods

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Abstract: We have investigated wave solutions of the Predator–Prey (PP) model with fractional derivative order by novel three modified mathematical methods with the help of the Mathematica platform. The derived solutions are in the form of distinct functions such as trigonometric, hyperbolic, exponential and rational functional. For the physical phenomena of fractional model, some solutions are plotted in 2-dimensional and 3-dimensional by inserting specific values to attached parameters under sufficient condition on each solution. Hence, proposed schemes are enormously superbly mathematical tools to review wave solutions of several fractional models in nonlinear science.

Keywords: Predator–Prey systemp; modified mathematical methods; exact and solitary solutions

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1. Introduction

The co-occurrence and meddlesome of biological sorts has been one of the chief trepidations in wildlife investments since the origin of time. It is in this observation that today; the modeling of natural phenomena has become a essential to well know wildlife collaborations. In most cases, inquiries concentrated on the bio-mathematical area see refs. [1,2]. Evidently, the employment of mathematical models telling the performance of these spectacles is a main benefit in bio-mathematics, but the resolution of these systems leftovers a chief concern. In this way, additional lately the well-known PP model has been examined by espousing two integration schemes. Some extra works have surveyed in refs. [1,3,4].

There are among others, Auxiliary Equation Method, Sine–Gordon Expansion Method, Rational Method, Generalized $exp[-\phi(\xi)]$-Expansion Method, Sub–ODE Equation Method, Sine–Cosine Method, Sinh–Expansion Method, (G'/G)-Expansion Method, Kudryashov Method, New sub-ODE Method, Homogenous Balance Method, New Extended Algebraic Method, Rational Hyperbolic Function Method, Hirota's Bilinear Method, Darboux Transformation Method and Homoclinic Breather Limit Method, Reproducing Kernel Hilbert Space Method and it is different modification see refs. [5–51].

Let the PP model with fractional derivative order be

$$\begin{aligned} U_t^{\alpha} &= U_{xx}^{2\alpha} - \beta U + \left(K + \frac{1}{\sqrt{\delta}}\right)U^2 - U^3 - HE, \\ E_t^{\alpha} &= E_{xx}^{2\alpha} + KUE - mE - \delta E^3 \end{aligned} \tag{1}$$

The animated of the natural crusade Predator–Prey mathematical model is demarcated such as: $m - \beta = 0$ and $\frac{1}{\sqrt{\delta}} + K - m = 1$. Here, U and E are used for Prey population and

Predator population size, respectively. For information about the basic definitions of the derivative order see ref. [8]. we have employed three mathematical methods on PP model, see details in ref. [52].

The remaining of part of the research is managed as: In Section 2, we demonstrate the sequence of our proposed methods. In Section 3, we implement these schemes to investigate solitary solutions of Equation (1). Conclusion is declared in Section 4.

2. Formation of Methods

Let the general NLFDE have following form:

$$L_1\left(U, D_t^{\alpha}U, D_x^{\beta}U, D_x^{\gamma}U, \ldots, D_t^{\alpha}D_t^{\alpha}U, D_t^{\alpha}D_x^{\beta}U, D_x^{\beta}D_x^{\beta}U, D_x^{\beta}D_x^{\gamma}U, \ldots\right) = 0, 0 < \alpha, \beta, \gamma \leq 1. \tag{2}$$

Let the fractional transformation be

$$U = U(\xi), \xi = \frac{\mu_1 t^{\alpha}}{\Gamma(\alpha+1)} + \frac{\mu_2 x^{\beta}}{\Gamma(\beta+1)} + \frac{\mu_3 y^{\gamma}}{\Gamma(\gamma+1)} \tag{3}$$

Put Equation (3) into Equation (2),

$$L_2(U, U', U'', U''', \ldots) = 0, \tag{4}$$

2.1. Modified Extended Auxiliary Equation Mapping Method

Let solution (4) be

$$U = \sum_{i=0}^{N} A_i \Psi^i + \sum_{i=-1}^{-N} B_{-i} \Psi^i + \sum_{i=2}^{N} C_i \Psi^{i-2} \Psi' + \sum_{i=1}^{N} D_i \left(\frac{\Psi'}{\Psi}\right)^i \tag{5}$$

Let Ψ satisfy

$$\Psi' = \sqrt{\beta_1 \Psi^2 + \beta_2 \Psi^3 + \beta_3 \Psi^4} \tag{6}$$

Put (5) with (6) in (4), solve for required destination of Equation (2).

2.2. Extended Simple Equation Method

Let (4) have the solution,

$$U(\xi) = \sum_{i=-N}^{N} A_i \Psi^i(\xi) \tag{7}$$

Let Ψ satisfy

$$\Psi' = c_0 + c_1 \Psi + c_2 \Psi^2 + c_3 \Psi^3 \tag{8}$$

Put (7) with (8) in (4). Solve the achieved system for required the solution of Equation (2).

2.3. Modified F-Expansion Method

Let the solution of (4) be

$$U = a_0 + \sum_{i=1}^{N} a_i F^i(\xi) + \sum_{i=1}^{N} b_i F^{-i}(\xi) \tag{9}$$

Let

$$F' = A + BF + CF^2. \tag{10}$$

Put (9) with (10) in (4). Solve obtained system to establish the required solution of (2).

3. Applications

Consider that

$$U = U(\xi), \xi = \frac{x^{\alpha}}{\alpha} - \frac{\mu t^{\alpha}}{\alpha} \tag{11}$$

Putting Equation (11) into Equation (1),

$$\begin{aligned} &U'' + \mu U' - \beta U + \left(K + \frac{1}{\sqrt{\delta}}\right)U^2 - U^3 - HE = 0, \\ &E'' + \mu E' + KUE - mE - \delta E^3 = 0 \end{aligned} \tag{12}$$

Let $E = \frac{U}{\sqrt{\delta}}$; the achieved the nonlinear ODE is

$$-\beta U + KU^2 - U^3 + U'' + \mu U' = 0 \tag{13}$$

3.1. Application of Modified Extended Auxiliary Equation Mapping Method

Let the solution of (13) be

$$U = A_1\Psi + A_0 + \frac{B_1}{\Psi} + \frac{D_1\Psi'}{\Psi} \tag{14}$$

Put (14) with (6) in (13),

$$\begin{aligned} &A_0 = \frac{\sqrt{2\beta + \beta_1}}{\sqrt{2}}, A_1 = \frac{\sqrt{\beta_3}}{\sqrt{2}}, B_1 = 0, D_1 = \frac{1}{\sqrt{2}}, \mu = \frac{(2\beta + \beta_1)^{3/2} - 2\beta\sqrt{2\beta + \beta_1} + \sqrt{2\beta + \beta_1}\beta_1}{2\beta_1}, \\ &K = \frac{-\sqrt{2}(2\beta + \beta_1)^{3/2} + 2\sqrt{2}\beta\sqrt{2\beta + \beta_1} + 5\sqrt{2}\sqrt{2\beta + \beta_1}\beta_1}{4\beta_1} \end{aligned} \tag{15}$$

Putting Equation (15) in (14),

CASE I:

$$\begin{aligned} U_1 = &-\left(\frac{\sqrt{\beta_3}\left(\beta_1\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi + \tau)\right) + 1\right)\right)}{\sqrt{2}\beta_2}\right) + \left(\frac{\beta_1^{3/2}\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}(\xi + \tau)\right)}{\frac{\sqrt{2}\left((2\beta_2)\left(-\beta_1\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\tau)\right)+1\right)\right)\right)}{\beta_2}}\right) \\ &+ \left(\frac{\sqrt{2\beta + \beta_1}}{\sqrt{2}}\right), \beta_1 > 0, \beta_2^2 - 4\beta_1\beta_3 = 0. \end{aligned} \tag{16}$$

$$E_1 = \left(\frac{U_1}{\sqrt{\delta}}\right), \beta_1 > 0, \beta_2^2 - 4\beta_1\beta_3 = 0. \tag{17}$$

CASE II:

$$\begin{aligned} U_2 = &-\left(\frac{\sqrt{\frac{\beta_1}{\beta_3}}\left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi+\tau)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\tau)\right)+\eta} - \frac{\sqrt{\beta_1}\epsilon\sinh^2\left(\sqrt{\beta_1}(\xi+\tau)\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\tau)\right)+\eta\right)^2}\right)}{\sqrt{2}\left(2\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\tau)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\tau)\right)+\eta} + 1\right)\right)\right)}\right) + \left(\frac{\sqrt{2\beta + \beta_1}}{\sqrt{2}}\right) + \\ &\left(\frac{\sqrt{\beta_3}\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\tau)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\tau)\right)+\eta} + 1\right)\right)}{\sqrt{2}}\right), \beta_1 > 0, \beta_3 > 0, \beta_2 = (4\beta_1\beta_3)^{1/2}. \end{aligned} \tag{18}$$

$$E_2 = \left(\frac{U_2}{\sqrt{\delta}}\right), \beta_1 > 0, \beta_3 > 0, \beta_2 = (4\beta_1\beta_3)^{1/2}. \tag{19}$$

CASE III:

$$U_3 = -\left(\frac{\beta_1\left(\frac{\sqrt{\beta_1}\epsilon\cosh\left(\sqrt{\beta_1}(\xi+\tau)\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\tau)\right)+\eta\sqrt{P^2+1}} - \frac{\sqrt{\beta_1}\epsilon\sinh\left(\sqrt{\beta_1}(\xi+\tau)\right)\left(\sinh\left(\sqrt{\beta_1}(\xi+\tau)\right)+P\right)}{\left(\cosh\left(\sqrt{\beta_1}(\xi+\tau)\right)+\eta\sqrt{P^2+1}\right)^2} \right)}{\frac{\sqrt{2}\left(\beta_2\left(-\beta_1\left(\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(\xi+\tau)\right)+P\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\tau)\right)+\eta\sqrt{P^2+1}}+1\right)\right)\right)}{\beta_2}} \right) + \left(\frac{\sqrt{2\beta+\beta_1}}{\sqrt{2}} \right)$$
$$\left(\frac{\sqrt{\beta_3}\left(-\beta_1\left(\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(\xi+\tau)\right)+P\right)}{\cosh\left(\sqrt{\beta_1}(\xi+\tau)\right)+\eta\sqrt{P^2+1}}+1\right)\right)}{\sqrt{2}\beta_2} \right), \beta_1 > 0. \tag{20}$$

$$E_3 = \left(\frac{U_3}{\sqrt{\delta}} \right), \beta_1 > 0. \tag{21}$$

For the physical demonstration of the model, the profiles of some solution are plotted in 2-dimensional and 3-dimensional by assigning particular values to attached parameters (Figures 1–5).

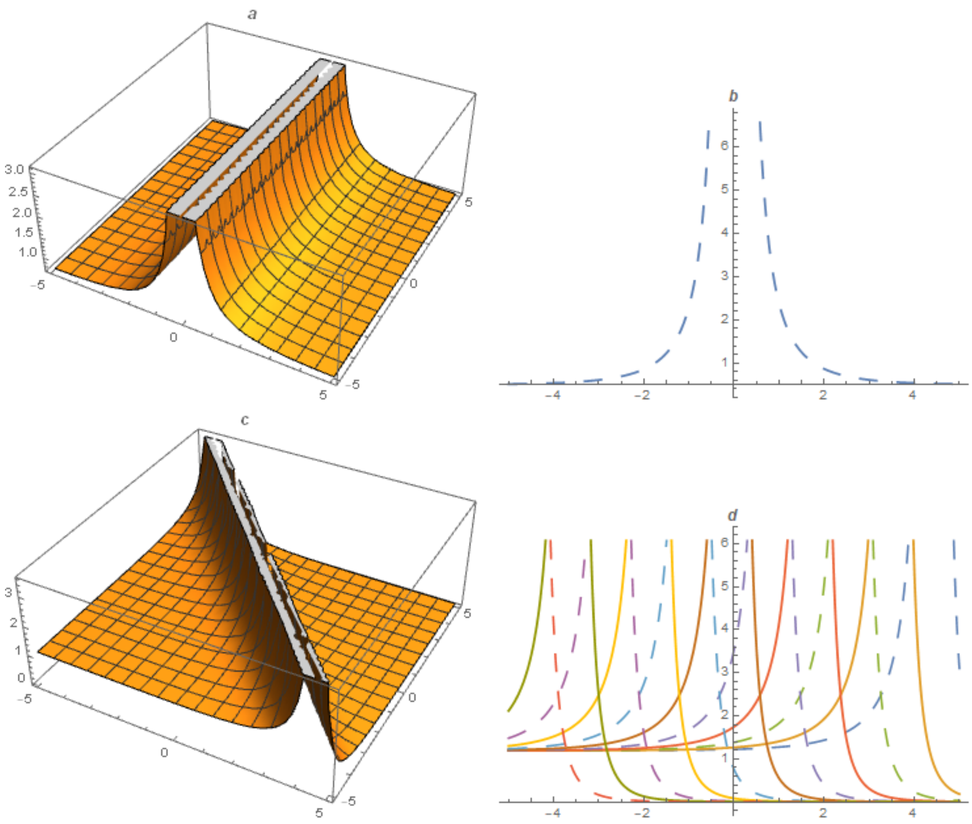

Figure 1. Solutions U_1 (**a**,**b**) and E_1 (**c**,**d**) with $\alpha = \beta_1 = \beta_3 = 1\beta_2 = -2,, \beta = -0.5, \tau = -0.01$, $\epsilon = -1$ and $\alpha = \beta_1 = \beta_3 = 1, \beta_2 = -2, \beta = -0.1, \delta = 1.5, \tau = -0.01, \epsilon = -1$, respectively.

3.2. Application of Extended Simple Equation Method

Let the solution of (13) be

$$U = \frac{A_{-1}}{\Psi} + A_1\Psi + A_0 \tag{22}$$

Put (22) with (8) in (13),
CASE 1: $c_3 = 0$,

FAMILY-I

$$A_{-1}=0, A_1=-\frac{\sqrt{c_1^2-\sqrt{c_1^4-4c_0c_1^2c_2}-2c_0c_2}\left(c_1^2+\sqrt{c_1^4-4c_0c_1^2c_2}\right)}{2c_0c_1}, A_0=-\sqrt{c_1^2-\sqrt{c_1^2(c_1^2-4c_0c_2)}-2c_0c_2},$$
$$K=-\frac{\sqrt{c_1^2-\sqrt{c_1^4-4c_0c_1^2c_2}-2c_0c_2}\left(c_1^2+\sqrt{c_1^4-4c_0c_1^2c_2}-4c_0c_2\right)(\beta+2c_1^2-8c_0c_2)}{4c_0c_2(4c_0c_2-c_1^2)},$$
$$\mu=\frac{\sqrt{c_1^4-4c_0c_1^2c_2}(\beta-c_1^2+4c_0c_2)}{c_1^3-4c_0c_1c_2} \tag{23}$$

Put (23) in (22),

$$U_4=\left(-\frac{\left(\sqrt{c_1^2-\sqrt{c_1^4-4c_0c_1^2c_2}-2c_0c_2}\left(c_1^2+\sqrt{c_1^4-4c_0c_1^2c_2}\right)\right)\left(-\left(c_1-\sqrt{4c_2c_0-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0-c_1^2}(\xi+\tau)\right)\right)\right)}{(2c_0c_1)(2c_2)}\right)$$
$$-\left(\sqrt{c_1^2-\sqrt{c_1^2(c_1^2-4c_0c_2)}-2c_0c_2}\right), 4c_0c_2>c_1^2. \tag{24}$$

$$E_4=\left(\frac{U_4}{\sqrt{\delta}}\right), 4c_0c_2>c_1^2. \tag{25}$$

FAMILY-II

$$A_{-1}=-\frac{\sqrt{c_1^2-\sqrt{c_1^4-4c_0c_1^2c_2}-2c_0c_2}\left(c_1^2+\sqrt{c_1^4-4c_0c_1^2c_2}\right)}{2c_1c_2}, A_0=-\sqrt{c_1^2-\sqrt{c_1^2(c_1^2-4c_0c_2)}-2c_0c_2},$$
$$K=-\frac{\sqrt{c_1^2-\sqrt{c_1^4-4c_0c_1^2c_2}-2c_0c_2}\left(c_1^2+\sqrt{c_1^4-4c_0c_1^2c_2}-4c_0c_2\right)(\beta+2c_1^2-8c_0c_2)}{4c_0c_2(4c_0c_2-c_1^2)},$$
$$A_1=0, \mu=\frac{\sqrt{c_1^4-4c_0c_1^2c_2}(-\beta+c_1^2-4c_0c_2)}{c_1^3-4c_0c_1c_2} \tag{26}$$

Substitute (26) in (22),

$$U_5=\left(-\frac{\left(\sqrt{c_1^2-\sqrt{c_1^4-4c_0c_1^2c_2}-2c_0c_2}\left(c_1^2+\sqrt{c_1^4-4c_0c_1^2c_2}\right)\right)(-2c_2)}{(2c_1c_2)\left(c_1-\sqrt{4c_2c_0-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0-c_1^2}(\xi+\tau)\right)\right)}\right)$$
$$-\left(\sqrt{c_1^2-\sqrt{c_1^2(c_1^2-4c_0c_2)}-2c_0c_2}\right), 4c_0c_2>c_1^2 \tag{27}$$

$$E_5=\left(\frac{U_5}{\sqrt{\delta}}\right), 4c_0c_2>c_1^2. \tag{28}$$

CASE 2: $c_0=0, c_3=0$,

$$A_{-1}=0, A_1=\sqrt{2}c_2, A_0=0, \mu=\frac{\beta-c_1^2}{c_1}, K=\frac{\left(-\sqrt{2}\right)\beta-2\sqrt{2}c_1^2}{2c_1} \tag{29}$$

Put (29) in (22),

$$U_6=\left(\frac{\sqrt{2}c_2(c_1\exp(c_1(\xi+\tau)))}{1-c_2\exp(c_1(\xi+\tau))}\right), c_1>0. \tag{30}$$

$$E_6 = \left(\frac{U_6}{\sqrt{\delta}} \right), c_1 > 0. \tag{31}$$

$$U_7 = \left(\frac{\sqrt{2}c_2(-c_1 \exp(c_1(\xi + \tau)))}{c_2 \exp(c_1(\xi + \tau)) + 1} \right), c_1 < 0. \tag{32}$$

$$E_7 = \left(\frac{U_7}{\sqrt{\delta}} \right), c_1 < 0. \tag{33}$$

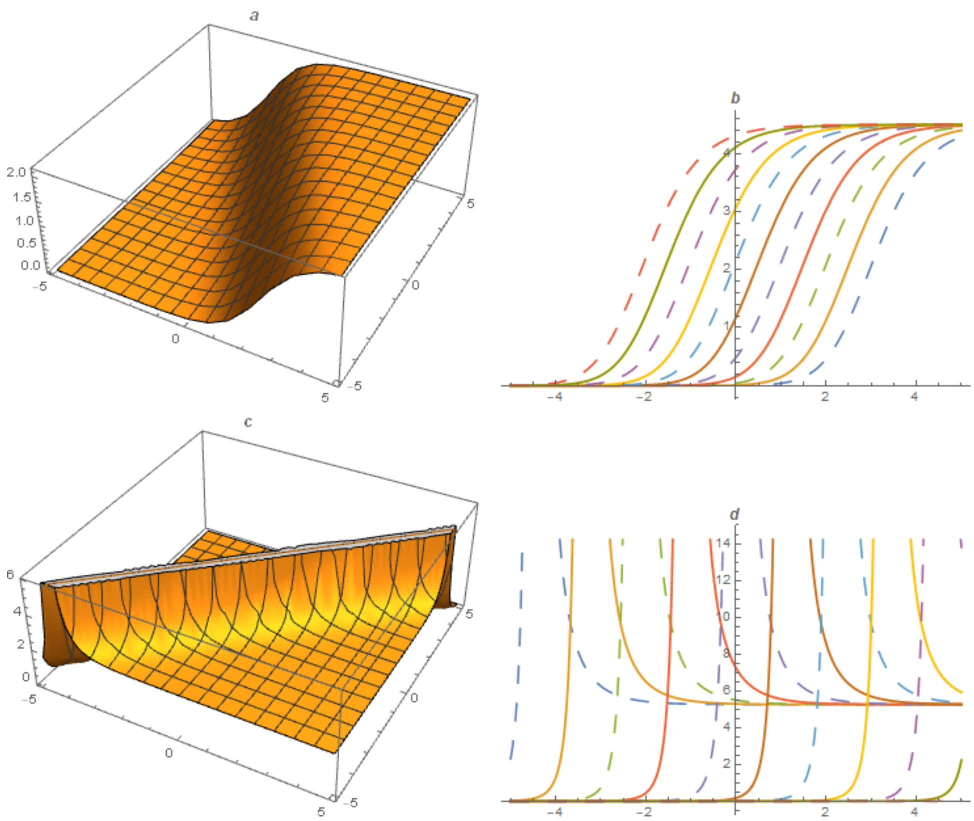

Figure 2. Solutions of U_6 (**a**,**b**) and E_6 (**c**,**d**) with $\alpha = 1, \beta = 3.01, c_1 = 1.5, c_2 = -0.01, \tau = 3.1$ and $\alpha = 1, \beta = 1, c_1 = 1.7, c_2 = 1, \delta = -1.1, \tau = -1.1$, respectively.

CASE 3: $c_1 = 0, c_3 = 0$,
FAMILY-I

$$A_1 = -\sqrt{2}c_2, A_0 = \sqrt{\beta - 2c_0c_2}, \mu = \frac{\sqrt{2}(\beta - 2c_0c_2)^{3/2} + \left(-\sqrt{2}\right)\beta\sqrt{\beta - 2c_0c_2} - 2\sqrt{2}c_0c_2\sqrt{\beta - 2c_0c_2}}{4c_0c_2},$$
$$K = \frac{(\beta - 2c_0c_2)^{3/2} - \beta\sqrt{\beta - 2c_0c_2} + 10c_0c_2\sqrt{\beta - 2c_0c_2}}{4c_0c_2}, A_{-1} = 0, \tag{34}$$

Put (34) in (22),

$$U_8 = \left(\sqrt{\beta - 2c_0c_2} - \frac{\sqrt{2}c_2(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi + \tau)))}{c_2} \right), c_0c_2 > 0, \tag{35}$$

$$E_8 = \left(\frac{U_8}{\sqrt{\delta}} \right), c_0c_2 > 0, \tag{36}$$

$$U_9 = \left(\sqrt{\beta - 2c_0c_2} - \frac{\sqrt{2}c_2(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\tau)))}{c_2} \right), c_0c_2 < 0. \tag{37}$$

$$E_9 = \left(\frac{U_9}{\sqrt{\delta}} \right), c_0c_2 < 0, \tag{38}$$

FAMILY-II

$$\begin{aligned} A_{-1} &= \sqrt{2}c_0, A_0 = \sqrt{\beta - 2c_0c_2}, \mu = \frac{\sqrt{2}(\beta - 2c_0c_2)^{3/2} + \left(-\sqrt{2}\right)\beta\sqrt{\beta - 2c_0c_2} - 2\sqrt{2}c_0c_2\sqrt{\beta - 2c_0c_2}}{4c_0c_2}, \\ K &= \frac{(\beta - 2c_0c_2)^{3/2} - \beta\sqrt{\beta - 2c_0c_2} + 10c_0c_2\sqrt{\beta - 2c_0c_2}}{4c_0c_2}, A_1 = 0. \end{aligned} \tag{39}$$

Put (39) in (22),

$$U_{10} = \left(\sqrt{\beta - 2c_0c_2} + \frac{\sqrt{2}c_0}{\frac{\sqrt{c_0c_2}\tan\left(\sqrt{c_0c_2}(\xi+\tau)\right)}{c_2}} \right), c_0c_2 > 0, \tag{40}$$

$$E_{10} = \left(\frac{U_{10}}{\sqrt{\delta}} \right), c_0c_2 > 0, \tag{41}$$

$$U_{11} = \left(\sqrt{\beta - 2c_0c_2} + \frac{\sqrt{2}c_0}{\frac{\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\tau))}{c_2}} \right), c_0c_2 < 0. \tag{42}$$

$$E_{11} = \left(\frac{U_{11}}{\sqrt{\delta}} \right), c_0c_2 < 0. \tag{43}$$

FAMILY-III

$$\begin{aligned} A_1 &= \sqrt{2}c_2, A_0 = \sqrt{\beta - 8c_0c_2}, \mu = \frac{-\sqrt{2}(\beta - 8c_0c_2)^{3/2} + \sqrt{2}\beta\sqrt{\beta - 8c_0c_2} + 8\sqrt{2}c_0c_2\sqrt{\beta - 8c_0c_2}}{16c_0c_2}, \\ K &= \frac{(\beta - 8c_0c_2)^{3/2} - \beta\sqrt{\beta - 8c_0c_2} + 40c_0c_2\sqrt{\beta - 8c_0c_2}}{16c_0c_2}, A_{-1} = -\sqrt{2}c_0, \end{aligned} \tag{44}$$

Put (44) in (22),

$$U_{12} = \left(\sqrt{\beta - 8c_0c_2} + \frac{\sqrt{2}c_2(\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\tau)))}{c_2} - \frac{\sqrt{2}c_0}{\frac{\sqrt{c_0c_2}\tan\left(\sqrt{c_0c_2}(\xi+\tau)\right)}{c_2}} \right), c_0c_2 > 0, \tag{45}$$

$$E_{12} = \left(\frac{U_{12}}{\sqrt{\delta}} \right), c_0c_2 > 0. \tag{46}$$

$$U_{13} = \left(\sqrt{\beta - 8c_0c_2} + \frac{\sqrt{2}c_2(\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\tau)))}{c_2} - \frac{\sqrt{2}c_0}{\frac{\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\tau))}{c_2}} \right), c_0c_2 < 0. \tag{47}$$

$$E_{13} = \left(\frac{U_{13}}{\sqrt{\delta}} \right), c_0c_2 < 0. \tag{48}$$

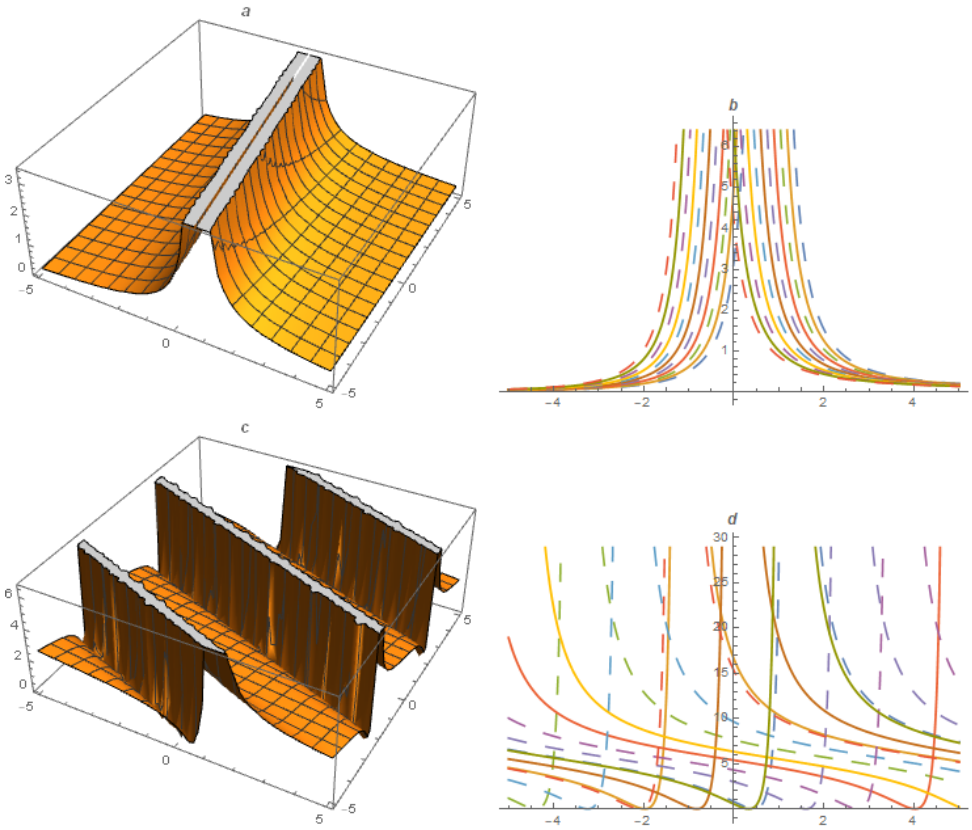

Figure 3. Solutions of U_{13} (**a**,**b**) and E_{13} (**c**,**d**) with $\alpha = 1, \beta = 0.01, c_0 = 1.1, c_2 = -0.0001, \tau = -0.01$ and $\alpha = 1, \beta = 3.1, c_0 = -2.1, c_2 = -0.01, \delta = -0.5, \tau = 0.01$, respectively.

3.3. Application of Modified F-Expansion Method

Let (13) have solution

$$U = a_0 + a_1 F + \frac{b_1}{F} \tag{49}$$

Put (49) in (13) with (10).

A = 0, B = 1, C = −1,

$$a_1 = \sqrt{2}, a_0 = 0, b_1 = 0, \mu = \beta - 1, K = \frac{1}{2}\left(\sqrt{2}\beta + 2\sqrt{2}\right) \tag{50}$$

Put (50) in (49),

$$U_{14} = \sqrt{2}\left(\frac{1}{2}\tanh\left(\frac{\xi}{2}\right) + \frac{1}{2}\right) \tag{51}$$

$$E_{14} = \left(\frac{U_{14}}{\sqrt{\delta}}\right) \tag{52}$$

A = 0, C = 1, B = −1,

$$a_1 = \sqrt{2}, a_0 = 0, b_1 = 0, \mu = 1 - \beta, K = \frac{1}{2}\left(\sqrt{2}\beta + 2\sqrt{2}\right) \tag{53}$$

Put (53) into (49),

$$U_{15} = \sqrt{2}\left(\frac{1}{2} - \frac{1}{2}\coth\left(\frac{\xi}{2}\right)\right) \tag{54}$$

$$E_{15} = \left(\frac{U_{15}}{\sqrt{\delta}}\right) \tag{55}$$

A = 1/2, B = 0, C = −1/2
FAMILY-I

$$a_1 = -\frac{1}{\sqrt{2}}, a_0 = -\frac{1}{\sqrt{2}}, b_1 = 0, \mu = \beta - 1, K = \frac{1}{2}\left(\sqrt{2}(-\beta) - 2\sqrt{2}\right) \tag{56}$$

Substitute (56) into (49),

$$U_{16,1} = \left(-\frac{\cot(\xi) + \csc(\xi)}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) \tag{57}$$

$$E_{16,1} = \left(\frac{U_{16,1}}{\sqrt{\delta}}\right) \tag{58}$$

FAMILY-II

$$a_1 = 0, a_0 = \frac{1}{\sqrt{2}}, b_1 = \frac{1}{\sqrt{2}}, \mu = \beta - 1, K = \frac{1}{2}\left(\sqrt{2}\beta + 2\sqrt{2}\right) \tag{59}$$

Put (59) in (49),

$$U_{16,2} = \left(\frac{1}{\sqrt{2}(\cot(\xi) + \csc(\xi))}\right) + \frac{1}{\sqrt{2}} \tag{60}$$

$$E_{16,2} = \left(\frac{U_{16,2}}{\sqrt{\delta}}\right) \tag{61}$$

FAMILY-III

$$a_1 = \frac{1}{\sqrt{2}}, a_0 = \sqrt{2}, b_1 = \frac{1}{\sqrt{2}}, \mu = \frac{\beta - 4}{2}, K = \frac{1}{4}\left(\sqrt{2}\beta + 8\sqrt{2}\right) \tag{62}$$

Put (63) in (49),

$$U_{16,3} = \frac{(\cot(\xi) + \csc(\xi)) + \frac{1}{\cot(\xi)+\csc(\xi)}}{\sqrt{2}} + \sqrt{2} \tag{63}$$

$$E_{16,3} = \left(\frac{U_{16,3}}{\sqrt{\delta}}\right) \tag{64}$$

A = 1, B = 0, C = −1
FAMILY-I

$$a_1 = \sqrt{2}, a_0 = -\sqrt{2}, b_1 = 0, \mu = \frac{4 - \beta}{2}, K = \frac{1}{4}\left(\sqrt{2}(-\beta) - 8\sqrt{2}\right) \tag{65}$$

Put (65) in (49),

$$U_{17,1} = \sqrt{2}\tanh(\xi) - \sqrt{2} \tag{66}$$

$$E_{17,1} = \left(\frac{U_{17,1}}{\sqrt{\delta}}\right) \tag{67}$$

FAMILY-II

$$a_1 = 0, a_0 = -\sqrt{2}, b_1 = \sqrt{2}, \mu = \frac{4-\beta}{2}, K = \frac{1}{4}\left(\sqrt{2}(-\beta) - 8\sqrt{2}\right) \tag{68}$$

Put (68) in (49),

$$U_{17,2} = \frac{\sqrt{2}}{\tanh(\xi)} - \sqrt{2} \tag{69}$$

$$E_{17,2} = \left(\frac{U_{17,2}}{\sqrt{\delta}}\right) \tag{70}$$

FAMILY-III

$$a_1 = \sqrt{2}, a_0 = 2\sqrt{2}, b_1 = \sqrt{2}, \mu = \frac{\beta-16}{4}, K = \frac{1}{8}\left(\sqrt{2}\beta + 32\sqrt{2}\right) \tag{71}$$

$$E_{13} = \left(\frac{U_{13}}{\sqrt{\delta}}\right), c_0c_2 < 0. \tag{72}$$

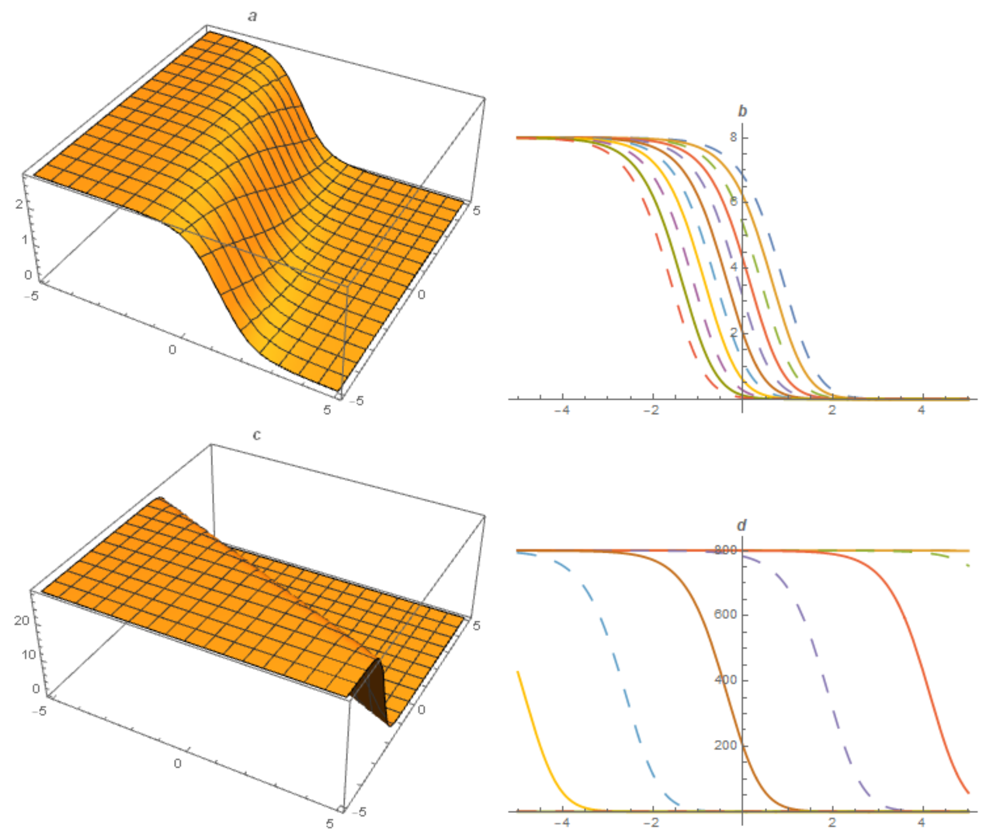

Figure 4. Solutions of $U_{17,1}$ (**a**,**b**) and $E_{17,1}$ (**c**,**d**) with $\alpha = 1, \beta = 3.5$ and $\alpha = 1, \beta = -0.5, \delta = -0.01$, respectively.

Put (71) in (49),

$$U_{17,3} = \sqrt{2}\left(\tanh(\xi) + \frac{1}{\tanh(\xi)}\right) + 2\sqrt{2} \tag{73}$$

$$E_{17,3} = \left(\frac{U_{17,3}}{\sqrt{\delta}}\right) \tag{74}$$

A = C = 1/2, B = 0,

FAMILY-I

$$\begin{aligned} &a_1 = \frac{1}{\sqrt{2}}, a_0 = \frac{\sqrt{2\beta-1}}{\sqrt{2}}, b_1 = 0, \mu = \frac{1}{2}\left(-(2\beta-1)^{3/2} + 2\beta\sqrt{2\beta-1} + \sqrt{2\beta-1}\right), \\ &K = \frac{1}{2}\left(\frac{(2\beta-1)^{3/2}}{\sqrt{2}} - \sqrt{2}\beta\sqrt{2\beta-1} + \frac{5\sqrt{2\beta-1}}{\sqrt{2}}\right) \end{aligned} \tag{75}$$

Put (75) in (49),

$$U_{18,1} = \frac{\sqrt{2\beta-1}}{\sqrt{2}} + \frac{\tan(\xi) + \sec(\xi)}{\sqrt{2}} \tag{76}$$

$$E_{18,1} = \left(\frac{U_{18,1}}{\sqrt{\delta}}\right) \tag{77}$$

FAMILY-II

$$\begin{aligned} &a_1 = 0, a_0 = \frac{\sqrt{2\beta-1}}{\sqrt{2}}, b_1 = \frac{1}{\sqrt{2}}, \mu = \frac{1}{2}\left((2\beta-1)^{3/2} - 2\beta\sqrt{2\beta-1} - \sqrt{2\beta-1}\right), \\ &K = \frac{1}{2}\left(\frac{(2\beta-1)^{3/2}}{\sqrt{2}} - \sqrt{2}\beta\sqrt{2\beta-1} + \frac{5\sqrt{2\beta-1}}{\sqrt{2}}\right) \end{aligned} \tag{78}$$

Put (78) in (49),

$$U_{18,2} = \frac{\sqrt{2\beta-1}}{\sqrt{2}} + \frac{1}{\sqrt{2}(\tan(\xi) + \sec(\xi))} \tag{79}$$

$$E_{18,2} = \left(\frac{U_{18,2}}{\sqrt{\delta}}\right) \tag{80}$$

FAMILY-III

$$\begin{aligned} &a_1 = -\frac{1}{\sqrt{2}}, a_0 = \sqrt{\beta-2}, b_1 = \frac{1}{\sqrt{2}}, \mu = \frac{1}{2}\left(\frac{(\beta-2)^{3/2}}{\sqrt{2}} - \sqrt{2}\sqrt{\beta-2} - \frac{\sqrt{\beta-2}\beta}{\sqrt{2}}\right), \\ &K = \frac{1}{4}\left((\beta-2)^{3/2} - \sqrt{\beta-2}\beta + 10\sqrt{\beta-2}\right) \end{aligned} \tag{81}$$

Put (81) in (49),

$$U_{18,3} = \sqrt{\beta-2} - \frac{(\tan(\xi) + \sec(\xi)) - \frac{1}{\tan(\xi)+\sec(\xi)}}{\sqrt{2}} \tag{82}$$

$$E_{18,3} = \left(\frac{U_{18,3}}{\sqrt{\delta}}\right) \tag{83}$$

A = C = −1/2, B = 0,

FAMILY-I

$$a_1 = \frac{1}{\sqrt{2}}, a_0 = \frac{\sqrt{2\beta-1}}{\sqrt{2}}, b_1 = 0, \mu = \frac{1}{2}\left((2\beta-1)^{3/2} - 2\beta\sqrt{2\beta-1} - \sqrt{2\beta-1}\right),$$
$$K = \frac{1}{2}\left(\frac{(2\beta-1)^{3/2}}{\sqrt{2}} - \sqrt{2}\beta\sqrt{2\beta-1} + \frac{5\sqrt{2\beta-1}}{\sqrt{2}}\right) \tag{84}$$

Put (84) in (49),

$$U_{19,1} = \frac{\sqrt{2\beta-1}}{\sqrt{2}} + \frac{\sec(\xi) - \tan(\xi)}{\sqrt{2}} \tag{85}$$

$$E_{19,1} = \left(\frac{U_{19,1}}{\sqrt{\delta}}\right) \tag{86}$$

FAMILY-II

$$a_1 = 0, a_0 = \frac{\sqrt{2\beta-1}}{\sqrt{2}}, b_1 = -\frac{1}{\sqrt{2}}, \mu = \frac{1}{2}\left((2\beta-1)^{3/2} - 2\beta\sqrt{2\beta-1} - \sqrt{2\beta-1}\right),$$
$$K = \frac{1}{2}\left(\frac{(2\beta-1)^{3/2}}{\sqrt{2}} - \sqrt{2}\beta\sqrt{2\beta-1} + \frac{5\sqrt{2\beta-1}}{\sqrt{2}}\right) \tag{87}$$

Put (87) in (49),

$$U_{19,2} = \frac{\sqrt{2\beta-1}}{\sqrt{2}} - \frac{1}{\sqrt{2}(\sec(\xi) - \tan(\xi))} \tag{88}$$

$$E_{19,2} = \left(\frac{U_{19,2}}{\sqrt{\delta}}\right) \tag{89}$$

FAMILY-III

$$a_1 = \frac{1}{\sqrt{2}}, a_0 = \sqrt{\beta-2}, b_1 = -\frac{1}{\sqrt{2}}, \mu = \frac{1}{2}\left(\frac{(\beta-2)^{3/2}}{\sqrt{2}} - \sqrt{2}\sqrt{\beta-2} - \frac{\sqrt{\beta-2}\beta}{\sqrt{2}}\right),$$
$$K = \frac{1}{4}\left((\beta-2)^{3/2} - \sqrt{\beta-2}\beta + 10\sqrt{\beta-2}\right) \tag{90}$$

Put (90) in (49),

$$U_{19,3} = \sqrt{\beta-2} + \frac{(\sec(\xi) - \tan(\xi)) - \frac{1}{\sec(\xi)-\tan(\xi)}}{\sqrt{2}} \tag{91}$$

$$E_{19,3} = \left(\frac{U_{19,3}}{\sqrt{\delta}}\right) \tag{92}$$

A = C = −1, B = 0,
FAMILY-I

$$a_1 = -\sqrt{2}, a_0 = -\sqrt{\beta-2}, b_1 = 0, \mu = \frac{1}{4}\left(\sqrt{2}(\beta-2)^{3/2} - \sqrt{2}\sqrt{\beta-2}\beta - 2\sqrt{2}\sqrt{\beta-2}\right),$$
$$K = \frac{1}{4}\left(-(\beta-2)^{3/2} + \beta\sqrt{\beta-2} - 10\sqrt{\beta-2}\right) \tag{93}$$

Put (93) in (49),

$$U_{20,1} = -\sqrt{\beta - 2} - \sqrt{2}\tan(\xi) \tag{94}$$

$$E_{20,1} = \left(\frac{U_{20,1}}{\sqrt{\delta}}\right) \tag{95}$$

FAMILY-II

$$a_1 = 0, a_0 = \sqrt{\beta - 2}, b_1 = \sqrt{2}, \mu = \frac{1}{4}\left(-\sqrt{2}(\beta - 2)^{3/2} + \sqrt{2}\beta\sqrt{\beta - 2} + 2\sqrt{2}\sqrt{\beta - 2}\right),$$
$$K = \frac{1}{4}\left((\beta - 2)^{3/2} - \sqrt{\beta - 2}\beta + 10\sqrt{\beta - 2}\right) \tag{96}$$

Put (96) in (49),

$$U_{20,2} = \frac{\sqrt{2}}{\tan(\xi)} \tag{97}$$

$$E_{20,2} = \left(\frac{U_{20,2}}{\sqrt{\delta}}\right) \tag{98}$$

FAMILY-III

$$a_1 = -\sqrt{2}, a_0 = -\sqrt{\beta - 8}, b_1 = \sqrt{2}, \mu = \frac{1}{16}\left(\sqrt{2}(\beta - 8)^{3/2} - \sqrt{2}\sqrt{\beta - 8}\beta - 8\sqrt{2}\sqrt{\beta - 8}\right),$$
$$K = \frac{1}{16}\left(-(\beta - 8)^{3/2} + \beta\sqrt{\beta - 8} - 40\sqrt{\beta - 8}\right) \tag{99}$$

Put (99) in (49),

$$U_{20,3} = -\sqrt{\beta - 8} - \sqrt{2}\left(\tan(\xi) - \frac{1}{\tan(\xi)}\right) \tag{100}$$

$$E_{20,3} = \left(\frac{U_{20,3}}{\sqrt{\delta}}\right) \tag{101}$$

A = B = 0

$$a_1 = \sqrt{2}C, a_0 = \sqrt{\beta}, b_1 = 0, \mu = \sqrt{2}\sqrt{\beta}, K = 2\sqrt{\beta} \tag{102}$$

Put (102) in (49),

$$U_{21} = \sqrt{\beta} + \sqrt{2}c_{\frac{1}{C\xi+\eta}} \tag{103}$$

$$E_{21} = \left(\frac{U_{21}}{\sqrt{\delta}}\right) \tag{104}$$

B = C = 0

$$a_1 = 0, a_0 = \sqrt{\beta}, b_1 = \sqrt{2}A, \mu = -\sqrt{2}\sqrt{\beta}, K = 2\sqrt{\beta} \tag{105}$$

Put (105) in (49),

$$U_{22} = \frac{\sqrt{2}A}{A\xi} + \sqrt{\beta} \tag{106}$$

$$E_{22} = \left(\frac{U_{22}}{\sqrt{\delta}} \right) \tag{107}$$

C = 0

$$a_1 = 0, a_0 = -\sqrt{2}B, b_1 = \frac{\sqrt{2}A\beta - 4\sqrt{2}AB^2}{4B^2 - \beta}, \mu = \frac{\beta - B^2}{B},$$
$$K = \frac{2\sqrt{2}\beta^2 - 16\sqrt{2}B^4 + \frac{\sqrt{2}\beta^3}{4B^2-\beta} - \frac{6\sqrt{2}\beta^2 B^2}{4B^2-\beta} - 4\sqrt{2}\beta B^2 + \frac{32\sqrt{2}B^6}{4B^2-\beta}}{2(4B^3 - \beta B)} \tag{108}$$

Put (108) in (49),

$$U_{23} = \frac{\sqrt{2}A\beta - 4\sqrt{2}AB^2}{\frac{(4B^2-\beta)(\exp(B\xi)-A)}{B}} - \frac{\sqrt{2}B(\exp(B\xi) - A)}{B} \tag{109}$$

$$E_{23} = \left(\frac{U_{23}}{\sqrt{\delta}} \right) \tag{110}$$

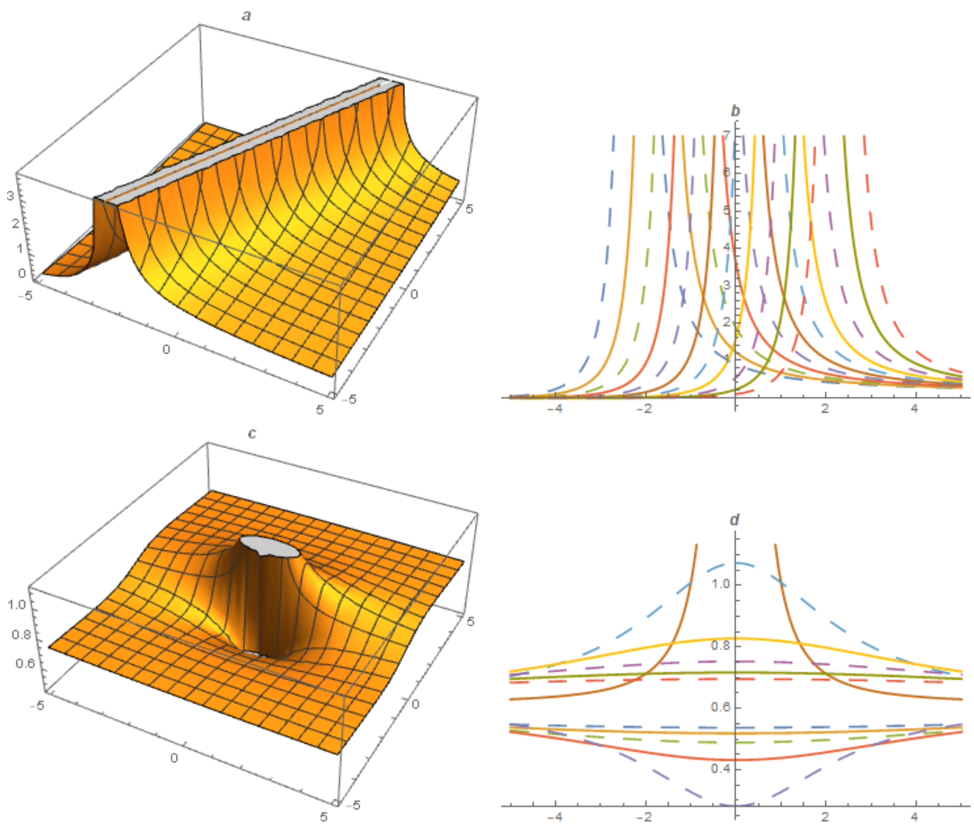

Figure 5. Solutions of U_{22} (**a**,**b**) and E_{22} (**c**,**d**) with $\alpha = 1, \beta = 0.1$ and $\alpha = 1, \beta = -3.1, \delta = -5.05$, respectively.

4. Conclusions

We have explored progressive and efficient solitary wave solutions of f PP system via successfully implementation of three mathematical methods. For the physical demonstration of the model, the profiles of some solution are plotted in 2-dimensional and 3-dimensional by assigning particular values to attached parameters. Hence, the offered techniques are meritoriously pertinent for advance studies for other NFPDEs.

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