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Research on Effective Thermal Conductivity in Porous Media Embedded with Randomly Distributed Damaged Tree-like Bifurcation Networks

Yihao Shao ^{1,2}, Xiuya Guo ^{1,2}, Huili Wang ^{1,2}, Limei Zhu ^{1,2} and Qian Zheng ^{1,2,*}

¹ School of Mathematical and Physical Sciences, Wuhan Textile University, Wuhan 430073, China; h852195193@163.com (Y.S.); xyguo@wtu.edu.cn (X.G.); hlwang@wtu.edu.cn (H.W.); lmzhu@wtu.edu.cn (L.Z.)

² Research Center for Applied Mathematics and Interdisciplinary Sciences, Wuhan Textile University, Wuhan 430073, China

* Correspondence: 2012025@wtu.edu.cn or tbftbfzhengqian@163.com

Abstract: Due to the complexity of the microstructure of porous media, it is of great significance to explore the heat transport mechanism in porous media in many engineering applications. In this study, an expression for effective thermal conductivity (ETC) of porous media embedded with randomly distributed damaged tree-like bifurcation networks is derived based on the theory of thermodynamics and fractal features of tree-like bifurcation networks. We investigate the effect of heat conduction and heat convection in porous media embedded with randomly distributed damaged tree-like bifurcation networks on the ETC of the porous media. It is found that our fractal model has good consistency with the existing available experimental data. In addition, the influence of the microstructural parameters of the model on heat transfer in the porous media is analyzed in detail. The research results can provide significant theoretical guidance for the development and design of heat transfer systems.

Keywords: effective thermal conductivity; fractals; porous media; damaged tree-like bifurcation network



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1. Introduction

The metric known as ETC plays a crucial role in quantifying the heat transfer properties inside porous material. The estimation of ETC in porous media has extensive use in several domains such as thermoelectric materials, porous construction materials, the chemical industry, petroleum extraction, and related sectors [1–6]. It is well known that porous media are usually composed of pore space and solid matrix, and the microstructure exhibits a high degree of complexity and chaos. Therefore, describing it using normal means poses challenges. The application of fractal geometry theory enables the characterization of pore properties in porous media when the microstructure of porous media has self-similar characteristics [7] and offers a novel approach for investigating the challenges associated with transportation in intricate and disorganized porous materials [8–10]. At present, many scholars use fractal theory to explore gas flow [11–13], gas diffusion [14–16], and seepage characteristics [17,18] in porous media. In addition, the investigation of heat transfer in porous media can be conducted through the utilization of fractal geometry theory [19–21]. For example, Xiao [22] introduced a novel ETC model that incorporates a microscale effect, leveraging the fractal properties exhibited by porous media. Shen et al. [23] developed a three-phase unsaturated porous media model based on fractal theory, employing thermoelectric simulation and a capillary bundle model. The study investigated the impact of liquid saturation and porosity on the ETC of porous media, elucidating the underlying influence mechanism.

The tree-like bifurcation network structure exhibits distinct transport characteristics; therefore, it has attracted the interest and attention of a large number of scholars, and

it has been widely used in related practical applications, such as microelectronic chip cooling systems and production engineering, etc. [24,25]. Chen and Cheng [26] discussed a comparative analysis to examine the distinctions between traditional parallel pipes and rectangular tree-like bifurcation networks in the context of heat convection. They discovered that the heat dissipation performance of pipes can be significantly enhanced by employing rectangular tree-like bifurcation networks, in contrast to the conventional parallel pipe configurations. Yu and Li [27] conducted a study on the ETC of composites that contain tree-like bifurcation networks, focusing on the self-similarity properties of these networks. The results show that the ETC of each component embedded in the networks exerts a substantial impact on the ETC of the composite [28]. However, the above research only focuses on the symmetric tree-like bifurcation networks and does not involve the damage of branches in the networks. Miao et al. [29] conducted a study on the heat transport and fluid flow, focusing on the fractal self-similarity exhibited by tree-like bifurcation networks. Xiao et al. [30] published a study to investigate the impact of pore surface roughness on the ETC of damaged tree-like bifurcation networks, building upon the research conducted by Miao et al. [29]. The model considered the effects of roughness and damaged structure of the pipeline on ETC; however, the impact of heat convection resulting from the movement of liquid was not taken into account. Based on these premises, Shao et al. [31] introduced an ETC model for porous media that incorporates a damaged tree-like branching network, taking into account the impact of roughness. The development of this model takes into account the distinctive features of tree-like bifurcation networks that have been damaged and conducts separate analyses on the impacts of heat convection and heat conduction on the ETC of porous media. The underlying assumption of the model posits the presence of a solitary tree-like bifurcation network within the porous media. However, in reality, a porous medium is usually composed of tree-like bifurcation networks that conform to fractal scale distribution. Xia et al. [32] studied the influence of heat convection caused by liquid flow on ETC by establishing the joint expression of heat conduction and heat convection in a tree branch network with rough surfaces. Nevertheless, the impact of branch damage within the tree-like bifurcation networks was not considered in the scope of this research [33].

The study conducted by Valvano et al. [34] introduces an improved methodology for the *in vivo* assessment of thermal conductivity, thermal diffusivity, and perfusion. This is achieved through the utilization of a self-heated spherical thermistor probe. The methodology encompasses the resolution of a time-dependent thermal model that is coupled with the probe–tissue system. This model is utilized to quantify the “effective” thermal conductivity and “effective” thermal diffusivity, which serve as representative measures of the thermal properties of perfused tissue. This study introduces a highly reproducible thermal conductivity probe with precise temperature control, emphasizing its application to measure and understand the influence of moisture content on ETC in a diverse biological cell. Afterwards, Liang et al. [35] introduced the highly repeatable thermal conductivity probe with precise temperature control to measure and understand the impact of water content on thermal conductivity in different biological tissues. Bhattacharya et al. [36] employed a new transient technique, using specialized thermal resistance wire probes to experimentally establish thermal conductivity of cells, revealing reversible changes within a specific temperature range, and emphasizing the significant impact of blood perfusion on enhancing heat transfer in living tissues. Valvano et al. [34] employed various methodologies to assess the thermal conductivity and thermal conductivity resulting from thermal convection in organisms. However, the intricate microstructure of porous media poses challenges in experimentally elucidating the influence of this complexity on thermal transport properties.

In the above brief literature research, the current research considering the effect of heat convection on the ETC did not meditate on The damaged structure of randomly distributed tree-like bifurcation networks. In our previous study [31], we exclusively examined the phenomena of heat convection and heat conduction within a solitary tree-like branching

network that was integrated within a porous medium. The pore structure of the main pipe with random distribution was not considered. As a result, we will talk about heat conduction and heat convection in our model, which has bifurcation networks that resemble damaged trees and are scattered at random. A fractal model of ETC will be built, and the influence of microstructural parameters of the composite on thermal transport will be studied. The outcome of this study can contribute to the comprehension of the heat transfer mechanism occurring within a porous medium that contains tree-like bifurcation networks that have been damaged.

2. Fractal Characteristics of Porous Media

Creating a mathematical model that faithfully captures the ETC of porous media embedded with randomly distributed damaged tree-like bifurcation networks is the main goal of this work. It is postulated that a significant quantity of impaired tree-like bifurcation networks is uniformly dispersed inside porous media, with the primary diameters of said networks following the principles of fractal scaling. Refer to Figure 1a.

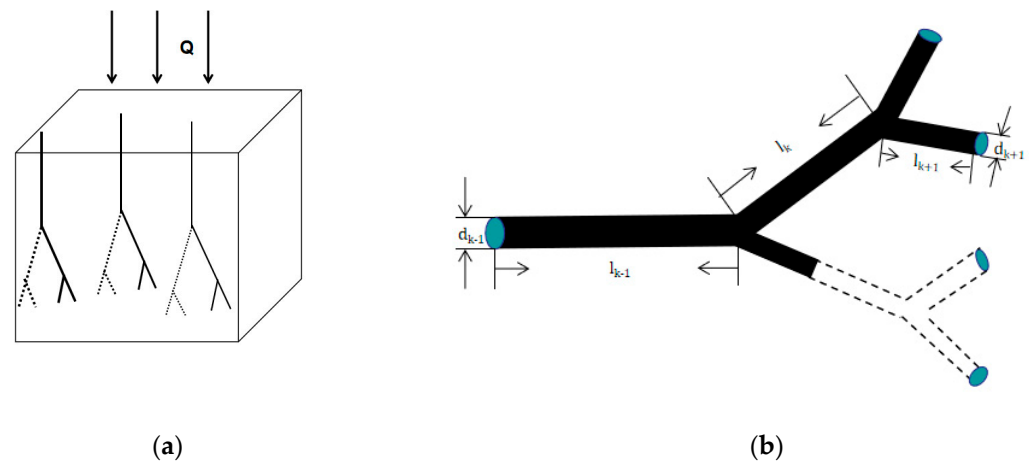


Figure 1. (a) Diagrammatic schematic of the porous media with randomly distributed damaged tree-like bifurcation networks; (b) diagrammatic schematic of a single damaged tree-like bifurcation network.

Therefore, the total number N of tree-like bifurcation networks with a diameter scale L larger than or equal to the diameter of the main channel d_0 can be expressed as [8]:

$$N(L \geq d_0) = \left(\frac{d_{0\max}}{d} \right)^{D_f} \quad (1)$$

where $d_{0\max}$ is the maximum diameter of the main channels, and D_f is the fractal dimension of the main channels. The D_f can be obtained by [8]:

$$D_f = D_E - \frac{\ln \phi_f}{\ln(d_{0\min}/d_{0\max})} \quad (2)$$

where ϕ_f is areal porosity, $d_{0\min}$ is the minimum diameter of the main channels, and D_E is the Euclidean dimension. $D_E = 2$ in two-dimensional space, and $D_E = 3$ in three-dimensional space. Since there are many tree-like bifurcation networks in the porous media, Equation (1) can be regarded as a continuous and differentiable function. The number of main channels with sizes ranging from d_0 to $d_0 + \Delta d$ can be calculated by differentiating with d_0 based on Equation (1) [7–9]:

$$-dN = D_f d_{0\max}^{D_f} d_0^{-(D_f+1)} dd_0 \quad (3)$$

where $-dN > 0$; the negative sign of the Equation (3) indicates that the number of main channels decreases with the increase in the diameter of the main channels. Therefore,

when d_0 is equal to the minimum diameter of the main channels, the total number of main channels in porous media can be expressed as [8]:

$$N_t(L \geq d_{0\min}) = \left(\frac{d_{0\max}}{d_{0\min}} \right)^{D_f} \quad (4)$$

Dividing Equation (3) by Equation (4), we have [7–9]:

$$-\frac{dN}{N_t} = D_f d_{0\min}^{D_f} d_0^{-(D_f+1)} dd_0 = f(d_0) dd_0 \quad (5)$$

where $f(d_0)$ is the probability function for pore size distribution and can be given by [7]:

$$f(d_0) = D_f d_{0\min}^{D_f} d_0^{-(D_f+1)} \quad (6)$$

According to the basic theory of probability theory, Equation (6) should meet the normalization condition; please see Ref. [18].

Equations (1)–(6) describe the randomly fractal distribution of tree-like bifurcation networks with mother channel diameter d_0 , as shown in Figure 1a. In the following, we will introduce the geometric structure of the single damaged tree-like bifurcation network (see Figure 1b), which is made up of “point to line” Y-shaped networks. In our model, we suppose that each branch in the bifurcation networks is a cylindrical tube, and the roughness and the thickness of the tube wall are ignored [25]. Figure 1b illustrates the utilization of l_k and d_k to denote the length and diameter, respectively, of the k th branching level (where k ranges from 0 to m). The network bifurcation numbers, denoted as m , represent the maximum number of branches that occur in a network. At each level of the network, every tube undergoes a bifurcation, resulting in the formation of n new tubes. In order to elucidate the geometric configuration of a bifurcation network resembling that of a tree, next, we proceed to present two scale variables, namely the length ratio α and the diameter ratio β [33].

$$A = \frac{l_{k+1}}{l_k} \quad (7)$$

$$B = \frac{d_{k+1}}{d_k} \quad (8)$$

So we obtain:

$$L_k = l_0 \alpha^k \quad (9)$$

$$D_k = d_0 \beta^k \quad (10)$$

where l_0 and d_0 are the length and diameter of the 0th branching level, respectively.

In order to consider an asymmetric tree-like bifurcation network, we assume that p channels of the k th branching level have been damaged, but the other parts of the network are intact. For example, if a branch of the tree-like bifurcation network in Figure 1b is damaged, the damaged part will not generate new branches, and the undamaged part will not be affected.

These equations above provide a theoretical basis for the analysis of heat transfer in the porous media embedded with randomly distributed damaged tree-like bifurcation networks.

3. Fractal Model of Effective Thermal Conductivity of Porous Media

3.1. The Effective Thermal Conductivity of Heat Conduction

In general, the heat transport process in the damaged tree-like bifurcation networks is different from that in the symmetric tree-like bifurcation networks.

According to Fourier's law and the thermal-electrical analogy technique, for a single damaged tree-like bifurcation network, the thermal resistance of the damaged tree-like bifurcation network can be written as [31]:

$$R_k = \frac{4l_0}{\lambda_1 \pi d_0^2} Z_1 \quad (11)$$

where $Z_1 = \frac{\left[1 - \left(\frac{\alpha}{n\beta^2}\right)^k\right] + \left(\frac{\alpha}{n\beta^2}\right)^k \frac{1}{1 - n^{-k}P} \left[1 - \left(\frac{\alpha}{n\beta^2}\right)^{m-k+1}\right]}{\left(1 - \frac{\alpha}{n\beta^2}\right)}$, λ_1 is the thermal conductivity of the fluid, n is the branching number of the tree-like branching network, m is total numbers of branching levels, and p is the numbers of damaged channels. With the help of Equations (4) and (11), the reciprocal of the total thermal resistance of the entire randomly distributed damaged tree-like bifurcation networks can be calculated by the following relation:

$$\frac{1}{R_k} = - \int_{d_{0\min}}^{d_{0\max}} \frac{1}{r_k} dN = \frac{\lambda_1 \pi d_{0\max}^2 D_f (1 - \phi_f)}{4l_0 (2 - D_f)} \frac{1}{Z_1} \quad (12)$$

Here, we add up the length of each branch of a single tree-like branching network as a straight pipe and consider the length of this straight pipe as the equivalent length. Then, the equivalent length of a single damaged tree-like bifurcation network, l_e , can be written as [31]:

$$L_e = \sum_{k=0}^m l_k = l_0 \frac{1 - \alpha^{m+1}}{1 - \alpha} \quad (13)$$

The total volume of the single damaged tree-like bifurcation network, V , can be expressed as [31]:

$$\begin{aligned} V &= \sum_{i=0}^{k-1} n^i V_i + \sum_{i=k}^m (n^i - n^{i-k}P) V_I \\ &= \frac{\pi d_0^2 l_0}{4} \frac{1 - (n\beta^2\alpha)^k}{1 - n\beta^2\alpha} + \frac{\pi d_0^2 l_0}{4} (n\beta^2\alpha)^k (1 - n^{-k}P) \frac{1 - (n\beta^2\alpha)^{m-k+1}}{1 - n\beta^2\alpha} \\ &= \frac{\pi d_0^2 l_0}{4} Z_2 \end{aligned} \quad (14)$$

where $Z_2 = \frac{[1 - (n\beta^2\alpha)^k] + (n\beta^2\alpha)^k (1 - n^{-k}P) [1 - (n\beta^2\alpha)^{m-k+1}]}{(1 - n\beta^2\alpha)}$, and V_I is the volume of the single pipe in the i th branching level. In the present study, we consider the singularly impaired tree-like bifurcation network as an exemplary model characterized by a solitary conduit exhibiting a consistent volume. The ETC of the single channel that is equivalent to the entire network is equivalent to the ETC of the entire network. Hence, utilizing Equations (13) and (14), the effective cross-sectional area of the individual tree-like bifurcation network, denoted as a_e , can be determined [31]:

$$a_e = \frac{V}{l_e} = \frac{\pi d_0^2}{4} \frac{1 - \alpha}{1 - \alpha^{m+1}} Z_2 \quad (15)$$

Due to the distribution of the diameter of main pipes conforming to the fractal scaling law, the effective cross-sectional area of the total porous media can be calculated from Equations (2), (4), and (15):

$$A_e = - \int_{d_{0\min}}^{d_{0\max}} a_e dN = \frac{\pi D_f d_{0\max}^2 (1 - \phi_f)}{4(2 - D_f)} \frac{1 - \alpha}{1 - \alpha^{m+1}} Z_2 \quad (16)$$

when $m = 0$, $n = 0$, and $p = 0$, the effective cross-sectional area is equal to the total cross-sectional area of main channels, A_0 . At this point, a single tree-like branching network can be seen as an undamaged channel, while the porous medium at this time can be seen as

a porous medium with a fractal distribution of the diameter of the main pipe, which is similar to in Yu's study [10]:

$$A_0 = \frac{\pi D_f d_{0\max}^2 (1 - \phi_f)}{4(2 - D_f)} \quad (17)$$

The area of the complete cross-section of the media, A , according to the notion of porosity, A_a , is determined by:

$$A_a = \frac{A_0}{\phi_f} = \frac{\pi D_f d_{0\max}^2 (1 - \phi_f)}{4(2 - D_f) \phi_f} \quad (18)$$

Likely, the thermal resistance of the media matrix of porous media, R_s , is given by Fourier's law and can be calculated as follows:

$$R_s = \frac{l_e}{A_a \lambda_s (1 - \phi_f)} \quad (19)$$

where λ_s is the thermal conductivity of the media matrix.

According to Fourier's law, the ETC of the randomly distributed damaged tree-like bifurcation network part, $K_{d,k}$, and the ETC of the media matrix part, $K_{d,s}$, can be respectively described as [31]:

$$K_{d,k} = \frac{l_e}{A_a R_k} \quad (20)$$

$$K_{d,s} = \frac{l_e}{A_a R_s} \quad (21)$$

According to Fourier's law and the series-parallel model, the ETC of the whole porous media, K_d , can be divided into two components: the randomly distributed damaged tree-like bifurcation network component and the media matrix component, which can be described as [31]:

$$K_d = K_{d,k} + K_{d,s} = \frac{l_e}{A_a} \left(\frac{1}{R_k} + \frac{1}{R_s} \right) \quad (22)$$

Inserting Equations (12), (13), (18), and (19) into Equation (22), the ETC of the whole porous media, K_d , can be calculated as:

$$K_d = \frac{1 - \alpha^{m+1}}{1 - \alpha} \frac{\lambda_l \phi_f \left(1 - \frac{\alpha}{n\beta^2} \right)}{Z_1} + \lambda_s (1 - \phi_f) \quad (23)$$

3.2. The Effective Thermal Conductivity of Heat Convection

The derivation of the ETC of the whole porous media is presented in Section 3.1. Nevertheless, the convective heat transfer occurring between the fluid and the wall plays a crucial role in facilitating the transportation of heat within porous media. This section primarily focuses on the determination of the ETC associated with heat convection K_{cv} .

According to Chen and Cheng [26], the fluid dynamics within the tree-like bifurcation networks exhibit laminar flow, and it is seen that the Nusselt number remains constant across each layer. Consequently, the coefficient of heat convection, H_r , can be made explicit as [26]:

$$H_r = \frac{Nu \cdot \lambda_l}{d} \quad (24)$$

The pipe's characteristic length, denoted as d , is equivalent to the diameter of the pipe. The Nusselt number, denoted as Nu , represents the dimensionless ratio of convective

heat transfer to conductive heat transfer. Hence, the heat convection coefficient of the i th branching level of the tree-like bifurcation network, denoted as H_i , can be determined [26]:

$$H_i = H_0 \beta^{-i} \quad (25)$$

where H_0 is the heat convection coefficient of the single main pipe of the tree-like bifurcation network, which can be written as [26]:

$$H_0 = \frac{Nu \cdot \lambda_l}{d_0} \quad (26)$$

where d_0 is the diameter of the 0th branching level. In line with Newton's cooling formula, the convective heat transfer rate in a solitary primary pipe, q_0 , can be written as:

$$q_0 = H_0 s_{s,0} \Delta T \quad (27)$$

where ΔT is temperature difference. $S_{s,0}$ is the heat convection area of a single main pipe of the tree-like bifurcation network, which can be determined by:

$$S_{s,0} = \pi d_0 l_0 \quad (28)$$

where l_0 is the length of the 0th branching level. According to Chen and Cheng [26], the temperature difference between tree-like bifurcation networks at different levels is invariable. Then, by means of Equations (25)–(28), the flow of the heat convection of the undamaged part of the single damaged tree-like bifurcation network before the k th branching level, $\Delta q_{1,a}$, can be modified as [31]:

$$\begin{aligned} \Delta q_{1,a} &= \sum_{i=0}^{k-1} n^i H_i s_{s,i} \Delta T \\ &= \pi d_0 l_0 H_0 \Delta T \sum_{i=0}^{k-1} n^i \beta^{-i} \alpha^i \beta^i \\ &= Nu \pi \lambda_l l_0 \frac{1 - (n\alpha)^k}{1 - n\alpha} \Delta T \end{aligned} \quad (29)$$

where $s_{s,i}$ is the heat convection area of the i th branching level in the tree-like bifurcation network.

It is likely that the flow of the heat convection of the damaged part of a single damaged tree-like bifurcation network, $\Delta q_{2,a}$, can be modified as [31]:

$$\begin{aligned} \Delta q_{2,a} &= \sum_{i=k}^m (n^i - n^{i-k} p) H_i s_{s,i} \Delta T \\ &= \pi d_0 l_0 H_0 \Delta T \sum_{i=k}^m (n^i - n^{i-k} p) \beta^{-i} \alpha^i \beta^i \\ &= Nu \pi \lambda_l l_0 (n\alpha)^k (1 - n^{-k} p) \frac{1 - (n\alpha)^{m-k+1}}{1 - n\alpha} \Delta T \end{aligned} \quad (30)$$

The heat convection flow of a singular damaged tree-like bifurcation network with smooth surfaces, denoted as q_a , can be computed using Equations (29) and (30) [31]:

$$\begin{aligned} q_a &= \Delta q_{1,a} + \Delta q_{2,a} \\ &= \sum_{i=0}^{k-1} n^i H_i s_{s,i} \Delta T + \sum_{i=k}^m (n^i - n^{i-k} p) H_i s_{s,i} \Delta T \\ &= \pi d_0 l_0 H_0 \Delta T \sum_{i=0}^{k-1} n^i \beta^{-i} \alpha^i \beta^i + \pi d_0 l_0 H_0 \Delta T \sum_{i=k}^m (n^i - n^{i-k} p) \beta^{-i} \alpha^i \beta^i \\ &= Nu \pi \lambda_l l_0 \frac{1 - (n\alpha)^k}{1 - n\alpha} \Delta T + Nu \pi \lambda_l l_0 (n\alpha)^k (1 - n^{-k} p) \frac{1 - (n\alpha)^{m-k+1}}{1 - n\alpha} \Delta T \end{aligned} \quad (31)$$

The heat convection area of the intact portion of a singular damaged tree-like bifurcation network prior to the k th level of branching, denoted as $\Delta S_{1,a}$, can be expressed in a similar manner:

$$\begin{aligned} \Delta S_{1,a} &= \sum_{i=0}^{k-1} n^i s_{s,i} \\ &= \pi d_0 l_0 \sum_{i=0}^{k-1} n^i \alpha^i \beta^i \\ &= \pi d_0 l_0 \frac{1 - (n\alpha\beta)^k}{1 - n\alpha\beta} \end{aligned} \quad (32)$$

The heat convection area of the damaged part of a single damaged tree-like bifurcation network, $\Delta S_{2,a}$, is:

$$\begin{aligned}\Delta S_{2,a} &= \sum_{i=k}^m (n^i - n^{i-k}P) s_{s,i} \\ &= \pi d_0 l_0 \sum_{i=k}^m (n^i - n^{i-k}P) \alpha^i \beta^i \\ &= \pi d_0 l_0 (n\alpha\beta)^k (1 - n^{-k}P) \frac{1 - (n\alpha\beta)^{m-k+1}}{1 - n\alpha\beta}\end{aligned}\quad (33)$$

The calculation of the heat convection area of a single damaged tree-like bifurcation network, denoted as S_a , may be performed using Equations (32) and (33).

$$\begin{aligned}S_a &= \Delta S_{1,a} + \Delta S_{2,a} \\ &= \sum_{i=0}^{k-1} n^i s_{s,i} + \sum_{i=k}^m (n^i - n^{i-k}P) s_{s,i} \\ &= \pi d_0 l_0 \sum_{i=0}^{k-1} n^i \alpha^i \beta^i + \pi d_0 l_0 \sum_{i=k}^m (n^i - n^{i-k}P) \alpha^i \beta^i \\ &= \pi d_0 l_0 \frac{1 - (n\alpha\beta)^k}{1 - n\alpha\beta} + \pi d_0 l_0 (n\alpha\beta)^k (1 - n^{-k}P) \frac{1 - (n\alpha\beta)^{m-k+1}}{1 - n\alpha\beta}\end{aligned}\quad (34)$$

The diameter distribution of the main pipes exhibits fractal scaling, according to the underlying principle, the heat flow, Q_{cv} , and the heat convection area, S_{cv} , can be obtained by respectively integrating the individual heat flow, q_a , and the individual heat convection area, S_a .

$$\begin{aligned}Q_{cv} &= - \int_{d_{0min}}^{d_{0max}} q_a dN \\ &= - \int_{d_{0min}}^{d_{0max}} Nu \pi l_0 \frac{1 - (n\alpha)^k}{1 - n\alpha} \Delta T + Nu \pi l_0 (n\alpha)^k (1 - n^{-k}P) \frac{1 - (n\alpha)^{m-k+1}}{1 - n\alpha} \Delta T dN \\ &= Nu \pi l_0 \left[\frac{1 - (n\alpha)^k}{1 - n\alpha} + (n\alpha)^k (1 - n^{-k}P) \frac{1 - (n\alpha)^{m-k+1}}{1 - n\alpha} \right] d_{0max}^{D_f} (d_{0min}^{-D_f} - d_{0max}^{-D_f}) \Delta T\end{aligned}\quad (35)$$

$$\begin{aligned}S_{cv} &= - \int_{d_{0min}}^{d_{0max}} S_a dN \\ &= - \int_{d_{0min}}^{d_{0max}} \pi d_0 l_0 \frac{1 - (n\alpha\beta)^k}{1 - n\alpha\beta} + \pi d_0 l_0 (n\alpha\beta)^k (1 - n^{-k}P) \frac{1 - (n\alpha\beta)^{m-k+1}}{1 - n\alpha\beta} dN \\ &= \pi l_0 \frac{D_f}{D_f - 1} \left[\frac{1 - (n\alpha\beta)^k}{1 - n\alpha\beta} + (n\alpha\beta)^k (1 - n^{-k}P) \frac{1 - (n\alpha\beta)^{m-k+1}}{1 - n\alpha\beta} \right] d_{0max}^{D_f} (d_{0min}^{1-D_f} - d_{0max}^{1-D_f})\end{aligned}\quad (36)$$

In accordance with Fourier's law, the thermal conductivity of heat convection resulting from fluid flow in porous media, denoted as K_{cv} , is [31]:

$$K_{cv} = \frac{Q_{cv}}{S_{cv} \frac{\Delta T}{\delta_T}} \quad (37)$$

The thickness of the thermal boundary layer in thermal convection, denoted as δ_T , is primarily influenced by the fluid's characteristics.

Inserting Equations (35) and (36) into Equation (37), the thermal conductivity of heat convection caused by fluid flow in the porous media, K_{cv} , is:

$$K_{cv} = Nu \delta_T l_0 \frac{1 - n\alpha\beta}{1 - n\alpha} \frac{1 - (n\alpha)^k + (n\alpha)^k (1 - n^{-k}P) [1 - (n\alpha)^{m-k+1}]}{1 - (n\alpha\beta)^k + (n\alpha\beta)^k (1 - n^{-k}P) [1 - (n\alpha\beta)^{m-k+1}]} \frac{D_f - 1}{D_f} (d_{0min})^{-1} \frac{1 - \left(\frac{d_{0min}}{d_{0max}}\right)^{D_f}}{1 - \left(\frac{d_{0min}}{d_{0max}}\right)^{D_f-1}} \quad (38)$$

According to the research of Yu et al. [10], only when $\left(\frac{d_{0min}}{d_{0max}}\right)^{D_f} = 0$ does the diameter of the main channels of damaged tree-like bifurcation networks, d_0 , conform to fractal scaling law. Therefore, the thermal conductivity of heat convection caused by fluid flow in the porous media K_{cv} can be simplified as:

$$K_{cv} = Nu \delta_T l_0 \frac{1 - n\alpha\beta}{1 - n\alpha} \frac{1 - (n\alpha)^k + (n\alpha)^k (1 - n^{-k}P) [1 - (n\alpha)^{m-k+1}]}{1 - (n\alpha\beta)^k + (n\alpha\beta)^k (1 - n^{-k}P) [1 - (n\alpha\beta)^{m-k+1}]} \frac{D_f - 1}{D_f} (d_{0min})^{-1} \frac{1}{1 - \left(\frac{d_{0min}}{d_{0max}}\right)^{D_f-1}} \quad (39)$$

The average diameter, $\overline{d_0}$, of the main channels of randomly distributed tree-like bifurcation networks can be obtained from Equation (6) [32]:

$$\overline{D_0} = \int_{d_{0\min}}^{d_{0\max}} d_0 f(d_0) dd_0 = \frac{D_f}{D_f - 1} d_{0\min} \left[1 - \left(\frac{d_{0\min}}{d_{0\max}} \right)^{D_f - 1} \right] \quad (40)$$

Inserting Equation (40) into Equation (39), the thermal conductivity of heat convection caused by fluid flow in the porous media can be simplified again as:

$$K_{cv} = Nu \delta_T \lambda_l \frac{1 - n\alpha\beta}{1 - n\alpha} \frac{1 - (n\alpha)^k + (n\alpha)^k (1 - n^{-k}P) [1 - (n\alpha)^{m-k+1}]}{1 - (n\alpha\beta)^k + (n\alpha\beta)^k (1 - n^{-k}P) [1 - (n\alpha\beta)^{m-k+1}]} \frac{1}{\overline{d_0}} \quad (41)$$

3.3. The Total Effective Thermal Conductivity of Porous Media

The development of the thermal conductivity model for heat conduction and the thermal conductivity model for heat convection in porous media was conducted in Sections 3.1 and 3.2, respectively. The current investigation proposes that the heat transfer mechanism in bifurcation networks resembling trees is facilitated by the combined effects of heat conduction and heat convection. In alternative terms, the equation representing the effective thermal conductivity (ETC) of porous media, which encompasses randomly distributed damaged tree-like bifurcation networks written as K_{eff} , can be formulated as [31]:

$$K_{eff} = K_d + K_{cv} \quad (42)$$

With respect to Equations (23), (41), and (42), the ETC of whole porous media can be derived as:

$$K_{eff} = \frac{1 - \alpha^{m+1}}{1 - \alpha} \frac{\lambda_l \phi_f \left(1 - \frac{\alpha}{n\beta^2} \right)^k}{\left\{ \left[1 - \left(\frac{\alpha}{n\beta^2} \right)^k \right] + \frac{\left(\frac{\alpha}{n\beta^2} \right)^k}{1 - n^{-k}P} \left[1 - \left(\frac{\alpha}{n\beta^2} \right)^{m-k+1} \right] \right\}} + \lambda_s (1 - \phi_f) + Nu \delta_T \lambda_l \frac{1 - n\alpha\beta}{1 - n\alpha} \frac{1 - (n\alpha)^k + (n\alpha)^k (1 - n^{-k}P) [1 - (n\alpha)^{m-k+1}]}{1 - (n\alpha\beta)^k + (n\alpha\beta)^k (1 - n^{-k}P) [1 - (n\alpha\beta)^{m-k+1}]} \frac{1}{\overline{d_0}} \quad (43)$$

The dimensionless ETC of porous media with randomly distributed damaged tree-like bifurcation networks is defined by $K^+ = \frac{K_{eff}}{\lambda_l}$, and it can be expressed as:

$$K^+ = \frac{1 - \alpha^{m+1}}{1 - \alpha} \frac{\phi_f \left(1 - \frac{\alpha}{n\beta^2} \right)^k}{\left\{ \left[1 - \left(\frac{\alpha}{n\beta^2} \right)^k \right] + \frac{\left(\frac{\alpha}{n\beta^2} \right)^k}{1 - n^{-k}P} \left[1 - \left(\frac{\alpha}{n\beta^2} \right)^{m-k+1} \right] \right\}} + \frac{\lambda_s}{\lambda_l} (1 - \phi_f) + Nu \delta_T \frac{1 - n\alpha\beta}{1 - n\alpha} \frac{1 - (n\alpha)^k + (n\alpha)^k (1 - n^{-k}P) [1 - (n\alpha)^{m-k+1}]}{1 - (n\alpha\beta)^k + (n\alpha\beta)^k (1 - n^{-k}P) [1 - (n\alpha\beta)^{m-k+1}]} \frac{1}{\overline{d_0}} \quad (44)$$

Equation (44) is the dimensionless coefficient of heat conductivity for the whole porous media. It is also a theoretical model, which can be used for analyzing the effect of structural parameters of tree-like bifurcation networks ($\overline{d_0}$, α , β , n , m , P , k), the porosity, ϕ_f , the thermal conductivity of porous media matrix, λ_s , and the thermal conductivity of fluid, λ_l , on the dimensionless thermal conductivity coefficient. In Equation (44), all parameters have a clear physical meaning, and there are not any empirical constants.

4. Results and Discussion

Figure 2 presents a comparative analysis of the experimental data obtained from the study conducted by Liang et al. [34–36] and ETC, K_{eff} , versus λ_s for varying quantities of impaired channels, as determined by Equation (43). The values of relevant parameters are all from experimental information from Liang et al. [34–36]. In Figure 2, the permeable me-

dia's thermal conductivity fluid is $\lambda_l = 0.5$, which is given by the experimental information from Liang et al. [35]. The porosity used in the experimental information of Valvano et al. is 0.0041–0.1645. The porosity used in the experimental data of Liang et al. is 0.1, and the porosity used in the experimental data of Bhattacharya et al. is 0.013–0.1645 [34–36]. Therefore, the porosity we select in the Figure 2 is 0.1, and the experiment data from Liang et al. gives the length ratio and diameter ratio of the arterial vascular tree, which are 1.30 and 1.25 [34–36]. So, we select $\alpha = 1/1.30 = 0.77$, $\beta = 1/1.25 = 0.8$. The structural parameters of pores in reality, such as n , and m , are difficult to obtain in experiments. In Tien's research [37], the Nusselt number, denoted as Nu , characterizes the convective heat transfer in a porous medium confined between two parallel plates in the fully developed regime. This analysis is based on the Darcy flow model, specifically considering slug flow. For the case of constant wall temperature, the Nusselt number is represented by $Nu = 4.93$. Conversely, for constant wall heat flux, the Nusselt number is given by $Nu = 6.0$. In this study, we assume that heat transfer in porous media occurs under constant temperature conditions. Due to the complex microporous structure of the tree-like branching network, which is difficult to measure, we arbitrarily select $m = 5$, $n = 2$. So, we take $m = 5$, $n = 2$, $Nu = 4.93$. Consistent with the experimental data, we choose the thickness of the thermal boundary layer as $\delta_T = 2.5 \times 10^{-5}$ and the average diameter of the randomly distributed tree-like network as $\overline{d_0} = 1 \times 10^{-2}$ m [34–36]. The picture reveals that the ETC of porous media containing randomly distributed damaged tree-like bifurcation networks, as predicted using the ETC model described by Equation (43), exhibits a notable level of concordance with the experimental findings. It has also been observed that the ETC exhibits a positive correlation with the thermal conductivity of the media matrix while displaying a negative correlation with the number of damaged channels. The observed outcome can be rationalized by considering that an augmentation in the number of impaired channels results in a reduction in both the effective cross-sectional area and thermal convection area. Consequently, the dimensionless ETC experiences a decline as the number of damaged channels increases.

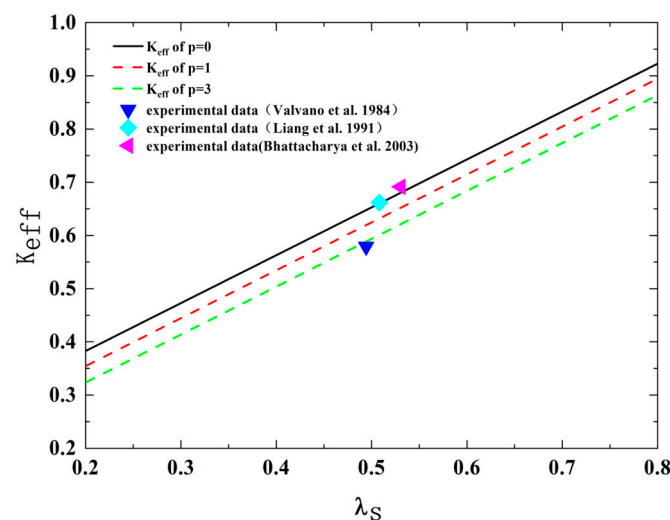


Figure 2. The comparison between experimental data and ETC K_{eff} versus λ_s at $\alpha = 0.77$, $\beta = 0.8$, $k = 2$, $m = 5$, $n = 2$, $\phi_f = 0.1$, $\lambda_l = 0.5$, $\overline{d_0} = 1 \times 10^{-2}$ m, $Nu = 4.93$, $\delta_T = 2.5 \times 10^{-5}$ m [34–36].

Figure 3 illustrates the relationship between the dimensionless ETC and the length ratio for varying numbers of damaged channels. The data presented in the figure demonstrate a positive correlation between the dimensionless ETC and the length ratio, indicating that as the length ratio increases, the ETC also increases. Conversely, there is a negative correlation between the dimensionless ETC and the number of damaged channels, suggesting that as the number of damaged channels increases, the ETC decreases. The reason for this

phenomenon is that as the length ratio increases, the heat convection area of the bifurcation network pipeline also increases, thereby leading to an increase in the overall ETC. It is worth noting that when $p = 4$, the dimensionless ETC hardly changes with the change of the length ratio. The reason for this result is that when all branches of the networks are damaged, only the heat conduction of matrix part exists in porous media. At this time, the porous medium can be regarded as a solid medium with all the pipes damaged. The length ratio hardly influences the dimensionless ETC of porous media.

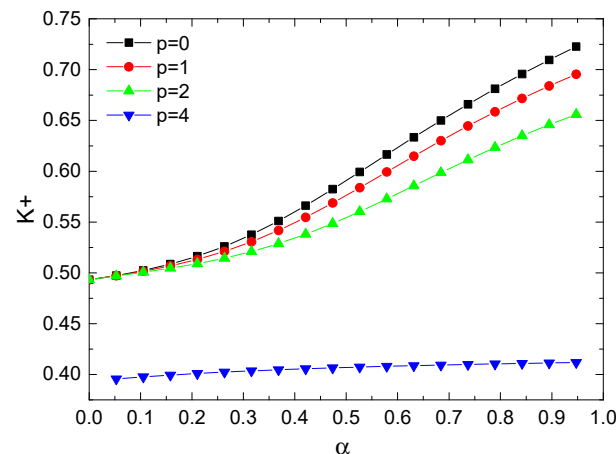


Figure 3. The dimensionless ETC K^+ versus α and p at $\beta = 0.8$, $n = 2$, $k = 2$, $\phi_f = 0.1$, $\lambda_l = 0.5$, $\lambda_s = 0.15$, $\bar{d}_0 = 1 \times 10^{-2}$ m, $Nu = 4.93$, $\delta_T = 2.5 \times 10^{-5}$ m.

Figure 4 illustrates the impact of the diameter ratio, denoted as β , on the dimensionless ETC for varying numbers of damaged channels and compares it with the data in Ref. [31]. The rationale behind this phenomenon is that an increase in the diameter ratio results in a decrease in the overall thermal resistance. Consequently, the ETC decreases while the length ratio remains constant. It can be observed from the figure that the dimensionless ETC exhibits a decreasing trend as the diameter ratio increases. Additionally, it is observed that the dimensionless ETC exhibits a minor variation with the change in the diameter ratio when $p = 4$. The aforementioned statement aligns with the result presented in Figure 4. Interestingly, by comparing the data with Ref. [31] ($m = 5$, $\alpha = 0.77$, $n = 2$, $k = 2$), it can be seen that when $p = 2$, our model has a higher ETC, but the trends of the two are basically consistent.

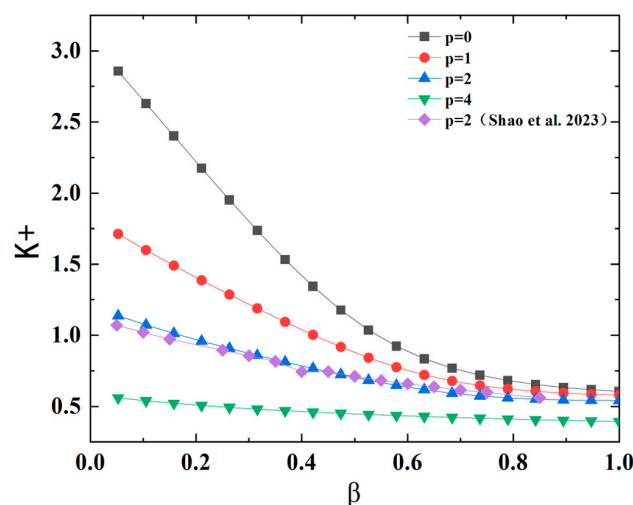


Figure 4. The dimensionless conductivity K^+ versus β and p at $\alpha = 0.77$, $n = 2$, $k = 2$, $\phi_f = 0.1$, $\lambda_l = 0.5$, $\lambda_s = 0.15$, $\bar{d}_0 = 1 \times 10^{-2}$ m, $Nu = 4.93$, $\delta_T = 2.5 \times 10^{-5}$ m [31].

Figure 5 illustrates the relationship between the ETC, K_{eff} , the ETC of the network part, $K_{d,1}$, the ETC of the matrix part, $K_{d,2}$, and the heat convection ETC, K_{cv} , with respect to the fractal dimension of the porous media. This relationship is examined for various numbers of damaged channels. The research indicates that there is a positive correlation between the fractal dimension of porous media and the ETC. Specifically, as the fractal dimension increases, the ETC also increases. On the other hand, the ETC of the randomly distributed damaged tree-like bifurcation network component falls when the fractal dimension of porous media increases. The influence of the fractal dimension of porous media on ETC resulting from heat convection is negligible. The ETC of the matrix exhibits a decrease as the fractal dimension of the porous media increases. The observed phenomenon can be attributed to the positive correlation between the fractal dimension of pores and the porosity of porous media. As the fractal dimension of holes increases, the porosity of the media also increases, thereby resulting in a decrease in the matrix composition of the porous media. The determination of the total ETC of the porous media may be observed from Equation (41), which indicates that it is influenced by the ETC of the three constituent parts. The dominance of heat transmission in porous media by the heat conduction of the randomly distributed damaged tree-like bifurcation network component as the fractal dimension grows can be observed in Figure 4.

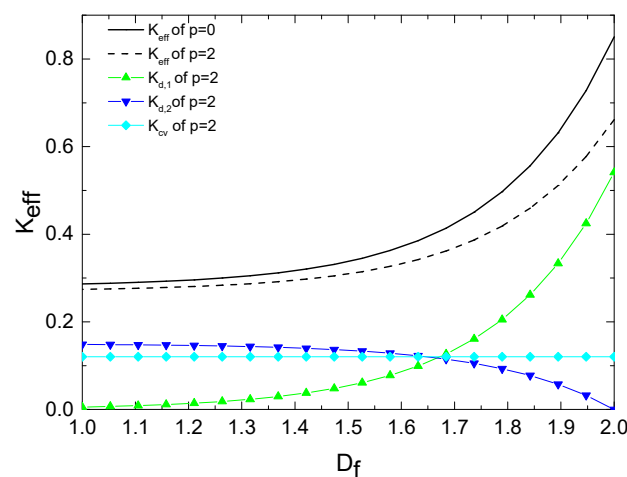


Figure 5. The ETC of each component K_{eff} , $K_{d,1}$, $K_{d,2}$, K_{cv} versus D_f , and p at $\alpha = 0.77$, $\beta = 0.8$, $k = 2$, $m = 5$, $n = 2$, $\lambda_s = 0.15$, $\lambda_l = 0.5$, $\bar{d}_0 = 1 \times 10^{-2}$ m, $Nu = 4.93$, $\delta_T = 2.5 \times 10^{-5}$ m.

Figure 6 illustrates the impact of the ratio between the ETC of the wall and the ETC of the fluid on the dimensionless ETC of the porous media considering various fractal dimensions. Based on the analysis of Figure 6, it can be inferred that an increase in the ratio of the ETC of the wall to that of the fluid results in a corresponding increase in the dimensionless ETC. It is noteworthy to mention that the dimensionless ETC remains consistent across various fractal dimensions, provided that the ratio between the ETC of the wall and that of the fluid is approximately 1.1. When the ratio is below 1.1, there is an observed rise in the dimensionless ETC as the fractal dimension increases. However, when the ratio exceeds 1.1, a decrease in the fractal dimension is associated with an increase in the dimensionless ETC. The relationship between porosity and fractal dimension can be elucidated by observing that an increase in fractal dimension corresponds to an increase in porosity. In cases when the ETC of the fluid surpasses that of the matrix component, an increase in porosity leads to a more significant involvement of tree-like bifurcation networks in the heat conduction process inside porous media. In contrast, if the ETC of the fluid is lower than that of the matrix, an increase in porosity will result in a reduction in the heat transfer area of the matrix. Consequently, the overall ETC will fall.

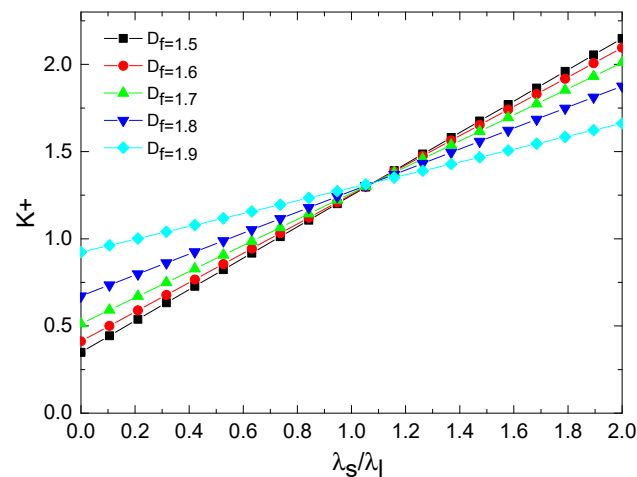


Figure 6. The dimensionless ETC K^+ versus $\frac{\lambda_s}{\lambda_l}$ and D_f at $\alpha = 0.77$, $\beta = 0.8$, $k = 2$, $m = 5$, $n = 2$, $p = 2$, $\bar{d}_0 = 1 \times 10^{-2}$ m, $Nu = 4.93$, $\delta_T = 2.5 \times 10^{-5}$ m.

5. Conclusions

This research presents the derivation of a dimensionless equation for the ETC of porous media including randomly distributed damaged tree-like bifurcation networks. Furthermore, the impact of the structural features of the porous media on the ETC is investigated. In this study, we took into account not only the heat conduction of the bifurcation network and matrix components, but also the heat convection resulting from liquid flow. Our findings indicate that when the ratio of ETC of the wall to that of the fluid is less than 1.1, there is an observed increase in the dimensionless ETC as the fractal dimension increases. Conversely, when the ratio exceeds 1.1, a decrease in the fractal dimension corresponds to an increase in the dimensionless ETC. Additionally, it was observed that an increase in the fractal dimension results in the dominance of heat conduction in the heat transfer process of porous medium, namely, inside the randomly distributed damaged tree-like bifurcation network segment. Furthermore, by a comparative analysis of the available experimental data and the fractal model suggested in our study, it was shown that the ETC model for porous media presented in this research has favorable concurrence with the experimental data. The proposed model aims to enhance the understanding of heat transport mechanisms in porous media and offer valuable insights for a wide range of engineering applications. The omission of considering the impact of capillary surface roughness on the effective heat conductivity of porous media is acknowledged in this study. Hence, our forthcoming research will focus on investigating the impact of surface roughness on the effective heat conductivity.

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Nomenclature

Symbols	Description		
N	branching number	H_r	the coefficient of heat convection
L	diameter scale	R_s	the thermal resistance of the media matrix
l_k	the length of the kth branching level	R_k	the thermal resistance of the network
d_k	the diameter of the kth branching level	ϕ_a	the porosity of the cross-section of the media
D_f	dimension of the main channels' fractal	ΔT	temperature difference
ϕ_f	areal porosity	ϕ_a	the porosity of the cross-section of the media
D_E	Euclidean dimension	$R_{d,a}$	the total thermal resistance of the network
n	the branching number	ΔT	temperature difference
D	fractal dimension	Q_0	the heat convection flow of a single main pipe
D_E	Euclidean dimension	$S_{s,0}$	the tree-like bifurcation network's single main pipe's heat convection area
d_{0max}	the primary channels' largest diameter	$\Delta q_{1,a}$	the heat convection flow in the undamaged part
d_{0min}	the primary channels' minimum diameter	$\Delta q_{2,a}$	the heat convection flow in the damaged part
α	the length ratio	$\Delta S_{1,a}$	the heat convection area of the undamaged part
β	the diameter ratio	$\Delta S_{2,a}$	the heat convection area of the damaged part
R_k	the kth level channel's single channel's thermal resistance	$K_{a,1}$	the effective thermal conductivity of the media matrix part
m	total number of levels in a branch	$K_{a,2}$	the effective thermal conductivity of the network part
P	the number of damaged channels	$\overline{d_0}$	the average diameter of the main channels
l_e	the equivalent length	k_{cv}	the thermal conductivity of heat convection
V	the total volume	Q_{cv}	the total heat flow
V_i	the single pipe's volume at the branching level	S_{cv}	the total heat convection area
λ_l	the thermal conductivity of the fluid part	q_a	the individual heat flow
		S_a	the individual heat convection area
λ_s	the thermal conductivity of the media matrix		the thickness of the thermal boundary layer of thermal convection caused by fluid flow is mainly related to the characteristics of the fluid
A_0	the overall main channel cross-sectional area	δ_T	
A_a	the area encompassing the whole spectrum of media	H_r	the coefficient of heat convection
$K_{d,k}$	the ETC of network part	K_{eff}	the effective thermal conductivity
$K_{d,s}$	the ETC of the media matrix part	k^+	the dimensionless thermal conductivity
K_d	the ETC of the whole porous media		
Nu	the Nusselt number		
k_{cv}	the thermal conductivity of heat convection		

References

- He, Y.L.; Xie, T. Advances of thermal conductivity model of nanoscale silica aerogel insulation material. *Appl. Therm. Eng.* **2015**, *81*, 28–50. [\[CrossRef\]](#)
- Mao, N.; Ye, J.; Quan, Z.Z.; Zhang, H.N.; Wu, D.Q.; Qin, X.H.; Wang, R.W.; Yu, J.Y. Tree-like structure driven water transfer in 1D fiber assemblies for Functional Moisture-Wicking Fabrics. *Mater. Des.* **2020**, *186*, 108305. [\[CrossRef\]](#)
- Liang, M.; Fu, C.; Xiao, B.; Luo, L.; Wang, Z. A fractal study for the effective electrolyte diffusion through charged porous media. *Int. J. Heat Mass Transf.* **2019**, *137*, 365–371. [\[CrossRef\]](#)
- Zhang, N.; Wang, Z.Y. Review of soil thermal conductivity and predictive models. *Int. J. Therm. Sci.* **2017**, *117*, 172–183. [\[CrossRef\]](#)
- Liu, H.; Zhao, X. Thermal conductivity analysis of high porosity structures with open and closed pores. *Int. J. Heat Mass Transfer* **2022**, *183*, 122089. [\[CrossRef\]](#)
- Pan, Y.; Zhao, R.; Nian, Y. Study on the flow and heat transfer characteristics of pin-fin manifold microchannel heat sink. *Int. J. Heat Mass Transfer* **2022**, *183*, 122052. [\[CrossRef\]](#)
- Mandelbrot, B.B. *The Fractal Geometry of Nature*; W H Freeman: San Francisco, CA, USA, 1982.
- Yu, B.M.; Li, J. Some fractal characters of porous media. *Fractals* **2001**, *9*, 365–372. [\[CrossRef\]](#)
- Yu, B.M.; Lee, L.J.; Cao, H. A fractal in-plane permeability model for fabrics. *Polym. Compos.* **2002**, *23*, 201–221. [\[CrossRef\]](#)
- Yu, B.M.; Cheng, P. A fractal model for permeability of bi-dispersed porous media. *Int. J. Heat Mass Transfer* **2002**, *45*, 2983–2993. [\[CrossRef\]](#)
- Zheng, Q.; Wang, H.L.; Guo, X.Y. Research on the effect of surface roughness on gas diffusion coefficient of porous media embedded with a fractal-like tree network. *Fractals* **2021**, *29*, 2150195. [\[CrossRef\]](#)

12. Miao, T.J.; Chen, A.M.; Zhang, L.W. A novel fractal model for permeability of damaged tree-like branching networks. *Int. J. Heat Mass Transfer* **2018**, *127*, 278–285. [\[CrossRef\]](#)
13. Kou, J.L.; Chen, Y.Y.; Zhou, X.Y.; Lu, H.J.; Wu, F.M.; Fan, J.T. Optimal structure of tree-like branching networks for fluid flow. *Phys. A* **2014**, *393*, 527–534. [\[CrossRef\]](#)
14. Zheng, Q.; Wang, H.L.; Jiang, J.; Xu, C. Fractal analysis of surface roughness effects on gas diffusion in porous nanofibers. *Fractals* **2020**, *28*, 2050125. [\[CrossRef\]](#)
15. Zheng, Q.; Yu, B.M.; Wang, S.F.; Luo, L. A diffusivity model for gas diffusion through fractal porous media. *Chem. Eng. Sci.* **2020**, *68*, 650–655. [\[CrossRef\]](#)
16. Zheng, Q.; Xu, J.; Yang, B.; Yu, B.M. Research on the effective gas diffusion coefficient in dry porous media embedded with a fractal-like tree network. *Phys. A* **2013**, *392*, 1557–1566. [\[CrossRef\]](#)
17. Wang, F.Y.; Cheng, H. A fractal permeability model for 2D complex tortuous fractured porous media. *J. Pet. Sci. Eng.* **2020**, *188*, 106938. [\[CrossRef\]](#)
18. Zheng, Q.; Yu, B.M. A fractal permeability model for gas flow through dual-porosity media. *J. Appl. Phys.* **2012**, *111*, 024316. [\[CrossRef\]](#)
19. Zhang, J.; Raza, A.; Khan, U.; Ali, Q.; Zaib, A.; Weera, W.; Galal, A. Thermophysical Study of Oldroyd-B Hybrid Nanofluid with Sinusoidal Conditions and Permeability: A Prabhakar Fractional Approach. *Fractal Fract.* **2022**, *6*, 357. [\[CrossRef\]](#)
20. Xie, B.; Hu, Y.H.; Liu, L.T. Thermal conduction model of asymmetric structural aramid nanofiber aerogel membranes based on fractal theory. *Int. J. Heat Mass Transfer* **2023**, *208*, 124086. [\[CrossRef\]](#)
21. Shi, Q.T.; Feng, C.; Li, B.; Ming, P.W.; Zhang, C.M. Fractal model for the effective thermal conductivity of microporous layer. *Int. J. Heat Mass Transfer* **2023**, *205*, 123884. [\[CrossRef\]](#)
22. Xiao, B.Q. A fractal model for predicting the effective thermal conductivity of roughened porous media with microscale conductivity. *Fractals* **2021**, *29*, 2150114. [\[CrossRef\]](#)
23. Shen, Y.Q.; Xu, P. A generalized thermal conductivity model for unsaturated porous media with fractal geometry. *Int. J. Heat Mass Transfer* **2020**, *152*, 119540. [\[CrossRef\]](#)
24. Jing, D.L.; He, L.; Wang, X.M. Optimization analysis of fractal tree-like microchannel network for electroviscous flow to realize minimum hydraulic resistance. *Int. J. Heat Mass Transfer* **2018**, *125*, 749–752. [\[CrossRef\]](#)
25. Xu, L.; Xu, Y.; Gu, H. Thermal-hydraulic performance of flat-plate microchannel with fractal tree-like structure and self-affine rough wall. *Eng. Appl. Comput. Fluid Mech.* **2023**, *17*, e2153174. [\[CrossRef\]](#)
26. Chen, Y.P.; Cheng, P. Heat transfer and pressure drop in fractal tree-like microchannel nets. *Int. J. Heat Mass Transfer* **2002**, *45*, 2643–2648. [\[CrossRef\]](#)
27. Li, Y.; Yu, B.M. Study of the starting pressure gradient in branching network. *Sci. China Technol. Sci.* **2010**, *53*, 2397–2403. [\[CrossRef\]](#)
28. Zhang, J.; Wang, Y.Z.; Lou, G.; Kou, J.L. A fractal model for effective thermal conductivity of dual-porosity media with randomly distributed tree-like networks. *Fractals* **2021**, *29*, 2150146. [\[CrossRef\]](#)
29. Miao, T.J.; Chen, A.M.; Xu, Y.; Yang, S.S.; Yu, B.M. Optimal structure of damaged tree-like branching networks for the equivalent thermal conductivity. *Therm. Sci.* **2016**, *102*, 89–99. [\[CrossRef\]](#)
30. Xiao, B.Q.; Fang, J.; Long, G.B.; Tao, Y.Z.; Huang, Z.J. Analysis Of Thermal Conductivity Of Damaged Tree-Like Bifurcation Network With Fractal Roughened Surfaces. *Fractals* **2022**, *30*, 1–13. [\[CrossRef\]](#)
31. Shao, Y.H.; Yang, H.; Guo, X.Y.; Wang, H.L.; Zhu, L.M.; Ma, X.; Chen, R.J.; Ruan, S.F.; Ren, L.L.; Zheng, Q. Thermal conductivity model of porous media embedded with a damaged tree-like branching network considering the influence of roughness. *Fractal Fract.* **2023**, *7*, 5. [\[CrossRef\]](#)
32. Xia, Y.L.; Zheng, S.; Yang, S.S.; Yi, S. Analysis on effective thermal conductivity model of porous media with rough fractal-like tree network. *J. Cent. China Norm. Univ.* **2022**, *2*, 255–261.
33. Xu, P.; Yu, B.M.; Yun, M.J.; Zou, M.Q. Heat conduction in fractal tree-like branched networks. *Int. J. Heat Mass Transfer* **2006**, *49*, 3746–3751. [\[CrossRef\]](#)
34. Valvano, J.W.; Allen, J.T.; Bowman, H.F. The simultaneous measurement of thermal conductivity, thermal diffusivity, and perfusion in small volumes of tissue. *J. Biomech. Eng.* **1984**, *106*, 192–197. [\[CrossRef\]](#) [\[PubMed\]](#)
35. Liang, X.G.; Ge, X.S.; Zhang, Y.P. A convenient method of measuring the thermal conductivity of biological tissue. *Phys. Med. Biol.* **1991**, *36*, 1599. [\[CrossRef\]](#) [\[PubMed\]](#)
36. Bhattacharya, A.; Mahajan, R.L. Temperature dependence of thermal conductivity of biological tissues. *Physiol. Meas.* **2003**, *24*, 769–783. [\[CrossRef\]](#) [\[PubMed\]](#)
37. Tien, C.L.; Vafai, K. Convective and radiative heat transfer in porous media. *Adv. Appl. Mech.* **1989**, *27*, 225–281.

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